Optimal Energy Efficient Design for Passive Distributed Radar Systems

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***Abstract*—In this paper, we address the energy efficiency maximization problem for a distributed passive radar system, with application in signal classification. Two energy efficiency maximization strategies are studied. Firstly, the total power consumption of the network is minimized, while maintaining a required classification quality. We show that the resulting optimization problem has a similar solution structure to the famous water-filling algorithm and can be obtained analytically. Secondly, the energy efficiency of the network is viewed as the ratio of the observed useful information to the total energy consumption of the network for each estimation process. The optimal solution for the latter case is achieved by converting the original problem into an iterative convex feasibility check with a guaranteed convergence to optimality. Finally, the optimal behavior of the defined system in terms of energy efficiency is examined with respect to different system parameters and design approaches by performing extensive numerical simulations.**

I. Introduction

Sensor networks are nowadays more and more applied in various fields. Their importance is growing since the technological proceedings permit the development of evermore smaller sized sensor nodes (SNs). A decreased size of SNs enables in turn the realization of high-density sensor networks with a large number of nodes. This is associated with more demand for electrical energy, which is consumed by all SNs. In this way, a smart power allocation in sensor networks receives more attention than ever. Many different approaches are proposed by scientists for special use cases. For the 'IceCube Neutrino Observatory', see [1], the publications [2] and [3] provide an optimal solution for the power allocation problem in closed-form. This solution is afterwards extended with a fast algorithm in [4]. In case of active radars, the power allocation in sensor networks is investigated for the region of high signal-to-noise ratio in [5] and then in [6], for the general noise conditions. Other approaches like [7] explicitly try to maximize the lifetime of a battery powered sensor network while in [8] the complexity of algorithms for an optimal sensor selection is studied.

Contribution: As the first contribution, we apply an accurate model for the network power consumption which incorporates important parts of power loss in our system. Afterwards the corresponding optimization problem, to minimize the total power consumption while fulfilling a given estimation quality, is formulated and converted into a convex form using the results of [2], and [4]. We then show that the KarushKuhnTucker (KKT) conditions of optimality results in a solution algorithm with water-filling (WF) structure which provides an analytic optimal solution. In the next step, as another energy-efficiency maximization approach, the ratio of the observed useful information to the total energy consumption in the network is maximized following a convex optimization framework. In the end, numerical simulations reveal important characteristics of the system behavior in the sense of energy efficiency.

Paper Organization: The remaining parts of the paper is organized as follows. In Section II, the system model is defined. Our energy efficiency maximization approaches are then presented in Sections III and IV. In Section V we present the numerical simulation results. We conclude this paper by summarizing the main results in Section VI.

II. System Model

We investigate a network of K amplify-and-forward (AF) passive sensor nodes (SNs), cooperating to achieve a single global observation via a fusion center (FC). Both communication and sensing channels (Rayleigh frequency-flat fading) are assumed to be wireless and static during the observation process. The final goal of each observation process is to classify (or detect) a target signal $r \in \mathbb{C}$. Each observation process can be segmented into three parts: sensing process, communication process and information fusion. The detailed description of the system function is presented in [2, Section II].

A. Operation of SNs

If a target signal $r \in \mathbb{C}$ is present, each SN receives and amplifies the incoming signal using an amplification coefficient $u_k \in \mathbb{C}$. The communication with FC is performed by using orthogonal waveforms for each SN so that data from different SNs can be separated and processed in FC. The process of each SN can be described as

$$x_k := (r \cdot g_k + m_k)\, u_k \tag{1}$$

and

$$X_k := \mathcal{E}\{|x_k|^2\} = |u_k|^2 \left(R|g_k|^2 + M_k\right), \quad R := \mathcal{E}\{|r|^2\}, \tag{2}$$

where $\mathcal{E}\{\cdot\}$ represents mathematical expectation. The sensing channel coefficient, communication signal and its power from the SN with index k is respectively denoted by $g_k \in \mathbb{C}$, $x_k \in \mathbb{C}$ and X_k. The additive white Gaussian noise (AWGN) on the sensing process and its variance are respectively denoted as $m_k \in \mathbb{C}$ and M_k. The function of each SN is limited by a maximum allowed individual average power consumption. The

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power consumption model of each SN as well as the corresponding limit will be discussed in more details in Subsection II.C.

B. Fusion Center

The transmitted signal from each SN passes through the communication channel, with coefficients $h_k \in \mathbb{C}$, and arrives at the FC combined with an AWGN component $n_k \in \mathbb{C}$, with variance N_k. A linear combination rule with weights $v_k \in \mathbb{C}$ is then applied at the FC to achieve an estimation, $\tilde{r}$, from the observed target signal. This is described as

$$y_k := (h_k x_k + n_k)\, v_k, \tag{3}$$

and results in

$$\tilde{r} := \sum_{k=1}^{K} y_k = r \sum_{k=1}^{K} g_k u_k h_k v_k + \sum_{k=1}^{K} (m_k u_k h_k + n_k)\, v_k. \tag{4}$$

Although linear processing and fusion strategies are not necessarily optimal, they are very simple and facilitate an analytic solution approach.

C. Power Consumption

In reality, the desired transmit power of each SN, is not the only part of the consumed power. For realistic modeling of the consumed power in each node, we should as well incorporate the dissipation power, and the operating power of the transmit and receiver chains. A detailed elaboration on the consumption of the involved elements can be found in [9]. The consumed power at each SN can be hence modeled with a good approximation as

$$\begin{aligned} P_k &= P_{k,\min} + P_{k,\mathrm{diss}} + X_k, \quad P_{k,\mathrm{diss}} \approx \eta_k X_k, \\ \Rightarrow\; P_k &\approx P_{k,\min} + {\eta_k}' \cdot X_k, \quad {\eta_k}' := 1 + \eta_k, \end{aligned} \tag{5}$$

where P_k represents the total consumed power in the node k and $P_{k,\min}$ represents the minimum required power to keep the node alive. The dissipation power (mainly at power amplifiers), which is proportional to the actual transmit power is denoted as $P_{k,\mathrm{diss}}$ where $\eta_k \in \mathbb{R}^+$ is the loss factor, relating the dissipation power to the actual transmit power from the SN. In order to limit the average power consumption of each node, we define

$$P_k \leq P_{k,\max} \;\; \Leftrightarrow \;\; X_k \leq X_{k,\max}, \tag{6}$$

where $P_{k,\max}$ represents the maximum average power consumption for SN with index k, and $X_{k,\max} := (P_{k,\max} - P_{k,\min}) / \eta'_k$.

D. Remarks

In the present work, we assume the availability of perfect channel information for both sensing and communication channels. In general, it is rather difficult to estimate the sensing channel in an accurate way unless the channel has a highly stationary nature (e.g., [1]). Hence, for scenarios where the sensing channel is not stationary, the results of this paper can be treated as theoretical limits. In the following parts of this paper, we aim at providing energy-efficient designs of the system parameters. Table 1 presents the used notations for different signals and system parameters.

TABLE I: Used symbols and notations

Notation	Description
K	number of all SNs
$r,\ R$	target (reference) signal and its power
$\tilde{r}$	the estimate of r
g_k, h_k	complex-valued sensing and communication channel coefficients
m_k, n_k	complex-valued zero-mean AWGN at each SN and at FC
M_k, N_k	variances of m_k and n_k
u_k, v_k	complex-valued amplification factors and fusion weights
X_k	communication power of k^{th} SN
$P_k, P_{k,\max}$	consumed power in a SN and its maximum allowed value
$P_{k,\min}$	minimum required power to keep the SN in its operational region
$P_{k,\mathrm{diss}}$	dissipated power
η_k	the loss factor, relating the actual transmit power to $P_{k,\mathrm{diss}}$
$\mathbb{F}_K$	the index-set of all K nodes
$\mathbb{K}_0$	the index-set of all inactive nodes
$\mathbb{K}_{\mathrm{sat}}$	the index-set of all nodes operating with maximum power
$\mathbb{K}$	the index-set of all active nodes (not saturated and not inactive)

III. Minimum Power Consumption Design for a Required Estimation Quality

In this section we provide an optimal analytic design with minimum power consumption while a required estimation quality is fulfilled.

A. Optimization problem

Our goal is to minimize the total power consumption of the network, while satisfying an application-dependent required estimation quality. In order to evaluate the estimation quality, we choose the mean squared error (MSE) for the unbiased class of estimators as our criteria which can be defined as

$$V := \mathcal{E}\{|\tilde{r} - r|^2\} = \sum_{k=1}^{K} |v_k|^2 \left(M_k |u_k|^2 |h_k|^2 + N_k \right), \tag{7}$$

where the unbiased estimation condition, recalling (4), can be formulated as

$$\sum_{k=1}^{K} g_k h_k u_k v_k = 1. \tag{8}$$

By incorporating the defined constraint on the average power consumption for each SN, we formulate our optimization problem as

$$\begin{aligned} \min_{u_k, v_k,\, k \in \mathbb{F}_K} \; & \sum_{k \in \mathbb{F}_K} P_k \\ \text{s.t.} \; & \sum_{k \in \mathbb{F}_K} g_k h_k u_k v_k = 1, \;\; V \leq V_{\max}, \\ & P_k \leq P_{k,\max}, \;\; \forall k \in \mathbb{F}_k, \end{aligned} \tag{9}$$

where $V_{\max}$ is the maximum tolerable MSE in our estimation process, corresponding to a required estimation quality.

B. Minimum Power Consumption Design

An optimization problem to maximize the estimation quality with simplified power consumption models is studied in [2] and [4], with constraints on total and individual transmit power at the SNs. We should note that in the special case where $\eta_k = 0,\ \forall k \in \mathbb{F}_K$, our problem can be converted into an equivalent form of the studied MSE minimization problem. Nevertheless, the more accurate power consumption model (5), changes the nature of our problem in the general case. In the first step, we summarize useful results from [2] and [4] in the following lemma, which reveals a direct relation between the transmit power at SNs and the achievable estimation quality at the FC.

Lemma 1: Let $X_1, X_2, \cdots, X_K \in \mathbb{R}^+$ be the transmit power values from the SNs. The minimum achievable MSE, corresponding to the highest estimation quality, for the defined system (1)-(8) is

$$V_{\min} = \left(\sum_{k \in \mathbb{F}_K} J_k (X_k), \right)^{-1}, \quad J_k (X_k) := \frac{X_k \alpha_k^2}{X_k + \beta_k^2}, \quad (10)$$

where $\alpha_k := \sqrt{\frac{|g_k|^2}{M_k}}$ and $\beta_k := \sqrt{\frac{N_k(R|g_k|^2+M_k)}{M_k|h_k|^2}}$. The corresponding values for $u_k, v_k,\ k \in \mathbb{F}_K$ that achieve this quality can be obtained as

$$u_k = \sqrt{\frac{X_k}{R|g_k|^2 + M}}, \quad (11)$$

$$v_k = V_{\min} \frac{(g_k h_k)^* \cdot u_k}{M_k u_k^2 |h_k|^2 + N_k}, \quad (12)$$

where $(\cdot)^*$ represents mathematical conjugation, $V_{\min}$ is the minimum achievable MSE for the given transmit power values, and $J_k(X_k)$ can be interpreted as the contribution function of each SN to the resulting estimation quality.

Proof: Please see [4, eq. (11)-(21)]. ∎

The importance of the above Lemma lies in the fact that it provides a direct relation between the consumed power at the SNs, and the minimum achievable MSE (estimation quality). This simplifies our problem into finding a set of transmit powers that result in minimum total consumed power, while the estimation quality constraint is fulfilled:

$$\min_{X_k \in \mathbb{R}, k \in \mathbb{F}_K} \sum_{k \in \mathbb{F}_K} P_{k,\min} + \sum_{k \in \mathbb{F}_K} {\eta_k}' X_k, \quad (13a)$$

$$\text{s.t.} \sum_{k \in \mathbb{F}_K} J_k(X_k) \geq V_{\max}^{-1}, \quad (13b)$$

$$0 \leq X_k \leq X_{k,\max}, \quad k \in \mathbb{F}_K, \quad (13c)$$

where (13b) represents a sub-set of transmit power values that can achieve the required estimation quality according to (10).

Lemma 2: The optimization problem (13) is convex.

Proof: It can be easily verified that all contribution functions, $J_k(X_k),\ k \in \mathbb{F}_K$, are increasing and concave with respect to X_k as it is shown in [4, eq. (39)-(40)]. As a result, (13b) constitutes a convex feasible set over $X_k,\ k \in \mathbb{F}_K$, while the objective function is affine and the power constraint (13c) is the intersection of $2K$ half-spaces, and hence convex [10]. This concludes the convex nature of our problem. ∎

It is worth mentioning that for the special case ${\eta_k}' = 1,\ \forall\, k \in \mathbb{F}_K$, above problem is convertible into an equivalent form of the previously investigated problem [4, eq. (22)]. Nevertheless, in the general case the problem holds a different structure and the arguments in [4, Lemma. 3-5] do not hold. In order to provide a general solution, we study the well-known KKT conditions of optimality, see [10], for (13). The corresponding Lagrangian function can be written as

$$L(X_k, \lambda, \gamma_k, \zeta_k) := \sum_{k \in \mathbb{F}_K} (P_{k,\min} + {\eta_k}' X_k) - \sum_{k \in \mathbb{F}_K} \gamma_k X_k + \lambda \left(V_{\max}^{-1} - \sum_{k \in \mathbb{F}_K} J_k(X_k) \right) + \sum_{k \in \mathbb{F}_K} \zeta_k (X_k - X_{k,\max}), \quad (14)$$

where $\lambda, \gamma_k,$ and ζ_k represent slack variables. The KKT optimality conditions can be subsequently expressed as

$$\gamma_k^\star \geq 0, \quad k \in \mathbb{F}_K, \quad (15a)$$

$$\zeta_k^\star \geq 0, \quad k \in \mathbb{F}_K, \quad (15b)$$

$$\gamma_k^\star X_k^\star = 0, \quad k \in \mathbb{F}_K, \quad (15c)$$

$$\zeta_k^\star (X_k^\star - X_{k,\max}) = 0, \quad k \in \mathbb{F}_K, \quad (15d)$$

$$\lambda^\star \left(V_{\max}^{-1} - \sum_{k \in \mathbb{F}_K} J_k(X_k^\star) \right) = 0,\ \lambda^\star \geq 0, \quad (15e)$$

$$0 \leq X_k^\star \leq X_{k,\max}, \quad k \in \mathbb{F}_K, \quad (15f)$$

$$\sum_{k \in \mathbb{F}_K} J_k(X_k^\star) \geq V_{\max}^{-1}, \quad (15g)$$

and

$$\frac{\partial}{\partial X_k} L\left(X_k^\star, \lambda^\star, \gamma_k^\star, \zeta_k^\star\right) = 0 \quad \Leftrightarrow$$

$${\eta_k}' - \lambda^\star J_k'(X_k^\star) - \gamma_k^\star + \zeta_k^\star = 0 \quad \Leftrightarrow$$

$$\frac{J_k'(X_k^\star)}{{\eta_k}'} = \frac{1}{\lambda^\star} \left(1 + \frac{\zeta_k^\star - \gamma_k^\star}{{\eta_k}'} \right), \quad (15h)$$

where ${J_k}'(X_k) := \frac{\partial}{\partial X_k} J_k(X_k)$, and $(\cdot)^\star$ indicates optimality. It is important to note that due to the convex nature of (13) the KKT conditions are the necessary and sufficient conditions for the global optimality of $X_k^\star,\ k \in \mathbb{F}_K$, and the slack variables $\lambda^\star$ and $\gamma_k^\star, \zeta_k^\star\ k \in \mathbb{F}_K$. In the following, we provide few observations on the conditions (15a)-(15h) which lead us to the final solution.

Lemma 3: The MSE constraint (13b) is active in the optimality.

Proof: It is easy to verify that $J_k'(X_k)$ is continuous and bounded within the feasible region of X_k as

$$J_k'(X_k) = \frac{\partial}{\partial X_k} J_k(X_k) = \frac{\alpha_k^2 \beta_k^2}{(X_k + \beta_k^2)^2}, \quad (16)$$

due to (10). As the result we have $\lambda^\star \neq 0$ from (15h) which leads to an active MSE constraint according to the complementary slackness condition (15e). ∎

As it is apparent from (15h), (15c) and (15d), the optimality conditions can be separately studied for three possibilities regarding the optimal allocated power at each SN. At the optimality, a SN can be allocated either with no power (*inactive* status, $X_k^\star = 0$), with maximum allowed power (*saturated* status, $X_k^\star = X_{k,\max}$) or with a power between these two extreme cases (*active* status, $0 < X_k^\star < X_{k,\max}$). The explicit optimality conditions for the aforementioned cases are discussed in the following lemma. We respectively denote the index set of all nodes with active, inactive, and saturated status by $\mathbb{K}, \mathbb{K}_0$ and $\mathbb{K}_{\text{sat}}$ hereinafter.

Lemma 4: The following conditional arguments hold at the optimality:

$$X_k^\star = 0 \;\Leftrightarrow\; J_k^{'}(0)/\eta_k{'} \;\le\; 1/\lambda^\star, \tag{17a}$$
$$X_k^\star = X_{k,\max} \;\Leftrightarrow\; J_k^{'}(X_{k,\max})/\eta_k{'} \;\ge\; 1/\lambda^\star, \tag{17b}$$
$$0 < X_k^\star < X_{k,\max} \;\Rightarrow\; J_k^{'}(X_k^\star)/\eta_k{'} \;=\; 1/\lambda^\star. \tag{17c}$$

In order to emphasize the special role of $1/\lambda^\star$ in (17a)-(17c) we name it as *water-level* hereinafter.

Proof: The proof is achieved by studying the derived KKT conditions for each of the (17a), (17b), and (17c), individually.

Proof to (17a): If $X_k^\star = 0$ then we have $\zeta_k^\star = 0$ due to (15d) and $\gamma_k^\star \ge 0$ due to (15a). This concludes $J_k^{'}(0)/\eta_k{'} = \frac{1}{\lambda^\star}\left(1 - \gamma_k^\star/\eta_k{'}\right) \le 1/\lambda^\star$ according to (15h). The proof for the reverse direction is achieved via contradiction: If $J_k^{'}(0)/\eta_k{'} \le 1/\lambda^\star$ and $X_k^\star \ne 0$, then $X_k^\star > 0$ due to (15f). Then we have $J_k^{'}(X_k^\star)/\eta_k{'} < 1/\lambda^\star$ and hence $\zeta_k^\star - \gamma_k^\star < 0$ due to the decreasing nature of $J_k^{'}(X_k)$ in X_k, see (16). This concludes $\gamma_k^\star > 0$ and consequently $X_k^\star = 0$ from (15c).

Proof to (17b): Similar to that of (17a), by exchanging the role of (15a) with (15b) and the role of (15c) with (15d).

Proof to (17c): If $0 < X_k^\star < X_{k,\max}$ we have $\zeta_k^\star = \gamma_k^\star = 0$ from (15c) and (15d). The identity $J_k^{'}(X_k^\star)/\eta_k{'} = 1/\lambda^\star$ can be then concluded from (15h). ∎

The importance of Lemma 4 lies in the fact that it defines clear borders, on how the water-level, i.e., value of $\frac{1}{\lambda^\star}$, is related to the classification of the node as active, inactive, or saturated. As a result, for a correct classification of the nodes, the water-level is positioned such that

$$\max_{l \in \mathbb{K}_0} \left\{ \frac{J_l^{'}(0)}{\eta_l^{'}} \right\} \le \frac{1}{\lambda^\star} \le \min_{k \in \mathbb{K}_{\text{sat}}} \left\{ \frac{J_k^{'}(X_{k,\max})}{\eta_k{'}} \right\}. \tag{18}$$

As it reasonably arises, our solution strategy is to choose $\frac{1}{\lambda^\star}$ as a search variable, and use the results of Lemma 4 to identify the correct status for all SNs. By obtaining the defined borders in (17a) and (17b) and sorting them as an increasing sequence, see Fig.1, we obtain $2K+1$ incremental regions to look for the optimal water-level value. Nevertheless in order to construct our search procedure, we still need an explicit criteria to determine if a value of $\frac{1}{\lambda^\star}$ fits into a selected region. By incorporating the results of the Lemma 3 and 4 we formulate the active MSE constraint as

$$\underbrace{\sum_{k\in\mathbb{K}_0} J_k(0)}_{=0} + \sum_{k\in\mathbb{K}_{\text{sat}}} J_k(X_{k,\max}) + \sum_{k\in\mathbb{K}} J_k(X_k^\star) = V_{\max}^{-1}. \tag{19}$$

On the other hand using the identities (10) and (16) we obtain

$$J_k(X_k^\star) = \alpha_k^2 - \frac{\alpha_k^2\beta_k^2}{X_k^\star + \beta_k^2} = \alpha_k^2 - \sqrt{\frac{J_k^{'}(X_k^\star)}{\eta_k{'}}} \cdot \sqrt{\eta_k{'}} \cdot \alpha_k\beta_k, \tag{20}$$

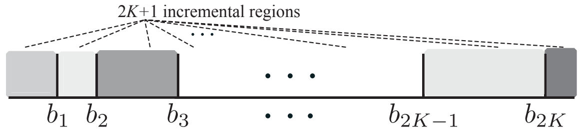

Fig. 1: We obtain $2K+1$ incremental regions in order to search for the value of water-level, i.e., $\frac{1}{\lambda^\star}$, where $b_1 \le \cdots \le b_{2K} \;\leftarrow\; \text{sort}\left\{ J_k^{'}(0)/\eta_k{'},\; J_k^{'}(X_{k,\max})/\eta_k{'},\; \forall k \in \mathbb{F}_K \right\}$.

which due to (17c), for the nodes with active status results in

$$J_k(X_k^\star) = \alpha_k^2 - \alpha_k\beta_k\sqrt{\eta_k{'}} \cdot \sqrt{\frac{1}{\lambda^\star}}, \;\; k \in \mathbb{K},$$
$$X_k^\star = \alpha_k\beta_k\sqrt{\frac{\lambda^\star}{\eta_k{'}}} - \beta_k^2, \;\; k \in \mathbb{K}. \tag{21}$$

From (21) together with (19) we conclude

$$\frac{1}{\sqrt{\lambda^\star}} = \frac{\sum_{k\in\mathbb{K}}\alpha_k^2 + \sum_{k\in\mathbb{K}_{\text{sat}}} J_k(X_{k,\max}) - V_{\max}^{-1}}{\sum_{k\in\mathbb{K}} \alpha_k\beta_k\sqrt{\eta_k{'}}}. \tag{22}$$

It is worth mentioning that the nominator in (22) is always positive for any $V_{\max}^{-1} \le \sum_{k\in\mathbb{F}_K} J_k(X_{k,\max})$ which is the feasibility condition for a required estimation quality, $V_{\max}^{-1}$. The significance of (22) is the fact that it provides an explicit relation between the status of the nodes (active, inactive, and saturated) and the resulting value for $\frac{1}{\lambda^\star}$. Algorithm 1 defines a procedure which finally provides optimal transmit power values, and consequently, the optimal system parameters, $u_k, v_k,\; k \in \mathbb{F}_K$, see (11) and (12).

C. Algorithm description

The procedure in Algorithm 1 is based on a bi-section search on the obtained incremental regions for the position of $\frac{1}{\lambda^\star}$, see Fig. 1. For any selected region, the correct status of all nodes are determined according to Lemma 4. On the other hand, for the obtained status of SNs we achieve the corresponding value of water-level, i.e., $\frac{1}{\lambda^\star}$, via (22) which indicates whether the selected region is correct, too big or too small. The number of required iterations for obtaining the correct region is upper-bounded by $\log_2(2K+1)+1$, following the bi-section search steps. At the end, the optimal value of water-level, along with the optimal transmit power values and the subsets $\mathbb{K}, \mathbb{K}_0$ and $\mathbb{K}_{\text{sat}}$ are determined.

IV. Optimal Energy Efficient Design

In the last Section, we have proposed a minimum energy consumption design for scenarios, where a required minimum estimation quality is compulsory. In this part, we investigate a more general energy efficiency criterion and focus on the maximum network throughput per energy unit. Since the behavior of the target and the properties of its signal r are fixed and not adjustable, we aim at maximizing the ratio of the theoretical channel capacity of this specific sensor network to its corresponding power consumption. This idea helps the network to operate at the highest possible observation rate with minimum energy. Since the received signal at the FC is determined with (4) and (8) by

$$\tilde{r} = r + \sum_{k=1}^{K} (m_k u_k h_k + n_k) v_k = r + e\,, \tag{23}$$

we can model the whole estimation process and the entire sensor network by a discrete-time additive white noise channel, where $e \sim \mathcal{N}(0, V)$ is the equivalent zero-mean Gaussian noise. The Gaussian nature of e results from the linear combination of multiple Gaussian random variables, $n_k, m_k,\ k \in \mathbb{F}_K$ with an overall effective variance of V, see (1)-(8). The minimum achievable variance of e has been presented in connection with the average transmit power values in Lemma 1, (10). The mutual information for such a channel is obtained as

$$\mathrm{MI}\left(r, \tilde{r}\right) = W \log_2 \left(1 + \frac{R}{V}\right), \tag{24}$$

where the values R, V are defined in (2) and (7), and $W := \frac{1}{\mathcal{T}_{ob}}$, where $\mathcal{T}_{ob}$ is the time period between two consequent observation cycles in the network. It is important to note that the above identity can be only achieved for a Gaussian distribution of signal and error. In our scenario, while the error is zero mean and Gaussian distributed for a given set of channel coefficients, this is not generally the case for the reference signal r. Hence we treat (24) as an approximation and express our energy efficiency criteria as

$$\begin{aligned} \mathrm{EE} :&= \frac{\mathrm{MI}\left(r, \tilde{r}\right)}{\sum_{\in \mathbb{F}_K} P_k} = \frac{W \log_2 \left(1 + R \cdot V^{-1}\right)}{\sum_{\in \mathbb{F}_K} P_k} \\ &= \frac{W \log_2 \left(1 + R \cdot \sum_{k \in \mathbb{F}_K} \frac{X_k \alpha_k^2}{X_k + \beta_k^2}\right)}{\sum_{k \in \mathbb{F}_K} P_{k,\min} + \sum_{k \in \mathbb{F}_K} {\eta_k}' \cdot X_k} \end{aligned} \tag{25}$$

where EE is the defined energy-efficiency metric which should be maximized. The corresponding optimization problem can be hence formulated as

$$\max_{X_k \in \mathbb{R}, k \in \mathbb{F}_K} \ \mathrm{EE}, \quad \text{s.t. } 0 \leq X_k \leq X_{k,\max}, \quad k \in \mathbb{F}_K, \tag{26}$$

or equivalently as

$$\begin{aligned} &\max_{\tau \in \mathbb{R},\ X_k \in \mathbb{R}, k \in \mathbb{F}_K} \quad \tau \\ &\qquad \text{s.t. } \mathrm{EE} \geq \tau, \\ &\qquad\quad 0 \leq X_k \leq X_{k,\max}, \quad k \in \mathbb{F}_K. \end{aligned} \tag{27}$$

By applying a bi-section search on the values of τ, our problem turns into the following convex feasibility check for each value of τ

$$\begin{aligned} &\text{find } X_k, \quad k \in \mathbb{F}_K, \\ &\text{s.t. } W \log_2 \left(1 + R \cdot V^{-1}\right) - \tau \cdot \sum_{k \in \mathbb{F}_K} P_k \geq 0, \qquad (28a) \\ &\qquad 0 \leq X_k \leq X_{k,\max}, \quad k \in \mathbb{F}_K. \qquad (28b) \end{aligned}$$

This problem can be determined with certainty, and within a polynomial time, using powerful numerical solvers [10]. To observe the convex nature of (28) we note that the mutual information function, is a concave (logarithmic) composition of the concave and non-decreasing function, i.e., $\sum_{k \in \mathbb{F}_K} \frac{X_k \alpha_k^2}{X_k + \beta_k^2}$, with respect to $X_k,\ k \in \mathbb{F}_K$. This concludes the concavity of $\mathrm{MI}(\cdot)$ function with respect to $X_k,\ k \in \mathbb{F}_K$ according to [10, Section 3.2.4], and consequently the convexity of the feasible set corresponding to (28a). Furthermore, the total power consumption of the network is an affine combination of $X_k,\ k \in \mathbb{F}_K$. This results in the convexity of the feasible set of (28), as an intersection of multiple convex sets defined by (28a) and (28b). The feasibility of (28) determines in each step whether the chosen τ is too large, corresponding to a non-achievable energy-efficiency value, or too small. In order to confine the bi-section search region, we observe that EE is necessarily non-negative on the feasible domain of $X_k,\ k \in \mathbb{F}_K$. Furthermore, due to the increasing nature of the $J_k(\cdot)$ function, we achieve the following bounds for the value of EE

$$0 \leq \mathrm{EE} \leq \frac{W \log_2 \left(1 + R \cdot \left(\sum_{k \in \mathbb{F}_K} \frac{X_{k,\max} \alpha_k^2}{X_{k,\max} + \beta_k^2}, \right)\right)}{\sum_{k \in \mathbb{F}_K} P_{k,\min}}. \tag{29}$$

Both bounds will be later used as the high and low ends of our bi-section search. The iterations of bi-section search over τ in (28) must be continued until a desired solution accuracy is achieved. In the next section we study the behavior of the defined optimal energy efficient design with respect to different system variables via numerical simulations.

Algorithm 1 A water-filling algorithm to achieve an optimal energy-efficient design for a required estimation quality.

$b_1 \leq \cdots \leq b_{2K} \ \leftarrow\ \mathrm{sort}\left\{ J_k^{'}(0)/{\eta_k}',\ J_k^{'}(X_{k,\max})/{\eta_k}',\ \forall k \in \mathbb{F}_K \right\}$ ▷ see (16)
$i_{\min} \leftarrow 1, \quad i_{\max} \leftarrow 2K + 1$
repeat
  $i \leftarrow \lfloor \frac{i_{\min} + i_{\max}}{2} \rfloor$
  $\mathbb{K}_0 \leftarrow \left\{ k \in \mathbb{F}_K \mid b_i \geq J_k^{'}(0)/{\eta_k}' \right\}$ ▷ see (17a)
  $\mathbb{K}_{\mathrm{sat}} \leftarrow \left\{ k \in \mathbb{F}_K \mid b_{i+1} \leq J_k^{'}(X_{k,\max})/{\eta_k}' \right\}$ ▷ see (17b)
  $\mathbb{K} \leftarrow \mathbb{F}_K \setminus (\mathbb{K}_{\mathrm{sat}} \cup \mathbb{K}_0)$
  $\tilde{V}_{\mathrm{remain}} \leftarrow V_{\max}^{-1} - \sum_{k \in \mathbb{K}_{\mathrm{sat}}} J_k(X_{k,\max})$
  if $\mathbb{K} = \emptyset$ **then**
    if $\tilde{V}_{\mathrm{remain}} = 0$ **then**
      break
    else if $\tilde{V}_{\mathrm{remain}} < 0$ **then**
      $i_{\min} \leftarrow i$
    else if $\tilde{V}_{\mathrm{remain}} > 0$ **then**
      $i_{\max} \leftarrow i$
    end if
  else
    $\lambda^\star \leftarrow \left(\frac{\sum_{k \in \mathbb{K}} \alpha_k \beta_k \sqrt{{\eta_k}'}}{V_{\max}^{-1} - \sum_{k \in \mathbb{K}_{\mathrm{sat}}} J_k(X_{k,\max}) - \sum_{k \in \mathbb{K}} \alpha_k^2} \right)^2$ ▷ see (22)
    if $\frac{1}{\lambda^\star} > b_{i+1}$ **or** $V_{\max}^{-1} < \sum_{k \in \mathbb{K}_{\mathrm{sat}}} J_k(X_{k,\max})$ **then**
      $i_{\min} \leftarrow i$
    else
      $i_{\max} \leftarrow i$
    end if
  end if
until $\left(\frac{1}{\lambda^\star} > b_i \text{ and } \frac{1}{\lambda^\star} < b_{i+1}\right)$
$X_k^\star \leftarrow\ \alpha_k \beta_k \sqrt{\frac{\lambda^\star}{{\eta_k}'}} - \beta_k^2,\ k \in \mathbb{K}$ ▷ see (21)
return $(\mathbb{K}_{\mathrm{sat}}, \mathbb{K}_0,\ X_k^\star, k \in \mathbb{K})$

V. Simulation Results

In this part we investigate the optimal behavior of the proposed energy efficient designs via Monte-Carlo simulations. We assume that all channels are accurately known

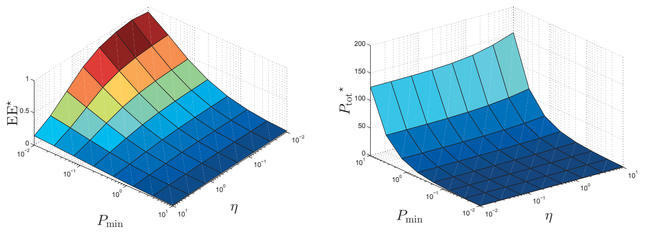

Fig. 2: The optimal energy efficiency, EE* [bits/Joule] and the corresponding power consumption with respect to $P_{\min}$ [Watt] and η. Significant effect of the power loss parameters on EE* is observable.

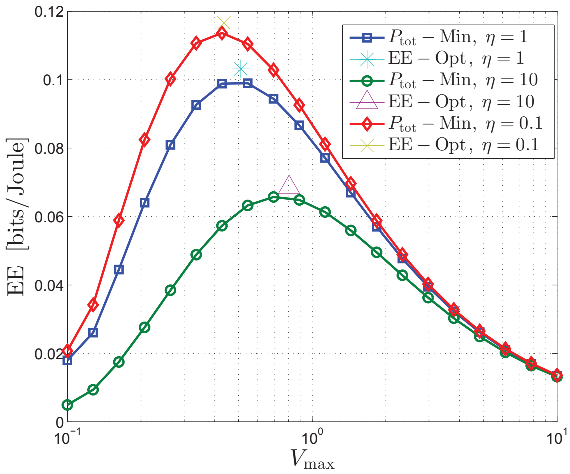

Fig. 3: EE [bits/Joule] vs. maximum tolerable MSE, $V_{\max}$, using the minimum power consumption design. The single points represent the maximum achievable EE for the system, see Section IV. Different required estimation qualities result in significantly different EE.

and follow the uncorrelated Rayleigh flat-fading model. We apply the proposed designs in Section III, regarding the power minimization with a given estimation quality constraint, and the energy efficiency (EE) maximization in Section IV, and average our results over several channel realizations. Unless stated otherwise, we use the following values as the default system parameters: $N_k = N = 1$ [Watt], $M_k = M = 1$ [Watt], $\eta_k = \eta = 1$, $P_{k,\max} = P_{\max} = +\infty$, $P_{k,\min} = P_{\min} = 1$ [Watt], $K = 10$, $\mathcal{E}\{|g_k|^2\} = \mathcal{E}\{|h_k|^2\} = 1$, $\forall k \in \mathbb{F}_K$, $\mathcal{T}_{ob} = 1$ [sec]. In Fig. 2 the result of the energy efficient design in Section IV is illustrated while in Fig. 4 the minimum system power consumption is depicted for different estimation qualities and system noise levels. As it is clear from Fig. 2, the power loss parameter plays a significant role on the optimal EE of the system and the corresponding total power consumption at the optimum point. In Fig. 3, '$\mathrm{EE}-\mathrm{Opt}$' and '$P_{\mathrm{tot}}-\mathrm{Min}$' represent the proposed method in Section IV and the minimum power consumption design, respectively. While the proposed method in Section IV provides a single optimal operational point for the system, the resulting EE of the power minimization method in Section III is significantly variable for different required $V_{\max}$. It is clear that the two methods converge for an optimal single point of $V_{\max}$, as it can be observed from Fig. 3.

VI. Conclusion

In order to achieve an energy efficient function in a distributed passive radar system, nodes with smaller power

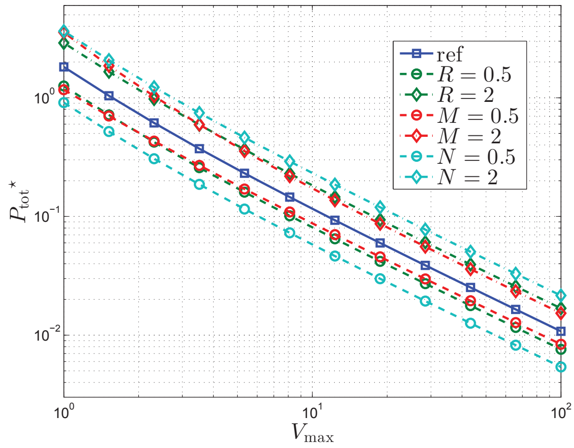

Fig. 4: The minimum required power consumption P_{tot} [Watt], $P_{\mathrm{tot}} := \sum P_k$, for different required estimation qualities, $V_{\max}$. $K = 15$, $P_{\min} = 0$ [Watt].

loss and better communication and sensing channel should be allocated with more power. In this work we have studied the optimal power allocation in such a system, concerning energy efficiency. Two separated approaches are considered. Firstly, the total power consumption of the network is minimized while satisfying a given estimation quality. In the second approach, the efficiency of the system is maximized with no constraint on the estimation quality. While both of the methods aim at providing an energy efficient system operation, it is shown that the solution has a different nature in each case. The first approach, answers the question *how can we fulfill our requirements most energy-efficiently*, while the second approach studies *what is the highest efficiency that can be obtained for a given system.*

References

[1] R. Abbasi, "Icecube neutrino observatory," *International Journal of Modern Physics D*, vol. 19, no. 06, pp. 1041–1048, 2010.

[2] G. Alirezaei, M. Reyer, and R. Mathar, "Optimum power allocation in sensor networks for passive radar applications," *IEEE Transactions on Wireless Communications*, vol. 13, no. 6, pp. 3222–3231, Jun. 2014.

[3] G. Alirezaei, O. Taghizadeh, and R. Mathar, "Optimum power allocation with sensitivity analysis for passive radar applications," *Sensors Journal, IEEE*, Nov 2014.

[4] O. Taghizadeh, G. Alirezaei, and R. Mathar, "Complexity-reduced optimal power allocation in passive distributed radar systems," in *The Eleventh International Symposium on Wireless Communication Systems*, Barcelona, Spain, Sep. 2014.

[5] G. Alirezaei and R. Mathar, "Optimum power allocation for sensor networks that perform object classification," *IEEE Sensors Journal*, vol. 14, no. 11, pp. 3862–3873, Nov. 2014.

[6] G. Alirezaei, O. Taghizadeh, and R. Mathar, "Optimum power allocation in sensor networks for active radar applications," *Wireless Communications, IEEE Transactions on*, 2015.

[7] Q. Dong, "Maximizing system lifetime in wireless sensor networks," in *Information Processing in Sensor Networks, 2005. IPSN 2005. Fourth International Symposium on.* IEEE, 2005, pp. 13–19.

[8] F. Bian, D. Kempe, and R. Govindan, "Utility based sensor selection," in *Proceedings of the 5th international conference on Information processing in sensor networks.* ACM, 2006, pp. 11–18.

[9] C. Shuguang, A. Goldsmith, and A. Bahai, "Energy-efficiency of MIMO and cooperative MIMO techniques in sensor networks," *Selected Areas in Communications, IEEE Journal on*, Aug. 2004.

[10] S. P. Boyd and L. Vandenberghe, *Convex optimization.* Cambridge University Press, 2004.