Fast Power Control for Amplify-and-Forward Multiple-Antenna Bidirectional Relays

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***Abstract*—Cooperative relay communication is an interesting area of research, since it is to enhance data rate and coverage in wireless networks. In this paper, we propose a method to optimize power allocation strategy in an amplify-and-forward relay system. Our objective is to attain a max–min fairness within co-channel users. To reduce the complexity we maximize the upper bound on the performance. The reason is that this bound is observed to be tight in low noise conditions. Therefore, one achievement of this work is separating the problem of relay precoding design from power allocation. Then, the power control is done iteratively via the proposed low-complexity algorithm whose convergence is proven. The relay precoder can be subsequently designed via existing methods, such as semidefinite programming, nonetheless, it is beyond the main focus of the current work. The optimality of the proposed power allocation algorithm is very hard to mathematically prove. Yet, the simulation results are promising.**

***Index Terms*—iterative power allocation, non-alternating joint design, max–min fairness, multi-user, multi-antenna relay**

I. Introduction

This paper considers a two-way relay system (TWRS) with a multiple-antenna half-duplex relay station (RS) and several single-antenna mobile stations (MS). Such a system is expected to offer a better performance compared to non-cooperative systems. First, in the medium access (MAC) phase, all users transmit data to RS which amplifies and then forwards the received signal in the next phase, known as broadcast (BC). The back propagated self-interference is known at each node and can be easily removed [1], since we assume all nodes have perfect knowledge of the channel coefficients. The main goal of this paper is to maximize the signal-to-interference-plus-noise ratio (SINR) of the weakest user link subject to a given power constraint at relay and a sum-power constraint on users.

Related Work: Cooperative systems were first introduced in systems with one relay node in which only one-way transmission is possible. [2] proposes an optimal solution for a system consisting of one transmitting node and one receiving node via multiple one-way single-antenna relay stations. The bottleneck of this system compared to TWRS is the need of four time hops for a complete bidirectional communication which leads to capacity loss of $\frac{1}{2}$. Contrarily, TWRS needs only two hops and gains thus doubled throughput of the former case. Such a scenario for a single pair of users is studied in several works such as [3]–[7]. Later, multiuser TWRS (MU-TWRS) with several transmitting pair of users, was studied in many works. For instance [8] and [9] consider multiple single-antenna relays, while in [1], [10] and [11] the system is equipped with one multiple-input multiple-output (MIMO) relay node. One famous method in these works is to maximize minimum SINR of all users [1], [8] and [9]. The corresponding optimization problem is known as max–min SINR optimization which is non-convex and $\mathcal{NP}$-hard, in general [12]. Most proposed algorithms are oriented around convexifying this problem by means of various relaxations/approximations [1].

While previous works consider symmetric relay systems, many others investigate the asymmetric case where a MIMO base station, a MIMO RS and several single-antenna users exist [13]–[17]. None of these works offer any optimal solutions and come up with sub-optimal methods. A main concern of ours is to avoid the so-called alternating optimization (AO) which is commonly used in many of the mentioned works. It provides no guarantee on the optimality, in spite of increased complexity.

Contribution: First, we propose an optimization problem which is different from the original problem. The proposed problem tries to maximize the so-called minimax bound on the original SINR maximization problem. This bound is tight in high SNR, based on the simulation results, and more importantly independent of the relay beamformer matrix, [18]. Therefore, the problems of relay beamformer and user power allocation are separated. Then, a fast iterative method is proposed to optimize power of users. We show afterwards, that the original max–min fairness formulation in bidirectional relays can be reformulated such that it is bounded by a relaxed representation. This relax representation corresponds to a similar classical fairness problem in MIMO systems in [19]. We further show that the proposed sequence for power allocation always converges to the optimal power of the relaxed problem, i.e., classical MIMO fairness problem, which is a lower bound of the proposed *minimax bound optimization* and also an upper bound on the original SINR optimization in relay systems.

None of this indicates the optimality with respect to the original SINR maximization in relay systems, since we only try to maximize the minimax bound whose optimum does not necessarily coincide with the original problem. In fact, we compromise on the optimality to reduce the complexity.

Once, the power is optimized, RS precoder can be found by any existing methods. The main benefit of the present work over AO is that, it induces no outer (alternating) iterations and the joint design is carried out in only one iteration.

This paper is organized as follows: Section II describes the system model. Section III formulates the joint optimization of relay and power control. Section IV proposes an algorithm for power control, while section V discusses its convergence. Numerical results along with the complexity analysis is given in Section VI. Finally, Section VII concludes the paper.

Notations: Along this paper, upper and lower case boldface symbols denote matrices and vectors, respectively. While $\mathbb{R}_+^n$ shows the set of all non-negative real vectors of size n, the set of all complex matrices of size $k \times n$ is denoted by $\mathbb{C}^{k \times n}$. Trace of a matrix is shown by $\text{Tr}(\,\cdot\,)$. Also, $[\mathbf{x}]_m$ refers to the m^{th} element of vector $\mathbf{x}$. We use $\mathbf{I}_n$ to show the identity matrix of size $n \times n$. Moreover, $(\,\cdot\,)^H$, $(\,\cdot\,)^T$ and $(\,\cdot\,)^*$ are hermitian, transpose and complex conjugate operations, respectively. $\mathcal{E}(\,\cdot\,)$ is the expected value and $\otimes$ notifies the Kronecker product. The sign $\succeq$ means positive semi-definite matrix. Also, $\lambda_{max}(\,\cdot\,)$, $\mathbf{0}_n$ and $\mathbf{1}_n$ refer to maximum eigenvalue of a matrix, all-zero and all-one vectors of size n, respectively. $\text{vec}(\mathbf{A})$ vectorizes a matrix by stacking its columns into one vector. The Kronecker delta function is shown by

$$\delta_{lm} = \begin{cases} 1 & , \; l = m \\ 0 & , \; l \neq m \end{cases},$$

while $||\cdot||_n$ denotes norm n of a vector. Finally, $\mathcal{O}$ stands for big O notation.

II. Data Model and System Setup

This paper considers a system consisting of one RS with N antennas and $M \geq 1$ single-antenna users. Without loss of generality, we assume that M is an even number such that users are communicating in pairs in a multi-pair fashion. For simplicity, let us assume there exits no line-of-sight (LoS) or specular link between relay and users. We also assume that channel state information (CSI) is perfectly known at each node. Figure 1 depicts the setting of the system of consideration. In MAC phase the received signal at RS is determined by

$$\mathbf{s}_r = \sum_{m \in \mathcal{M}} \mathbf{h}_m \sqrt{p_m}\, x_m + \mathbf{n}_r \tag{1}$$

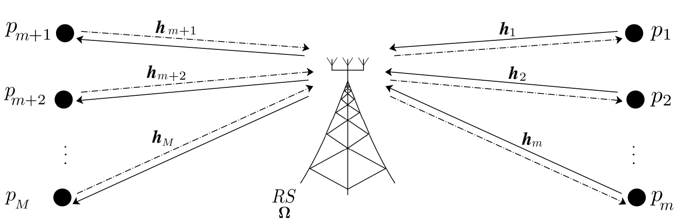

Fig. 1. System setup of the considered network consisting of a multiple-antenna bidirectional RS and multiple single-antenna users.

where $\mathbf{h}_m \in \mathbb{C}^N$ shows the propagation channel from m^{th} MS to RS, while p_m and $x_m \in \mathbb{C}$ denote the transmit power and signal of that MS, respectively. Note, $\mathbf{p} = [p_1 \cdots, p_M]^T \in \mathbb{R}_+^M$ corresponds to the transmit power of users. Let $\mathcal{M} = \{1, \ldots, M\}$ be the set of all mobile nodes with elements indexed by m. The additive noise at relay antennas is also notified with $\mathbf{n}_r \in \mathbb{C}^N$.

The reverse channel from RS to the m^{th} is represented by $\mathbf{h}_m^T$ in the slow fading conditions. We further assume data symbols are independently and identically distributed (i.i.d) with unit power, such that $\mathcal{E}(x_l x_m^*) = \delta_{lm}$. The noise $\mathbf{n}_r \sim \mathcal{CN}(\mathbf{0}_N, \sigma_r^2 \mathbf{I}_N)$ is assumed to be zero-mean circular complex Gaussian noise (CCGN), i.e.,

$$\mathcal{E}(\mathbf{n}_r \mathbf{n}_r^H) = \sigma_r^2 \mathbf{I}_N. \tag{2}$$

After performing amplify-and-forward precoding, the RS broadcasts the signal $\mathbf{x}_r = \boldsymbol{\Omega}\mathbf{s}_r$ in BC phase, where $\boldsymbol{\Omega} \in \mathbb{C}^{N \times N}$ is the corresponding RS precoding matrix. Let the index $\overline{m}(m) \in \mathcal{M}$ be defined by

$$\overline{m}(m) = \begin{cases} m + \dfrac{M}{2} & , \qquad\quad 1 \leq m \leq \dfrac{M}{2} \\ m - \dfrac{M}{2} & , \quad \dfrac{M}{2} + 1 \leq m \leq M, \end{cases} \tag{3}$$

which will be hereafter noted in short from by $\overline{m}$ instead of $\overline{m}(m)$. In fact, index $\overline{m}$ corresponds the desired node to whom user m transmits signal x_m. Assuming a narrowband transmission over the channel along with superposition of noise and interference, user $\bar{l}$ receives the following signal sent by user l:

$$s_{\bar{l}} = \mathbf{h}_{\bar{l}}^T\, \mathbf{x}_r + n_{\bar{l}} = \mathbf{h}_{\bar{l}}^T \boldsymbol{\Omega} (\sum_{m \in \mathcal{M}} \mathbf{h}_m \sqrt{p_m}\, x_m + \mathbf{n}_r) + n_{\bar{l}} \tag{4}$$

in which $n_{\bar{l}}$ is the zero-mean Gaussian additive noise with the variance of σ^2. For all users, the noise term is assumed to be i.i.d , i.e., $\mathcal{E}(n_l n_m^*) = \sigma^2 \delta_{lm}$. Following the aforementioned properties of the data symbols and noise terms, the power of the signal in (4) can be written as

$$\mathcal{E}(s_{\bar{l}} s_{\bar{l}}^*) = \sum_{m \in \mathcal{M}} p_m \mathbf{h}_m^H \boldsymbol{\Omega}^H \mathbf{h}_{\bar{l}}^* \mathbf{h}_{\bar{l}}^T \boldsymbol{\Omega} \mathbf{h}_m + \sigma_r^2 \; \boldsymbol{\Omega}^H \mathbf{h}_{\bar{l}}^* \mathbf{h}_{\bar{l}}^T \boldsymbol{\Omega} + \sigma^2. \tag{5}$$

Since $\mathbf{h}_l^T \boldsymbol{\Omega} \mathbf{h}_m$ is a scalar value, which means $\text{vec}(\mathbf{h}_l^T \boldsymbol{\Omega} \mathbf{h}_m) = \mathbf{h}_l^T \boldsymbol{\Omega} \mathbf{h}_m$, we use the identities below, see [20],

$$\text{vec}(\mathbf{XYZ}) = (\mathbf{Z}^T \otimes \mathbf{X})\, \text{vec}(\mathbf{Y}), \tag{6}$$

$$(\mathbf{X} \otimes \mathbf{Y})^H (\mathbf{X} \otimes \mathbf{Y}) = \mathbf{X}^H \mathbf{X} \; \otimes \; \mathbf{Y}^H \mathbf{Y} \tag{7}$$

to simplify (5) to the following form

$$\begin{aligned} \mathcal{E}(s_{\bar{l}} s_{\bar{l}}^*) = & \sum_{m \in \mathcal{M}} p_m \boldsymbol{\omega}^H (\mathbf{h}_m^H \mathbf{h}_m \otimes \mathbf{h}_{\bar{l}}^H \mathbf{h}_{\bar{l}}) \boldsymbol{\omega} \\ & + \boldsymbol{\omega}^H (\sigma_r^2 \mathbf{I}_N \otimes \mathbf{h}_{\bar{l}}^H \mathbf{h}_{\bar{l}}) \boldsymbol{\omega} + \sigma^2 \end{aligned} \tag{8}$$

where $\boldsymbol{\omega} = \text{vec}(\boldsymbol{\Omega}^*)$. Note that each MS is aware of its back-propagated signal, and also all channels, which enables for self-interference suppression, [1]. Consequently, the SINR of

$$\gamma_{\bar{l}}(\mathbf{p},\boldsymbol{\omega}) = \frac{p_l\ \boldsymbol{\omega}^H(\mathbf{h}_l^H\mathbf{h}_l \otimes \mathbf{h}_{\bar{l}}^H\mathbf{h}_{\bar{l}})\boldsymbol{\omega}}{\sum\limits_{m\in\mathcal{M}\setminus\{l,\bar{l}\}} p_m\boldsymbol{\omega}^H(\mathbf{h}_m^H\mathbf{h}_m \otimes \mathbf{h}_{\bar{l}}^H\mathbf{h}_{\bar{l}})\boldsymbol{\omega}\ + \boldsymbol{\omega}^H(\sigma_r^2\mathbf{I}_N \otimes \mathbf{h}_{\bar{l}}^H\mathbf{h}_{\bar{l}})\boldsymbol{\omega} + \sigma^2}. \tag{9}$$

$$\gamma_{\bar{l}}(\mathbf{p},\boldsymbol{\omega}) = \frac{\boldsymbol{\omega}^H\ p_l\ \overbrace{(\mathbf{h}_l^H\mathbf{h}_l \otimes \mathbf{h}_{\bar{l}}^H\mathbf{h}_{\bar{l}})}^{=:\mathbf{A}_l}\ \boldsymbol{\omega}}{\boldsymbol{\omega}^H\left(\underbrace{\sigma_r^2[(\mathbf{I}_N \otimes \mathbf{h}_{\bar{l}}^H\mathbf{h}_{\bar{l}}) + (\frac{\sigma^2}{P_r(\mathbf{p})}\mathbf{I}_{N^2})]}_{=:\mathbf{C}_l} + \sum\limits_{m\in\mathcal{M}} p_m\underbrace{[\alpha_{lm}(\mathbf{h}_m^H\mathbf{h}_m \otimes \mathbf{h}_{\bar{l}}^H\mathbf{h}_{\bar{l}}) + \frac{\sigma^2}{P_r(\mathbf{p})}(\mathbf{h}_m^H\mathbf{h}_m \otimes \mathbf{I}_N)]}_{=:\mathbf{B}_{lm}}\right)\boldsymbol{\omega}}. \tag{10}$$

the transmitted signal by l^{th} MS at its corresponding node, i.e., $\gamma_{\bar{l}}(\mathbf{p},\boldsymbol{\omega})$ or in short form $\gamma_{\bar{l}}$, is expressed by (9).

In this paper, we consider two power constraints with respect to the given power budget $\overline{P}$: first a sum-power constraint on all users, i.e., $||\mathbf{p}||_1 \leq \overline{P}$, and second a power constraint at RS, i.e., $P_r(\mathbf{p}) = \mathcal{E}(\mathbf{x}_r^H\mathbf{x}_r) \leq \overline{P}$. Note, the relay power depends also on the power of all the users. Therefore, relay power is denoted by $P_r(\mathbf{p})$ instead of P_r.

Using (6) and (7) and similar to (8), the total relay power can be represented by

$$P_r(\mathbf{p}) = \boldsymbol{\omega}^H\big(\underbrace{(\sigma_r^2\mathbf{I}_N \otimes \mathbf{I}_N)}_{=\sigma_r^2\mathbf{I}_{N^2}} + \sum_{m\in\mathcal{M}} p_m(\mathbf{h}_m^H\mathbf{h}_m \otimes \mathbf{I}_N)\big)\boldsymbol{\omega}, \tag{11}$$

or equivalently by

$$P_r(\mathbf{p}) = \boldsymbol{\omega}^H\mathbf{Z}\boldsymbol{\omega},\ \mathbf{Z} \coloneqq \sigma_r^2\mathbf{I}_{N^2} + \sum_{m\in\mathcal{M}} p_m(\mathbf{h}_m^H\mathbf{h}_m \otimes \mathbf{I}_N). \tag{12}$$

Straightforwardly, there always exists a non-negative slack variable ΔP for which $P_r(\mathbf{p}) = \overline{P} - \Delta P$. This means the following mathematical statement is true:

$$\exists \Delta P \geq 0 \Rightarrow P_r(\mathbf{p}) = \overline{P} - \Delta P \leq \overline{P}, \tag{13}$$

Thus, we can multiply the noise term σ^2 in the denominator of (9) by

$$1 = \frac{\boldsymbol{\omega}^H\big(\sigma_r^2\mathbf{I}_{N^2} + \sum\limits_{m\in\mathcal{M}} p_m(\mathbf{h}_m^H\mathbf{h}_m \otimes \mathbf{I}_N)\big)\boldsymbol{\omega}}{\overline{P} - \Delta P}. \tag{14}$$

Then, the resulting equation can be recast into (10) in which

$$\alpha_{lm} = \begin{cases} 0 & ,\ m \in \{l,\bar{l}\} \\ 1 & ,\ m \in \mathcal{M}\setminus\{l,\bar{l}\} \end{cases}. \tag{15}$$

Let us denote $\mathbf{C}_l + \sum\limits_{m\in\mathcal{M}} p_m\mathbf{B}_{lm}$ by $\mathbf{D}_l(\mathbf{p})$. Then, we can simplify equation (10) into the following:

$$\gamma_{\bar{l}}(\mathbf{p},\boldsymbol{\omega}) = p_l\ \frac{\boldsymbol{\omega}^H\mathbf{A}_l\boldsymbol{\omega}}{\boldsymbol{\omega}^H\mathbf{D}_l(\mathbf{p})\boldsymbol{\omega}}\quad,\ \bar{l} \in \mathcal{M}. \tag{16}$$

It must be admitted that the steps to derive (16) are rather complicated. On the other hand, one sees that (16) is rather simple and represents SINR as an explicit function of $\mathbf{p}$.

III. Joint Optimization

The max–min optimization that we consider is

$$\gamma^\star = \gamma(\mathbf{p}^\star,\boldsymbol{\omega}^\star) = \max_{\mathbf{p},\boldsymbol{\omega}}\ \min_{\bar{l}}\ \gamma_{\bar{l}}(\mathbf{p},\boldsymbol{\omega}) \tag{17}$$

$$\text{s.t.}\quad P_r(\mathbf{p}) \leq \overline{P} \tag{18}$$

$$||\mathbf{p}||_1 \leq \overline{P}. \tag{19}$$

This problem is non-convex and $\mathcal{NP}$-hard due to its quadratic fractional objective function, [21]. It is also an extension to the problem studied in our previous work [18], which optimizes only relay beamformer with a given sum power constraint on the relay and a fixed power allocation strategy among users. More precisely, for a given fixed user power allocation $\mathbf{p}$, [18] considers the following problem

$$\gamma(\mathbf{p},\boldsymbol{\omega}^\star(\mathbf{p})) = \max_{\boldsymbol{\omega}}\ \min_{\bar{l}}\ \gamma_{\bar{l}}(\mathbf{p},\boldsymbol{\omega}) \tag{20}$$

$$\text{s.t.}\quad P_r(\mathbf{p}) \leq \overline{P}. \tag{21}$$

Note that $\boldsymbol{\omega}^\star(\mathbf{p})$ indicates that for each given vector $\mathbf{p}$, the relay beamformer can be optimized, individually. The corresponding optimum, i.e., $\gamma^\star(\mathbf{p}) = \gamma(\mathbf{p},\boldsymbol{\omega}^\star(\mathbf{p}))$, differs if the power allocation vector changes. One can easily observe that (17) is equivalent to

$$\gamma^\star = \max_{\mathbf{p}} \gamma^\star(\mathbf{p}) \tag{22}$$

$$\text{s.t.}\quad ||\mathbf{p}||_1 \leq \overline{P}. \tag{23}$$

In [18], it is proved that $\gamma^\star(\mathbf{p})$ is upper-bounded by the so-called minimax upper bound $\hat{\gamma}(\mathbf{p})$,

$$\hat{\gamma}(\mathbf{p}) = \min_{l\in\mathcal{M}}\ \lambda_{\max}\big(p_l\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\big). \tag{24}$$

This bound is used in what follows for the power allocation.

Proposition 1. *At the optimal solution of* (20)*, i.e.,* $\gamma^\star(\mathbf{p})$*, the relay power constraint is satisfied with the equality, i.e,* $P_r(\mathbf{p}) = \overline{P}$*. The proof is given in the appendix.*

This deduces that $\Delta P = 0$ in the equation (14). Thus, we replace $P_r(\mathbf{p})$ with $\overline{P}$ in (10) and accordingly in (16) .

It is important to note that for all $l \in \mathcal{M}$, the matrices $\mathbf{A}_l$ are rank-1 and positive-semidefinite. On the other hand, the matrices $\mathbf{D}_l^{-1}(\mathbf{p})$ are full-rank, positive-definite and thus invertible. Consequently, the matrices $\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l$ are rank-1 and positive-semidefinite. Therefore, the maximum eigenvalue and trace of these matrices are equal. Hence, it is valid to write

$$\hat{\gamma}(\mathbf{p}) = \min_{l\in\mathcal{M}}\ f_l(\mathbf{p}), \tag{25}$$

where

$$f_l(\mathbf{p}) = p_l \operatorname{Tr}\left(\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\right) = \lambda_{max}\left(p_l\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\right). \quad (26)$$

Now, the main questions arises; "how does $\hat{\gamma}(\mathbf{p})$ helps us to solve the power allocation problem?". Even though it is hard to mathematically prove, we have seen from numerous simulations that the minimax upper bound is very tight in high SNR regimes in the BC phase. More precisely, $\gamma^\star(\mathbf{p}) \to \hat{\gamma}(\mathbf{p})$, if $\sigma \to 0$ (for a fixed value of σ_r). This can be, for instance, well evidenced in [18, Figure 3]. Therefore and based on this last observation, we assume that σ is infinitesimally small and aim at solving the following problem instead of (22):

$$\max_{\mathbf{p}} \hat{\gamma}(\mathbf{p}) \quad (27)$$

$$\text{s.t.} \quad ||\mathbf{p}||_1 \leq \overline{P}. \quad (28)$$

Apparently, we cannot expect to achieve the optimum of (22), even if (27) can be optimally solved. But, there is a big advantage in doing so: $\hat{\gamma}(\mathbf{p})$ is apparently independent of relay beamformer, i.e., $\boldsymbol{\omega}$, which means relay beamforming problem is segregated from power allocation problem. This avoids the need of AO method, at the cost of sub-optimality.

IV. Power Control

By replacing (25) into (27), we form our proposed power optimization problem as

$$f^\star = \max_{\mathbf{p}} \min_{l\in\mathcal{M}} f_l(\mathbf{p}) \quad (29)$$

$$\text{s.t.} \quad ||\mathbf{p}||_1 \leq \overline{P}. \quad (30)$$

This problem is also non-convex and very hard to solve. But, we manage to solve it intuitively with a very simple assumption which seems to be effective in the simulations. Since (29) is a power allocation problem, it is expected at optimum the sum power in (30) holds with equality, and all functions $f_l(p)$ are equalized with the optimal value $f^\star$, [22]. This means $||\mathbf{p}||_1 = \overline{P}$, and also $f_l(\mathbf{p}^\star) = f^\star, l \in \mathcal{M}$. Let $\mathcal{P} = \{\mathbf{p} \mid ||\mathbf{p}||_1 \leq \overline{P}, \mathbf{p} \in \mathbb{R}_+^M\}$ be the feasible set of power, then the following statement for all $l \in \mathcal{M}$ is correct:

$$\forall \mathbf{p} \in \mathcal{P},\ \exists \Delta_l(\mathbf{p}) \geq 0 \Rightarrow f_l(\mathbf{p}) = f^\star - \Delta_l(\mathbf{p}). \quad (31)$$

The intuitive assumption is assuming that we are close to the optimum, i.e., $\mathbf{p} \to \mathbf{p}^\star$. Thus, $f_l(\mathbf{p}) \to f^\star$ and $\Delta_l(\mathbf{p}) \to 0$. Hence, usinf definition in (26) we can write

$$p_l = \frac{f^\star}{\operatorname{Tr}\left(\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\right)}. \quad (32)$$

To satisfy power constraint $||\mathbf{p}||_1 \leq \overline{P}$ with equality, we write:

$$p_l = \overline{P}\frac{\frac{f^\star}{\operatorname{Tr}\left(\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\right)}}{\sum_{i\in\mathcal{M}}\frac{f^\star}{\operatorname{Tr}\left(\mathbf{D}_i^{-1}(\mathbf{p})\mathbf{A}_i\right)}} = \overline{P}\frac{\frac{1}{\operatorname{Tr}\left(\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\right)}}{\sum_{i\in\mathcal{M}}\frac{1}{\operatorname{Tr}\left(\mathbf{D}_i^{-1}(\mathbf{p})\mathbf{A}_i\right)}}. \quad (33)$$

This equation not only satisfies the power constraint, but also requires no prior information of the optimal solution $f^\star$. Now, we update the power iteratively using (33). Let k denote the iteration index, then $p_l(k+1)$ is determined by

$$p_l(k+1) = \overline{P}\,\frac{p_l(k)}{f_l(k)}\left(\sum_{i\in\mathcal{M}}\frac{p_i(k)}{f_i(k)}\right)^{-1}. \quad (34)$$

Algorithm 1 shows our proposed method for joint design of relay and power control. In Section V we prove the convergence of (34) based on a related problem.

Algorithm 1 Joint power allocation and relay design

```
randomize p(0) = p_0, define precision ε > 0, set k = 0
while var(p) ≥ ε do
    calculate p_l(k+1) by (34)
    k ← k + 1
end while
return p (solve (20) with the returned p, see [1], [18])
```

It is significant to highlight, that in the simulation we will choose $\mathbf{p}_0$ completely randomly, without any a priori knowledge of $\mathbf{p}^\star$.

V. Convergence Analysis

First, we need to point out few existing facts in order to understand the convergence of Algorithm 1.

Fact 2. *By replacing the variable $\mathbf{y} = (\frac{1}{\overline{P}}\mathbf{Z})^{\frac{1}{2}}\boldsymbol{\omega}$ into* (10) *and* (16)*, SINR can be equivalently expressed by*

$$\zeta_l(\mathbf{p},\mathbf{y}) = \frac{\mathbf{y}^H\mathbf{P}_l\mathbf{y}}{\sum_{m\in\mathcal{M}}\mathbf{y}^H\mathbf{Q}_{lm}\mathbf{y} + n_l},\ \forall l \in \mathcal{M} \quad (35)$$

where $\mathbf{P}_l = p_l\ \mathbf{Z}^{-\frac{1}{2}}\mathbf{A}_l\mathbf{Z}^{-\frac{1}{2}}$, $n_l = \mathbf{Z}^{-\frac{1}{2}}\mathbf{C}_l\mathbf{Z}^{-\frac{1}{2}}$ and $\mathbf{Q}_{lm} = p_m\mathbf{Z}^{-\frac{1}{2}}\mathbf{B}_{lm}\mathbf{Z}^{-\frac{1}{2}}$. Similarly, the relay power in (12) *can be identically described by*

$$P_r(\mathbf{p}) = \boldsymbol{\omega}^H\mathbf{Z}\boldsymbol{\omega} = \overline{P}\ (\mathbf{Z}^{-\frac{1}{2}}\mathbf{y})^H\mathbf{Z}(\mathbf{Z}^{-\frac{1}{2}}\mathbf{y}) \quad (36)$$

and since $\mathbf{Z}^H = \mathbf{Z}$, we have

$$P_r(\mathbf{p}) = \overline{P}\ \mathbf{y}^H\mathbf{y} \leq \overline{P} \to \mathbf{y}^H\mathbf{y} \leq 1. \quad (37)$$

Thus, the original optimization problem in (17) *can be subsequently reformulated into the equivalent form below:*

$$\gamma^\star = \max_{\mathbf{p},\mathbf{y}} \min_{l\in\mathcal{M}} \zeta_l(\mathbf{p},\mathbf{y}) \quad (38)$$

$$s.t. \quad ||\mathbf{p}||_1 \leq \overline{P} \quad (39)$$

$$\mathbf{y}^H\mathbf{y} = 1. \quad (40)$$

Fact 3. *The original optimization* (17)*, or identically* (38)*, is upper-bounded by the following relaxed version*

$$\gamma^\star \leq \zeta^\star = \max_{\mathbf{p}} \max_{\mathbf{v}_1,\cdots,\mathbf{v}_M} \min_{l\in\mathcal{M}} \zeta_l(\mathbf{p},\mathbf{v}_l) \quad (41)$$

$$s.t. \quad ||\mathbf{p}||_1 \leq \overline{P} \quad (42)$$

$$\mathbf{v}_l^H\mathbf{v}_l = 1,\ l \in \mathcal{M}. \quad (43)$$

The reason is simple, the problem in (41) *has one beamforming vector, $\mathbf{v}_l$, for each user. While in our problem relay has only one beamformer for all the user, i.e., $\boldsymbol{\omega}$ or $\mathbf{y}$.*

In fact, (41) is a special case of the classical max–min SINR in MIMO systems which was first studied by [19]. It is also mathematically identical to the dual problem of multicell downlink beamforming, [23]. Later in [24], Cai et al. proved convergence of the proposed method in [19]. Indeed, (41) corresponds to SIMO case of [24] with the parameters $\boldsymbol{\beta} = \mathbf{w} = \mathbf{1}_M$, therein. The sequence for updating the power proposed in [24, eq. (26)], is the same as ours in (34). Let first $\zeta(\mathbf{p})$ be the optimum of the following problem

$$\zeta(\mathbf{p}) = \max_{\mathbf{v}_1,\cdots,\mathbf{v}_M} \min_{l\in\mathcal{M}} \zeta_l(\mathbf{p},\mathbf{v}_l) \tag{44}$$

$$\text{s.t.} \quad \mathbf{v}_l^H\mathbf{v}_l = 1,\ l \in \mathcal{M}. \tag{45}$$

and then the power vector achieved by Algorithm 1 be denoted by $\tilde{\mathbf{p}}^\star$, then it is correct to write $\zeta(\tilde{\mathbf{p}}^\star) = \max_{\mathbf{p}\in\mathcal{P}} \zeta(\mathbf{p}) = \zeta^\star$.

Remark 4. *It is important to mention, that we are not interested in beamforming vectors, i.e., $\mathbf{v}_l,\ l \in \mathcal{M}$. The only reason for optimization over these vectors in* (44) *is to show* $\zeta(\tilde{\mathbf{p}}^\star) = \zeta^\star$.

Proposition 5. *As it is proved in the appendix, $\zeta^\star$ is upper bounded by the optimal value of* (29)*, i.e., $\zeta^\star \le f^\star$.*

Corollary 6. *Subsequent to the Proposition 5 and also previous facts, one can deduce $\gamma^\star \le \zeta(\tilde{\mathbf{p}}^\star) = \zeta^\star \le f^\star$.*

Remark 7. *Indeed, Corollary 6 states that sequence* (34) *converges to the optimal power of the problem* (41) *which is a lower bound of the proposed power allocation problem in* (29)*. The convergence of* (34) *is guaranteed based on the proof provided in [24]. Since the sequence* (34) *is identical to the one in [24], we deduce our proposed power sequence always converges to the optimum of* (41)*, i.e., $\zeta(\tilde{\mathbf{p}}^\star) = \zeta^\star$.*

Remark 8. *Nevertheless, the proposed power sequence does not necessarily achieve the optimum of the either of the optimization problems in* (22) *and* (29)*. The optimality with respect to the problem* (29) *depends on the gap $f^\star - \zeta^\star$.*

If $f^\star - \zeta^\star = 0$ the optimum of (29) *is achieved, but even this does not imply the optimality with respect to the joint optimization problem in* (17)*, or equivalently* (22) *and* (38)*. The reason is that we have separated the relay beamforming problem from power allocation. More precisely,* (34) *tries to optimize power with respect to the upper bound of $\gamma^\star(\mathbf{p})$, i.e., $\hat{\gamma}(\mathbf{p})$, instead of $\gamma^\star(\mathbf{p}) = \gamma(\mathbf{p}, \boldsymbol{\omega}^\star(\mathbf{p}))$ itself.*

Remark 9. *Despite the satisfactory performance which is evidenced by simulation results, authors do not claim any optimality of the proposed power allocation in* (34)*. Anyhow, the convergence is guaranteed. Most importantly, the convergence is fast which might be considered as the main contribution of the current work.*

VI. Simulation and Complexity Analysis

To justify the results, we have performed simulations for 1000 different realizations of the channels. The presented results are averaged over all attained data. We have assumed that $N = M = 6$, $\overline{P} = 100$ and a fixed $\sigma_r = 1$ while varying σ from 0.01 to 1. The channel coefficients are Rayleigh distributed with real and imaginary parts that are i.i.d Gaussian random variables with zero mean and unit variance.

Figure 2 shows the minimax bound $\hat{\gamma}(\mathbf{p})$ with and without power control. In the figure we can observe how power control can increase the bound compared to the case that power is equally distributed among users, i.e., $p_l = \frac{\overline{P}}{M},\ l \in \mathcal{M}$. For comparison, (29) is solved by Optimization toolbox of Matlab which is based on Lagrange multipliers method. Interestingly and due to selected parameter which impose limited precision on the Matlab solver, (34) outperforms Matlab. Also, the upper bound of SINR after relay precoder optimization, using SDP-bisection is shown in the figure, see [18] for more details. Note that the tightness hypothesis of minimax bound in high SNR, even after power control, is verified.

As discussed earlier, the proposed method always converges, regardless of the choice of initial power vector $\mathbf{p}_0$. For each realization of the channel and SNR value the proposed power sequence is simulated with 100 different values of $\mathbf{p}_0$. The values of $\mathbf{p}_0$ are chosen randomly with the uniform distribution. The variance of the 100 resulting values of $\hat{\gamma}(\mathbf{p})$ for each realization of channel and each SNR value is calculated. This variance is in the order of 10^{-10} for $\epsilon = 10^{-12}$, which justifies the convergence of the proposed method.

Also, simulations show that (34) converges fast. In order to provide a rough estimate of complexity, the runtime of the proposed method is compared with the one of the Optimization toolbox. The simulations are done with the same computer. The execution time of our sequence is about 400 ms, which is almost 3% of the one required by Matlab's Optimization toolbox. This is not unexpected, since the only difficulty of (34) is matrix inversion whose complexity is $\mathcal{O}(n^3)$. Besides, the number of iterations for the proposed sequence to converge is between $7 - 8$ in average for $\epsilon = 10^{-12}$.

VII. Conclusion

This paper proposes a fast power allocation algorithm for mobile nodes in TWRS. The proposed method is based on

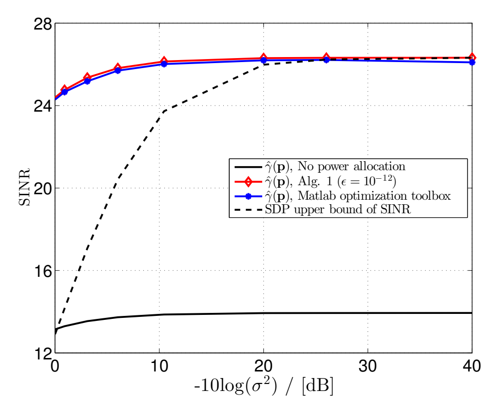

Fig. 2. The minimax bound $\hat{\gamma}(\mathbf{p})$ optimized by, i.e., 1) proposed method (red), 2) Optimization toolbox of Matlab (blue) and 3) equal power allocation (solid black). Also dashed black line shows SDP bound (without randomization), see [18].

maximizing the minimax bound on SINR which is shown by simulations to be tight in high SNR. Even though the proposed optimization problem is non-convex and hard to solve, we have proposed a fast sequence. We have shown that the proposed power allocation, regardless of the choice of initial power vector, convergences to a value which is a lower bound of the proposed optimization problem (29).

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APPENDIX

Proof of Proposition 1. Similar to [2], it can be shown that SINR of each and every user, is increasing as $P_r(\mathbf{p})$ increases. This fact can also be justified obviously, since the denominator of (10) is monotonically decreasing with respect to the relay power. Subsequently, the optimality of (20) is gained when the relay power reaches its maximum $\overline{P}$. □

Proof of Proposition 5. Following the same idea presented in [18, Lemma 1 & Proposition 1] for finding minimax bound (24), we exchange the order of max and min in (44):

$$\zeta(\mathbf{p}) \leq \min_{l\in\mathcal{M}} \max_{\mathbf{v}_1,\cdots,\mathbf{v}_M} \zeta_l(\mathbf{p},\mathbf{v}_l) \tag{46}$$

$$\text{s.t.} \quad \mathbf{v}_l^H\mathbf{v}_l = 1,\ l\in\mathcal{M}. \tag{47}$$

Since

$$\begin{aligned}
&\max_{\substack{||\mathbf{v}_k||_2=1\\ k\in\mathcal{M}}} \zeta_l(\mathbf{p},\mathbf{v}_l) = \max_{\substack{||\mathbf{v}_k||_2=1\\ k\in\mathcal{M}\setminus\{l\}}} \max_{||\mathbf{v}_l||_2=1} \zeta_l(\mathbf{p},\mathbf{v}_l)\\
&= \max_{\substack{||\mathbf{v}_k||_2=1\\ k\in\mathcal{M}\setminus\{l\}}} \max_{||\mathbf{y}||_2=1} \zeta_{\bar{l}}(\mathbf{p},\mathbf{y})\\
&= \max_{\substack{||\mathbf{v}_k||_2=1\\ k\in\mathcal{M}\setminus\{l\}}} \lambda_{\max}\big(p_l\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\big) = \lambda_{\max}\big(p_l\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\big),
\end{aligned} \tag{48}$$

it is obvious that

$$\zeta(\mathbf{p}) \leq \min_{l\in\mathcal{M}} \lambda_{\max}\big(p_l\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\big) \tag{49}$$

and consequently

$$\zeta^\star = \max_{\mathbf{p}\in\mathcal{P}} \zeta(\mathbf{p}) \leq \max_{\mathbf{p}\in\mathcal{P}} \min_{l} \lambda_{\max}\big(p_l\mathbf{D}_l^{-1}(\mathbf{p})\mathbf{A}_l\big) = f^\star, \tag{50}$$

using which, along with (26) and (29), the proof is complete. □

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