FAST AND EFFICIENT AUTOMATIC CHANNEL ALLOCATION IN CELLULAR RADIO NETWORKS

Rudolf Mathar and Martin Hellebrandt
Department of Stochastics
Aachen University of Technology
D-52056 Aachen, Germany

ABSTRACT

In this paper, a fully automated frequency planning tool is described, using integer linear programming as a basic building block. The algorithm itself is very versatile, and able to cope with a variety of demands. It aims at satisfying all requirements as well as minimizing the total sum of interference probabilities, regarding compatibilities, coherence bandwidth and non-supplied cells. Traffic and broadcast control channels are treated under different quality requirements. Practical experience shows that channel designs of highest quality are achieved which clearly outperform those generated by standard planning tools.

1. INTRODUCTION

Frequency planning is one of the most challenging and time consuming tasks in designing a cellular mobile network. On the other hand, optimal assignment of channels is most rewarding with respect to network capacity, call drop rate, and quality of transmission. Different benefit criteria should be taken into account when allocating channels to base stations.

First of all, the interference between each pair of cells must not exceed a certain maximum threshold. This can be expressed by corresponding interference probabilities when the same or adjacent frequencies are used. These values are transformed into compatibilities $c_{ij} \in \mathbb{N}_0$ representing the minimum allowable distance between channels in cell i and j. Channels should be allocated as to satisfy all traffic requirements per cell, while observing the compatibility constraints. This is first carried out by the presented algorithm.

Moreover, it is important to spread channels within individual cells as far as possible. Far distant channels are favorable for applying frequency hopping, in order to cancel out interference peaks between ongoing calls.

To this stage, any channel allocation satisfying the above criteria is equally acceptable. Nothing has been said so far about the total sum of pairwise interferences. As a new feature in frequency planning, our algorithm determines also a channel allocation that has the minimum total interference sum.

In existing cellular GSM-networks there are two classes of channels with different tasks and quality requirements. A moderate interference probability is acceptable for traffic channels (TCH), while broadcast control channels (BCCH) should be interference free to the highest achievable degree. This feature is also taken into account.

The input data of the present frequency planning algorithm are the numbers of required channels for each cell, and interference probabilities between each pair of cells using the same (co-channel interference) or adjacent channels (ad-channel interference) of a certain band. This data is usually provided by measurements or by simulation of radio wave propagation in the area of interest. The method itself is a hybrid, partially stochastic algorithm. Subproblems of moderate size are selected from the whole network, and optimized exactly after having been formulated as integer linear programs.

Our algorithm considers all above described aspects. It realizes an advanced way of automatic frequency planning, which to our experience significantly outperforms manual planning as well as 'old-fashioned' greedy heuristics. The presented method yields frequency designs of highest quality, capable of carrying heavy traffic loads at an excellent quality of service.

There is a vast literature on the channel assignment problem. Most of the presented algorithms take account only of the compatibilities. The used heuristics can be roughly classified into stochastic algorithms (see [1], [5], [8]), neural network approaches (see [4], [11]), and specialized algorithms assuming certain types of regular structures (see, e.g., [7], [8], [9]). Variants of local search and greedy type algorithms are [2], [10], [12], [13] . An excellent overview of network planning in practice is given in [6].

The paper is organized as follows. Section 2 provides the basic problem formulation by means of integer linear programming (ILP). In section 3 we suggest a reliable preprocessing method to speed up the ILPs. We also suggest a method to supply cells which, even under an optimal assignment, are still demanding for further channels (section 4). Traffic and broadcast control channels with the corresponding specific optimization problems are introduced in section 5. Fur-

0-7803-4984-9/98/$10.00 ©1998 IEEE.

thermore, we describe how a certain minimum distance between channels within a cell can be achieved without violating compatibilities. Finally, a method is presented to minimze the total interference sum.

2. CHANNEL ALLOCATION AND LINEAR PROGRAMMING

We first fix the subsequently used notation. Cells are identified with the supporting base stations (BS), and are denoted by $B_1, \dots, B_z$. Physical channels or frequencies are labeled $F_1, \dots, F_N$. These possibly come in bands, e.g., numbered $1, \dots, 10$, $30, \dots, 40$, $50, \dots, 70$. A channel design is a binary matrix

$$\boldsymbol{X} = (x_{ik})_{1 \le i \le z, 1 \le k \le N}, \quad x_{ik} \in \{0, 1\},$$

where $x_{ik} = 1$ means that channel k is allocated to cell i, and $x_{ik} = 0$, otherwise.
The aim now is to determine a channel design $\boldsymbol{X}$ which satisfies the following constraints.

1. Interference conditions given by the compatibily matrix $\boldsymbol{C} = (c_{ij})_{i,j=1,\dots,z}$. c_{ij} is the minimum channel distance between cells i and j. Formally, we have

   $$x_{ik} + x_{j\ell} \le 1$$

   for all $(i,k) \ne (j,\ell)$ with $|k - \ell| < c_{ij}$.

2. Channel requirements r_i, $i = 1, \dots, z$, i.e., $\sum_{k=1}^N x_{ik} = r_i$ for all $i = 1, \dots, z$.

3. Prescribed frequencies $x_{ik_i} = 1$ for certain $k_i \in \{1, \dots, N\}$.

4. Prohibited frequencies $x_{i\ell_i} = 0$ for certain $\ell_i \in \{1, \dots, N\}$.

In practice, prescribed and banned frequencies are often externally specified, e.g., by a neighboring network.
The channel assignment problem can now be formulated as follows. Given compatibilities $\boldsymbol{C} = (c_{ij})$, requirements $\boldsymbol{r} = (r_1, \dots, r_z)$, sets of banned channels V_i per cell, and presets $G_i \subset \{1, \dots, N\}$, $i = 1, \dots, z$. The aim is to maximize the number of assigned channels such that the above constraints are satisfied. This leads to the following integer linear program.

CAP

$$\text{maximize} \quad f(\boldsymbol{X}) = \sum_{i=1}^{z} \sum_{k=1}^{N} x_{ik}$$

such that

$$\begin{aligned} x_{ik} &\in \{0,1\} && \text{for all } i = 1, \dots, z,\ j = 1, \dots, N \\ x_{ik} + x_{j\ell} &\le 1 && \text{for all } (i,k) \ne (j,\ell) \text{ with } |k-\ell| < c_{ij} \\ \sum_{k=1}^{N} x_{ik} &\ge r_i && \text{for all } i = 1, \dots, z \\ x_{ik} &= 0 && \text{for all } k \in V_i \\ x_{ik} &= 1 && \text{for all } k \in G_i \end{aligned}$$

This problem is NP-hard in general (see [3]). For practically relevant examples the number of variables can be as large as 67 317 with 17 342 constraints.
The maximal feasible problem size for advanced linear programming software, however, ranges between 1000 and 2000 variables with only several hundreds of constraints. Even for problems of this size the running times were about several hours, sometimes days, on a SUN ultra.

3. PREPROCESSING AND SUBPROBLEM SELECTION

Preprocessing of the channel assignment problem is the key point to speed up procedure CAP. For this purpose, define a graph with vertex set $\{(i,k) \mid 1 \le i \le z,\ 1 \le k \le N\}$. (i,k) and (j,ℓ) are connected by an edge whenever $|k - \ell| < c_{ij}$. Let $U_1, \dots, U_q$ be a set of maximal cliques such that each edge is contained in at least one clique. It can be shown that the compatibility constraints in CAP may be equivalently substituted by the conditions $\sum_{(i,k) \in U_s} x_{ik} \le 1$ for all $s = 1, \dots, q$, hence significantly reducing the number of constraints. This approach results in distinctly shorter running times. The solution of assignment problems with more than 1000 variables was obtained within a few minutes.
However, for real networks the size of the ILPs is still too large. Since CAP is NP-hard, this is an inherent property of the problem, and cannot be avoided. We choose the following approach to sidestep this obstacle. First select a small number of base stations out of the whole network such that the number of variables in the corresponding CAP remains tractable. Interference restrictions to base stations outside of the selected subset are included as additional banning conditions. Then, this subproblem is solved exactly by procedure CAP, hence improving the network locally. This approach is iterated by choosing the next subproblem, adapting constraints accordingly, and optimizing locally again.
Several strategies for cutting off subproblems are conceivable. A direct approach would be to optimize the neighborhood of those cells which, after an initial assignment, still deserve channels to be allocated. An-

other, more stochastic optimization like setup is to select subproblems at random, and iterate until a certain stopping criterion is satisfied. For real networks we made excellent experience with the second strategy.

4. SUPPLYING CELLS WITH NO CHANNEL SO FAR

Compatibility constraints may be so hard that even after optimization of the above type there are cells which have got no channel assigned so far. Adding the constraints

$$\sum_{k=1}^{N} x_{ik} \geq 1 \quad \text{for all } i = 1, \ldots, z.$$

would avoid this problem. However, in many practical instances the corresponding CAP would not even possess a feasable solution. This problem can be circumvented by the following strategy: sacrifice at most γ allocated channels in the neighborhood of a cell in order to assign one frequency to this cell. This goal can be formalized by the following integer linear program. h_i are auxiliary variables, indicating by $h_i = 1$ that a channel has been assigned to cell i.

HTzero

$$\text{maximize} \quad f(\boldsymbol{X}) = \sum_{i=1}^{z} \sum_{k=1}^{N} x_{ik} + \gamma \sum_{i=1}^{z} h_i$$

such that

$$x_{ik} \in \{0,1\}, \quad h_i \in \{0,1\}$$
$$x_{ik} + x_{j\ell} \leq 1 \text{ for all } (i,k) \neq (j,\ell) \text{ with } |k - \ell| < c_{ij}$$
$$\sum_{k=1}^{N} x_{ik} \geq r_i \text{ for all } i = 1, \ldots, z$$
$$h_i \leq \sum_{k=1}^{N} x_{ik} \text{ for all } i = 1, \ldots, z$$

5. SOLVING REAL PROBLEMS

In this section we discuss which of the above described basic methods have to be extended when dealing with channel assignment in practice. First of all, there are two classes of channels: traffic channels (TCH), permitting moderate quality deterioration, and broadcast control channels (BCCH), demanding for nearly no interference by other frequencies. Consequently, three different compatibility matrices must be taken into account, namely

$$\boldsymbol{C}_{\text{BB}} = \left(c_{ij}^{(BB)}\right), \ \boldsymbol{C}_{\text{TT}} = \left(c_{ij}^{(TT)}\right), \ \boldsymbol{C}_{\text{BT}} = \left(c_{ij}^{(BT)}\right),$$

$i, j = 1, \ldots, z$. These matrices comprise inter- and intra-compatibilities between broadcast and traffic channels. Additionally, variables $x_{ik}, y_{ik} \in \{0,1\}$ are needed, with the meaning

$$\begin{aligned} x_{ik} &= 1 \text{ whenever TCH } k \text{ is assigned to cell } i, \\ &\quad \text{and } x_{ik} = 0 \text{ otherwise,} \\ y_{ik} &= 1 \text{ whenever BCCH } k \text{ is assigned to cell } i, \\ &\quad \text{and } y_{ik} = 0 \text{ otherwise.} \end{aligned}$$

Compatibility constraints for TCHs and BCCHS can be easily included in CAP, putting in the values $\left(c_{ij}^{(BB)}\right)$, $\left(c_{ij}^{(TT)}\right)$, and $\left(c_{ij}^{(BT)}\right)$.
It is often desirable, particularly when applying frequency hopping, to maintain a certain minimum distance k_i between channels in cell i. The following integer linear program is used to formulate this objective as a mathematical optimization problem. Auxiliary variables $h_{ik} \in \{0,1\}$ are employed, with the meaning $h_{ik} = 1$ iff channel k obeys the minimum distance requirement in cell i. Optimizing the coherence bandwidth then reads as follows.

COH

$$\text{maximize} \sum_{i=1}^{z} \sum_{k=1}^{N} h_{ik}$$

such that all compatibility restrictions apply, and

$$\begin{gathered} h_{ik} \leq x_{ik} + y_{ik} \text{ for all } (i,k) \\ h_{ik} + x_{i\ell} \leq 1 \text{ for all } (k,\ell) \text{ with } |k-\ell| < k_i \\ h_{ik} + y_{i\ell} \leq 1 \text{ for all } (k,\ell) \text{ with } |k-\ell| < k_i \\ i = 1, \ldots, z \end{gathered}$$

In order to minimize the total sum of interferences, the values $f_{ijk\ell}$ must be known. f_{ijhl} denotes the interference probability when channel j in cell i and channel k in cell ℓ are used simultaneously.
The corresponding objective function is given by

$$\text{minimize} \sum_{i,k} \sum_{j,\ell} f_{ijhl}(x_{ij} + y_{ij})(x_{k\ell} + y_{k\ell}).$$

This is a quadratic function in the binary variables x_{ij} and y_{ij}. Of course, the constraints describing minimal channel distance, and coherence bandwidth restrictions still apply. After selecting sufficiently small subproblems, the above target function can be minimized by standard algorithms of quadratic integer programming under linear constraints.

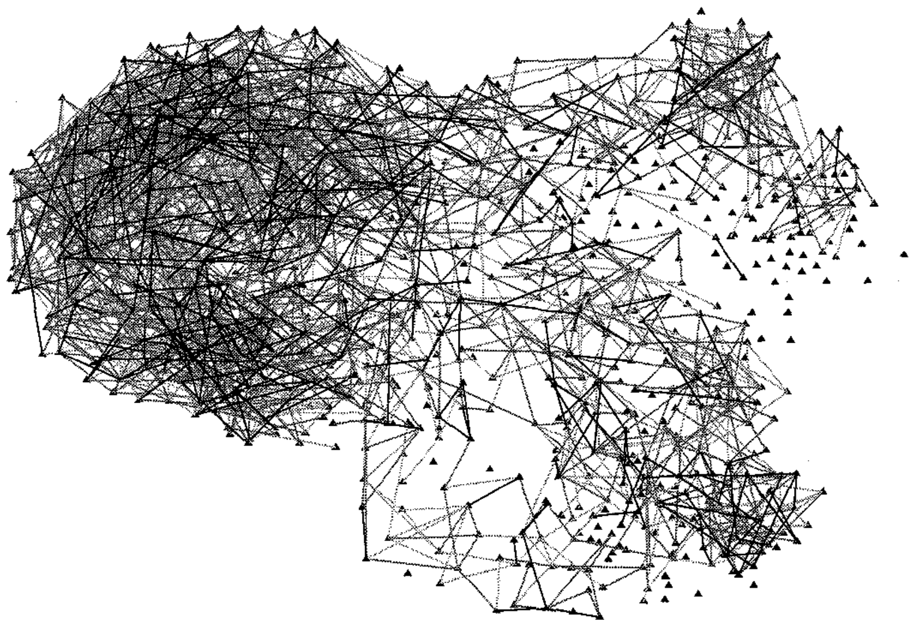

Figure 1: Remaining interferences, all below 6%. Labeling of interferences: solid 4–6%, dashed 2–4%, dotted 1–2%. Stop when all requirements are satisfied.

We have applied the above sophisticated procedure to real world problems. For a specific example, two corresponding graphical outputs are represented in Fig. 1 and Fig. 2. A channel design which fulfills all requirements per cell and observes all compatibilities was calculated. Lines connecting base stations indicate remaining interference probabilities greater than 1%. However, no interference is larger than 6%, according to the compatibility threshold. Furthermore, a channel distance of $k_i = 3$ applies within each cell. Amazingly, the total sum of interference can be reduced to about 30% of its original value when applying the above described procedure (see Fig 2).

In summary, the proposed automatic frequency planning tool proves to be most powerful, efficient and versatile. Running times for examples with about 1000 base stations were in the range of two hours on a SUN ultra. Future work will be devoted to parallelizing the algorithms. We claim that running times can be reduced to about 10% of its present value.

References

[1] M. Duque-Anton, D. Kunz, B., and Rüber. Channel assignment for cellular radio using simulated annealing. *IEEE Transactions on Vehicular Technology*, 42:14–21, 1993.

[2] A. Gamst. Homogeneous distribution of frequencies in a regular hexagonal cell system. *IEEE Transactions on Vehicular Technology*, 31:132–144, 1982.

[3] A. Gamst and K. Ralf. Computational complexity of some interference graph calculations. *IEEE Transactions on Vehicular Technology*, 39:140–149, 1990.

[4] D. Kunz. Channel assignment for cellular networks using neural networks. *IEEE Transactions on Vehicular Technology*, 40:188–193, 1991.

[5] W.K. Lai and G.G. Coghill. Channel assignment through evolutionary optimization. *IEEE Transactions on Vehicular Technology*, 45:91–95, 1996.

[6] F. Lamprecht and A. Baier. Methodik der Funknetzplanung für zellulare Mobilfunksysteme. In *ITG-Fachbericht 135, Mobile Kommunikation*, pages 517–524, Berlin, 1995. VDE-Verlag.

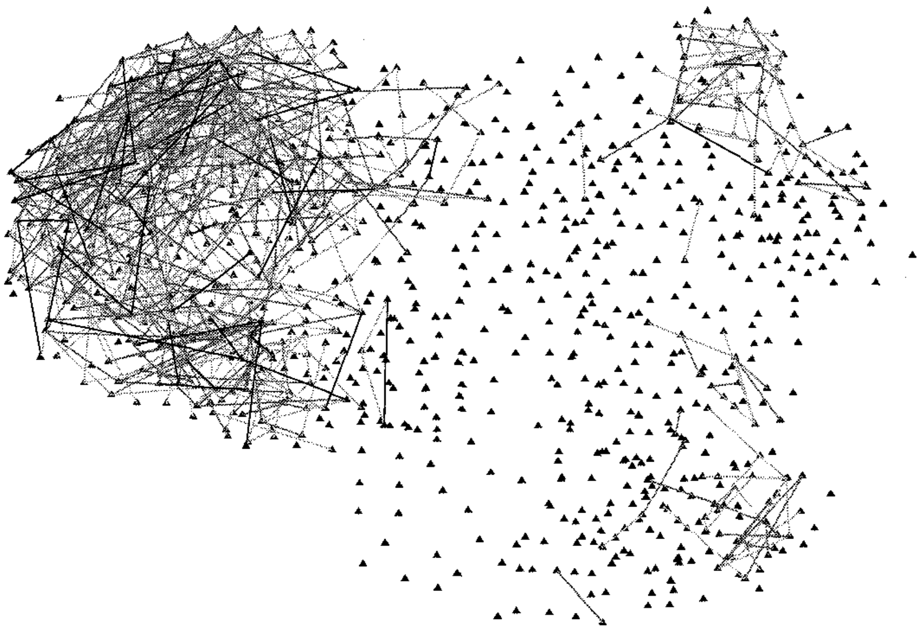

Figure 2: Remaining interferences, all below 6%. Labeling of interferences: solid 4–6%, dashed 2–4%, dotted 1–2%. All requirements are satisfied. Additionally coherence bandwidth and total interference minimization applies.

[7] R.A. Lese. Tiling methods for channel assignment in radio communication networks. *Zeitschrift für Angewandte Mathematik und Mechanik*, 76:303–306, 1996.

[8] R. Mathar and J. Mattfeldt. Channel assignment in cellular radio networks. *IEEE Transactions on Vehicular Technology*, 42:647–656, 1993.

[9] T. Niessen. Optimal channel allocation for several types of cellular radio networks. *Discrete Applied Mathematics*, 79:155–170, 1997.

[10] J. Plehn. Applied frequency assignment. In *IEEE 44th Vehicular Technology Conference*, pages 853–857, Stockholm, 1994.

[11] E. Del Re and R. Fantacci. A dynamic channel allocation technique based on Hopfield neural networks. *IEEE Transactions on Vehicular Technology*, 45:26–32, 1996.

[12] W. Wang and C.K. Rushforth. An adaptive local-search algorithm for the channel-assignment problem (CAP). *IEEE Transactions on Vehicular Technology*, 45:459–466, 1996.

[13] M. Zhang and T.P. Yum. The nonuniform compact pattern allocation algorithm for cellular mobile systems. *IEEE Transactions on Vehicular Technology*, 40:387–391, 1991.