A Capacity Utilizing Distributed Call Admission Control Scheme for CDMA with Power Constraints

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Abstract—We consider a stable CDMA cellular network of \( N \) users with given quality-of-transmission requirements, and ask the question if an additional user can be admitted to the network without violating any of the active quality-of-service thresholds. An important point is that, although the inclusion of a new user impinges on the power adjustment of all mobiles in the network, this decision is made on the basis of information locally collected at the base stations only. We achieve this goal by an appropriate agglomeration of outer- and inner-cell interference, and an according dimensionality reduction scheme. The corresponding admission control algorithm applies an interference prediction phase prior to making a decision about accepting further customers to the wireless network. Capacity is exploited in that users are admitted whenever there is room to. Individual power constraints are included in our system model, which makes the resulting admission control algorithm widely applicable for practical purposes.

I. INTRODUCTION

Wide-band code division multiple access (W-CDMA) is widely used for third generation mobile communication systems to meet the ever growing capacity demand. Smart radio resource management is an essential building block to ensure high system performance. This can be achieved by efficient call admission control (CAC) policies by adjusting the number of admitted calls to the actual interference conditions. An overview of existing CAC algorithms can be found in [1].

A particular challenge when implementing admission control for W-CDMA systems is the limited available information. Existing distributed CAC schemes rely on local interference measurements, and often require specialized algorithms executed at the mobiles, see [2]. The scheme presented in this paper is related to [3] in that it also utilizes standard distributed power control algorithms at the mobiles, as implemented for the inner loop power control in third generation CDMA networks. In the present paper, however, we apply a dimensionality reduction method instead of using a constant test power. Furthermore, an explicit form of the updated power allocation for all mobiles after admission of a new user is given. This allows for including individual power constraints in the admission decision.

The admission control scheme in the present paper extends the approach of [4] in giving further, far reaching analytical results. Moreover, the present CAC algorithm is completely redesigned and relies on the extended information from this analysis. All information required for an admission decision can be collected at the base stations and is usually already available in existing networks for hand-over purposes. The basic idea behind the algorithm is to predict the amount of interference caused by the inclusion of a new customer with given quality requirements in such a way that none of the current quality constraints is violated. The admission decision is based on measuring the total other-cell interference at each base station in a prediction phase.

We show that this admission decision is optimal and that neither in the prediction phase nor after the admission of the newly arriving customer any individual quality constraints are violated.

This paper is organized as follows. In Section II we present the system model and use a technique presented in [5] to reduce the dimensionality of a system of equations for obtaining a feasible power allocation. Admissibility conditions for a new user are formulated in Section III and power constraints are considered in Section IV. Section V describes and discusses our admission control algorithm and methods to collect the necessary information in a network. Section VI deals with simulating the case that only incomplete path gain information is available. We conclude this work with an overview of the results in Section VII.

II. DIMENSIONALITY REDUCTION

We consider the uplink of a W-CDMA cellular system. This is generally believed to be the restricting factor from a capacity point of view, regarding symmetric real-time traffic like voice or video telephony, see [6]. These services are sensitive against reduced transmission rates and are therefore critical from an admission point of view.

In the following we assume a network of \( N \) mobiles, \( 1, \ldots, N \), with a fixed allocation to \( K \) base stations, \( 1, \ldots, K \), expressed by an assignment function

\[
c : \{1, \ldots, N\} \to \{1, \ldots, K\} : i \mapsto k_i
\]

such that \( k_i \) denotes the base station serving mobile \( i \). The set of mobiles allocated to base station \( k \) is denoted by \( \mathcal{C}(k) = \{i \mid k_i = k\}, \ k = 1, \ldots, K \). Hence, the sets \( \mathcal{C}(1), \ldots, \mathcal{C}(K) \) form a partition of the set \( \{1, \ldots, N\} \).
Let $p_i$ denote the transmit power of mobile $i$, and $a_{ik} \in [0, 1]$ the transmission gain from mobile $i$ to base station $k$. We assume that $a_{ik} > 0$ for all $i \in C(k)$, which is obvious to avoid meaningless assignments. In our model $a_{ik}$ is subject to slow fading effects which are assumed to be known to the transmitter. Fast fading effects are not included, as CAC decisions must not rely on effects changing on a time scale of milliseconds.

The signal-to-interference-plus-noise ratio of user $i$ is then given as

$$\text{SINR}_i = \frac{a_{ik} p_i}{\sum_{j \neq i} a_{jk} p_j + \tau_{k}^{0}}, \quad (1)$$

where $\tau_{k}^{0} > 0$ denotes the general background and thermal receiver noise at base station $k_i$. The numerator $a_{ik} p_i$ represents the received power of mobile $i$ at the connecting base station $k_i$. The denominator $\sum_{j \neq i} a_{jk} p_j + \tau_{k}^{0}$ collects the received interference from all other mobiles.

Let $\gamma_i$ be the minimum acceptable SINR for user $i$. Hence,

$$\text{SINR}_i \geq \gamma_i \quad (2)$$

for all mobile stations $i \in \{1, \ldots, N\}$ is required.

The problem now is to determine the minimum transmit power for mobiles such that (2) is satisfied. Since the numerator of (1) is increasing in $p_i$ and the denominator is increasing in $p_j$, $j \neq i$, it is clear that the minimum is attained at a boundary point such that a solution $p = (p_i)_{1 \leq i \leq N}$ of the system

$$\frac{a_{ik} p_i}{\sum_{j \neq i} a_{jk} p_j + \tau_{k}^{0}} = \gamma_i, \quad i = 1, \ldots, N, \quad (3)$$

is sought. Equation (3) is easily converted into the following system of linear equations,

$$\frac{1}{\gamma_i} a_{ik} p_i - \sum_{j \neq i} a_{jk} p_j = \tau_{k}^{0}, \quad i = 1, \ldots, N. \quad (4)$$

The number of mobiles is usually large such that several hundred equations may be involved. Hence, dimensionality reduction is an important issue. The works [7] and [8] deal with this aspect. In [5], a different way of agglomerating interference is chosen, which turns out to be very fruitful for the purpose of admission control in the present paper.

Following this approach, some base station $k \in \{1, \ldots, K\}$ is selected and (4) is rewritten for $i \in C(k)$ as

$$\frac{1}{\gamma_i} a_{ik} p_i - \sum_{j \in C(k), j \neq i} a_{jk} p_j = \tau_k, \quad i \in C(k), \quad (5)$$

where $\tau_k = \tau_{k}^{0} + \sum_{j \notin C(k)} a_{jk} p_j$ is the interference at base station $k$, composed of the background noise and the interference from mobiles in other cells.

Proposition 1: If a solution of system (5) exists, it is given by

$$p_i = q_i(k) \tau_k, \quad i \in C(k), \quad (6)$$

where

$$q_i(k) = \left[ a_{ik} \left( \frac{1}{\gamma_i} + 1 \right) \left( 1 - \sum_{j \in C(k)} \frac{\gamma_j}{1 + \gamma_j} \right) \right]^{-1} \quad (7)$$

comprises the local path gain and QoS parameters.

The proof of Proposition 1 is given in [5].

The factor $q_i(k)$ multiplying $\tau_k$ in (6) depends merely on the minimum required SINR, and the path gains $a_{ik}$ to the serving base station, and is hence independent of the particular power assignment.

$$q_i(k)$$ is positive and finite only if

$$\sum_{j \in C(k)} \frac{\gamma_j}{1 + \gamma_j} < 1 \quad (8)$$

for all base stations $k \in \{1, \ldots, K\}$. Hence, a feasible power allocation may exist only if (8) holds, which is assumed in the following.

We proceed with a short outline of the dimensionality reduction steps. Using representation (6) the total interference $\tau_k$ may be written as

$$\begin{align*}
\tau_k &= \tau_{k}^{0} + \sum_{j \notin C(k)} a_{jk} p_j \\
&= \tau_{k}^{0} + \sum_{m \neq k} \left( \sum_{j \in C(m)} a_{jk} q_j(m) \right) \tau_m \\
&= \tau_{k}^{0} + \sum_{m \neq k} c_{km} \tau_m, \quad k = 1, \ldots, K \quad (9)
\end{align*}$$

with quantities $c_{km} = \sum_{j \in C(m)} a_{jk} q_j(m)$.

In order to obtain a compact representation of system (9) we define the nonnegative $K \times K$ matrix $C = (c_{km})_{k,m=1,\ldots,K}$, where $\delta_{km} = 1 - \delta_{km}$ denotes the complementary Kronecker delta such that $C$ has diagonal entries 0 and non-diagonal entries $c_{km}$. Then (9) reads as

$$\begin{bmatrix} I - C \end{bmatrix} \tau = \tau^{0} \quad (10)$$

with the obvious notation $\tau = (\tau_1, \ldots, \tau_K)'$ and $\tau^{0} = (\tau^{0}_1, \ldots, \tau^{0}_K)'$. Once $\tau$ is computed from (10) the power allocation to all mobiles is given explicitly by (6).

III. ADMISSION OF A NEW USER

In the context of admission control, the question whether equation (10) has a positive solution is crucial. By Perron-Frobenius’ theory a positive solution exists iff the spectral radius $\rho(C)$ satisfies $\rho(C) < 1$, provided $C$ is irreducible, see, e.g., [9].

In the following, we assume that (8) is fulfilled and that a solution $\tau^{0}$ of (10) exists for a network with $N$ users, i.e., $\rho(C) < 1$. Without loss of generality, we assume that a new user $N + 1$ entering the system is assigned to base station $K$, i.e. $k_{N + 1} = K$. Otherwise base stations may be renumbered appropriately. Let $\bar{C}(k)$, the set of mobiles allocated to base station $k$, be defined as above but with user $N + 1$ included. Assume that condition (8) holds for $N + 1$ users too. Otherwise, no feasible power allocation exists and user $N + 1$ can
be rejected straight away. Let \( \tilde{q}_i(k) \) denote the corresponding solution of (7) for \( N+1 \) users. Obviously, the factors (7) must be updated only for cell \( K \) and remain unchanged otherwise. Finally, let \( \tilde{C} = (\tilde{c}_{km})_{km} \) denote the receiver interference matrix for \( N+1 \) users. User \( N+1 \) can be admitted if there is positive solution \( \tau^\text{new} \) of

\[
(I - \tilde{C}) \tau = \tau^0. \tag{11}
\]

The vector of thermal background noise \( \tau^0 \) is assumed to be the same for both cases with \( N \) and \( N+1 \) users which is quite warrantable.

Define \( \Delta = \tilde{C} - C \) and write (11) as

\[
(I - C - \Delta) \tau = \tau^0 \tag{12}
\]

The existence of a solution of this system is characterized by the following Proposition.

**Proposition 2:** Let \( \rho(C) < 1 \). A solution of (11) and (12), respectively, exists if

\[
\rho((I - C)^{-1} \Delta) < 1.
\]

**Proof:** As \( \rho(C) < 1 \), a positive solution \( \tau^\text{old} \) of (10) exists. Since

\[
(I - C - \Delta) \tau = \tau^0 \iff (I - C) \tau - \Delta \tau = \tau^0 \]

\[
\iff \tau - (I - C)^{-1} \Delta \tau = (I - C)^{-1} \tau^0 \iff (I - (I - C)^{-1} \Delta) \tau = \tau^\text{old}.
\]

by Perron-Frobenius’ theory the system of equations in the last line has a solution for any positive right hand side iff \( \rho((I - C)^{-1} \Delta) < 1 \), which concludes the proof.

Observe that only the last column of \( \tilde{C} \) and \( C \) are different, the difference denoted by

\[
\tilde{C} - C = \Delta = \begin{pmatrix} 0 & \cdots & 0 & \delta_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \delta_K \end{pmatrix},
\]

with \( \delta_K = 0 \) and

\[
\delta_i = \sum_{j \in C(K)} a_{ji} \tilde{q}_j(K) - \sum_{j \in C(K)} a_{ji} q_j(K) = \sum_{j \in C(K)} a_{ji} (\tilde{q}_j(K) - q_j(K)) + a_{N+1,i} \tilde{q}_{N+1}(K) \geq 0.
\]

Hence, admitting a new user only affects a single column of the matrix \( C \). This observation leads to the following explicit representation of \( (I - (I - C)^{-1} \Delta) \), which is needed in the following.

**Proposition 3:** Let \( \delta = (\delta_1, \ldots, \delta_K)' \) and

\[
\lambda = (\lambda_1, \ldots, \lambda_K)' = (I - C)^{-1} \delta.
\]

It holds that

\[
\rho((I - C)^{-1} \Delta) = \lambda_K.
\]

Furthermore, if \( \lambda_K < 1 \), then

\[
(I - (I - C)^{-1} \Delta)^{-1} = I + \frac{1}{1 - \lambda_K} (0_{K \times K-1, \lambda}).
\]

**Proof:** Obviously, it holds that

\[
(I - C)^{-1} \Delta = \begin{pmatrix} 0 & \cdots & 0 & \lambda_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & \lambda_K \end{pmatrix} = 0_{K \times K-1, \lambda}.
\]

The only non-zero eigenvalue of this matrix is \( \lambda_K \), which proofs the first part of the proposition.

As one can easily verify, the inverse of \( I - (I - C)^{-1} \Delta \) is

\[
(I - (0_{K \times K-1, \lambda})^{-1} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \vdots & \vdots \\ \vdots & \ddots & 1 & \frac{\lambda_{K-1}}{1 - \lambda_K} \\ 0 & \cdots & 0 & \frac{1 - \lambda_K}{1 - \lambda_K} \end{pmatrix}.
\]

The assertion eventually follows since \( \frac{1}{1 - \lambda_K} = 1 + \frac{\lambda_K}{1 - \lambda_K} \).

Since \( \rho(C) < 1 \), the system of equations

\[
(I - C) \tau = \tau^0 + \varepsilon \delta \tag{13}
\]

has a solution \( \tau^\varepsilon \) for any \( \varepsilon > 0 \), namely

\[
\tau^\varepsilon = (I - C)^{-1} (\tau^0 + \varepsilon \delta) = \tau^\text{old} + \varepsilon \lambda.
\]

Now, by denoting

\[
\tau^\Delta = \tau^\varepsilon - \tau^\text{old} = \varepsilon \lambda
\]

we can formulate the following central result.

**Theorem 4:** The system of equations (11) with one more user assigned to cell \( K \) has a solution iff

\[
\frac{\tau^\Delta}{\varepsilon} < 1.
\]

In this case, the solution \( \tau^\text{new} \) is obtained as

\[
\tau^\text{new} = \tau^\text{old} + \frac{\tau^\text{old}}{\varepsilon - \tau^\Delta} \tau^\Delta.
\]

Furthermore, \( \tau^\text{new} > \tau^\text{old} \) holds.

**Proof:** Since \( \lambda_K = \frac{\tau^\Delta}{\varepsilon} \), the first part is a direct consequence of Propositions 3 and 2. The second part follows from

\[
\tau^\text{new} = (I - (I - C)^{-1} \Delta)^{-1} \tau^\text{old}
\]

\[
= \left( I + \frac{1}{1 - \lambda_K} (0_{K \times K-1, \lambda}) \right) \tau^\text{old}
\]

\[
= \tau^\text{old} + \frac{\tau^\text{old}}{1 - \lambda_K} \lambda.
\]
Finally, recall that $\lambda = (I - C)^{-1} \delta$. The inverse $(I - C)^{-1}$ has only positive entries since $C$ is irreducible. Hence, $\lambda > 0$ holds whenever $\delta \neq 0$.

IV. POWER CONSTRAINTS

Power constraints are not explicitly covered in the previous sections so far. However, in practical systems power is usually limited, particularly for small hand-held devices with small batteries. Individual power constraints are now included by assuming

$$p_i \leq \overline{p}_i \quad \text{for all users } i.$$  

A valid power allocation in the presence of power constraints is a solution to (4) subject to constraints (14). In the context of admission control, a newly arriving customer $N+1$ is admitted only if a valid power allocation can be found for all users, the $(N + 1)$st included. Obviously, the dimension reduction concept still works also in the presence of power constraints. The following proposition generalizes the above results to the case with limited power.

**Theorem 5:** Let $\tau_k = \min_{i \in C(k)} \frac{p_i}{q_i(k)}$ and

$$\tau = \min_{k; \delta_k \neq 0} \left( \frac{\tau_k^0}{\delta_k} \right) \cdot \min_k \left( \frac{\tau_k - \tau_k^{\text{old}}}{\tau_k^{\text{old}}} \right).$$

1) If $\tau_k = \tau_k^{\text{old}}$ for some $k$, then no new user can be admitted.

2) Let $0 < \varepsilon \leq \tau$. Then (13) has a solution $\tau^\varepsilon \leq \tau$ corresponding to a feasible power allocation.

3) For any $0 < \varepsilon \leq \tau$ a power allocation constrained by (14) for $N + 1$ users exists if

$$\frac{\tau_k^0}{\varepsilon} < 1 \quad \text{and} \quad \tau_k^{\text{new}} \leq \min_{i \in C(k)} \frac{p_i}{q_i(k)}$$

for all base stations $k$.

**Proof:** From Theorem 4 we recall that $\tau^{\text{new}} > \tau^{\text{old}}$. Hence, there is some index $k$ such that

$$\tau_k^{\text{new}} > \tau_k = \min_{i \in C(k)} \frac{p_i}{q_i(k)} \geq \min_{i \in C(k)} \frac{p_i}{q_i(k)},$$

a contradiction against (14).

To prove 2) we show that

$$\tau \geq \tau^\varepsilon = \tau^{\text{old}} + \varepsilon \lambda \quad \iff \quad \tau - \tau^{\text{old}} \geq \varepsilon \lambda.$$

If $\delta = 0$, the above obviously holds. Let $\delta \neq 0$ and $\varepsilon' = \min_k \frac{\tau_k^{\text{old}}}{\tau_k^{\text{new}}}$. Then

$$\tau - \tau^{\text{old}} \geq \varepsilon' \tau^{\text{old}} = \varepsilon' (I - C)^{-1} \tau^0$$

follows.

For $\varepsilon'' = \min_{k; \delta_k \neq 0} \frac{\tau_k^0}{\delta_k}$ we obtain

$$(I - C)^{-1} \tau^0 \geq \varepsilon'' (I - C)^{-1} \delta = \varepsilon'' \lambda.$$

Hence, for any $\varepsilon \leq \varepsilon'' \varepsilon'' = \tau$ assertion 2) follows.

Part 3) is a direct consequence of Theorem 5.

Theorem 5 forms the basis for the admission control algorithm described in the following section.

V. ADMISSION CONTROL ALGORITHM

**Algorithm 6:** Assume a new user $N + 1$ enters a stable CDMA system with a feasible power allocation for $N$ users.

1) User $N + 1$ reports its SINR requirement $\gamma_i$, and the path-loss values $a_{Ni+1,k}$ to base station $K$.

2) Each base station $k$ determines the current other cell interference $\tau_{k}^{\text{old}}$ and transmits these value together with $\tau_{k}$ to base station $K$.

3) Base station $K$ computes $\delta$ and $\tau$. If $\tau = 0$, the arriving user is rejected.

Base station $K$ selects some $0 < \varepsilon \leq \tau$ and reports to each base station $k$ the values $\varepsilon$ and $\delta_k$.

4) Each base station adjusts the SINR target values for its mobile stations, as if the background noise $\tau_k^0$ at this station rises to $\tau_k^0 + \varepsilon \delta_k$.

5) The distributed power control algorithm implemented in the network will adjust the powers for all users.

6) Each base station determines the new other cell interference $\tau_{k}^{\text{new}}$ and reports it to base station $K$.

7) If $\tau_{K}^{\text{new}} \geq \varepsilon$ the new user is rejected.

Otherwise, base station $K$ calculates $\tau_{K}^{\text{new}}$. If $\tau_{K}^{\text{new}} > \min_{i \in C(K)} \frac{p_i}{q_i}$, or $\tau_k > \tau_k$ for any $k \neq K$, the user is rejected, otherwise the user is admitted.

As proved in Theorem 5, the algorithm is optimal in the sense that a user is admitted if and only if there exists a feasible power allocation accounting for all power constraints. Additionally, it is assured that during the execution of steps 1 to 6 always a feasible power allocation is tuned.

The algorithm is distributed and only one or two numbers have to be exchanged between base station $K$ and the other base stations in each of the Steps 2, 3, and 6. Step 4 of the algorithm rests on the standard power control algorithm implemented in network. No further communication between the mobiles is necessary.

All other information necessary to run the algorithm is already known at the base station or is calculated or estimated from quantities that can be easily obtained in a real network.

The uplink transmission gain $a_{ik}$ from mobile $i$ to base station $k$ can be estimated in the following way. Each mobile $i$ measures the downlink transmission gain $a_{ik}$ from its neighboring base stations for handover purposes by analyzing their pilot signals. If the frequency used for up- and downlink are close-by, the path gains will essentially differ in fast fading effects only. Averaging $a_{ik}^m$ at the mobiles over sufficiently many values leads to a reliable estimate $a_{ik}$ for $a_{ik}$. The difference between the two is mainly due to fast fading effects. Admission control, however, will generally try to cope with fast fading effects by appropriate fading margins. An admission decision on the basis of fast fading is not desirable as the time scale or fast fading is much smaller than the average duration of a call or the average interarrival times for users.

Alternatively, the path gain can be estimated from the uplink path gain bit error rate. Under mild assumptions the BER is a strictly monotone function of the SINR, see e.g. [10]. The
knowledge of the SINR can be used to evaluate the path-loss as
\[
\text{SINR}_i = \frac{a_{ik} p_i}{\tau_k - a_{ik} p_i},
\]
leading to
\[
a_{ik} = \frac{1}{p_i} \text{SINR}_i \frac{\tau_k}{1 - \text{SINR}_i}.
\]

The \(k\)th component of the base station interference vector \(\tau^{\text{old}} = (\tau_{1}^{\text{old}}, \ldots, \tau_{K}^{\text{old}})/\) can be obtained by measurement and computation at each base station as follows. It is assumed that the total received power in the transmission band can be measured directly at base station \(k\). As described above, \(\gamma_i\) and \(a_{ik}\) are known for all mobiles served by base station \(k\). Therefore, one can compute \(\tau_k\) locally at receiver \(k\) without any communication to neighboring base stations, as
\[
\tau_k^{\text{old}} = \tau_k^{\text{tot}} - \sum_{i \in C(k)} a_{ik} p_i.
\]

In summary, all information necessary to compute \(\delta\) is available.

VI. Simulation Results

In a real network, not all components of \(\delta\) might be known as path gain information \(a_{ik}\) to very distant base stations are often unknown. The effect of a partially known vector \(\delta\) is investigated by simulations in the following.

The basic setup for the simulation consists of a regular hexagonal grid of 91 cells with a base station in the center of each cell. The distance between two neighboring base stations is 1000 m. Users are distributed according to a uniform distribution in the served area. We assume distance dependent path loss with log-normal fading. The system parameters are chosen for a UMTS like network, as listed in Table I.

![Figure 1](image-url) Simulation results. Probability of rejecting a user correctly, if some components of \(\delta\) are unknown.

VII. Conclusions

The main theme of the present paper is an admission control algorithm which decides if an additional user with a certain quality-of-transmission demand can be accommodated by a wireless network without violating any of the ongoing quality requirements. This goal has been achieved by a careful, dimensionality reducing agglomeration of interference into inner-cell and outer-cell effects. Using these terms we have characterized when a feasible power allocation exists, also incorporating individual power constraints. Finally, we have developed a call admission control algorithm, which in the decision phase never violates any quality-of-transmission requirements and admits new users exactly if feasible. We have shown that only local information is needed for the execution of this algorithm, and we have discussed how to compute and estimate the parameters in practice.

REFERENCES


