Fast Resource Allocation for OFDMA Downlink Utilizing An Efficient Bit Loading

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***Abstract*—In this paper, bit loading for orthogonal frequency division multiplexing (OFDM) is studied first. The continuous water-filling is modified to take the maximal rate constraint on each subcarrier into account. Its continuous outputs are optimally quantized by a non-iterative algorithm. By iteratively utilizing bit loading for OFDM, a class of resource allocation methods with low complexity for orthogonal frequency division multiple access (OFDMA) downlink is proposed. Subcarrier assignments for users are initialized independently, which may cause conflicts on arbitrary subcarriers. We suggest a method for conflict cancellation, which may be improved by using appropriate sorting criteria. The proposed method is compared to a newly suggested reference method by simulation. Within the simulation the performance loss is limited by about 5% whereas the complexity reduction is up to 95%.**

I. Introduction

To take advantage of channel diversity among users in different locations, different modulation schemes can be employed on subcarriers adapted to varying channel characteristics. For OFDM the margin maximization problem (MMP) to maximize the system performance margin while satisfying the data transmission requirements has been solved by water-filling [1]. However, when rates have to be discrete, water-filling has to be modified. Bit loading algorithms for OFDM in [2], [3] get the complexity of $\mathcal{O}(N\log(N))$, where N is the number of subcarriers.

A slight complexity reduction in bit loading may significantly speed up multiuser resource allocation, when bit loading for OFDM is utilized iteratively, e.g., [4], [5]. The resource allocation method for OFDMA downlink in [6] achieves near-optimal performance at expense of high computational complexity. Heuristic methods are suggested in [7], [8], [9] with some performance loss.

In this paper, we design a bit loading algorithm for OFDM to implement water-filling efficiently and quantize the continuous output rates of water-filling optimally. It has linear complexity as in [10], [11]. A class of resource allocation methods for OFDMA downlink is suggested, which iteratively utilizes bit loading for OFDM, while achieving comparable performance to the reference method taken from [4].

This paper is organized as follows. Section II formulates the MMP for OFDMA downlink. In Section III a bit loading algorithm for OFDM is presented. A class of heuristic methods for multiuser resource allocation is addressed in Section IV, which iteratively utilizes the bit loading algorithm. Numerical results are given in Section V. Finally, this paper is concluded.

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II. Problem Formulation

Consider an OFDMA downlink over N subcarriers with one transmitter and K users. It is assumed that transmissions of different users are subject to independent frequency selective fading and that perfect channel state information (CSI) is available at the transmitter. Let $G_k[n]$ denote the channel-to-noise ratio (CNR) on the nth subcarrier of user k as

$$G_k[n] = \frac{|H_k[n]|^2}{\Gamma_k \sigma^2_{\omega_k}},$$

where $H_k[n]$ refers to the frequency response on the nth subcarrier of user k and ω_k denotes the additive zero-mean complex Gaussian noise with variance $\sigma^2_{\omega_k}$. The signal-to-noise ratio (SNR) gap Γ_k is a function of the bit error rate (BER) required by user k.

The MMP for OFDMA downlink aims to minimize the total transmit power while satisfying the data rate and BER requirements. In mathematical terms it reads as

$$\min \sum_{k=1}^{K}\sum_{n=1}^{N} c_k[n]P_k[n] \tag{1}$$

s. t.

$$\begin{aligned}
C1: &\quad \sum_{n=1}^{N} c_k[n] r_k[n] \geq R_k, && \forall\, k \in \{1,\ldots,K\}\\
C2: &\quad 0 \leq r_k[n] \leq M,\ \forall\, k \in \{1,\ldots,K\}, && \forall\, n \in \{1,\ldots,N\}\\
C3: &\quad c[n] = \sum_{k=1}^{K} c_k[n] \leq 1, && \forall\, n \in \{1,\ldots,N\}\\
C4: &\quad r_k[n] = \log_2\left(1 + P_k[n]G_k[n]\right),
\end{aligned}$$

where $c_k[n] \in \{0,1\}$ with $c_k[n] = 1$ if the nth subcarrier is assigned to user k, and $c_k[n] = 0$ otherwise. A subcarrier assignment for user k is represented by the vector

$$\mathbf{C}_k = (\, c_k[1], \ldots, c_k[N]\,)$$

and $c_k = \sum_{n=1}^{N} c_k[n]$ subcarriers are used. The minimal data rate required by user k is denoted by R_k. Constraint $C1$ ensures the rate requirement to be fulfilled. The rate on every subcarrier has to be non-negative and is limited by M, which is expressed by $C2$. Constraint $C3$ illustrates that one subcarrier must not be assigned to more than one user at a specific time. In $C4$ the functionality between power $P_k[n]$ and rate $r_k[n]$ is given and referred to as the power-rate function. Further, we define the individual transmit power for user k

$$P_k = \sum_{n=1}^{N} c_k[n] P_k[n].$$

III. Optimal Bit Loading For OFDM

In this section, a bit loading algorithm for OFDM is addressed. For simplicity the user index k is suppressed within this section.

A. Water-Filling for OFDM

With perfect CSI, the optimal power and rate allocation for the single-user MMP has been solved by water-filling, see [1],

$$\begin{aligned} r[n] &= \log_2(\lambda G[n]) \text{ and} && (2)\\ P[n] &= \lambda - \frac{1}{G[n]}, && (3) \end{aligned}$$

where λ is the water level and is determined by the rate constraint. Convexity of the single-user MMP ensures that the optimal solution is achieved at equality in $C1$. It follows that the water level can be obtained as

$$\lambda = 2^{\frac{R}{d}} \left(\prod_{n \in \mathcal{D}} \frac{1}{G[n]} \right)^{\frac{1}{d}}, \quad (4)$$

where $\mathcal{D}$ is the set of d used subcarriers.

Algorithm 1 returns the optimal solution to the single-user MMP, which we call the strict water-filling (SWF), because constraint $C2$ on the maximal rate over each subcarrier is further met unlike the one in [1].

In practice, high data rates are often required and most of the subcarriers are used, so $\mathcal{D}$ is initialized as containing all subcarriers. If all power levels result in positive rates, which are limited M, Algorithm 1 is finished. Otherwise we first take out the subcarriers leading to non-positive rates, and then collect all subcarriers which achieve full rate M in the set $\mathcal{A}$ with order a. After each adaptation of the used subcarriers in set $\mathcal{D}$ the powers need to be recalculated. The modified water-filling can be expressed by

$$(\mathbf{r}, \lambda, R^Q, \mathcal{D}, \mathcal{A}) = \text{SWF}(\mathbf{C}),$$

where $\mathbf{r} = (r[1], \ldots, r[N])$ and R^Q denotes the amount of rates distributed on the subcarriers in $\mathcal{D}$.

The complexity of SWF depends on d, which is determined by R, N and CNRs. To simplify this matter, we assume that $d/2$ subcarriers are removed from $\mathcal{D}$ in each iteration. In such a case, if $\mathbf{C} = \mathbf{1}$, N multiply operations for $P^M[n]$, N log operations for $\mathbf{r}$ and $2N$ multiply operations plus $\log_2(N)$ exponential operations for λ must be executed. Besides these, only simple operations, like compare and minus, are needed. Hence, the complexity of SWF is $\mathcal{O}(N)$.

Algorithm 1 Strict Water-Filling (SWF)

initialization
  $\mathcal{D} \leftarrow \{n \mid c[n] = 1\}$
  $\mathcal{A} \leftarrow \emptyset$
  $P^M[n] \leftarrow \frac{1}{G[n]}(2^M - 1), n \in \mathcal{D}$
repeat
  $\lambda \leftarrow (4)$
  $P[n] \leftarrow (3)\ , n \in \mathcal{D}$
  $\mathcal{S} \leftarrow \{n \in \mathcal{D} \mid P[n] \leq 0\}$
  $\mathcal{L} \leftarrow \{n \in \mathcal{D} \mid P[n] > P^M[n]\}$
  if $\mathcal{S} \neq \emptyset$ **then**
    $\mathcal{D} \leftarrow \mathcal{D} \setminus \mathcal{S}$
  else if $\mathcal{L} \neq \emptyset$ **then**
    $\mathcal{A} \leftarrow \mathcal{A} \cup \mathcal{L}$
    $\mathcal{D} \leftarrow \{1, \ldots, N\} \setminus \mathcal{A}$
    $R \leftarrow R - M \times |\mathcal{L}|$
  end if
until $\mathcal{S} = \emptyset$ and $\mathcal{L} = \emptyset$
$R^Q \leftarrow R$
$r[n] \leftarrow (2)\ , n \in \mathcal{D}$
Output
  $\mathbf{r}$, λ, R^Q, $\mathcal{D}$ and $\mathcal{A}$

B. Rate Quantization

The rates, distributed on subcarriers by SWF, can be any real numbers in $[0, M]$. However, in practical transmission, only discrete rate distribution is realizable, where fractional rates may be feasible by channel coding. Let β denote the constant distance between two neighboring available rates.

Rounding up a continuous rate to an available rate is expressed as

$$r_Q^+[n] = r[n] + \Delta r^+[n]$$

and rounding down a continuous rate to an available rate is expressed as

$$r_Q^-[n] = r[n] - \Delta r^-[n],$$

where $\Delta r^+[n], \Delta r^-[n] \in [0, \beta)$ denote the bit increment and decrement by rounding up and down, respectively. Accordingly, with the inverse of the power-rate function the transmit power increment on each subcarrier can be expressed by

$$\Delta P^+[n] = \frac{1}{G[n]} 2^{r[n]} (2^{\Delta r^+[n]} - 1).$$

Assume that the rates on the lth and nth subcarriers are positive with $\Delta r^+[l] > \Delta r^+[n]$. It follows that the power increment on the lth subcarrier must be higher than the one on the nth subcarrier, shown as

$$\begin{aligned} \Delta P^+[l] &= \lambda(2^{\Delta r^+[l]} - 1)\\ &> \lambda(2^{\Delta r^+[n]} - 1) = \Delta P^+[n] \end{aligned}$$

due to (2) and that the exponential function is monotonically increasing. It can be concluded as follows.

Theorem 1: Given continuous rates on two subcarriers by SWF, the smaller bit increment results in a lower power increment than the larger one.

Recall that the number of bits per modulation symbol consecutively increases by step size β for all available modulation schemes. Rates on some subcarriers must be rounded up, while rates on the others must be rounded down, so that the data rate constraint $C3$ can be met after the rate quantization due to that

$$d > \frac{1}{\beta}\sum_{n\in\mathcal{D}}\Delta r^{+}[n] \quad \text{and}$$

$$d > \frac{1}{\beta}\sum_{n\in\mathcal{D}}\Delta r^{-}[n].$$

Based on the above analysis, it can be deduced that a continuous rate, given by SWF, cannot be rounded up by more than β bits and that subcarriers with rates zero from SWF cannot load any bits after the rate quantization either.

With the above theorem and deduction, a non-iterative rate quantization method for optimal bit loading is designed, as shown in Algorithm 2. Algorithm 1 is used to initialize the rate distribution on subcarriers, where continuous rates are allowed and the constraint on the maximal allowed rate over each subcarrier is met. The gap R^r between R^Q and the summation of all rounded down rates is calculated. Then the rounding up has to be performed to meet the rate requirement R^Q again. The number of subcarriers to be increased by rate β is obviously given by $\lceil \frac{R^r}{\beta} \rceil$. Using Theorem 1 we choose the set $\mathcal{B}$ with $\lceil \frac{R^r}{\beta} \rceil$ elements containing the subcarriers $n \in \mathcal{D}$ with the smallest $\Delta r^{+}[n]$. Finally, the rates on these subcarriers are rounded up and transmit power $P[n]$ on each used subcarrier is calculated. Only simple operations, like plus, minus and compare, are mainly required for rate quantization.

The efficient bit loading (EBL) for an arbitrary user k can be expressed as

$$(\mathbf{r}_k, \mathbf{P}_k) = \text{EBL}(\mathbf{C}_k), \tag{5}$$

where $\mathbf{P}_k = (P_k[1], \ldots, P_k[N])$, $\mathbf{r}_k = (r_k[1], \ldots, r_k[N])$ and $c_k[n] = 1$ only with $r_k[n] > 0, \forall n \in \{1, \ldots, N\}$, otherwise $c_k[n] = 0$.

Algorithm 2 Efficient Bit Loading (EBL)

initialization

$(\mathbf{r}, \lambda, R^Q, \mathcal{D}, \mathcal{A}) \leftarrow \text{SWF}(\mathbf{C})$

quantization

$R^r \leftarrow \sum_{n\in\mathcal{D}} \Delta r^{-}[n]$

$\mathcal{B} \leftarrow \{\lceil R^r/\beta \rceil$ subcarriers in $\mathcal{D}$ with smallest $\Delta r^{+}[n]\}$

$r[n] \leftarrow M,\ n \in \mathcal{A}$

$r[n] \leftarrow r_Q^{+}[n],\ n \in \mathcal{B}$

$r[n] \leftarrow r_Q^{-}[n],\ n \in \mathcal{D} \setminus \mathcal{B}$

$r[n] \leftarrow 0,\ n \in \{n \mid c[n] = 1\} \setminus (\mathcal{D} \cup \mathcal{A})$

$P[n] \leftarrow \frac{1}{G[n]}(2^{r[n]} - 1), n \in \{1, \ldots, N\}$

Output

$\mathbf{r}, \mathbf{P}$

C. Complexity Analysis

In [2], [3], [12], after rounding the continuous rates with a ceiling rate M, the total achieved rate may be larger (smaller) than R. The rates on the subcarriers in $\{1, \ldots, N\}$ with the smallest (largest) bit increments are rounded down (up) by β bits *iteratively* till meeting $C1$. Note that in our algorithm considering M in SWF allows that only subcarriers in $\mathcal{D}$ are considered. $\lceil R^r/\beta \rceil$ subcarriers with the smallest bit increments can be rounded up at one time *non-iteratively*. Rates on other subcarriers are rounded down. By using the order statistic selection algorithms [13], this step can be efficiently implemented with complexity $\mathcal{O}(N)$ in the worst case. Instead of the power increments used in [10], [11], we reduce the number of exponential operations from N to $N/2$ to obtain the transmit power after rounding up and down on subcarriers in $\mathcal{D}$ by comparing rate increments, when the average of d equal to $N/2$ is assumed. From the above mentioned, we derive the complexity of EBL as $\mathcal{O}(N)$.

IV. Heuristic Power and Rate Allocation for OFDMA downlink

By iteratively using bit loading, many algorithms have been developed for multiuser resource allocation, e.g., [5], [7]. In this section, methods, iteratively utilizing EBL, are devised and achieve a good balance between performance and complexity.

A. Efficient Utilization of EBL

For efficient usage of EBL it is important to have a deep look at two different types of varying a subcarrier assignment. One is to add a subcarrier to a subcarrier assignment. The other is to remove a subcarrier from a subcarrier assignment.

1) Adding a subcarrier to a subcarrier assignment: The optimal power allocation (3) can be employed to investigate the possibility of reducing transmit power by adding a subcarrier to a subcarrier assignment. For example, before changing $c_k[n]$ from zero to one, the inverse of its CNR is compared to the current water level first. If $\lambda_k > \frac{1}{G_k[n]}$, it is possible to reduce the transmit power, otherwise it is impossible. When $\lambda_k > \frac{1}{G_k[n]}$ holds, the water level decreases and Algorithm 1 is used to calculate the reduction of transmit power for user k, while negative rates may probably appear and not exceeding the maximal allowed rate over every subcarrier cannot be guaranteed. Therefore, the full process of Algorithm 1 must be executed, as shown in Fig. 1.

2) Removing a subcarrier from a subcarrier assignment: Removing a subcarrier from a subcarrier assignment can only result in increase of the water level and transmit power. It follows that comparison between a water level and an inverse

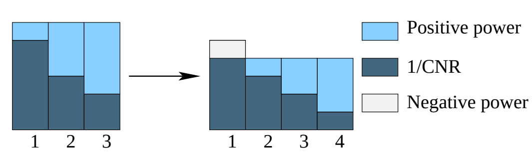

Fig. 1. An example of adding a subcarrier to a subcarrier assignment.

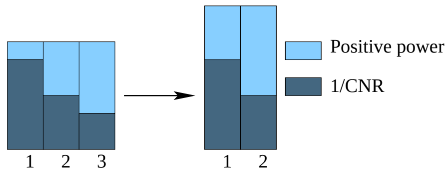

Fig. 2. An example of removing a subcarrier from a subcarrier assignment.

of CNR can be skipped and positive rates can be always ensured, which means that it is unnecessary to check if rates on all used subcarriers are positive in Algorithm 1 any more, for example, as shown in Fig. 2.

B. Re-Assignment of Conflicting Subcarriers (RACS)

When a large number of users is accommodated, a resource allocation scheme often becomes outdated after a short period of time, since resource allocation for all users needs to be updated even when the channel of only one user varies. Hence, complexity of resource allocation becomes more crucial in such a case. Existing methods for multiuser resource allocation contain two main steps: an initialization followed by an iterative process, e.g., [5], [7]. To exploit the abovementioned property of utilizing EBL, a class of methods is developed to provide suboptimal resource allocation for OFDMA downlink. This class mainly keeps removing subcarriers from subcarrier assignments and is specified in the following.

1) Initialization: Instead of the popular initialization of assigning a subcarrier to the user who has the largest CNR on this subcarrier, e.g. [5], [14], we initialize the subcarrier assignment for each user with all subcarriers by using a similar idea as in [15]. For each user Algorithm 1 is executed independently, as shown in Algorithm 3, where N_k^{min} denotes the least number of subcarriers for user k.

2) Conflict cancellation: We call subcarrier n a *conflicting subcarrier*, if $c[n] > 1$ holds. The set of all conflicting subcarriers is denoted by $\mathcal{F}$. If $\mathcal{F} = \emptyset$, $(P_1, \ldots, P_K)$ is the optimal power allocation, otherwise conflicts occur. To cancel such conflicts, we propose three consecutive steps, as shown in Algorithm 4, 5, 6. Removing a conflicting subcarrier from a subcarrier assignment results in an individual transmit power increment. In Algorithm 4 a conflicting subcarrier remains assigned to the user with the largest individual transmit power increment only. Conflicting subcarriers in $\mathcal{F}$ are processed successively in Algorithm 4.

Algorithm 3 User-Independent Initialization

$\mathcal{K} \leftarrow \{1, \ldots, K\}$
$\mathbf{C}_k \leftarrow \mathbf{1}, \ \forall k \in \mathcal{K}$
$(\mathbf{r}_k, \mathbf{P}_k) \leftarrow \text{EBL}(\mathbf{C}_k), \ \forall k \in \mathcal{K}$
$c[n] \leftarrow \sum_{k=1}^{K} c_k[n], \forall n \in \{1, \ldots, N\}$
$N_k^{min} \leftarrow \lceil \frac{R_k}{M} \rceil, \ \forall k \in \mathcal{K}$
$\mathcal{Y} \leftarrow \emptyset$
$\mathcal{L} \leftarrow \emptyset$

Algorithm 4 Greedy Conflict Cancellation

$\mathcal{F} \leftarrow \{n \in \{1, \ldots, N\} \mid c[n] > 1\}$
for each $\check{n} \in \mathcal{F}$ **do**
  $\mathcal{U} \leftarrow \{k \in \{1, \ldots, K\} \mid c_k[\check{n}] = 1\}$
  $c_k \leftarrow \sum_{n=1}^{N} c_k[n], \ \forall k \in \mathcal{U}$
  $\mathcal{V} \leftarrow \left\{k \in \mathcal{U} \mid c_k = N_k^{min}\right\}$
  $v \leftarrow |\mathcal{V}|$
  if $v = 0$ **then**
    $c_k[\check{n}] \leftarrow 0, \forall k \in \mathcal{U}$
    $(\check{\mathbf{r}}_k, \check{\mathbf{P}}_k) \leftarrow \text{EBL}(\mathbf{C}_k), \forall k \in \mathcal{U}$
    $k_m \leftarrow \text{argmax}_{k \in \mathcal{U}} \check{P}_k - P_k$
    $c_{k_m}[\check{n}] \leftarrow 1$
    $P_k \leftarrow \check{P}_k, \forall k \in \mathcal{U} \setminus \{k_m\}$
  else if $v = K$ **then**
    $\mathcal{Y} \leftarrow \mathcal{Y} \cup \{\check{n}\}$
  else
    $c_k[\check{n}] \leftarrow 0, \forall k \in \mathcal{U} \setminus \mathcal{V}$
    $(\mathbf{r}_k, \mathbf{P}_k) \leftarrow \text{EBL}(\mathbf{C}_k), \ \forall k \in \mathcal{U} \setminus \mathcal{V}$
    if $v \neq 1$ **then**
      $\mathcal{L} \leftarrow \mathcal{L} \cup \{\check{n}\}$
    end if
  end if
end for

It may happen that among the users intending to use a conflicting subcarrier, there are v users having only N_k^{min} subcarriers. We call such users *tough users*. If $v = 1$ holds, this conflicting subcarrier has to be assigned to the tough user only. If $v = K$ holds, this conflicting subcarrier is included in set $\mathcal{Y}$ and such conflicts are cancelled in Algorithm 6. For other cases of v, this conflicting subcarrier is included in the set $\mathcal{L}$ and such conflicts are cancelled in Algorithm 5.

In Algorithm 5 we first select the *non-tough user* with the smallest power-to-rate ratio for each conflicting subcarrier in $\mathcal{L}$. The set $\mathcal{S}$ contains the subcarriers used by this non-tough user but not used by a tough user. Then we find a subcarrier in $\mathcal{S}$, which can substitute this conflicting subcarrier for the tough user with the smallest increment of total transmit power. Once a conflict cannot be solved by Algorithm 5, this conflicting subcarrier would be added to set $\mathcal{Y}$.

For each conflicting subcarrier n in $\mathcal{Y}$, there must exist $|\mathcal{I}|$ remaining subcarriers with $|\mathcal{I}| \geq c[n] - 1$, which are not used by any users. Algorithm 6 finds the $c[n]-1$ remaining subcarriers to replace each conflicting subcarrier in $\mathcal{Y}$ for $c[n]-1$ tough users separately with the smallest increment of total transmit power. Obviously, Algorithm 6 is rarely activated, when a large number of users is accommodated in the considered system.

3) Sorting conflicting subcarriers: We call the above group of algorithms *RACS*, because it is designed to ***re-a**ssign **c**onflicting **s**ubcarriers*. Apparently, conflicting subcarriers are re-assigned following a random order with respect to their CNRs in Algorithm 4. Sorting conflicting subcarriers in $\mathcal{F}$ after the initialization can result in better performance. It can be performed according to the following criterion.

Algorithm 5 Smart Conflict Cancellation

> **for** each $\tilde{n} \in \mathcal{L}$ **do**
> $\mathcal{U} \leftarrow \{k \in \mathcal{K} \mid c_k[\tilde{n}] = 1\}$
>   **for** each $k \in \mathcal{U}$ **do**
> $\mathcal{Z} \leftarrow \{k \in \mathcal{K} \setminus \mathcal{U} \mid c_k = N_k^{min}\}$
>   **if** $|\mathcal{U}| + |\mathcal{Z}| < K$ **then**
> $\mathcal{Q} \leftarrow \{\tilde{k} \in \mathcal{K} \setminus (\mathcal{U} \cup \mathcal{Y}) \mid \exists\ n\ c_{\tilde{k}}[n] = 1 \wedge c_k[n] = 0\}$
> $k' \leftarrow \mathrm{argmin}_{k \in \mathcal{Q}} P_k / R_k$
> $\mathcal{S} \leftarrow \{n \in \{1, \ldots, N\} \mid c_{k'}[n] = 1 \wedge c_k[n] = 0\}$
>   **for** each $n \in \mathcal{S}$ **do**
> $(c_k[n], c_{k'}[n], c_k[\tilde{n}]) \leftarrow (1, 0, 0)$
> $(\mathbf{r}_{k',n}, \tilde{\mathbf{P}}_{k',n}) \leftarrow \mathrm{EBL}(\mathbf{C}_{k'})$
> $(\mathbf{r}_{k,n}, \tilde{\mathbf{P}}_{k,n}) \leftarrow \mathrm{EBL}(\mathbf{C}_k)$
> $(c_k[n], c_{k'}[n]) \leftarrow (0, 1)$
> $\Delta P_{k,n} = \tilde{P}_{k,n} + \tilde{P}_{k',n} - P_{k,n} - P_{k',n}$
>   **end for**
> $n_m \leftarrow \mathrm{argmin}_{n \in \mathcal{S}} \Delta P_{k,n}$
> $\Delta P_k \leftarrow \tilde{P}_{k,n_m} - P_{k,n_m}$
> $(c_k[n_m], c_{k'}[n_m]) \leftarrow (1, 0)$
> $(P_{k'}, P_k) \leftarrow (P_{k',n_m}, P_{k,n_m})$
>   **else**
> $\mathcal{Y} \leftarrow \mathcal{Y} \cup \{\tilde{n}\}$
>   break
>   **end if**
>   **end for**
> $k_m \leftarrow \mathrm{argmax}_{k \in \mathcal{U}} \Delta P_k$
> $c_{k_m}[\tilde{n}] \leftarrow 1$
> $(\mathbf{r}_{k_m}, \mathbf{P}_{k_m}) \leftarrow \mathrm{EBL}(\mathbf{C}_{k_m})$
> **end for**

The variability of the nth subcarrier over users is defined as

$$\mathrm{V}[n] = \sum_{k=1}^{K} c_k[n] \mid g[n] - G_k[n] \mid \tag{6}$$

with the average CNRs over users on conflicting subcarriers

$$g[n] = \frac{1}{c[n]} \sum_{k=1}^{K} c_k[n] G_k[n]. \tag{7}$$

Consider the extreme case that only one user can use the nth subcarrier and all the others have too low CNRs on this subcarrier to use it. Improperly assigning this subcarrier may hardly happen and $\mathrm{V}[n]$ is very large. On the contrary, when $\mathrm{V}[n]$ is small, users have similar CNRs on the nth subcarrier, which may be assigned to a wrong user with higher probability. Hence, the conflicting subcarriers with larger variabilities are supposed to be re-assigned earlier. The revised RACS, where conflicting subcarriers in $\mathcal{F}$ are re-assigned following a descending order of their variabilities by Algorithm 4, is called ***o****rdered* ***r****e-****a****ssignment of* ***c****onflicting* ***s****ubcarriers (ORACS).*

The variability in (6) may be improved by balancing users' different attenuations, BER requirements and noise powers. Alternatively, instead of CNRs used in (6) and (7), normalized CNRs over subcarriers are employed and defined as

$$\overline{G_k[n]} = G_k[n] / \Sigma_{n=1}^{N} G_k[n], \tag{8}$$

Algorithm 6 Occasional Conflict Cancellation

> **for** each $\hat{n} \in \mathcal{Y}$ **do**
> $\mathcal{U} \leftarrow \{k \in \mathcal{K} \mid c_k[\tilde{n}] = 1\}$
>   **for** each $k \in \mathcal{U}$ **do**
> $\mathcal{I} \leftarrow \{n \in \{1, \ldots, N\} \mid c[n] = 0\}$
>   **for** $n \in \mathcal{I}$ **do**
> $(c_k[n], c_k[\hat{n}]) \leftarrow (1, 0)$
> $(\mathbf{r}_{k,n}, \hat{\mathbf{P}}_{k,n}) \leftarrow \mathrm{EBL}(\mathbf{C}_k)$
> $c_k[n] \leftarrow 0$
>   **end for**
> $n_m \leftarrow \mathrm{argmin}_{n \in \mathcal{I}} \hat{P}_{k,n} - P_{k,n}$
> $(P_k, c_k[n_m]) \leftarrow (\hat{P}_{k,n_m}, 1)$
>   **end for**
> **end for**

then RACS can be further revised as ***n****ormalized* ***o****rdered* ***r****e-****a****ssignment of* ***c****onflicting* ***s****ubcarriers (NORACS).*

C. Complexity Analysis

To briefly analyze the complexity, we consider the worst case that all N subcarriers are conflicting and each is used by K users after the initialization, which implies that there exist almost no frequency selective fading. In the initialization EBL must be executed K times for all users, so the complexity of this step is $\Omega(KN)$ due to the linear complexity of EBL. EBL must be called at most KN times to process all conflicts in Algorithm 4. If it is assumed that $\hat{N}$ subcarriers are used by each user on average after the initialization, then the complexity of this step must be lower than $\Omega(KN\hat{N})$. Conflict cancellation in Algorithm 5 occurs not often, its complexity is $\Omega(K\hat{N}^2)$. The complexity of Algorithm 6 is $\Omega(N^2\hat{N})$ while it is called very rarely. From the abovementioned analysis the complexity of the suggested methods is bound by $\Omega(KN + KN\hat{N} + K\hat{N}^2 + N^2\hat{N})$.

V. Numerical Results

In this section, numerical results are given to compare the performance and complexity of RACS, ORACS and NORACS with the successive user integration algorithm (SUSI), which was newly suggested in [4] and can iteratively use EBL. SUSI has better performance than most of other heuristic methods, e.g., [5], [7], [8], [9].

The frequency selective channels of different users are assumed to be independent of each other. The channel behavior is derived by 16 independently Rayleigh distributed multipaths with an exponentially decaying profile. The maximal expected CNR on each subcarrier is set to be 5 dB, which exponentially

TABLE I
USERS IN THE SIMULATION

User type	Proportion	Rate(bits/OFDM symbol)	SNR gap (dB)
Video user	10%	32	7.5
Audio user	40%	8	8.8
Data user	50%	8 (mean)	9.5

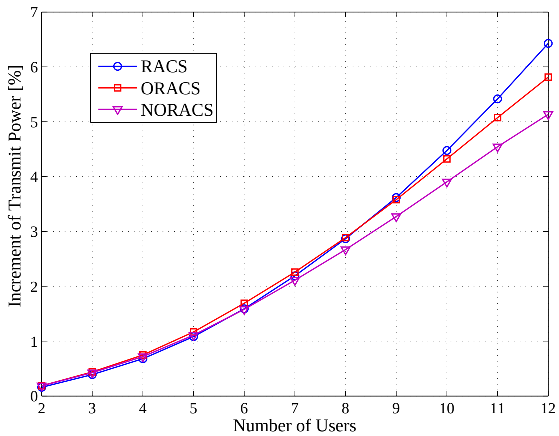

Fig. 3. Increment of total transmit power by using RACS, ORACS and NORACS compared to SUSI in percent.

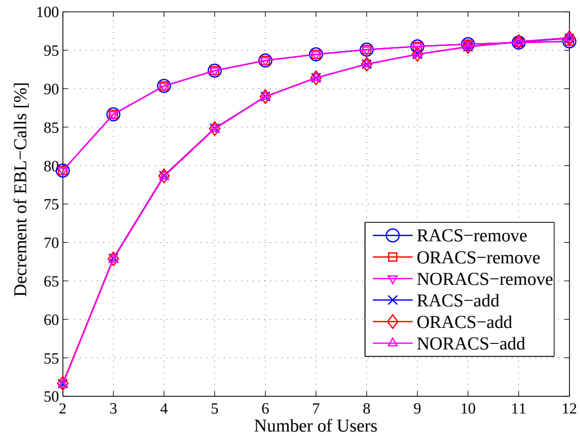

Fig. 4. Decrement of EBL-calls by adding or removing subcarriers while using RACS, ORACS, NORACS compared to SUSI in percent.

fades with the distance from the transmitter to the receiver with the attenuation coefficient equal to two. For a target BER of 10^{-6}, the numbers of bits per modulation symbol in the considered system may be $\{0, 1, 2, 3, 4, 5, 6\}$, which means that $M = 6$ bits at maximum can be transmitted over one subcarrier in one OFDM symbol and that the granularity β is equal to one. We consider an OFDMA system with 64 subcarriers and 2 to 12 users for simulations, which can serve three types of users, shown in Table I. The discrete rate of a data user is obtained by rounding a continuous rate, which is exponentially distributed with a maximal rate of 32 bits per OFDM symbol. Hence, the maximal sum rate of the system is 384 bits per OFDM symbol. 50000 channel samples are generated for each number of users.

Since the proposed methods provide suboptimal solutions to the MMP problem, performance loss cannot be avoided. Fig. 3 shows the increment of total transmit power by using the proposed methods compared to SUSI. By sorting conflicting subcarriers, the performance loss is reduced by around 0.5% and 1% with ORACS and NORACS, respectively.

In Fig. 4 the decrement of EBL-calls compared to SUSI is given. They are split into the calls, where a subcarrier is removed and where a subcarrier is added. It can be seen that the three proposed methods almost have the same running time. They reduce about 92% EBL-calls with a subcarrier removed and about 85% EBL-calls with a subcarrier added on average compared to SUSI.

VI. Conclusion

Efficient methods for multiuser resource allocation allow mobile networks to promptly adapt to fast-varying environments. In this paper, an optimal and efficient bit loading algorithm for OFDM has been proposed, which has the linear complexity of $\mathcal{O}(N)$. It is efficiently utilized in a class of resource allocation methods for OFDMA downlink. Sorting of conflicting subcarriers has been applied leading to effective performance improvement. Simulations have shown that our algorithm reduces the running time significantly while achieving comparable performance to the reference method.

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