ENERGY EFFICIENCY ON REAL TIME TRANSMISSION IN MULTIUSER OFDM DOWNLINK

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ABSTRACT

It has been shown that energy efficiency can be significantly improved by adaptive multicarrier transmission. When the water-filling strategy is employed, different rates are allocated to subcarriers. However, a large signalling overhead is induced, when the employed allocation scheme must be sent to receivers via signalling. In this paper, to enhance energy efficiency, we propose that a dynamic and symmetric rate is allocated to subcarriers of each user. Thereby, fewer bits are required to express each allocation scheme. The asymptotic limit of induced performance loss is given compared to the water-filling strategy. Furthermore, the proposed strategy brings convenience for heuristic design. Simulations demonstrate that the proposed method achieves higher energy efficiency than the water-filling strategy as allocation schemes are frequently updated in fast time-varying environments.

1. INTRODUCTION

Different powers and rates are allocated to subcarriers in adaptive orthogonal frequency division multiplexing (OFDM) according to channel conditions so that energy efficiency is enhanced. The data rate and the time duration are generally fixed for real time transmission. Hence, improving energy efficiency of such transmission, i.e., improving the product of power and time, is equivalent to minimizing transmission power at any specific time. In multiuser systems, users' channel knowledge may not be available at each receiver. Thus, power and rate allocation is executed at the transmitter, while the allocation scheme must be transmitted to receivers for data detection. In [1] the employed allocation scheme is included in the signalling overhead. On one hand, the faster channels vary in time, the more frequently the allocation scheme must be updated. On the other hand, in multiuser systems, even though the channel for only one user changes, the allocation scheme must be updated. Thus, resource allocation schemes must be updated frequently.

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To reduce the signalling overhead, all subcarriers are employed permanently in [2]. Subcarriers are clustered in [3, 4] and symmetric power and rate are allocated to each cluster. In other words, the transmission band is divided into fewer subcarriers. These methods severely worsen energy efficiency when transmission experiences strong frequency selective fading. In [5] the number of employed subcarriers is fixed and the same power and rate are allocated to them such that the receiver preliminarily knows the power and rate allocation scheme in the single-user case. This solution cannot be explicitly extended for the multiuser case, since the same set of subcarriers for different users may have good channel conditions. To reduce the computational complexity, the symmetric power is allocated to subcarriers in [6] and [7], while different rates are achieved over subcarriers. The large signalling overhead still exists.

The energy efficiency problem in this work aims at minimizing the energy for signalling and data transmission for multiuser real time transmission. It is proposed that a symmetric rate but different powers are allocated to subcarriers assigned to each user such that the signalling overhead is significantly reduced. Rates to subcarriers assigned to different users differ. Compared to the water-filling strategy, its performance loss tends to the theoretical limit as the required data rate increases. By using the proposed strategy, it is convenient to swap any two subcarriers between users employing these two subcarriers. This feature is utilized for designing heuristic methods. The energy per bit by the proposed heuristic method is similar to or even lower than that by the dual optimal multiuser water-filling [8].

The remainder of this paper is organized as follows. Section 2 formulates energy efficiency problems for the water-filling strategy and the proposed strategy while considering the signalling overhead, respectively. In Section 3, first, the limit of the induced instantaneous per-symbol performance loss is obtained for the single-user case. The heuristic method is then devised. The proposed strategy is compared to the water-filling strategy by simulations in Section 4. Finally, the content of this paper is concluded.

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2. PROBLEM FORMULATION

Consider OFDM downlink over N subcarriers from the base station (BS) to K users. Assume that perfect channel knowledge is available at the BS. The power and rate allocation is executed at the BS. Receivers are notified of the employed allocation scheme via signalling. One power and rate allocation scheme is only effective for L OFDM symbols due to time-varying channels. Rates are distinguished by M bits. In [1] if the water-filling strategy is used, the following amount of bits is required to express one power and rate allocation scheme, as

$$\lceil \log_2(K) \rceil N + MN$$

where $\lceil \log_2(K) \rceil$ bits are used to identify the user assigned with each subcarrier. The BS must do the best effort broadcasting for the first $\lceil \log_2(K) \rceil N$ bits to let receivers know which subcarriers are assigned to them. The associated energy consumption is *fixed* given the number of users and subcarriers. After that each receiver knows which subcarriers are assigned to it. The other MN bits can be unicasted separately so that each receiver knows which modulation scheme is employed over its subcarriers. Then, data symbols follow.

According to the setting above, energy consumption is related to the energy of the bits expressing modulation schemes and the energy of data symbols. The amount of data bits in L OFDM symbols over subcarrier n is $L\, r_{k,n} - M$, in which $r_{k,n}$ is the data rate to subcarrier n for user k. The energy efficiency problem can be written as

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} && (1) \\
\text{subject to} \quad & r_{k,n} = \log_2(1 + p_{k,n}\, g_{k,n}) - \frac{M}{L} \\
& \sum_{n=1}^{N} r_{k,n} \geq R_k, \quad k = 1, \ldots, K \\
& \sum_{n=1}^{N} r_{k,n}\, r_{l,n} = 0, \quad k, l = 1, \ldots, K, k \neq l
\end{aligned}$$

where $p_{k,n}$ is the power allocated to subcarrier n for user k. The power $p_{k,n}$ is related to $r_{k,n}$ via the first equality constraint function, where $g_{k,n}$ is the channel gain-to-noise ratio (CNR) of subcarrier n of user k. Once a subcarrier is employed, the rate M/L must be allocated for signalling. The rate required by user k is denoted by R_k and the associated power is referred to as $P_k^{(\text{WF})} = \sum_{n=1}^{N} p_{k,n}$. One subcarrier is assigned to at most one user at any specific time [9], represented by the last constraint.

To reduce the signalling overhead, we propose that an adaptive and symmetric rate is allocated to subcarriers assigned to each user. Intuitively, the amount of bits for expressing one power and rate allocation scheme is

$$\lceil \log_2(K) \rceil N + KM.$$

As K grows to N, the signalling overhead above approaches the previous one, while $K \leq N$ holds. With this setting, the energy efficiency problem is equivalent to

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} \sum_{n \in \mathcal{S}_k} p_{k,n} && (2) \\
\text{subject to} \quad & r_k = \log_2(1 + p_{k,n}\, g_{k,n}) \\
& s_k r_k \geq R_k + \frac{M}{L}, \quad k = 1, \ldots, K \\
& \sum_{n=1}^{N} p_{k,n}\, p_{l,n} = 0, \quad k, l = 1, \ldots, K, k \neq l
\end{aligned}$$

where only M bits are sufficient for expressing the employed modulation scheme for each user. The subcarrier assignment for user k is denoted by a set $\mathcal{S}_k$ with the cardinality s_k.

3. POWER AND COMPUTATIONAL EFFICIENCY

By the proposed strategy (1) changes to (2) in the section above. This offers convenience for designing a computationally efficient method in the following.

3.1. Water-Filling (WF)

Before solving the problem (1), let us first review a dual optimal solution of (1). Duality theory [10] is used to offer dual optima for resource allocation problems in [8]. Although the power-rate function is changed in (1) compared to the primal one in [8], the rate and power allocated to subcarriers given the dual variables are

$$\begin{aligned}
\forall\, k, n: \quad r_{k,n} &= \left[\log_2 \left(\frac{\lambda_k\, g_{k,n}}{\ln(2)} \right) - \frac{M}{L} \right]^+, \\
\forall\, k, n: \quad p_{k,n} &= \left[\frac{\lambda_k}{\ln(2)} - \frac{1}{g_{k,n}} \right]^+
\end{aligned}$$

where λ_k is the dual variable associated to R_k. Subcarrier n is assigned to user l as

$$l = \underset{k=1,\ldots,K}{\operatorname{argmin}}\; p_{k,n} - \lambda_k\, r_{k,n}\,.$$

The subcarrier assignment is determined.

As explicated in [10], the complementary slackness condition states that the constraint is met with equality if and only if the dual variable associated with the inequality is strictly greater than zero. Hence, the rate constraint must be satisfied with equality at optimum. The ellipsoid method is used to determine a dual optimum. Its complexity is $\mathcal{O}(NK^3)$. In each iteration, the logarithmic operation is executed KN times, which make the dual optimum computationally expensive. This may lead to large delay.

3.2. Single-User Symmetric Rate (SR)

Different from the strategy above, a symmetric rate r_k is allocated to subcarriers assigned to user k. This results in a simple solution to (2) when $\mathcal{S}_k$ is fixed. Given $\mathcal{S}_k$ the single-user symmetric rate allocation problem is extracted as

$$\begin{aligned} \text{minimize} \quad & \sum_{n\,\mathcal{S}_k} p_{k,n} \\ \text{subject to} \quad & r_k = \log_2(1 + p_{k,n}\, g_{k,n}) \\ & s_k r_k = R_k + \frac{M}{L} \end{aligned} \tag{3}$$

according to the complementary slackness condition as before. The symmetric rate allocated to subcarriers is

$$\forall\, n \in \mathcal{S}_k : \quad r_k = \frac{R_k + M/L}{s_k},$$

while the powers to subcarriers are different as

$$\forall\, n \in \mathcal{S}_k : \quad p_{k,n} = \frac{1}{g_{k,n}}(2^{r_k} - 1).$$

The sum power is obtained as $P_k^{(\mathrm{SR})} = s_k/H_k(2^{r_k} - 1)$ where H_k is the harmonic average of CNRs of subcarriers in $\mathcal{S}_k$.

The symmetric rate allocation above can be interpreted as a water-filling solution over s_k subcarriers with the same CNR H_k. If $\mathcal{S}_k$ changes, H_k varies and the power allocation is updated immediately. The induced performance loss is given without considering the signalling overhead (M/L is omitted in (3)) compared to the water-filling strategy as

$$\frac{P_k^{(\mathrm{SR})} - P_k^{(\mathrm{WF})}}{P_k^{(\mathrm{WF})}} = \frac{G_k - H_k}{H_k - G_k 2^{-\frac{R_k}{s_k}}} \tag{4}$$

where G_k is the geometric average of CNRs of subcarriers in $\mathcal{S}_k$. The inequality $H_k \le G_k$ always holds. The equality is met as transmission experiences flat fading and there exists no performance loss by using the proposed strategy. When R_k/s_k goes to infinity, the performance loss is

$$\lim_{R_k/s_k \to \infty} \frac{P_k^{(\mathrm{SR})} - P_k^{(\mathrm{WF})}}{P_k^{(\mathrm{WF})}} = \frac{G_k}{H_k} - 1. \tag{5}$$

It implies that the performance loss is limited when at least one of the required rate and the number of users is large. This condition may be satisfied in large scale systems, where many users are covered and each of them demands a large rate.

Fig. 1 shows that the performance loss tends to the limit (5) as the required rate increases, when the eight subcarriers are all employed. The maximum loss appears, when these two strategies employ different numbers of subcarriers. Note that the large loss 27.89% is caused by the channel with strong frequency selectivity in our example. In general, this asymptotic limit is low shown by simulations.

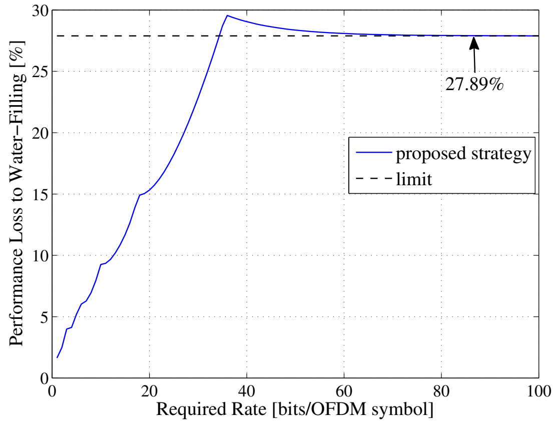

Fig. 1. Performance loss of the proposed strategy for a single user with $N = 8$ subcarriers $g_{k,n} = n, n = 1, \ldots, 8$.

3.3. Multiuser Symmetric Rate

As explained earlier, the single-user symmetric rate allocation is simple as the subcarrier assignment is fixed. The heuristic design can benefit from the following feature of the proposed strategy. Assume that subcarrier n is assigned to user k and subcarrier m is assigned to user l in the present subcarrier assignment. If these two subcarriers were swapped, i.e., subcarrier n were reassigned to user l while subcarrier m were reassigned to user k, the resulting variation $\Delta_m^n P(k,l)$ of the sum power would be equal to

$$\underbrace{(2^{\frac{R_k}{s_k}} - 1)}_{=Z_k}\Big(\sum_{i\in\mathcal{S}_k} \frac{1}{g_{k,i}} + \frac{1}{g_{k,m}} - \sum_{i\in\mathcal{S}_k} \frac{1}{g_{k,i}} - \frac{1}{g_{k,n}}\Big) + \underbrace{(2^{\frac{R_l}{s_l}} - 1)}_{=Z_l}\Big(\sum_{i\in\mathcal{S}_l} \frac{1}{g_{l,i}} + \frac{1}{g_{l,n}} - \sum_{i\in\mathcal{S}_l} \frac{1}{g_{l,i}} - \frac{1}{g_{l,m}}\Big)$$

which simplifies to

$$\Delta_m^n P(k,l) = Z_k\Big(\frac{1}{g_{k,m}} - \frac{1}{g_{k,n}}\Big) + Z_l\Big(\frac{1}{g_{l,n}} - \frac{1}{g_{l,m}}\Big)$$

where $\mathcal{S}_k$ and $\mathcal{S}_l$ denote the subcarrier assignments before swapping. Since Z_k is only determined by the cardinality of $\mathcal{S}_k$, it remains constant after swapping. If $\Delta_m^n P(k,l) < 0$ holds, we actually perform this swapping. Otherwise, we check another pair. The criterion for this adjustment is simplified to

$$Z_k\Big(\frac{1}{g_{k,m}} - \frac{1}{g_{k,n}}\Big) + Z_l\Big(\frac{1}{g_{l,n}} - \frac{1}{g_{l,m}}\Big) < 0. \tag{6}$$

Updating the subcarrier assignment where the symmetric rates are allocated requires only several simple operations as long as the number of subcarriers assigned to each user is fixed.

To take advantage of the feature above, an idea from [11] is used in Algorithm 1 for initializing the subcarrier assignment. First, Part 1 decides the number of subcarriers assigned

Algorithm 1 Initialization

Part 1:
$s_k \leftarrow 1, k = 1, \ldots, K$
for $n = 1, \ldots, N$ **do**
  $\tilde{k} \leftarrow \underset{k=1,\ldots,K}{\operatorname{argmin}} \frac{(s_k+1)}{g_k} 2^{R_k/(s_k+1)} - \frac{s_k}{g_k} 2^{R_k/s_k} - \frac{1}{g_k}$
  $s_{\tilde{k}} \leftarrow s_{\tilde{k}} + 1$
end for

Part 2:
$\mathcal{N} \leftarrow \{1, \ldots, N\}$
repeat
  for k = 1,...,K **do**
    $\mathcal{A} \leftarrow \{\hat{s}_k$ subcarriers with the largest $g_{k,n}, n \in \mathcal{N}\}$
    $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \mathcal{A}$
    $\mathcal{N} \leftarrow \mathcal{N} \setminus \mathcal{A}$
  end for
until $\mathcal{N} = \emptyset$

to each user $s_1, \ldots, s_K$, where the average CNR for each user $g_k = \sum_{n=1}^{N} g_{k,n} / N$ is used to simplify the procedure. Part 2 then iteratively assigns subcarriers to users according to the evaluated cardinalities from Part 1. In each iteration, the $\hat{s}_k = s_k / (\prod_{k=1}^{K} s_k)^{1/K}$ available subcarriers with the greatest CNRs are assigned to each user till all subcarriers are assigned. Here, the geometric average is chosen empirically. In doing so, every user has the opportunity of having subcarriers with relatively high CNRs for it. Subcarriers having high CNR for several users can be dispersed to these users. Finally, the subcarrier assignment $\mathcal{S}_1, \ldots, \mathcal{S}_K$ is initialized. Intuitively, the complexity of the initialization is $\mathcal{O}(KN)$.

After the initialization, the cardinalities are fixed. This means that $\{Z_k | k = 1, \ldots, K\}$ remains constant. For each subcarrier n, subcarriers $\{m \in \mathcal{N} \setminus \mathcal{S}_k\}$ that may be swapped with it are checked whether the associated swapping may improve performance or not, where (6) is used. This successive procedure repeats till no improvement can be made. Its complexity is $\mathcal{O}(N^2)$ and is not related to the number of users K.

The complexities of determining the dual optimum of (1) and the heuristic solution of (2) are listed in Table 1. For the dual optimum, the logarithmic operation is computed KN times in each iteration. In Algorithm 2, only simple operations are necessary. Therefore, even though $K < N$ may lead to $\mathcal{O}(NK^3) < \mathcal{O}(N^2)$, the computing time for calculating the proposed method is still much shorter.

Table 1. Complexity Comparison

Dual optimum of water-filling	proposed method
$\mathcal{O}(NK^3)\times$ logarithm	$\mathcal{O}(KN^2))\times$ addition

Algorithm 2 Subcarrier Adjustment

$\mathcal{N} \leftarrow \{1, \ldots, N\}$
repeat
  for $n = 1, \ldots, N$ **do**
    $k \leftarrow \{j = 1, \ldots, K | \mathcal{S}_k \cap \{n\} \neq \emptyset\}$
    for $m \in \mathcal{N} \setminus \mathcal{S}_k$ **do**
      $l \leftarrow \{j = 1, \ldots, K | \mathcal{S}_k \cap \{m\} \neq \emptyset\}$
      if $Z_k(\frac{1}{g_{k,m}} - \frac{1}{g_{k,n}}) + Z_l(\frac{1}{g_{l,n}} - \frac{1}{g_{l,m}}) < 0$ **then**
        $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \{m\} \setminus \{n\}$
        $\mathcal{S}_l \leftarrow \mathcal{S}_l \cup \{n\} \setminus \{m\}$
      end if
    end for
  end for
until no improvement

4. SIMULATION RESULTS

In this section, the proposed heuristic solution of (2) is compared to the dual optimum of (1). The simulation system is built with the parameters from the mobile worldwide interoperability for microwave access (WiMAX) [12]. It consists of $N = 128$ subcarriers. The time duration of one OFDM symbol is $102.9\,\mu$s. Each demanded data rate R_k is uniformly distributed within $[50, 100]$ bits per OFDM symbols. Each rate is expressed by $M = 6$ bits. The frequency selective channel is modeled as consisting of $N/8 = 16$ independently Rayleigh distributed multiple paths with an exponentially decaying profile. The expected channel gain of each subcarrier is normalized to one and the power of the additive Gaussian noise is set to be -5 dBW.

We evaluate the system performance via the average energy per bit, defined as $E_b = \sum_{k=1}^{K} P_k (\sum_{k=1}^{K} R_k)^{-1}$ where P_k is the transmission power for user k and R_k is the data rate required by user k. The frame length in WiMAX is $L = 48$ OFDM symbols. Fig. 2 draws the average energy per bit by the proposed heuristic solution of (2) and the dual optimum of (1). They have the similar performance. As explained before, the average computing time for calculating our method is much shorter than that for determining the dual optimum, seen from Table 2.

The smaller L becomes, the faster channels change in time so that more frequently the power and rate allocation scheme must be updated and vice versa. Shown by Fig. 3, when the primal L doubles to 96, our method still has close

Table 2. Average Computing Time [s/10]

K	4	6	8	10	12
dual optimum	5.75	16.15	34.46	64.09	105.5
proposed	2.78	2.83	2.86	2.89	2.93

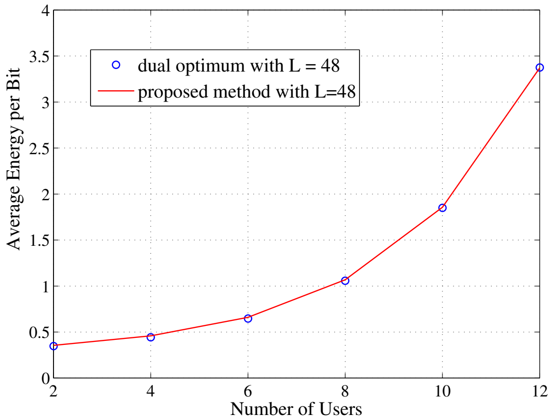

Fig. 2. Energy efficiency vs. number of user with $L = 48$.

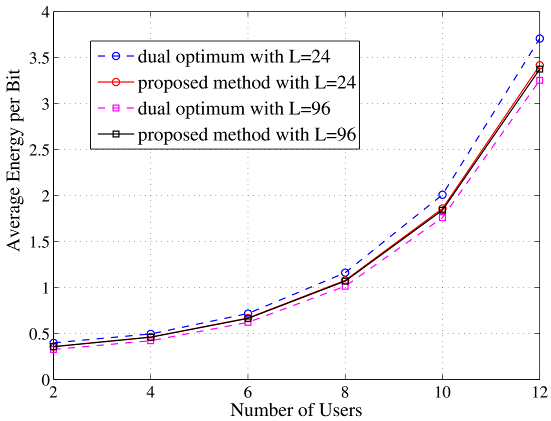

Fig. 3. Energy efficiency vs. number of user with $L = 24$ or $L = 96$.

performance to the dual optimum, whereas it has much better performance than the dual optimum as L reduces to 24.

5. CONCLUSION

In this paper, we have investigated energy efficiency on real time transmission with the signalling overhead additionally considered. To balance energy consumption on signalling and data transmission, we have proposed that a symmetric rate is allocated to subcarriers assigned to each user. The induced performance loss compared to the water-filling strategy has been analyzed. The heuristic method has been provided by utilizing the feature of the proposed strategy that it is simple to swap any two subcarriers between users employing these two subcarriers. Only simple operations are necessary in the proposed method. The simulation results have demonstrated that our method achieves a better compromise of energy consumption on signalling and data transmission so that energy efficiency is enhanced.

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