

Joint Receiver Design and Power Allocation Strategies for Multihop Wireless Sensor Networks

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Abstract—In this paper, we consider a multihop wireless sensor network (WSN) with multiple relay nodes for each hop where the amplify-and-forward (AF) scheme is employed. Our strategy is to jointly design the linear receiver and the power allocation parameters via an alternating optimization approach subject to global, local and individual power constraints. We derive constrained minimum mean-square error (MMSE) expressions for the linear receiver and the power allocation parameters that contain the optimal complex amplification coefficients for each relay node. Computer simulations show good performance of our proposed methods in terms of bit error rate (BER) compared to the method with equal power allocation.

I. INTRODUCTION

Recently, there has been a growing research interest in wireless sensor networks (WSNs) as their unique features allow a wide range of applications in the areas of defence, environment, health and home [1]. They are usually composed of a large number of densely deployed sensing devices which can transmit their data to the desired user through multihop relays [2]. Low complexity and high energy efficiency are the most important design characteristics of communication protocols [3] and physical layer techniques employed for WSNs. The performance and capacity of WSNs can be significantly enhanced through exploitation of spatial diversity with cooperation between the nodes [2]. In a cooperative WSN, nodes relay signals to each other in order to propagate redundant copies of the same signals to the destination nodes. Among the existing relaying schemes, the amplify-and-forward (AF) and the decode-and-forward (DF) are the most popular approaches [4].

Due to limitations in sensor node power, computational capacity and memory [1], some power allocation methods have been proposed for WSNs to obtain the best possible SNR or best possible quality of service (QoS) [5] at the destinations. The majority of the previous literature considers a source and destination pair, with one or more randomly placed relay nodes. These relay nodes are usually placed with uniform distribution [6], equal distance, or in line with the source and destination. The reason of these simple considerations is that they can simplify complex problems and obtain closed-form solutions. A single relay AF system using mean channel gain channel state information (CSI) is analyzed in [7], where the outage probability is the criterion used for optimization. For DF systems, a near-optimal power allocation strategy called the Fixed-Sum-Power with Equal-Ratio (FSP-ER) scheme

based on partial CSI has been developed in [6]. This near-optimal scheme allocates one half of the total power to the source node and splits the remaining half equally among selected relay nodes. A node is selected for relay if its mean channel gain to the destination is above a threshold. Simulation results show that this scheme significantly outperforms two traditional power allocation schemes. One is the 'Constant-Power scheme' where all nodes serve as relay nodes and all nodes including the source node and relay nodes transmit with the same power. The other one is the 'Best-Select scheme' where only one node with the largest mean channel gain to the destination is chosen as the relay node.

In this paper, we consider a general multihop wireless sensor network where the AF relaying scheme is employed. Our strategy is to jointly design the linear receiver (\mathbf{W}) and the power allocation parameter (\mathbf{a}) that contains the optimal complex amplification coefficients for each relay node via an alternating optimization approach. It can be considered as a constrained optimization problem where the objective function is the mean-square error (MSE) and the constraint is a bound on the power levels among the relay nodes. Then the constrained MMSE expressions for the linear receiver and the power allocation parameter can be derived. In this work, we present three strategies where the allocation of power level across the relay nodes is subject to global, local and individual power constraints. The major novelty in these strategies presented here is that they are applicable to general multihop WSNs with multi source nodes and destination nodes, as opposed to the simple 2-hop WSNs with one pair of source-destination nodes [5], [7], [8]. Further novelty in this work is that a closed form solution for the Lagrangian multiplier (λ) that arises in the expressions of the power allocation parameter can be achieved.

This paper is organized as follows. Section II describes the general multihop WSN system model. Section III develops three joint receiver design and power allocation strategies subject to three different power constraints. Section IV presents and discusses the simulation results, while Section V provides some concluding remarks.

II. COOPERATIVE WSN SYSTEM MODEL

Consider a general m -hop wireless sensor network (WSN) with multiple parallel relay nodes for each hop, as shown in Fig. 1. The WSN consists of N_0 source nodes, N_m destination nodes and N_r relay nodes which are separated into $m - 1$

groups: N_1, N_2, \dots, N_{m-1} . We will concentrate on a time division scheme with perfect synchronization, for which all signals are transmitted and received in separate time slots. The sources first broadcast the $N_0 \times 1$ signal vector \mathbf{s} to the first group of relay nodes. We consider an amplify-and-forward (AF) cooperation protocol in this paper. Each group of relay nodes receives the signal, amplifies and rebroadcasts them to the next group of relay nodes (or the destination nodes). In practice, we need to consider the constraints on the transmission policy. For example, each transmitting node would transmit during only one phase. In our WSN system, we assume that each group of relay nodes transmits the signal to the nearest group of relay nodes (or the destination nodes) directly. We can use a block diagram to indicate the multihop WSN system as shown in Fig. 2.

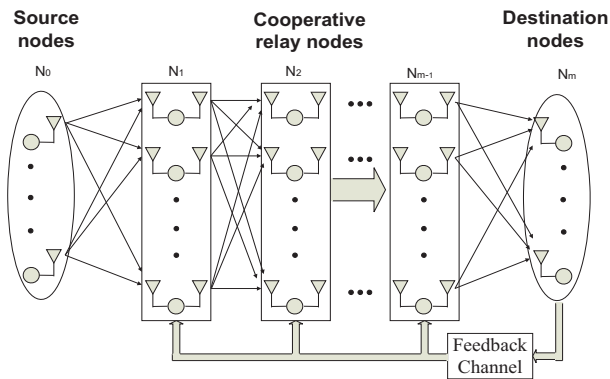


Fig. 1. m -hop WSN with N_0 sources, N_m destinations and N_r relays.

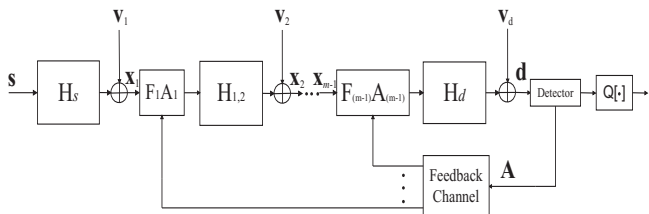


Fig. 2. Block diagram of the multihop WSN system.

Let \mathbf{H}_s denote the $N_1 \times N_0$ channel matrix between the source nodes and the first group of relay nodes, \mathbf{H}_d denote the $N_m \times N_{m-1}$ channel matrix between the $(m-1)$ th group of relay nodes and destination nodes, and $\mathbf{H}_{i-1,i}$ denote the $N_i \times N_{i-1}$ channel matrix between two groups of relay nodes as described by

$$\mathbf{H}_s = \begin{bmatrix} \mathbf{h}_{s,1} \\ \mathbf{h}_{s,2} \\ \vdots \\ \mathbf{h}_{s,N_1} \end{bmatrix}, \quad \mathbf{H}_d = \begin{bmatrix} \mathbf{h}_{m-1,1} \\ \mathbf{h}_{m-1,2} \\ \vdots \\ \mathbf{h}_{m-1,N_m} \end{bmatrix}, \quad \mathbf{H}_{i-1,i} = \begin{bmatrix} \mathbf{h}_{i-1,1} \\ \mathbf{h}_{i-1,2} \\ \vdots \\ \mathbf{h}_{i-1,N_i} \end{bmatrix},$$

where $\mathbf{h}_{s,j} = [h_{s,j,1}, h_{s,j,2}, \dots, h_{s,j,N_0}]$ for $j = 1, 2, \dots, N_1$ is a row vector between source nodes and the j th relay of the first group of relay nodes, $\mathbf{h}_{m-1,j} = [h_{m-1,j,1}, h_{m-1,j,2}, \dots, h_{m-1,j,N_{m-1}}]$ for $j = 1, 2, \dots, N_m$

is a row vector between the $(m-1)$ th group of relay nodes and the j th destination node and $\mathbf{h}_{i-1,j} = [h_{i-1,j,1}, h_{i-1,j,2}, \dots, h_{i-1,j,N_{i-1}}]$ for $j = 1, 2, \dots, N_i$ is a row vector between the $(i-1)$ th group of relay nodes and the j th relay of the i th group of relay nodes. The received signal at the i th group of relay nodes (\mathbf{x}_i) for each phase can be expressed as:

Phase 1:

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{H}_s \mathbf{s} + \mathbf{v}_1, \\ \mathbf{y}_1 &= \mathbf{F}_1 \mathbf{x}_1, \end{aligned}$$

Phase 2:

$$\begin{aligned} \mathbf{x}_2 &= \mathbf{H}_{1,2} \mathbf{A}_1 \mathbf{y}_1 + \mathbf{v}_2, \\ \mathbf{y}_2 &= \mathbf{F}_2 \mathbf{x}_2, \end{aligned}$$

⋮

Phase i : ($i = 2, 3, \dots, m-1$)

$$\begin{aligned} \mathbf{x}_i &= \mathbf{H}_{i-1,i} \mathbf{A}_{i-1} \mathbf{y}_{i-1} + \mathbf{v}_i, \\ \mathbf{y}_i &= \mathbf{F}_i \mathbf{x}_i, \end{aligned}$$

At the destination nodes, the received signal can be expressed as

$$\mathbf{d} = \mathbf{H}_d \mathbf{A}_{m-1} \mathbf{y}_{m-1} + \mathbf{v}_d,$$

where \mathbf{v} is a zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix $\sigma^2 \mathbf{I}$. $\mathbf{A}_i = \text{diag}\{a_{i,1}, a_{i,2}, \dots, a_{i,N_i}\}$ is a diagonal matrix whose elements represent the amplification coefficient of each relay of the i th group. $\mathbf{F}_i = \text{diag}\{E(|x_{i,1}|^2), E(|x_{i,2}|^2), \dots, E(|x_{i,N_i}|^2)\}^{-\frac{1}{2}}$ denotes the normalization matrix which can normalize the power of the received signal for each relay of the i th group of relays. Please note that the property of the matrix vector multiplication $\mathbf{A}\mathbf{y} = \mathbf{Y}\mathbf{a}$ will be used in the next section, where \mathbf{Y} is the diagonal matrix form of the vector \mathbf{y} and \mathbf{a} is the vector from of the diagonal matrix \mathbf{A} . In the receiver, a linear MMSE detector is considered where the optimal filter and optimal amplification coefficients are calculated. The optimal amplification coefficients are transmitted to the relays through the feedback channel. And the block marked with a $\mathbf{Q}[\cdot]$ represents a decision device.

III. PROPOSED JOINT MMSE DESIGN OF THE RECEIVER AND POWER ALLOCATION

In this section, three constrained optimization problems are proposed to describe the joint design of the linear receiver (\mathbf{W}) and power allocation parameter (\mathbf{a}) subject to a global, local and individual power constraints.

A. MMSE Design with a Global Power Constraint

We first consider the case where the total power of all the relay nodes is limited to P_T . The proposed method can be considered as the following optimization problem

$$\begin{aligned} [\mathbf{W}_{opt}, \mathbf{a}_{1,opt}, \dots, \mathbf{a}_{m-1,opt}] &= \arg \min_{\mathbf{W}, \mathbf{a}_1, \dots, \mathbf{a}_{m-1}} E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2], \\ \text{subject to } \sum_{i=1}^{m-1} P_i &= P_T \end{aligned}$$

where $(\cdot)^H$ denotes the complex-conjugate (Hermitian) transpose, P_i is the transmitted power of the i th group of relay nodes, and $P_i = N_{i+1} \mathbf{a}_i^H \mathbf{a}_i$.

To solve this constrained optimization problem, we can modify the MSE cost function using the method of Lagrange multipliers [9] which yields the following Lagrangian function

$$\begin{aligned} \mathcal{L} &= E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] + \lambda \left(\sum_{i=1}^{m-1} N_{i+1} \mathbf{a}_i^H \mathbf{a}_i - P_T \right) \\ &= E(\mathbf{s}^H \mathbf{s}) - E(\mathbf{d}^H \mathbf{W} \mathbf{s}) - E(\mathbf{s}^H \mathbf{W}^H \mathbf{d}) + E(\mathbf{d}^H \mathbf{W} \mathbf{W}^H \mathbf{d}) \\ &\quad + \lambda \left(\sum_{i=1}^{m-1} N_{i+1} \mathbf{a}_i^H \mathbf{a}_i - P_T \right) \end{aligned} \quad (1)$$

By fixing $\mathbf{a}_1, \dots, \mathbf{a}_{m-1}$ and setting the gradient of \mathcal{L} in (1) with respect to the filter \mathbf{W} equal to zero, we can get

$$\begin{aligned} \mathbf{W}_{opt} &= [E(\mathbf{d} \mathbf{d}^H)]^{-1} E(\mathbf{d} \mathbf{s}^H) \\ &= \left[\mathbf{H}_d \mathbf{A}_{m-1} E(\mathbf{y}_{m-1} \mathbf{y}_{m-1}^H) \mathbf{A}_{m-1}^H \mathbf{H}_d^H + \sigma_n^2 \mathbf{I} \right]^{-1} \\ &\quad \cdot \mathbf{H}_d \mathbf{A}_{m-1} E(\mathbf{y}_{m-1} \mathbf{s}^H). \end{aligned} \quad (2)$$

The optimal expression for \mathbf{a}_{m-1} is obtained by equating the partial derivative of \mathcal{L} with respect to \mathbf{a}_{m-1} to zero

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{a}_{m-1}} &= -2E\left(\frac{\partial \mathbf{d}}{\partial \mathbf{a}_{m-1}} \mathbf{W} \mathbf{s}\right) + 2E\left(\frac{\partial \mathbf{d}}{\partial \mathbf{a}_{m-1}} \mathbf{W} \mathbf{W}^H \mathbf{d}\right) \\ &\quad + 2N_m \lambda \mathbf{a}_{m-1} \\ &= -2E(\mathbf{Y}_{m-1}^H \mathbf{H}_d^H \mathbf{W} \mathbf{s}) + 2N_m \lambda \mathbf{a}_{m-1} \\ &\quad + 2E[\mathbf{Y}_{m-1}^H \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H (\mathbf{H}_d \mathbf{Y}_{m-1} \mathbf{a}_{m-1} + \mathbf{v}_d)] \\ &= \mathbf{0}. \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} \mathbf{a}_{m-1,opt} &= [E(\mathbf{Y}_{m-1}^H \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{Y}_{m-1}) + N_m \lambda \mathbf{I}]^{-1} \\ &\quad \cdot E(\mathbf{Y}_{m-1}^H \mathbf{H}_d^H \mathbf{W} \mathbf{s}) \\ &= [\mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \circ E(\mathbf{y}_{m-1} \mathbf{y}_{m-1}^H) + N_m \lambda \mathbf{I}]^{-1} \\ &\quad \cdot [\mathbf{H}_d^H \mathbf{W} \circ E(\mathbf{y}_{m-1} \mathbf{s}^H) * \mathbf{u}] \end{aligned} \quad (3)$$

where \circ denotes the Hadamard (element-wise) product, $(\cdot)^*$ denotes the complex-conjugate and $\mathbf{u} = [1, 1, \dots, 1]^T$.

Similarly, for $i = 2, 3, \dots, m-1$, we have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{a}_{i-1}} &= -2E\left(\frac{\partial \mathbf{d}}{\partial \mathbf{a}_{i-1}} \mathbf{W} \mathbf{s}\right) + 2\left(\frac{\partial \mathbf{d}}{\partial \mathbf{a}_{i-1}} \mathbf{W} \mathbf{W}^H \mathbf{d}\right) + 2N_i \lambda \mathbf{a}_{i-1} \\ &= \mathbf{0} \end{aligned}$$

where

$$\frac{\partial \mathbf{d}}{\partial \mathbf{a}_{i-1}} = \mathbf{Y}_{i-1}^H \left(\prod_{k=i}^{m-1} \mathbf{H}_{k-1,k}^H \mathbf{F}_k^H \mathbf{A}_k^H \right) \mathbf{H}_d^H.$$

Let

$$\mathbf{B}_{i-1} = \prod_{k=i}^{m-1} \mathbf{H}_{k-1,k}^H \mathbf{F}_k^H \mathbf{A}_k^H.$$

Then, we get

$$\begin{aligned} \mathbf{a}_{i-1,opt} &= [E(\mathbf{Y}_{i-1}^H \mathbf{B}_{i-1} \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{B}_{i-1} \mathbf{Y}_{i-1}) + N_i \lambda \mathbf{I}]^{-1} \\ &\quad \cdot E(\mathbf{Y}_{i-1}^H \mathbf{B}_{i-1} \mathbf{H}_d^H \mathbf{W} \mathbf{s}) \\ &= [\mathbf{B}_{i-1} \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{B}_{i-1} \circ E(\mathbf{y}_{i-1} \mathbf{y}_{i-1}^H) + N_i \lambda \mathbf{I}]^{-1} \\ &\quad \cdot [\mathbf{B}_{i-1} \mathbf{H}_d^H \mathbf{W} \circ E(\mathbf{y}_{i-1} \mathbf{s}^H) * \mathbf{u}]. \end{aligned} \quad (4)$$

From (3) and (4), we conclude that

$$\begin{aligned} \mathbf{a}_{i,opt} &= [E(\mathbf{Y}_i^H \mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{B}_i \mathbf{Y}_i) + N_{i+1} \lambda \mathbf{I}]^{-1} \\ &\quad \cdot E(\mathbf{Y}_i^H \mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{s}) \\ &= [\mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{B}_i \circ E(\mathbf{y}_i \mathbf{y}_i^H) + N_{i+1} \lambda \mathbf{I}]^{-1} \\ &\quad \cdot [\mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \circ E(\mathbf{y}_i \mathbf{s}^H) * \mathbf{u}] \end{aligned} \quad (5)$$

where

$$\mathbf{B}_i = \begin{cases} \prod_{k=i+1}^{m-1} \mathbf{H}_{k-1,k}^H \mathbf{F}_k^H \mathbf{A}_k^H, & \text{for } i = 1, 2, \dots, m-2, \\ \mathbf{I}, & \text{for } i = m-1. \end{cases}$$

Please see the appendix to find the expressions of \mathbf{F}_i , $E(\mathbf{y}_i \mathbf{y}_i^H)$, and $E(\mathbf{y}_i \mathbf{s}^H)$. The expressions in (2) and (5) depend on each other. Therefore it is necessary to iterate them with an initial value of \mathbf{a}_i ($i = 1, 2, \dots, m-1$) to obtain the solutions.

The Lagrange multiplier λ can be determined by solving

$$\sum_{i=1}^{m-1} N_{i+1} \mathbf{a}_{i,opt}^H \mathbf{a}_{i,opt} = P_T. \quad (6)$$

Let

$$\phi_i = E(\mathbf{Y}_i^H \mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{B}_i \mathbf{Y}_i) \quad (7)$$

and

$$\mathbf{z}_i = E(\mathbf{Y}_i^H \mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{s}) \quad (8)$$

Then, we get

$$\mathbf{a}_i = (\phi_i + N_{i+1} \lambda \mathbf{I})^{-1} \mathbf{z}_i$$

When λ is a real value,

$$[(\phi_i + N_{i+1} \lambda \mathbf{I})^{-1}]^H = [(\phi_i + N_{i+1} \lambda \mathbf{I})^H]^{-1} = (\phi_i + N_{i+1} \lambda \mathbf{I})^{-1}.$$

Equation (6) becomes

$$\sum_{i=1}^{m-1} N_{i+1} \mathbf{z}_i^H (\phi_i + N_{i+1} \lambda \mathbf{I})^{-1} (\phi_i + N_{i+1} \lambda \mathbf{I})^{-1} \mathbf{z}_i = P_T. \quad (9)$$

Using an eigendecomposition,

$$\phi_i = \mathbf{Q}_i \mathbf{\Lambda}_i \mathbf{Q}_i^{-1}$$

where $\mathbf{\Lambda}_i = \text{diag}\{\alpha_{i,1}, \alpha_{i,2}, \dots, \alpha_{i,M_i}, 0, \dots, 0\}$ consists of eigenvalues of ϕ_i and $M_i = \min\{N_0, N_i, N_m\}$. Then, we get

$$\phi_i + N_{i+1} \lambda \mathbf{I} = \mathbf{Q}_i (\mathbf{\Lambda}_i + N_{i+1} \lambda \mathbf{I}) \mathbf{Q}_i^{-1}.$$

Therefore, (9) can be expressed as

$$\sum_{i=1}^{m-1} N_{i+1} \mathbf{z}_i^H \mathbf{Q}_i (\mathbf{\Lambda}_i + N_{i+1} \lambda \mathbf{I})^{-2} \mathbf{Q}_i^{-1} \mathbf{z}_i = P_T. \quad (10)$$

Using the properties of the trace operation, (10) can be written as

$$\sum_{i=1}^{m-1} N_{i+1} \text{tr} \left((\mathbf{\Lambda}_i + N_{i+1} \lambda \mathbf{I})^{-2} \mathbf{Q}_i^{-1} \mathbf{z}_i \mathbf{z}_i^H \mathbf{Q}_i \right) = P_T. \quad (11)$$

Defining $\mathbf{C}_i = \mathbf{Q}_i^{-1} \mathbf{z}_i \mathbf{z}_i^H \mathbf{Q}_i$, (11) becomes

$$\sum_{i=1}^{m-1} \sum_{j=1}^{N_i} N_{i+1} (\alpha_{i,j} + N_{i+1} \lambda)^{-2} \mathbf{C}_i(j, j) = P_T.$$

Since ϕ_i is a matrix with at most rank M_i , only the first M_i columns of \mathbf{Q}_i span the column space of $E(\mathbf{Y}_i^H \mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{s})^H$ which causes that the last $(N_i - M_i)$ columns of $\mathbf{z}_i^H \mathbf{Q}_i$ are zero vectors and the last $(N_i - M_i)$ diagonal elements of \mathbf{C}_i are zero. Therefore, we can obtain the $\{\sum_{i=1}^{m-1} 2M_i\}$ -th-order polynomial

$$\sum_{i=1}^{m-1} \sum_{j=1}^{M_i} N_{i+1} (\alpha_{i,j} + N_{i+1} \lambda)^{-2} \mathbf{C}_i(j, j) = P_T.$$

B. MMSE Design with Local Power Constraints

Secondly, we consider the case where the total power of the relay nodes in each group is limited to some value $P_{T,i}$. The proposed method can be considered as the following optimization problem

$$[\mathbf{W}_{opt}, \mathbf{a}_{1,opt}, \dots, \mathbf{a}_{m-1,opt}] = \arg \min_{\mathbf{W}, \mathbf{a}_1, \dots, \mathbf{a}_{m-1}} E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2],$$

subject to $P_i = P_{T,i}$, $i = 1, 2, \dots, m-1$.

where P_i as defined above is the transmitted power of the i th group of relays, and $P_i = N_{i+1} \mathbf{a}_i^H \mathbf{a}_i$. Using the method of Lagrange multipliers, we obtain the following Lagrangian function

$$\mathcal{L} = E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] + \sum_{i=1}^{m-1} \lambda_i (N_{i+1} \mathbf{a}_i^H \mathbf{a}_i - P_{T,i}).$$

Following the same steps described in Section III.A, we get the same optimal expression for the \mathbf{W} as in (2), and the optimal expression for the power allocation vector \mathbf{a}_i

$$\mathbf{a}_{i,opt} = [\mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{B}_i^H \circ E(\mathbf{y}_i \mathbf{y}_i^H)^* + N_{i+1} \lambda_i \mathbf{I}]^{-1} \cdot [\mathbf{B}_i \mathbf{H}_d^H \mathbf{W} \circ E(\mathbf{y}_i \mathbf{s}^H)^* \mathbf{u}],$$

where

$$\mathbf{B}_i = \begin{cases} \prod_{k=i+1}^{m-1} \mathbf{H}_{k-1,k}^H \mathbf{F}_k^H \mathbf{A}_k^H, & \text{for } i = 1, 2, \dots, m-2, \\ \mathbf{I}, & \text{for } i = m-1. \end{cases}$$

The Lagrange multiplier λ_i can be determined by solving

$$N_{i+1} \mathbf{a}_{i,opt}^H \mathbf{a}_{i,opt} = P_{T,i} \quad i = 1, 2, \dots, m-1.$$

Following the same steps in Section III.A, we obtain $(m-1)$ $\{2M_i\}$ -th-order polynomials

$$\sum_{j=1}^{M_i} N_{i+1} (\alpha_{i,j} + N_{i+1} \lambda_i)^{-2} \mathbf{C}_i(j, j) = P_{T,i}, \quad i = 1, 2, \dots, m-1.$$

C. MMSE Design with Individual Power Constraints

Thirdly, we consider the case where the power of each relay node is limited to some value $P_{T,i,j}$. The proposed method can

be considered as the following optimization problem

$$[\mathbf{W}_{opt}, \mathbf{a}_{1,opt}, \dots, \mathbf{a}_{m-1,opt}] = \arg \min_{\mathbf{W}, \mathbf{a}_1, \dots, \mathbf{a}_{m-1}} E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2],$$

subject to $P_{i,j} = P_{T,i,j}$, $i = 1, 2, \dots, m-1$, $j = 1, 2, \dots, N_i$.

where $P_{i,j}$ is the transmitted power of the j th relay node in the i th group, and $P_{i,j} = N_{i+1} a_{i,j}^* a_{i,j}$. Using the method of Lagrange multipliers, we have the following Lagrangian function

$$\mathcal{L} = E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] + \sum_{i=1}^{m-1} \sum_{j=1}^{N_i} \lambda_{i,j} (N_{i+1} a_{i,j}^* a_{i,j} - P_{T,i,j}).$$

Following the same steps described in Section III.A, we get the same optimal expression for the \mathbf{W} as in (2), and the optimal expression for the $a_{i,j}$

$$a_{i,j,opt} = [\phi_i(j, j) + N_{i+1} \lambda_{i,j}]^{-1} [\mathbf{z}_i(j) - \sum_{l \in I, l \neq j} \phi_i(j, l) a_{i,l}]$$

where $I = \{1, 2, \dots, N_i\}$, ϕ_i and \mathbf{z}_i have the same expression as in (7) and (8). The Lagrange multiplier $\lambda_{i,j}$ can be determined by solving

$$N_{i+1} a_{i,j,opt}^* a_{i,j,opt} = P_{T,i,j} \quad i = 1, 2, \dots, m-1, \quad j = 1, 2, \dots, N_i$$

IV. SIMULATIONS

In this section, we numerically study the BER performance of our three proposed joint MMSE design of the receiver and power allocation methods and compare them with the equal power allocation method [6] which allocates the same power level equally for all links from the relay nodes. For the purpose of fairness, we assume that the total power for all relay nodes in the network is the same which can be indicated as $\sum_{i=1}^{m-1} P_{T,i} = \sum_{i=1}^{m-1} \sum_{j=1}^{N_i} P_{T,i,j} = P_T$. We consider a 3-hop ($m=3$) wireless sensor network. The number of source nodes (N_0), two groups of relay nodes (N_1, N_2) and destination nodes (N_3) are 1, 4, 2 respectively. We consider an AF cooperation protocol. The quasi-static fading channel (block fading channel) is considered in our simulations whose elements are Rayleigh random variables (with zero mean and unit variance) and assumed to be invariant during the transmission of each packet. In our simulations, the channel is assumed to be known at the destination nodes. For channel estimation algorithms for WSNs and other low-complexity parameter estimation algorithms, one can refer to [10] and [11]. During each phase, the sources transmit the QPSK modulated packets with 1500 symbols. The noise at the destination nodes is modeled as circularly symmetric complex Gaussian random variables with zero mean. When perfect (error free) feedback channel between destination nodes and relay nodes is assumed to transmit the amplification coefficients, it can be seen from Fig. 3 that our three proposed methods can achieve a better performance than the equal power allocation method. The method with global constraint has the best performance whereas the method with individual constraints has the worst performance. This result is what we expect because a global constraint provides the most freedom for allocating the power among the relay nodes whereas an individual constraint provides the least. In practice, the feedback channel can not be error free. In order to study the

impact of feedback channel errors on the performance, we employ the binary symmetric channel (BSC) as the model for the feedback channel and quantize each complex amplification coefficient to an 8-bit binary value (4 bits for the real part, 4 bits for the imaginary part). Vector quantization methods [12] can also be employed on increased spectral efficiency. The error probability (Pe) of BSC is fixed at 10^{-3} . The dashed curves in Fig. 3 show the performance degradation compared with the performance when using a perfect feedback channel. To show the performance tendency of the BSC for other values of Pe, we fix the SNR at 10 dB and choose Pe ranging from 0 to 10^{-2} . The performance curves are shown in Fig. 4, which indicates the BER performance versus Pe of our three proposed methods. It can be seen that along with the increase in Pe, their performance becomes worse.

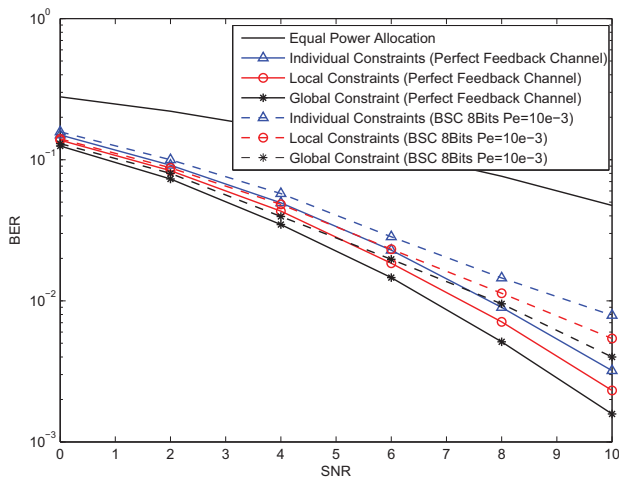


Fig. 3. BER performance versus SNR of our proposed joint strategies, compared with equal power allocation method.

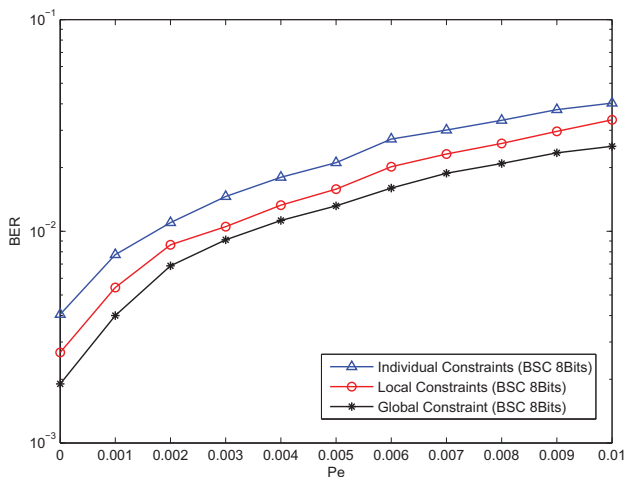


Fig. 4. BER performance versus Pe of our proposed joint strategies when employing BSC as the model for the feedback channel.

V. CONCLUSIONS

Three joint receiver design and power allocation strategies have been proposed for general multihop WSNs. It has been shown that our proposed strategies achieve a better performance than the equal power allocation method. Possible extensions to this work may include the design of non-linear receivers [13] and the study of the complexity and the requirement for the feedback channel.

APPENDIX

Here, we derive the the expressions of \mathbf{F}_i , $E(\mathbf{y}_i \mathbf{y}_i^H)$, and $E(\mathbf{y}_i \mathbf{s}^H)$ that are used in Section III.

$$\mathbf{F}_i = \text{diag}\{E(|x_{i,1}|^2), E(|x_{i,2}|^2), \dots, E(|x_{i,N_i}|^2)\}^{-\frac{1}{2}}$$

where

$$E(|x_{i,j}|^2) = \begin{cases} \sigma_s^2 |\mathbf{h}_{s,j}|^2 + \sigma_n^2, & \text{for } i = 1, \\ \mathbf{h}_{i-1,j} \mathbf{A}_{i-1} E(\mathbf{y}_{i-1} \mathbf{y}_{i-1}^H) \mathbf{A}_{i-1}^H \mathbf{h}_{i-1,j} + \sigma_n^2, & \text{for } i = 2, 3, \dots, m. \end{cases}$$

$$E(\mathbf{y}_i \mathbf{y}_i^H) = \begin{cases} \mathbf{F}_i (\sigma_s^2 \mathbf{H}_s \mathbf{H}_s^H + \sigma_n^2 \mathbf{I}) \mathbf{F}_i^H, & \text{for } i = 1, \\ \mathbf{F}_i [\mathbf{H}_{i-1,i} \mathbf{A}_{i-1} E(\mathbf{y}_{i-1} \mathbf{y}_{i-1}^H) \mathbf{A}_{i-1}^H \mathbf{H}_{i-1,i}^H + \sigma_n^2 \mathbf{I}] \mathbf{F}_i^H & \text{for } i = 2, 3, \dots, m. \end{cases}$$

$$E(\mathbf{y}_i \mathbf{s}^H) = \begin{cases} \sigma_s^2 \mathbf{F}_i \mathbf{H}_s, & \text{for } i = 1, \\ \mathbf{F}_i \mathbf{H}_{i-1,i} \mathbf{A}_{i-1} E(\mathbf{y}_{i-1} \mathbf{s}^H), & \text{for } i = 2, 3, \dots, m. \end{cases}$$

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