BEP of Fourier Transform and Discrete Wavelet Transform based OFDM

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***Abstract*—This paper studies the analytical bit error probability of Discrete Wavelet Transform and Fast Fourier Transform based Orthogonal Frequency Division Multiplexing (OFDM) systems. Closed form expressions of the bit error probabilities for both two methods are derived and validated. In the simulation, two channels with different impulse response are adopted. The results show that the Discrete Wavelet Transform based OFDM performs better than the classic OFDM in both channels.**

I. Introduction

OFDM is a very important scheme used in modern communication systems. It has succeeded in quite a few applications. Traditional OFDM methods apply the Inverse Fast Fourier Transform (IFFT) in the transmitting part and Fast Fourier Transform (FFT) in the receiving part. Nevertheless, wavelet transforms are adopted to substitute the Fourier transform in some recent researches. The advantage of the wavelet based OFDM is that the transmission scheme of the system has the ability to adapt to different environments. This flexibility meets the requirement of future communication systems. The wavelet transforms in these new systems are various, such as Discrete Wavelet Transform (DWT) [1], Wavelet Packet Transform (WPT) [2][3], Dual-Tree Complex Wavelet Transform (DTWT) [4] and so on.

The bit error rate (BER) performance of these methods are better than the traditional FFT to some extent, according to simulation results. However, the analytical bit error probability (BEP) expressions of them have yet been derived. In this paper, we consider the model where DWT is an alternative of FFT. And closed-form BEP expressions of both DWT and FFT based OFDM are derived and validated. The model for both FFT and DWT methods is shown in Figure 1. Based on this model, we achieve the closed-form BEP expressions for both DWT and FFT based OFDM.

The rest of the paper is organised as follows. The system model is analysed in Section II. From it, the noise influencing the demodulation is obtained. In Section III, the expressions of the noise variances for different carriers of FFT and DWT are obtained. By combining these two expressions of the variances with the theoretical bit error expression of the QAM modulation, the analytical BEP expressions of FFT and DWT based OFDM are derived. The expressions of BEP and variances are proved to be correct by the simulations in Section III, and the comparison between DWT and FFT based OFDM is conducted at the end of Section III, too.

II. Analysis of the Model

A. FFT Based model

If the model uses I-ary QAM modulation and has M carriers (which also means that the FFT transform size is M),

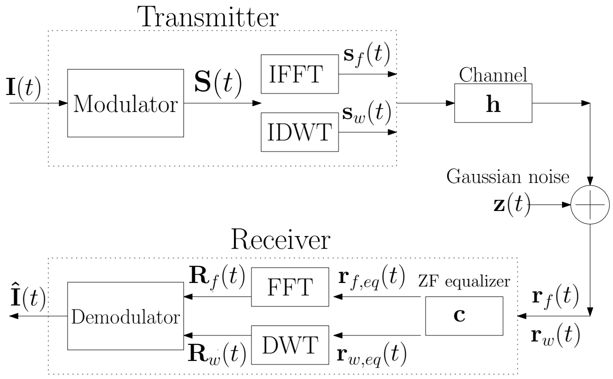

Fig. 1. The system model

the received signal before the channel equalization $\mathbf{r}_f(t)$ can be expressed as

$$\mathbf{r}_f(t) = \mathbf{s}_f(t) * \mathbf{h} + \mathbf{z}(t), \qquad t = 0, 1, 2, \ldots, T, \tag{1}$$

where $*$ denotes the linear convolution. $\mathbf{s}_f(t)$ is the transmitted signal, which is also the IFFT of the modulated signal $\mathbf{S}(t)$. The vector $\mathbf{h}$ is the channel impulse response for any time-invariant communication channel. The M independent and identically distributed (i.d.d) complex Gaussian noises are represented by the vector $\mathbf{z}(t)$. Each element of it is represented by $z_i(t)$ which is the noise adding to the i-th carrier. Thus, $z_i(t)$ is independent to other noises and has a zero-mean normal distribution over time t, denoted as $z_i(t) \sim N(0, \delta^2)$. T is the signalling interval. The boldface letter represents a M point vector.

In the zero-forcing process, the channel equalization coefficients $\mathbf{c}$ are obtained, which satisfy,

$$\mathbf{h} * \mathbf{c} = \delta. \tag{2}$$

Here, the δ represents the Dirac Delta function. Therefore, the received signal after equalization is

$$\begin{aligned} \mathbf{r}_{f,eq}(t) &= \mathbf{c} * \mathbf{r}_f(t) \\ &= \mathbf{s}_f(t) + \mathbf{c} * \mathbf{z}(t) \\ &= \mathbf{s}_f(t) + \widehat{\mathbf{z}}(t). \end{aligned} \tag{3}$$

By conducting FFT to the equation (3), the signal before the QAM demodulation $\mathbf{R}_f(t)$ can be given as:

$$\begin{aligned} \mathbf{R}_f(t) &= \mathrm{FFT}\left\{\mathbf{r}_{f,eq}(t)\right\} \\ &= \mathrm{FFT}\left\{\mathbf{s}_f(t) + \widehat{\mathbf{z}}(t)\right\} \\ &= \mathrm{FFT}\left\{\mathbf{s}_f(t)\right\} + \mathrm{FFT}\left\{\widehat{\mathbf{z}}(t)\right\} \\ &= \mathbf{S}(t) + \mathbf{Z}^f(t). \end{aligned} \tag{4}$$

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Obviously, from (4) we can see that the second term $\mathbf{Z}^f(t)$ in the equation is the noises influencing the demodulation.

B. DWT based model

The discrete wavelet transform is usually realised by passing the signal through a high pass filter and a low pass filter separately and subsampling the output of each filter by 2 [5]. We use h and g to indicate the high pass and low pass filter's impulse response respectively. These two filters are also known as a quadrature mirror filter pair. Therefore, the output of DWT contains two parts, Detail Coefficients (DC) from the high pass filter and Approximation Coefficients (AC) from the low pass filter. The procedure is shown in the block diagram, see Figure 2.

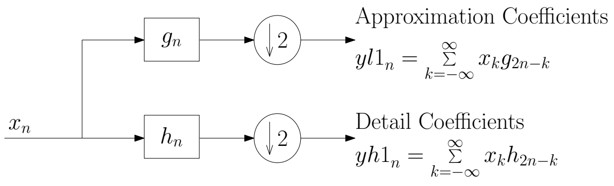

Fig. 2. The DWT

To conduct the multi-level DWT, the filter bank can be applied. At each level, the AC can be decomposed into the AC and DC for the next level. This tree structure is shown in Figure 3.

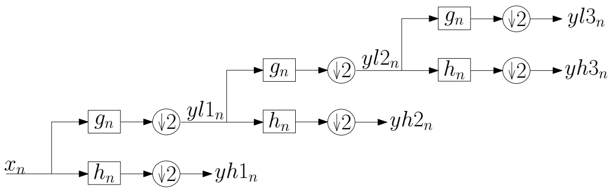

Fig. 3. The 3 levels DWT

Apparently, just like FFT, the DWT is also a linear transform. From the same model, the received modulated signal $\mathbf{R}_w(t)$ is

$$\begin{aligned}\mathbf{R}_w(t) &= \mathrm{DWT}\left\{\mathbf{r}_{w,eq}(t)\right\} \\ &= \mathrm{DWT}\left\{\mathbf{s}_w(t) + \widehat{\mathbf{z}}(t)\right\} \\ &= \mathrm{DWT}\left\{\mathbf{s}_w(t)\right\} + \mathrm{DWT}\left\{\widehat{\mathbf{z}}(t)\right\} \\ &= \mathbf{S}(t) + \mathbf{Z}^w(t).\end{aligned} \tag{5}$$

Now it is clear that only the term $\mathbf{Z}^w(t)$ of (5) will influence the demodulation process. Thus, the comparison between $\mathbf{Z}^f(t)$ with $\mathbf{Z}^w(t)$ must be studied.

III. Analytical derivation of BEP

The analytical BEP expression of the QAM modulation usually is a function of the variable E_b/N_0. Here, the N_0 is the variance of the channel noise received by the demodulator. However, for the OFDM system, this noise is clearly related to the FFT, DWT and the channel equalization. In order to get a more accurate BEP expression, the exact variances of the noises influencing the demodulation must be obtained. These will be discussed in the following sections.

A. FFT Model

For the FFT model, we know from the equation (4) that the received noise of the demodulator is $\mathbf{Z}^f(t)$. In order to use E_b/N_0 to compute the BEP, the N_0 of $\mathbf{Z}^f(t)$ should be acquired. In the section II, the expression of it is given as $\mathbf{Z}^f(t) = \mathrm{FFT}\left\{\widehat{\mathbf{z}}(t)\right\} = \mathrm{FFT}\left\{\mathbf{c} * \mathbf{z}(t)\right\}$. Thus, for each element of $\mathbf{Z}^f(t)$, we have

$$\begin{aligned}Z_m^f(t) &= \frac{1}{\sqrt{M}} \sum_{k=0}^{M} \widehat{z}_k(t) e^{-i\frac{2\pi}{M}km} \\ &= \frac{1}{\sqrt{M}} \sum_{k=0}^{M} \left(\sum_n c_{k-n} z_n(t)\right) e^{-i\frac{2\pi}{M}km} \\ &= \frac{1}{\sqrt{M}} \sum_n z_n(t) \sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km}\ ,\end{aligned} \tag{6}$$

where $m = 0, 1, \ldots, M-1$ indicates the subscript of the carriers. Consequently, the mean value of $\mathbf{Z}^f(t)$ is

$$\begin{aligned}E\left\{Z_m^f(t)\right\} &= E\left\{\frac{1}{\sqrt{M}} \sum_n z_n(t) \sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km}\right\} \\ &= \frac{1}{\sqrt{M}} \sum_n E\left\{z_n(t)\right\} \sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km} \\ &= \frac{1}{\sqrt{M}} \sum_n 0 \sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km} \\ &= 0,\end{aligned} \tag{7}$$

and the variance can be obtained as

$$\begin{aligned}N_m^F &= Var\left\{Z_m^f(t)\right\} \\ &= E\left\{\left(Z_m^f(t)\right)^2\right\} - \left(E\left\{Z_m^f(t)\right\}\right)^2 \\ &= E\left\{\left(\frac{1}{\sqrt{M}} \sum_n z_n(t) \sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km}\right)^2\right\} \\ &= \frac{1}{M} \cdot \sum_n E\left\{z_n^2(t)\right\} \left(\sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km}\right)^2 \\ &= \frac{\delta^2}{M} \sum_n \left(\sum_{k=0}^{M} c_{k-n} e^{-i\frac{2\pi}{M}km}\right)^2 .\end{aligned} \tag{8}$$

Thus, the variance of $\mathbf{Z}^f(t)$ is a function of m, which means that the noises added to the corresponding carriers are different in their variance. In another words, the power of the noise for each carrier is changed and not equal to each other anymore.

B. DWT Model

From the previous section, we know that using the DWT simply means passing the input through quadrature mirror filter banks. Hence, the convolution of the input signal and the filter must be applied. However, the linear convolution will introduce an excessive length of the output. In order to avoid this, we use the circular convolution in the DWT. We denote the length of the quadrature mirror filter as L. Thus, the circular convolution means adding the last $L/2$ elements of the normal convolution to the first $L/2$ elements. Because of the causality of the filter, the $L/2$ left circular shift must be conducted for the input of each DWT process in order to guarantee the perfect reconstruction of the signal.

As mentioned in the previous section, the expression of $\mathbf{Z}^w(t)$ is known as $\mathbf{Z}^w(t) = \mathrm{DWT}\left\{\widehat{\mathbf{z}}(t)\right\} = \mathrm{DWT}\left\{\mathbf{c} * \mathbf{z}(t)\right\}$. As the AC and DC parts of DWT are obtained by the same

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process with different filters, we only show the derivation of the AC part of DWT $\{\widehat{\mathbf{z}}(t)\}$ in the following.

1) Left circular shift of $L/2$ elements of the input: A circular shift can be expressed by a particular permutation. We define the operation in DWT as $\sigma(i, X) \equiv (i + L/2) \bmod X$. So, the each shifted element of the input can be given as

$$\begin{aligned} \bar{z}_m(t) &= \widehat{z}_{\sigma(m,M)}(t) \\ &= c_{\sigma(m,M)} * z_m(t) \\ &= \sum_n c_{\sigma(m,M)-n} z_n(t) \\ &= \sum_n c_{m+\sigma(-n,M)} z_n(t). \end{aligned} \tag{9}$$

2) Circular convolution of the shifted signal with quadrature mirror filter and downsampling of the output: As we use the AC part for the analysis, the filter used here is the low pass filter g. We define the circular convolution of two series $\mathbf{x}$ and $\mathbf{y}$ with period N as $x_n \otimes y_n^N = \sum_m x_m y_{n-m}^N$. We use $\downarrow$ to denote the downsampling operator. For any sequence y_n, we have $y_n \downarrow k = y_{kn}$. Thus, for the first level of the DWT, we have

$$\begin{aligned} xl1_m &= (\bar{z}_m(t) \otimes g_m) \downarrow 2 \\ &= \sum_n \bar{z}_n(t) \cdot g_{2m-n}^M \\ &= \sum_n \left(\sum_a c_{\sigma(n,M)-a} \cdot z_a(t) \right) g_{2m-n}^M \\ &= \sum_a z_a(t) \sum_n c_{\sigma(n,M)-a} g_{2m-n}^M. \end{aligned} \tag{10}$$

Consequently, the expectation and variance of $xl1_m$ can be obtained as,

$$\begin{aligned} E\{xl1_m\} &= E\left\{ \sum_a z_a(t) \sum_n c_{\sigma(n,M)-a} g_{2m-n}^M \right\} \\ &= \sum_a E\{z_a(t)\} \sum_n c_{\sigma(n,M)-a} g_{2m-n}^M \\ &= \sum_a 0 \sum_n c_{\sigma(n,M)-a} g_{2m-n}^M = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} Var\{xl1_m\} &= E\{xl1_m^2\} - (E\{xl1_m\})^2 = E\{xl1_m^2\} \\ &= \sum_a E\{z_a^2(t)\} \left(\sum_n c_{\sigma(n,M)-a} g_{2m-n}^M \right)^2 \\ &= \delta^2 \sum_a \left(\sum_n c_{\sigma(n,M)-a} g_{-n}^M \right)^2 \\ &= \delta^2 \sum_a \left(\sum_n c_{a+\sigma(-n,M)} g_n^M \right)^2 \\ &= \delta^2 \sum_a \left(\sum_n c_{\sigma(a,M)-n} g_n^M \right)^2 \\ &= \delta^2 \sum_a \left(c_{\sigma(a,M)} \otimes y_a^M \right)^2. \end{aligned} \tag{12}$$

Now we can obtain the expectation and variance of the multilevel DWT. For the second level, we have,

$$\begin{aligned} xh2_m &= \left(xl1_{\sigma(m,\frac{M}{2})} \otimes h_m^{\frac{M}{2}} \right) \downarrow_2 \\ &= \sum_b xl1_{\sigma(b,\frac{M}{2})} \cdot h_{2m-b}^{\frac{M}{2}} \\ &= \sum_b \left(\sum_n \left(\sum_a c_{\sigma(n,M)-a} z_a(t) \right) g_{2\sigma(b,\frac{M}{2})-n}^M \right) h_{2m-b}^{\frac{M}{2}} \\ &= \sum_a z_a(t) \sum_n c_{\sigma(n,M)-a} \sum_b g_{2\sigma(b,\frac{M}{2})-n}^M h_{2m-b}^{\frac{M}{2}}. \end{aligned} \tag{13}$$

Similar to $xl1_m$, the expectation of $xh2_m$ is 0, too. And the variance of it is

$$\begin{aligned} Var\{xh2_m\} &= E\{xh2_m^2\} \\ &= \sum_a E\{z_a^2(t)\} \\ &\quad \left(\sum_n c_{\sigma(n,M)-a} \sum_b g_{2\sigma(b,\frac{M}{2})-n}^M h_{2m-b}^{\frac{M}{2}} \right)^2 \\ &= \delta^2 \sum_a \left(\sum_n c_{\sigma(n,M)-a} \sum_b g_{2\sigma(b,\frac{M}{2})-n}^M h_{-b}^{\frac{M}{2}} \right)^2 \\ &= \delta^2 \sum_a \left(\sum_n c_{\sigma(-n,M)-a} \underbrace{\sum_b g_{n+2\sigma(-b,\frac{M}{2})}^M h_b^{\frac{M}{2}}}_{s_n} \right)^2 \\ &= \delta^2 \sum_a \left(\sum_n c_{a+\sigma(-n,M)} s_n \right)^2 \\ &= \delta^2 \sum_a \left(c_{\sigma(a,M)} * s_a \right)^2 \end{aligned} \tag{14}$$

In order to compute s_n, we introduce a kind of special convolution, called separated convolution. Suppose there are two sequences, x and y. The length of x is twice the length of y, denoted as $2N$ and N. Then the sequence x is partitioned into two parts, x^e and x^o, where $x^e = [x_0, x_2, x_4 \dots x_{2N-2}]$ and $x^o = [x_1, x_3, x_5 \dots x_{2N-1}]$. By conducting the circular convolution of x^e and x^o with y respectively, we get,

$$\begin{aligned} z_k^e &= x_k^e \otimes y_k^N = \sum_n x_{k-n}^e y_n^N \\ z_k^o &= x_k^o \otimes y_k^N = \sum_n x_{k-n}^o y_n^N. \end{aligned} \tag{15}$$

By combining z^e with z^o we can get the sequence z, which is $z = [z_0^e, z_0^o, z_1^e, z_1^o, \dots z_{N-1}^e, z_{N-1}^o]$. This is the result of the separated convolution and can be expressed in a simple formula. By using the symbol $\odot$ to denote this operation, we have

$$\begin{aligned} z_a &= x_a \odot y_b \\ &= \sum_n x_{a-2n} \cdot y_n^N, \end{aligned} \tag{16}$$

with $a = 0, 1, 2 \ldots 2N-1; b = 0, 1, 2 \ldots N-1$. It is evident that the equation (16) has the same expression with s_n of equation (14), except for the circular shift. The circular shift of (14) actually performs a circular shift of x_k^e and x_k^o in the equation (15), respectively. The left circular shift has length $L/2$. So, the sequence z will be left circular shifted with length L instead of $L/2$. We define a function $\bar{\sigma}(i, X) \equiv (i + L) \bmod X$ to indicate this L left shift, thus,

$$\begin{aligned} z_{\bar{\sigma}(a,X)} &= x_{\bar{\sigma}(a,X)} \odot y_b \\ &= \sum_n x_{\bar{\sigma}(a,X)-2n} \cdot y_n^{\frac{X}{2}} \\ &= \sum_n x_{a+2\sigma\left(-n,\frac{X}{2}\right)} \cdot y_n^{\frac{X}{2}} \ . \end{aligned} \tag{17}$$

From (17) we can rewrite s_n in (14) as

$$\begin{aligned} s_n &= \sum_b g^M_{n+2\sigma\left(-b,\frac{M}{2}\right)} h_b^{\frac{M}{2}} \\ &= g_{\bar{\sigma}(n,M)} \odot h_k \ . \end{aligned} \tag{18}$$

By substituting (18) into (14), we obtain the variance of the DWT's DC part of the second level, which is

$$Var\{xh2_m\} = \delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot h_k^{\frac{M}{2}}\right)\right)^2 . \tag{19}$$

Analogously, we can get the variance of the coefficients of the third level, which can be expressed as

$$\begin{aligned} Var\{xh3_m\} &= \delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot \left(g_{\bar{\sigma}\left(b,\frac{M}{2}\right)} \odot h_k^{\frac{M}{4}}\right)\right)\right)^2 , \\ Var\{xg3_m\} &= \delta^2 \sum_a \left(c_{\sigma(a,M)} * \left(g_{\bar{\sigma}(a,M)} \odot \left(g_{\bar{\sigma}\left(b,\frac{M}{2}\right)} \odot g_k^{\frac{M}{4}}\right)\right)\right)^2 . \end{aligned} \tag{20}$$

From the equations about the variance above, we can see that the variances of each carrier within DC or AC in the same level are constant. In other words, their variances over the M carriers form a step function. Based on the tree structure of the DWT, we use the function $VW(m, M, J)$ to represent this step function, where J is the DWT level. For example, for J=3, M=64 we have

$$VW(m, 64, 3) = \begin{cases} Var\{xg1_m\} & 0 \leq m < 32 \\ Var\{xg2_m\} & 32 \leq m < 48 \\ Var\{xg3_m\} & 48 \leq m < 56 \\ Var\{xh3_m\} & 56 \leq m < 64 \ . \end{cases} \tag{21}$$

Therefore, the variance of $\mathbf{Z}^w(t)$ can be calculated easily, which is

$$N_m^W = VW(m, M, J). \tag{22}$$

C. BEP derivation

For arbitrary I-ary rectangular QAM modulation with even number of bits per symbol, the theoretical BEP of it can be calculated approximately by

$$\mathrm{Pb} \approx \frac{\sqrt{I}-1}{\sqrt{I}\log_2\sqrt{I}} \mathrm{erfc}\left[\sqrt{\frac{3\log_2 I}{2(I-1)} \cdot \frac{E_b}{N_0}}\right], \text{ see [6]}. \tag{23}$$

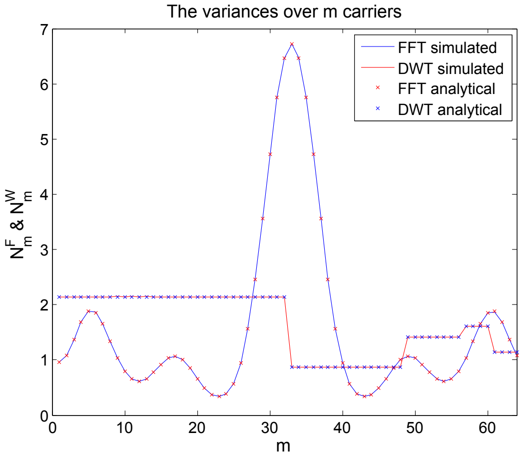

Fig. 4. The test of the noise variances expressions of DWT and FFT

It is clear that after using the FFT and DWT, the N_0 of this equation for each carrier is no longer the same. If we combine (8) with (23), the BEP of each carrier after FFT is

$$\mathrm{Pb}_m^F \approx \frac{\sqrt{I}-1}{\sqrt{I}\log_2\sqrt{I}} \mathrm{erfc}\left[\sqrt{\frac{3\log_2 I}{2(I-1)} \cdot \frac{E_b}{N_m^F}}\right]. \tag{24}$$

Therefore, the average BEP over m carriers is

$$\widehat{\mathrm{Pb}}^F = \frac{1}{M}\sum_m \mathrm{Pb}_m^F. \tag{25}$$

Similarly, the BEP for each carrier after DWT, Pb_m^W, can be obtained by replacing N_m^F in equation (24) with N_m^W. Thus, the average BEP of the DWT based OFDM system is

$$\widehat{\mathrm{Pb}}^W = \frac{1}{M}\sum_m \mathrm{Pb}_m^W. \tag{26}$$

IV. Simulation and Evaluation

In this section, first we will show that the derived variances after FFT and DWT are correct by conducting simulation. Second the BEP expressions of both methods are tested. Third we will evaluate the BEP performance of FFT and DWT.

A. Variances simulation

In the simulation of the variances of the noises after the FFT (N_m^F) and DWT (N_m^W), we use 64 as the carriers size, and the channel ($ch1$) [7] introduced here has an impulse response of [0.04, -0.05, 0.07, -0.21, -0.5, 0.72, 0.36, 0, 0.21, 0.03, 0.07]. The ZF equalizer has 21 taps. The Gaussian noise added to each carrier has a power of 0 dB. The DWT level is 4. Figure 4 shows the result of the simulation with 500000 runs. From it, we can see that the simulation result fits the analytical expression well.

B. Simulated and analytical BEP

As we have already shown that the expressions of the variances over each carrier after FFT and DWT are correct, we now conduct the simulation to test the BEP expressions (25) and (26) that are derived from the variances. In this simulation, 16-QAM is applied for the modulation and the carriers size is 64. The wavelet used here is the Haar wavelet with a DWT level of 2. The result is shown in Figure 5, which demonstrates that the analytical BEP fits the simulated BEP

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well. The theoretical BEP of 16-QAM is also plotted in the figure as a baseline by using equation (23). The N_0 for it is the mean value of N_m^F.

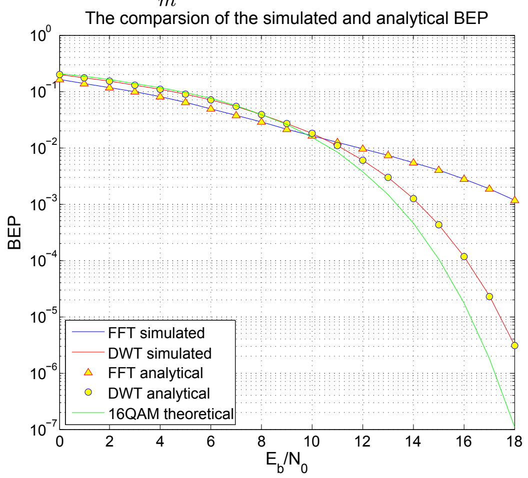

Fig. 5. The simulated and analytical BEP comparison over $ch1$

C. FFT and DWT evaluation based on analytical BEP

In this part, we look into the BEP performance of DWT and FFT based OFDM systems depending on the analytical expression. Not only the channel $ch1$, but also a frequency selective channel $ch2$ [8] is used for the comparison. The impulse response of $ch2$ is [0.08, 0.18, 0.08, 0.3, 0.28, -0.005, 0.05, -0.005, -0.18, 0.2, 0.12, -0.3, -0.28, 0.14, 0.04].

First of all, the DWT performance with different decomposition levels J are compared. The Daubechies wavelet (db4) is used in the calculation over $ch1$. Figure 6 shows that the DWT level does not influence the performance significantly. In fact, the calculated data from the equation proves that the BEP difference between them are negligible.

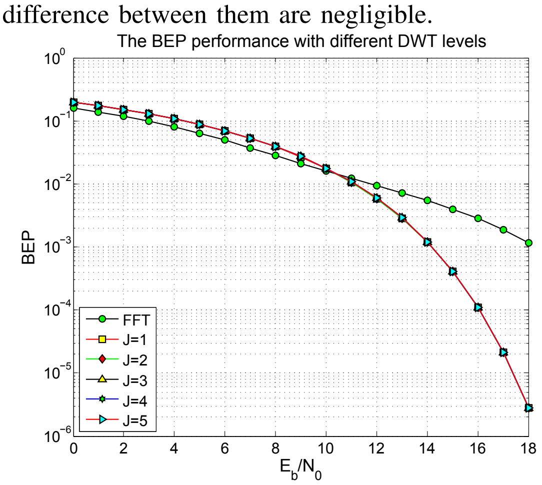

Fig. 6. The BEP performance of different DWT levels over $ch1$

Second, the comparison of different wavelets over both $ch1$ and $ch2$ are conducted. The wavelets used for the comparison include Haar, Daubechies (db10), Coiflets (coif4), Symlets (sym8) and Discrete Meyer (dmey)[9]. As shown in Figure 7, there is no big difference between each kind of wavelet for $ch1$. For $ch2$, the Haar wavelet performs slightly better than the other wavelets in higher E_b/N_0, see Figure 8.

V. CONCLUSION

In this paper, we analysed the DWT and FFT based OFDM systems. We proposed a new method to achieve the BEP for

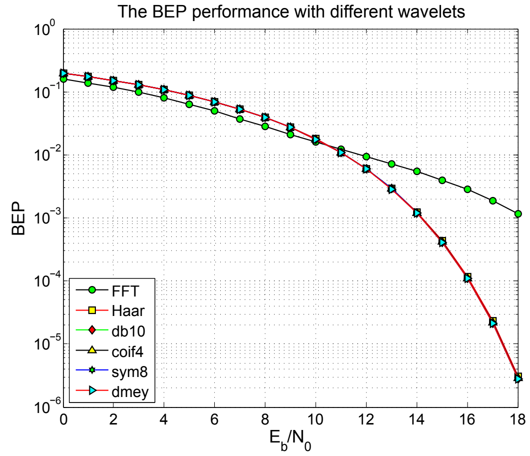

Fig. 7. The BEP performance of different wavelets over $ch1$

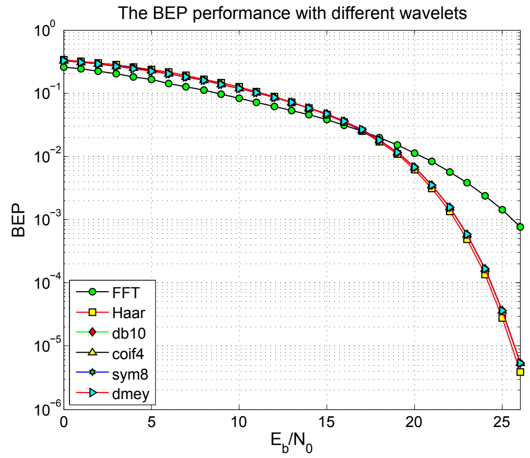

Fig. 8. The BEP performance of different wavelets over $ch2$

both QAM modulated OFDM systems with ZF equalizer. Our model and the expressions of the BEP are validated by a comparison between simulations and analytical computation. According to these BEP expressions, we shown that the DWT based OFDM performs better than the traditional OFDM in higher E_b/N_0 for both frequency selective and nonselective channels.

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