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http://wicat.poly.edu
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Resource Management under Elastic Distortion Constraints: Power Management over a Wireless Link

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Outline

- Motivation: why utility functions?
- A family of plausible utility functions
- The "right amount" of distortion when avoiding it is "costly"
- Distortion and power management over wireless link
- Conclusions

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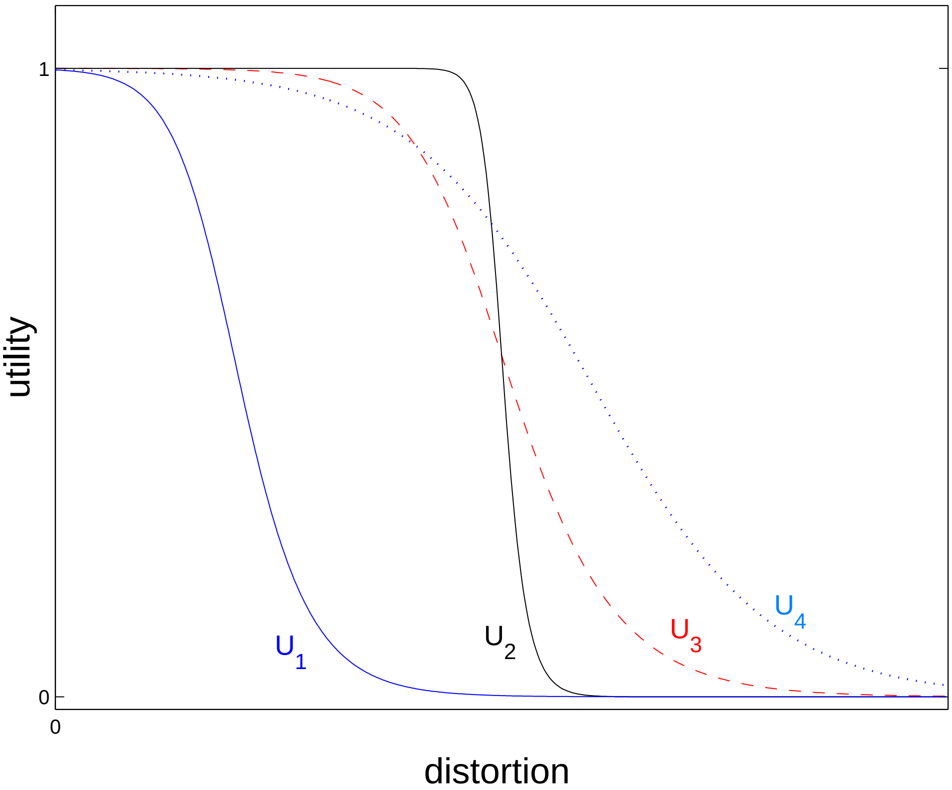
- The literature typically assumes that, up to a level, distortion has no effect on signal quality, but beyond that level it makes the signal useless. The proposed family of utility functions contains this "hard threshold" as a special case. (See U_2 below).



Beauty is in the eyes of the beholder

The “right amount” of distortion

- Equally valuable media files are offered
- Each file can be acquired at any desired distortion, $y \in [0, \overline{D}]$
- Cost of file ($$, Joules, etc) is $c(y)$, which is *decreasing* in y
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- Answer: Maximize total utility: $n \times u(y)$ with $n = B/c(y)$
- That is, maximize $u(y)/c(y)$ (quality/Joule, quality/$$, etc.)

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- Assume linear cost : $c(y) = \bar{c} \cdot (\bar{D} - y) \equiv \bar{c}x$ with $x = \bar{D} - y$
- Maximize $u(y)/(\bar{D} - y) \equiv u(\bar{D} - x)/x \equiv s(x)/x$. $u(\bar{D} - x)$ is the “reversed and right-shifted” version of $u()$; that is, a “standard S-curve”

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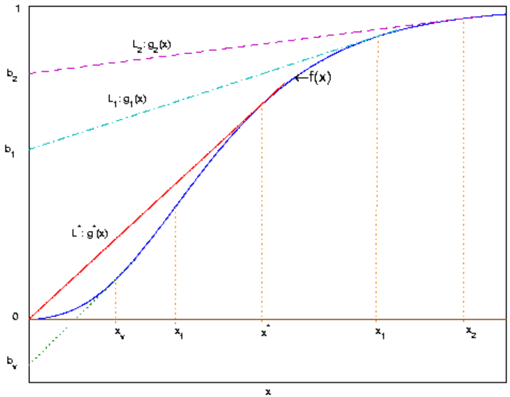

- Maximizer of $s(x)/x$ is x^*, which is found, as shown in the figure, by drawing a tangent to the curve from origin (proved elsewhere)

Media Files over a wireless link

- Transmitter, whose utility function on distortion is u, faces:
  - many equally important media files to transfer (assume simple b/w images, 1 bit per pixel)
  - limited energy, E
  - fixed transmission rate R bps
  - N packets per file, L info bits, M total bits
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  - power $\longrightarrow$ SIR, $\gamma \longrightarrow$ BER, $\varepsilon(\gamma)$
  - With indep. errors, prob of k bit errors ($k \leq M$) is: $\binom{M}{k}\varepsilon^k(1-\varepsilon)^{M-k}$
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  - Assume, pessimistically, that if $m+1$ to L errors occur, each will cause an error among info bits.
  - Then, ***expected utility*** of a file $U_E(\gamma)$ is

$$u(0)\underbrace{\left(\sum_{k=0}^{m}\binom{M}{k}\varepsilon^k(1-\varepsilon)^{M-k}\right)}_{\text{Prob of 0 to m bit errors}}+\sum_{k=m+1}^{L}\binom{M}{k}\varepsilon^k(1-\varepsilon)^{M-k}u(k)$$

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- Development follows from the properties of a general family of utility functions ("reversed" S-curves)
- Solutions are described as clearly identifiable points in the graph of the utility function, assumed known, or of the expected utility function, which is derived as a function of the SIR
- Because this family of utility functions contains as special case the "sharp threshold" assumed in the literature when dealing with distortion, this formulation adds to the literature without taking anything away.

THANK YOU!!!

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