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Optimal Power and Data Rate Assignment when Power-Limited Data and Media Terminals share a 3G CDMA cell: The 3 Terminal Scenario

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Outline of research program

- Analytical Core
- Single-user applications: choose power, data rate and/or coding rate for data, image, video transmission
- Decentralized multi-user applications:
 - Game formulation
 - Mechanism design
- Centralized data throughput maximization
 - Without noise
 - With noise and media terminals present

Overview of our analytical framework

- Many radio-resource optimizations share a common analytical core, which enables robust and tractable analysis and provides clear answers in fairly general scenarios
- It involves
 - A tractable abstraction of the physical layer
 - A tractable abstraction of the human visual system (TOMORROW!)
 - A fundamental result: maximize f(x)/x with f an "S-curve".
- Problems to which this framework applies:
 - Power and coding rate choice for media files (images, video)
 - Choosing the "right amount" of media distortion
 - Decentralized power control for 3G CDMA
 - Data rate and power allocation for maximal cell throughput when data and media terminals share a CDMA cell

Outline of the remainder of this presentation

- Motivation : Variable Spread Gain CDMA
- Simple Model for Single-Cell CDMA Data
- An abstraction of the physical layer
- Optimization model (objective function and constraints), and first-order optimizing conditions (FONOC)
- Solution
- Discussion/extensions





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Motivation : VSG-CDMA

- Modern (3G) wireless nets are expected to accommodate terminals operating at very different data transmission rates.
- Variable Spreading Gain CDMA can accommodate terminals operating at dissimilar bit rates
- In a VSG CDMA system, chip rate is common, but each terminal's spreading (processing) gain is the ratio of the common chip rate to the terminal's bit rate
- In previous works, we have jointly optimized power and data rates to maximize the (weighted) data throughput of a VSG-cell.
- However, we have neglected (i) "noise" (out-of-cell interference), (ii) power limits, and (iii) the presence of "media" terminals with fixed data rates and SIR requirements. Now, we consider these 3 items.



The 3-Terminal Scenario

- One power-limited media terminal share a VSG-CDMA cell with two data terminals
- Media terminal has an inflexible QoS requirement (fixed data rate and SIR)
- Data terminals are delay-tolerant, and their powers and data rates can be assigned at will within specified limits
- One data terminal is "important" in the sense that the network weighs its throughput more heavily
- The data terminals have "plenty of power" but the media terminal is power limited
- We assign the power of each terminal, and the data rate of each data terminal, to maximize the cell weighted data throughput while honoring the QoS commitment made to the media terminal (bit rate, SIR)



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Fundamental quantities of interest

- Terminals send data to a base station
- R_c : chip rate ; R_i : data rate ; $G_i = R_c/R_i$: Spread Gain
- $G_i \ge G_0 \ge 1$ is required $(R_i \le R_{MAX} \le R_C)$
- $f_s(\gamma_i)$: probability of correct reception of a data packet, in terms of received SIR. The properties of the physical layer (modulation, channel characteristics, FEC, diversity, etc.) are embodied into the FSF
- $\gamma_i := G_i \alpha_i$ is the SIR with α_i the CIR given by

$$\alpha_i = \frac{h_i P_i}{\sum_{\substack{j=1\\j\neq i}}^N h_j P_j + \sigma^2} := \frac{Q_i}{\sum_{\substack{j=1\\j\neq i}}^N Q_j + \sigma^2}$$

- h_i : gain ("path loss"); $h_i P_i := Q_i$: received power
- Throughput of terminal *i*, $T_i(G_i, \alpha_i) \propto R_i f(G_i \alpha_i) \propto f(G_i \alpha_i) / G_i$



An abstraction of the physical layer

- To accommodate most physical layers of interest, all that is assumed on the FSF is that it is a smooth "S-curve":
 - "starts out" convex.
 - smoothly transitions to concave, as it approaches an asymptote.



• Ex: for non-coherent FSK with packet size M=80, no FEC, and perfect error detection, the FSF is $f_s(x) = \left[1 - \frac{1}{2} \exp\left(-\frac{x}{2}\right)\right]^{80}$

Optimization Model

Power levels can be determined through power ratios, α_i (CIR); and data rates determined through optimal spread gains, G_i . BUT CIR's need to be constrained so that they lead to feasible power levels for all 3 terminals

 $\max_{G_i,\alpha_i} T_1(G_1,\alpha_1) + \beta T_2(G_2,\alpha_2)$

subject to :

$$egin{array}{rcl} lpha_1, lpha_2) &\in \ {\cal R}_0 \ G_i &\geq \ G_0 \ , \, i \in \{1,2\} \ G_3 &= \ ar{G}_3 \ lpha_3 &= \ ar{G}_3 \ lpha_3 &= \ ar{\gamma}_3 \ ar{G}_3 := ar{lpha}_3 \end{array}$$



Acceptable region for the CIR's

• For CIR's α_1 , α_2 , α_3 the "matching" set of power levels is s.t.

$$Q_i = \frac{\sigma^2}{1-s_3}a_i$$
 with $a_i := \frac{\alpha_i}{1+\alpha_i}$ and $s_N := \sum_{i=1}^N a_i$

- Two possible problems: (i) Q_i may be "too large" and (ii) Q_i may be "too small" (i.e. negative!)
- Clearly, $s_3 < 1$ prevents problem (ii). To prevent problem (i), we must set

$$Q_3 = \frac{\sigma^2}{1 - s_3} \bar{a}_3 \le \bar{Q}_3 \Longrightarrow s_3 \le 1 - \frac{\bar{a}_3}{\bar{Q}_3/\sigma^2} \Longrightarrow s_2 \le 1 - \bar{a}_3 - \frac{\bar{a}_3}{\bar{Q}_3/\sigma^2}$$

• Therefore, desired region is

$$s_2 = \frac{\alpha_1}{1+\alpha_1} + \frac{\alpha_2}{1+\alpha_2} \le 1-\bar{\varepsilon}_3$$



Optimization Model re-stated

It is easy to see that $s_2 = 1 - \bar{\epsilon}_3$ is necessary for maximal throughput

 $\max_{G_i,\alpha_i} T_1(G_1,\alpha_1) + \beta T_2(G_2,\alpha_2)$

subject to :

$$\frac{\alpha_1}{1+\alpha_1} + \frac{\alpha_2}{1+\alpha_2} = 1 - \bar{\epsilon}_3$$

$$G_i \geq G_0, i \in \{1,2\}$$

$$G_3 = \bar{G}_3$$

$$\alpha_3 = \frac{\bar{\gamma}_3}{\bar{G}_3} := \bar{\alpha}_3$$

The Lagrangian is $\phi(G_1, G_2, \alpha_1, \alpha_2) :=$

$$T_1(G_1\alpha_1) + \beta T(G_2\alpha_2) + \lambda \left(\sum_{i=1}^2 \frac{\alpha_i}{1 + \alpha_i} - 1 + \varepsilon_3\right) + \sum_{i=1}^2 \mu_i (G_0 - G_i)$$





First-Order Necessary Optimizing Conditions

$$\begin{bmatrix} \partial T_1(G_1, \alpha_1) / \partial G_1 - \mu_1 \\ \beta \partial T_2(G_2, \alpha_2) / \partial G_2 - \mu_2 \\ f'(\gamma_1) + \lambda(1 + \alpha_1)^{-2} \\ \beta f'(\gamma_2) + \lambda(1 + \alpha_2)^{-2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

with

$$\frac{\alpha_1}{1+\alpha_1} + \frac{\alpha_2}{1+\alpha_2} = 1 - \varepsilon_3$$

$$\mu_1(G_0 - G_1) = 0$$

$$\mu_2(G_0 - G_2) = 0$$

Notice that

$$\frac{\partial T_i(G_i, \alpha_i)}{\partial G_i} = \frac{\gamma_i f'(\gamma_i) - f(\gamma_i)}{G_i^2}$$



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Known about solutions to FONOC

On the basis of previous work we know that:

- Each physical layer has a "preferred" SIR, γ_0 (see fig.).
- An interior solution to FONOC with all terminals operating with an SIR of γ_0 generally exists, but is always suboptimal (a "saddle point"). It is "fair" in some sense.
- Hence, some terminals ("favored") must operate at highest available data rate. Those not operating at this rate, must achieve the "preferred" SIR of γ_0 .
- γ_0 is a respectable value. For example, for the simple FSF corresponding to non-coherent FSK, $f(\gamma_0) = 0.83$.
- With many terminals, a basic rationale is needed to search all the possible "boundary" solutions to FONOC (see WCNC-04).



Examining various solutions to FONOC

- The optimal allocation will involve some terminals (favored ones) operating at the highest SIR. But which/how many?
- Consider first a "single favorite" solution (SFBS) in which only the "important" terminal operates at maximal data rate. Then examine the dual-favorite boundary solution (DFBS), with both terminals operating at maximal data rate.
- Existence of SFBS requires that either highest data rate be "very high", or the weight of important terminal is "very heavy". With only 2 active terminals, SFBS is a maximizer, if it exists.





Dual-Favorite Boundary Solution

- With x and y the SIR of, respect., the important and ordinary terminal, FONOC requires $\beta h(x) = h(y)$, with $h(t) = f'(t) (1 + t/G_0)^2$.
- Any of the pairs (x_1, y_1) , (x_2, y_2) , (x_1, y_2) , or (x_2, y_1) (top of fig) satisfies this equation, but may not be feasible. We plot all such points, which reveals an "X-shaped" graph.
- On the same axes, we plot the hyperbolic curves (dotted) which represent the constraint equation obtained from $s_2 = 1 - \bar{\epsilon}_3$. The intersection of the hyperbola with the NE leg of the X yields the maximizer.
- The term ε_3 (representing the resources used up by the media terminal) has the effect of "pulling down" the hyperbola, whereas G_0 tends to raise it.
- When G_0 is sufficiently low or ε_3 is sufficiently high, the hyperbola may *only* intersect the SW leg of the X-curve, which leads to a minimum.



x (SIR)

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Discussion

- We have allocated data rate and power for maximal weighted throughput in the presence of a power-limited media terminal. Analysis is relevant to 3G CDMA (VSG).
- Weights may denote "importance", utility, or price paid per bit delivered.
- Physical layer enters in model through frame-success function (FSF). To accommodate most/all physical layers, we assume that all that is known about FSF is that it is an "S-curve".
- Each physical layer has one clearly specified "preferred" SIR.
- One or both terminals should operate at highest data rate; a terminal not operating at this rate, should achieve the preferred SIR.





- With both data terminals at maximal bit rate,
 - the intersection of an X-shaped graph (FONOC) and an L-shaped graph (from SIR-feasibility constraint) determines the SIR of the terminals
 - The main effect of the media terminal is to reduce the "search space", by "pulling down" the constraint curve.
 - If the media terminal is "too demanding" and/or the maximal data rate is "too high", the hyperbola may *only* intersect the SW leg of the X-curve, which leads to a minimum!! In this case, the ordinary terminal should NOT transmit at maximal bit rate.
- This analysis should be extended to consider many terminals. QoS requirements and power limits for the data terminals may also be considered.



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