Power Allocation for Social Benefit Through Price-taking Behaviour on a CDMA Reverse Link Shared by Energy-constrained and Energy-sufficient Data Terminals

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6th Inter. Symp. on Wireless Comm. Systems
Siena-Tuscany, Italy, 7–10 September, 2009

Executive Summary

- A “central planer” allocates power to maximise “social benefit”, in the uplink of a CDMA cell with heterogeneous data terminals, with limited and limitless energy supplies
- In available decentralised schemes, terminal’s interdependent choices $\Rightarrow$“games” $\Rightarrow$ PROBLEMS!
- To reach social optimum WITHOUT “games”, price: a terminal’s fraction of the total power at receiver
- The optimal price “clears the market”, and is common for a given energy class; energy-limited terminal pays by *the square* of its power fraction
- Related work (VTC Spr’09):
  - Network sets individual price, to force each terminal to maximise “revenue per Watt”.
  - Netw. price is higher than planner’s; an active terminal “consumes less”, thus more terminals may be served.

Power control in the cellular up-link

- Why is power control important?
  - 3G nets are based on CDMA, which is interference limited
  - a terminal's power creates interference for the others
  - power control increases capacity by limiting interference
  - it also extends battery life
- Decentralised solutions are preferable because of:
  - Complexity/cost of central controllers
  - Signalling overhead
  - Certain application scenarios are **inherently** decentralised (e.g. ad-hoc nets)
- For CDMA, many useful decentralised algorithms are based on on per-Watt pricing, which leads to "games"
- Games have some problems!

Why another paper?: “Games” have some problems!

- Games creates both *technological* and *marketing* problems
  - Terminals’ choices depend on one another (complex!)
  - Solution concept is the Nash equilibrium (each terminal’s choice is its “best response” to the choices by the others) which presents important challenges:
    - is in general inefficient
    - may NOT exist, or there may be many of them
    - even if uniquely exists, it is often unclear: (a) *how* will the players reach it, and (b) after how many “iterations”
    - In network, terminals “don’t know” one another, and enter/exit at arbitrary times, which further aggravates
    - If “true” billing is based on per-Watt pricing, consumers may resist it (one’s “utility” depends on everyone else’s choice!)
- Below we provide a “de-coupled” solution: for given price, terminal’s performance depends solely on OWN choice

Feasibility of key power ratios

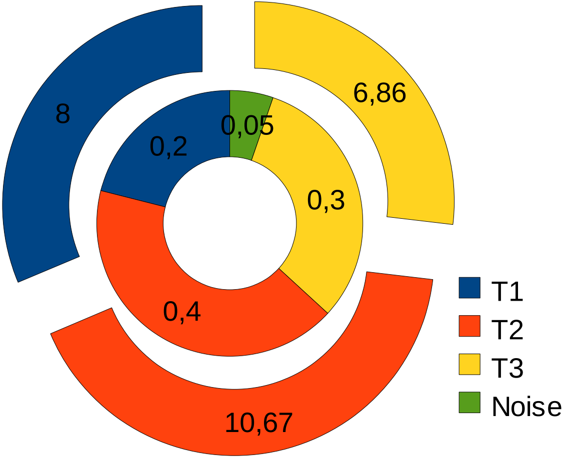
- Let p_i and G_i denote terminal i's received power, and spreading gain, with p_0 the Gaussian noise
- carrier-to-interference ratio (CIR): $\kappa_i := p_i / Y_i$ where $Y_i = p_0 + \sum_{k \neq i} p_i$ (total noise plus interference)
- signal-to-interference ratio (SIR): $\sigma_i = G_i \kappa_i$
- Known fact: each i can enjoy SIR σ_i only if

$$\sum \frac{\kappa_\mathbf{i}}{\mathbf{1} + \kappa_\mathbf{i}} \equiv \leq \mathbf{1} - \mathbf{d} \text{ for some } d \in (0,1)$$

- $\pi_i := \kappa_i/(1+\kappa_i)$ is i's share of total received power:

$$\frac{\kappa_i}{1+\kappa_i} \equiv \frac{p_i/Y_i}{p_i/Y_i + 1} \equiv \frac{p_i}{p_i + Y_i} := \frac{p_i}{\Pi}$$

Power allocation as “pie cutting”



- To allocate power, assign to each terminal a fraction of the “pie” $p_0 + \sum p_i$
- i's SIR: $\sigma_i = G_i \pi_i / (1 - \pi_i)$ with
  - G_i: spread gain
  - $\pi_i = p_i / (p_0 + \sum_j p_j)$
- Illustrated:

i	π_i	κ_i	G_i	σ_i
1	$\frac{1}{5}$	$\frac{1}{4}$	32	8,0
2	$\frac{2}{5}$	$\frac{2}{3}$	16	10,7
3	$\frac{3}{10}$	$\frac{3}{7}$	16	6,9
0	$\frac{1}{20}$	-	-	-

Central planner problem I

- Planner maximises the sum of the “benefit” that each gets
- For each terminal, benefit is the “value” of information bits transferred over a period of interest
  - An energy-limited terminal, focuses on battery life (“bits/Joule”)
  - An energy-sufficient terminal focuses on the time unit (“bits/sec”)

Socially-optimal allocation

With V_i i's benefit function, planner solves

$$\text{maximise:} \quad \sum_{i=1}^{N} \mathscr{V}_i(\pi_i) \tag{1}$$

subject to,

$$\sum_{i=1}^{N} \pi_i = 1 - d \tag{2}$$

$$\pi_i \geq 0 \tag{3}$$

The necessary optimising conditions are:

$$\mathscr{V}_i'(\pi_i) - \mu_0 \leq 0 \text{ with equality for } \pi_i > 0 \tag{4}$$

with μ_0 a Lagrange multiplier

Power fraction pricing

- The optimising condition for non-zero π_i is $\mathscr{V}_i'(\pi_i) = \mu_0$ with μ_0 a Lagrange multiplier.
- If i is allowed to freely choose π_i for a cost $c\pi_i$, the maximiser of $\mathscr{V}_i(\pi_i) - c\pi_i$ satisfies $\mathscr{V}_i'(\pi_i) = c$.
- Thus, the planner can lead the terminals to the optimum in a decentralised manner by setting the "right" price for π_i; that is, a price that coincides with μ_0.
- Notice that for given π_i, terminal i can obtain directly its CIR $\kappa_i = \pi_i/(1-\pi_i)$ and hence its SIR, $\sigma_i = G_i\kappa_i$
- Thus, the terminal can make its optimal choice independently of choices made by others!
- If planner sets the right price, ordered "slices" will equal "pie size".

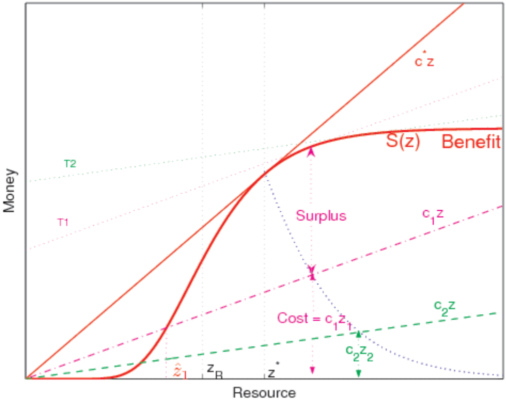
Choice by an energy-sufficient terminal I

- Terminal maximises benefit minus cost over reference period T
- Benefit is $v_i B_i$, with B_i the total number of information bits uploaded in T
- $B_i(\pi_i) = (L_i/M_i) R_i f_i(G_i \kappa(\pi_i)) T$ with f_i frame-success rate
- Terminal's cost is $c_i \pi_i T$
- The terminal chooses π to maximise :

$$\left(v_i \frac{L_i}{M_i} R_i f_i(G_i \kappa(\pi)) - c_i \pi \right)$$

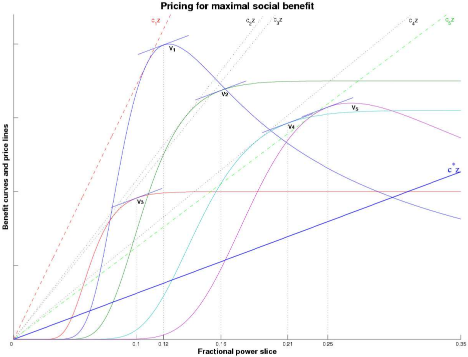
- f_i is an S-curve, and so is $f_i(\kappa(\pi))$ as a function of π. Thus, the optimal π is the maximiser of $S(z) - cz$ with S some S-curve

Choice by an energy-sufficient terminal II



With a power share z, the terminal
$\max S(z) - cz$. 1st order cond.: $S'(z) = c$.
The largest acceptable c is the slope of the tangenu of S.

Finding the optimal price



- The planner sweeps a price line, from vertical to horizontal.
- If $c \geq c_1$ (line left of $c_1 z$) no one buys.
- When $c = c_1$, terminal 1 chooses to operate.
- As price drops more, more terminals become active
- Planner stops when the sum of “slices” equals $1 - d$.

Optimal price, II

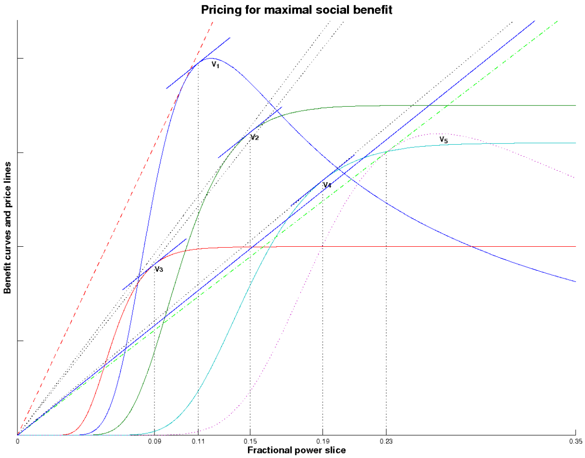


Figure: Bell and S curves are benefit graphs. The solid blue line represents the socially optimal price. Terminal 5 is left out when the resource is 0,54.

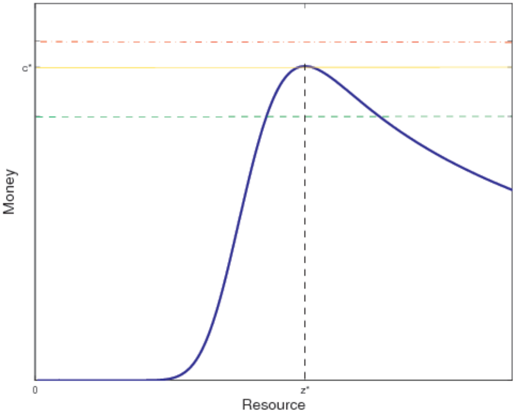
Recapitulation

- We characterise the power allocation that maximises the sum of terminals “benefits” the uplink of a CDMA cell, and describe how to reach the solution distributively via price-taking behaviour.
- By pricing a terminal’s fraction of the total power at the receiver ($p_i/(\sum p_i + p_0$ with p_0 denoting noise), we avoid the many problems of “games”.
- This fraction solely determines the terminal’s performance. Thus, for given price, each terminal can make its own optimal choice independently from the others
- Each data terminal has own bit rate, channel gain, willingness to pay, and link-layer configuration; energy supplies are limited only for some
- A terminal’s benefit function depends on whether its energy budget is finite or infinite

Choice by an energy-constrained terminal I

- Terminal maximises benefit minus cost over battery life T_i
- Benefit is $v_i B_i$, with B_i the total number of information bits uploaded in T_i
- $B_i(\pi_i) = (L_i/M_i) R_i f_i(G_i \kappa(\pi_i)) T_i$
- For π_i the corresponding transmission power is $P_i = p_i/h_i \equiv \pi_i \Pi / h_i$
- With energy E_i, battery life is $T_i = E_i/P_i \equiv E_i h_i / (\pi_i \Pi)$
- Terminal's cost is $c_i \pi_i T_i \equiv c_i E_i h_i / \Pi$ (π_i drops out!)
- The terminal chooses π to maximise total benefit minus total cost:
$$\frac{E_i h_i}{\Pi} \left(\frac{L_i}{M_i} v_i R_i \frac{f_i(G_i \kappa(\pi))}{\pi} - c_i \right)$$
- Optimal π is the maximiser of $\mathscr{B}(\pi) := f_i(G_i \kappa(\pi))/\pi$

Choice by an energy-constrained terminal II



For $c \leq c^*$ the e-terminal chooses z^*; else $z = 0$ is optimal.