Power allocation through revenue-maximising pricing on a CDMA reverse link shared by energy-constrained and energy-sufficient heterogeneous data terminals

Virgilio Rodriguez[1] , Friedrich Jondral[2] , Rudolf Mathar[1]
[1]Institute for Theoretical Information Tech., RWTH Aachen, Germany
[2]Institut für Nachrichtentechnik, Universität Karlsruhe (TH), Germany
email: vr <at> ieee.org

IEEE 69th Vehicular Technology Conference
Barcelona, Spain, 26–29 April 2009

Executive Summary

- We study the important issue of power control, in the uplink of a CDMA cell with homogeneous data terminals, with limited and boundless energy supplies
- Decentralised solutions have many advantages, but typically involve “games” in which terminals choices depend on one another
- Our solution is both decentralised and “decoupled”, which has important technical and social advantages
- We accomplish it by pricing the terminal’s fraction of the total power at the receiver. Because this fraction directly determines performance, each terminal can choose independently
- If “orders” exceed “capacity”, the network chooses the set of terminals that maximises revenue
- Our scheme outperforms a game in which terminals costlessly choose power, and the gap grows steadily as the number of active terminals increases

Decentralised power control in the cellular up-link

- Why is power control important?
  - 3G nets are based on CDMA, which is interference limited
  - a terminal's power creates interference for the others
  - power control increases capacity by limiting interference
  - it also extends battery life
- Decentralised solutions are preferable because of:
  - Complexity/cost of central controllers
  - Signalling overhead
  - Certain application scenarios are **inherently** decentralised (e.g. ad-hoc nets)
- For CDMA, many useful decentralised algorithms are based on on per-Watt pricing, which leads to "games"
- Games have some problems!

Why another paper?: "Games" have some problems!

- Games creates both *technological* and *marketing* problems
  - Terminals' choices depend on one another (complex!)
  - Solution concept is the Nash equilibrium (each terminal's choice is its "best response" to the choices by the others) which presents important challenges:
    - is in general inefficient
    - may NOT exist, or there may be many of them
    - even if uniquely exists, it is often unclear: (a) *how* will the players reach it, and (b) after how many "iterations"
    - In network, terminals "don't know" one another, and enter/exit at arbitrary times, which further aggravates
    - If "true" billing is based on per-Watt pricing, consumers may resist it (one's "utility" depends on everyone else's choice!)
- Below we provide a "de-coupled" solution: for given price, terminal's performance depends solely on OWN choice

Feasibility of key power ratios

- Let p_i and G_i denote terminal i's received power, and spreading gain, with p_0 the Gaussian noise
- signal-to-interference ratio (SIR): $\sigma_i = G_i \kappa_i$
- carrier-to-interference ratio (CIR): $\kappa_i := p_i / Y_i$
- $Y_i = p_0 + \sum_{k \neq i} p_i$ (total noise plus interference)
- Known fact: Each i can enjoy SIR σ_i only if

$$\sum \frac{\kappa_\mathbf{i}}{\mathbf{1} + \kappa_\mathbf{i}} \equiv \sum \frac{\sigma_\mathbf{i}}{\mathbf{G_i} + \sigma_\mathbf{i}} \leq \mathbf{1} - \mathbf{d}$$

- If $p_0 \approx 0$ (interference limited cell), condition is $\sum \pi_\mathbf{i} = \mathbf{1}$

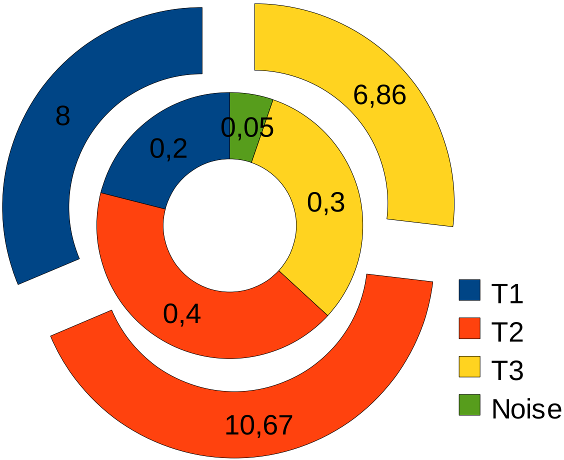
Power allocation as "cutting a pie"

- As discussed above, each i can enjoy SIR $\sigma_i = G_i \kappa_i$ only if $\sum \kappa_i/(1+\kappa_i) \leq 1-d$
- Let $\pi_i := \kappa_i/(\kappa_i+1) \equiv \sigma_i/(\sigma_i+G_i)$
- Notice that

$$\pi_i := \frac{\kappa_i}{1+\kappa_i} \equiv \frac{p_i/Y_i}{p_i/Y_i+1} \equiv \frac{p_i}{p_i+Y_i} := \frac{p_i}{\Pi}$$

- $\Pi := p_i + Y_i \equiv p_0 + \sum p_i \Rightarrow$ total power at receiver (a "pie")
- π_i is i's "fractional slice" of the pie
- Network can view uplink power allocation as assigning to each terminal a fraction of a fixed resource (dividing the "pie" $\Pi = p_0 + \sum p_i$ among the terminals)

Illustration of power allocation as “pie cutting”



- i's SIR:
  $\sigma_i = G_i \pi_i / (1 - \pi_i)$ with
  - G_i: spread gain
  - $\pi_i = p_i / (p_0 + \sum_j p_j)$
- Illustrated:

i	π_i	G_i	σ_i
1	0,2	32	8,0
2	0,4	16	10,7
3	0,3	16	6,7
0	0,05	-	-

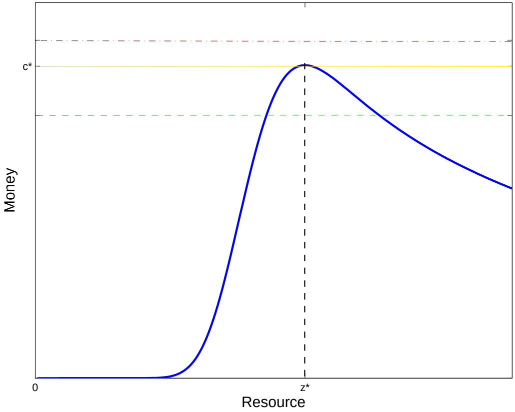
Power fraction pricing

- Network can set price c_i at which terminal i can "buy" π_i
- For given π_i, terminal can obtain directly its CIR $\kappa_i = \pi_i/(1-\pi_i)$ and hence its SIR, $\sigma_i = G_i\kappa_i$
- Thus, the terminal can make its optimal choice independently of choices made by others!
- if ordered "slices" exceed "pie size", network follows a "knapsack" approach to find the revenue-maximising set of terminals

Choice by an energy-constrained terminal I

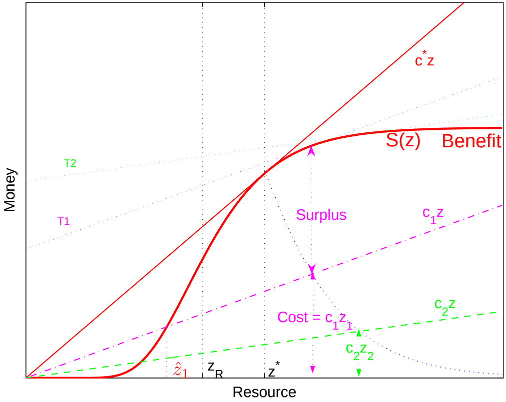
- Terminal maximises benefit minus cost over battery life T_i
- Benefit is $v_i B_i$, with B_i the total number of information bits uploaded in T_i
- $B_i(\pi_i) = (L_i/M_i) R_i f_i(G_i \kappa(\pi_i)) T_i$
- For π_i the corresponding transmission power is $P_i = p_i/h_i \equiv \pi_i \Pi / h_i$
- With energy E_i, battery life is $T_i = E_i/P_i \equiv E_i h_i/(\pi_i \Pi)$
- Terminal's cost is $c_i \pi_i T_i \equiv c_i E_i h_i / \Pi$ (π_i drops out!)
- The terminal chooses π to maximise total benefit minus total cost:
$$\frac{E_i h_i}{\Pi} \left(\frac{L_i}{M_i} v_i R_i \frac{f_i(G_i \kappa(\pi))}{\pi} - c_i \right)$$
- Optimal π is the maximiser of $\mathscr{B}(\pi) := f_i(G_i \kappa(\pi))/\pi$

Choice by an energy-constrained terminal II



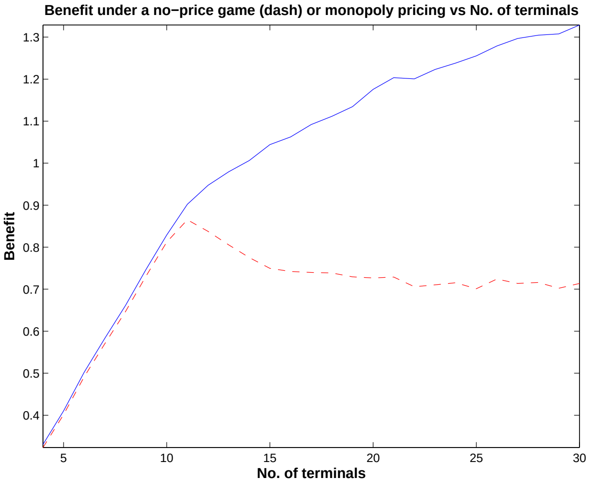
For $c \leq c^*$ the e-terminal chooses z^*; else $z = 0$ is optimal.

Choice by an energy-sufficient terminal



With a power share z, the terminal maximises $S(z) - c(z)$. The largest acceptable c is the slope of the tangenu of S.

No-cost game vs. power-share pricing



Recapitulation

- We propose a decentralised decoupled power control solution for the uplink of a CDMA cell
- Each data terminal has own bit rate, channel gain, willingness to pay, and link-layer configuration; energy supplies are limited only for some
- We price the terminal's fraction of the total power at the receiver ($p_i/(\sum p_i + p_0$ with p_0 denoting noise).
- This fraction solely determines the terminal's performance. Thus, for given price, each terminal can make its own optimal choice independently from the others
- The network follows a "knapsack approach" to select the set of terminals that maximises revenue
- As a base line for performance, we study a game in which each terminal can choose its power level without cost
- With few active terminals, our scheme outperforms the game only slightly, but the performance gap grows steadily with the number of terminals, to 2 to 1 and beyond