

# Optimum Resource Allocation for Heterogeneous Wireless OFDM Networks

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# Preface

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Chunhui Liu



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# 1 Introduction

As the number of wireless systems and services has experienced exponential growth over the last decade, wireless communications is the fastest growing segment of the communications industry. Orthogonal frequency division multiplexing (OFDM) [39,47] is a promising technique to combat multipath fading in wireless channels. It is preferred for the physical layer [4]. Its popularity is evident by the present standards, e.g., for digital video broadcasting-terrestrial (DVB-T) [22] and wireless local area networks (WLAN) [66], and the next-generation broadband wireless standards, e.g., for the third generation partnership project long term evolution (3GPP LTE) [73] and worldwide interoperability for microwave access (WiMAX) [2]. In OFDM systems, two principle transmission resources are provided, i.e., subcarriers and transmission power. The primal challenge for designing the wireless networks is to use the transmission resources as efficiently as possible while satisfying requirements of data transmission.

## 1.1 Related work

In communication networks, unicasting means to send messages to a single network destination identified by a unique address. Future cellular networks are expected to provide a large variety of services with diverse quality-of-service requirements, for example, delay-tolerant applications (non-real time transmission, e.g., online movie and data downloading) and delay-sensitive ones (real time transmission, e.g., voice and video phone calls). Hence, resource allocation problems in single-user OFDM systems are generally classified into two groups [47]. In one group, the transmission rate for non-real time transmission is maximized subject to limited transmission power, called the rate-adaptive (RA) resource allocation problem. In the other group, the transmission power is minimized while satisfying a fixed data rate for real time transmission, called the margin-adaptive (MA) resource allocation problem. For convenience, we call users requiring non-real time transmission RA users and users demanding real time transmission MA users.

In multiuser OFDM systems, performance can be further improved by taking advantage of the user diversity. When there are only multiple RA users in an OFDM system, the sum rate for these RA users is maximized subject to limited transmission power. If only multiple MA users appear in an OFDM system, the transmission power for these MA users is minimized while meeting the fixed data rates required by MA users. The two groups of single-user resource allocation problems are extended to the multiuser RA resource allocation problem [39,80] and the multiuser MA resource allo-

allocation problem [9, 40, 84], respectively. Dual optima are determined for both problems by duality theory in [72, 89].

For the multiuser RA resource allocation, one subcarrier should be assigned to the user with the greatest channel gain-to-noise ratio (CNR) of this subcarrier to achieve the maximal throughput, when the total transmission power is limited and there are not other constraints. This optimality has been proved in [36]. However, some users may be located far from the base station (BS) or behind an obstruction, which leads to deep or shadow fading. They would be seldom served by the BS. To maintain the fairness for these users, the data rate for each has been individually lower bounded in [86]. The lower bounds are the smallest acceptable data rates for RA users, e.g., the minimal transmission rate for keeping an online movie fluent. The achieved rate for each user has been weighted in [94] to balance the rate achievement among users. Heuristic solutions have been suggested in these works.

Alternatively, for the multiuser MA resource allocation problem, heuristic methods have been suggested in [10, 11, 23, 25, 26, 40, 41, 83, 84, 90], where computational complexity and system performance are balanced. Furthermore, resource allocation has been studied for relay-assisted OFDM systems in [37, 38, 69]. Previous works [13, 34, 45, 93] have concentrated on resource allocation for homogeneous unicasting, i.e., either multiuser RA resource allocation or multiuser MA resource allocation. However, a mixture of real time and non-real time applications must be supported in practice, called heterogeneous unicasting. In [79] resource allocation is sequentially performed for MA and RA users and the resulting performance loss is large.

Different from the aforementioned works, this thesis investigates resource allocation for heterogeneous unicasting from one or multiple BSs to multiple RA and MA users. Resource allocation is jointly considered for RA users (non-real time transmission) and MA users (real time transmission), so called heterogeneous resource allocation. For heterogeneous resource allocation the weighted sum rate for RA users is maximized, while the constraints on the minimum data rates required by RA users and the fixed data rates required by MA users are satisfied and while the total transmission power for all users is limited. The suggested solutions can also be used for the resource allocation problems mentioned earlier with small modifications. Moreover, they can be extended to other resource allocation problems and to other multicarrier systems.

## 1.2 Outline

Figure 1.1 shows the diagram of this thesis. Solutions have been suggested for single-user RA and MA resource allocation problems in [47], well known as the water-filling solution. As mentioned earlier, dual optima and heuristic solutions have been given for multiuser RA or MA resource allocation problems. The dashed boxes in Figure 1.1 represent these previous works. Different from these works, we concentrate on resource allocation for heterogeneous unicasting in this thesis, which is a mixture of multiuser RA and MA resource allocation problems. We classify the content of this thesis into blocks, which are denoted by the solid boxes in Figure 1.1. The dependency among them and the previous works is also clarified there.

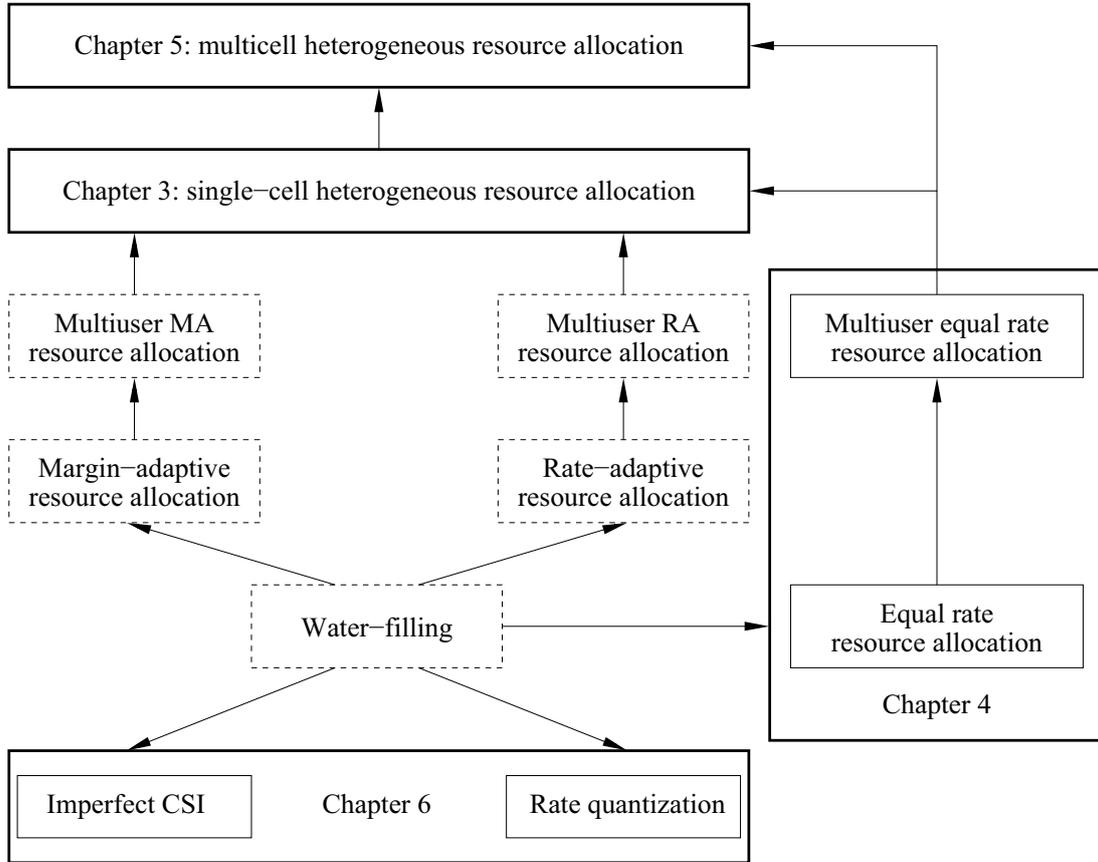


Figure 1.1: A dependency diagram.

## Chapter 2: prerequisites

Chapter 2 introduces the prerequisites used throughout this thesis. First, it summarizes the mathematical concepts of convex optimization in Section 2.1. It then simply models wireless channels and OFDM systems in Section 2.2 and Section 2.3 relying on several approximations, respectively. These approximations do not confine the solutions for heterogeneous resource allocation suggested in the following chapters. In the last section, it formulates the resource allocation problem for heterogeneous unicasting by a single BS and then extends this formulation to heterogeneous unicasting by multiple BSs.

## Chapter 3: heterogeneous resource allocation

Chapter 3 studies resource allocation for heterogeneous unicasting by a single BS. With duality theory a dual optimum is determined for heterogeneous resource allocation in Section 3.1. It is compared to the primal optimum in Section 3.2. The gap between the dual and primal optima decreases approximately exponentially in the number of

subcarriers, which means that the derived dual optimum is qualified to be a reference for assessing heuristic solutions.

After that we aim at designing heuristic methods for the considered problem in Section 3.3. A simple procedure is employed to make the proposed heuristic method also suitable for other resource allocation problems in other multicarrier systems, e.g., relay-aided OFDM, cognitive radio and energy efficiency problems. To accelerate this procedure, efficient approaches are developed for updating the objective after changing the subcarrier assignment in Subsection 3.3.1. Furthermore, two techniques are integrated into this procedure in Subsection 3.3.3 to improve the balance between performance and computing time. Verified by simulations in Section 3.4, the suboptimal solutions by the suggested heuristic methods are close to the dual optimum and the computing time is increasing approximately linearly in the number of users and subcarriers. Moreover, the performance is effectively improved by the two integrated techniques, while the computing time is significantly reduced.

## Chapter 4: equal rate per user resource allocation

In Chapter 4 we still focus on resource allocation for heterogeneous unicasting by a single BS. This chapter proposes a new strategy of resource allocation that differs from the water-filling strategy. Since wireless channels are subject to rapid variation in time, resource allocation schemes may have to be frequently updated via the signalling overhead. Receivers are then notified of the employed power and rate allocation scheme so that data detection can be performed. In a water-filling solution, different powers and rates are allocated to subcarriers depending on channel conditions. A large signalling overhead is required for representing a water-filling solution [28]. Hence, system performance may deteriorate significantly in fast time-varying environments.

To combat the effect above, we propose that an equal rate is allocated to subcarriers assigned to each user according to channel qualities. In doing so, the signalling overhead is reduced and a better balance of energy consumption is achieved for signalling and data transmission. With the signalling overhead additionally considered, resource allocation problems are reformulated in Section 4.1. By using the basic idea of the golden section search, an upgraded bisection method is designed in Section 4.2 to determine the subcarrier assignment for single-user equal rate resource allocation. Furthermore, the asymptotic limits are deduced for the instantaneous per-symbol performance loss of the proposed strategy compared to the water-filling strategy.

With the proposed strategy applied, it is much easier to update the objective after changing the subcarrier assignment. By using the framework of the heuristic method presented in Chapter 3, Section 4.3 gives heuristic solutions for multiuser equal rate resource allocation. The proposed strategy and heuristic method are thoroughly evaluated by simulations in Section 4.4. Compared to the water-filling strategy, the average instantaneous per-symbol performance loss of the proposed strategy is limited. The proposed strategy achieves better energy efficiency when the resource allocation scheme is frequently updated.

## **Chapter 5: multicell resource allocation**

Chapter 5 deals with resource allocation for heterogeneous unicasting by multiple BSs. Resource allocation is performed over three dimensions, i.e., different BSs, users and subcarriers. Two different scenarios are considered here. On one hand, when signals received from different BSs are synchronized, each user may be served by more than one BS. The primal optimum for this multicell heterogeneous resource allocation is computationally intractable even for a simulation system consisting of a small number of BSs, users and subcarriers. The dual optimum and the heuristic solutions for single-cell heterogeneous resource allocation offered in Chapter 3 are extended for the purpose of multicell resource allocation in Section 5.1 and Section 5.2, respectively.

On the other hand, if synchronization of signals received from different BSs cannot be achieved, each user can be covered by only one BS. In other words, a specific BS must be selected for each user, so called BS selection. Even if a user is served by an inappropriate BS, the resulting subcarrier assignment significantly differs from the optimal one. Thus, the heuristic design become more challenging for this scenario. The remainder of this chapter suggests a dual optimum and a heuristic solution for joint BS selection and subcarrier assignment. It concludes by presenting simulation results, which demonstrate that the suboptima by the proposed heuristic methods are near to the dual optima. The two techniques developed in Subsection 3.3.3 can still effectively improve performance and significantly reduce computing time for the proposed heuristic solutions.

## **Chapter 6: imperfect CSI and rate quantization**

Chapter 6 investigates two important issues on resource allocation. In the previous chapters, perfect channel knowledge is assumed at the transmitter. In practice, channels are measured at receivers and then channel knowledge is fed back to the transmitter. Section 6.1 quantifies the imperfection of channel knowledge while considering noisy channel estimation and channel variation during unavoidable feedback delay. Section 6.2 then studies resource allocation in the presence of imperfect channel state information (CSI).

Moreover, the practical transmission rate must be discrete because of a limited number of available coding and mapping schemes. Section 6.3 proposes a non-iterative method for optimally quantizing the output rates of a water-filling solution. It can be directly attached to the methods suggested previously.

## **Chapter 7: conclusions**

Finally, Chapter 7 summarizes the contribution of this thesis. The offered methods are not limited to the considered systems or the concerned resource allocation problems. They may be used for other resource allocation problems with small modifications.

Sections of this thesis and related topics have already been published in [49–60, 81, 82].



## 2 Prerequisites

This chapter introduces some important characteristics of wireless OFDM transmission concerning resource allocation. First, Section 2.1 gives an overview of a few important concepts of convex optimization and duality theory in [6], which will be used in the following chapters. Thereafter, Section 2.2 models wireless channels. Section 2.3 then explains the transmitter and the receiver of OFDM systems. Finally, the last section formulates the resource allocation problems under consideration.

### 2.1 Mathematical preliminaries

In this section, we review some geometric properties of sets and functions, which are the fundamental tools for dealing with resource allocation problems. The variables and sets defined in this section do not have any physical meaning.

#### 2.1.1 Convex sets and functions

Definitions and properties of convex sets and functions are generally introduced.

**Definition 2.1.**

A set  $\mathcal{C} \subseteq \mathbb{R}^n$  is called *convex* if the line segment between any two points in  $\mathcal{C}$  lies in  $\mathcal{C}$ , i.e., if

$$\forall \mathbf{x}_1, \mathbf{x}_2 \in \mathcal{C}, \lambda \in [0, 1] : \quad \lambda \mathbf{x}_1 + (1 - \lambda) \mathbf{x}_2 \in \mathcal{C}.$$

Examples of convex and non-convex sets are given in Figure 2.1. To establish convexity of a set  $\mathcal{C}$ , the definition above can be applied. We can also show that  $\mathcal{C}$  is derived from simple convex sets by operations that preserve convexity, e.g., intersection, perspective functions and linear-fractional functions.

**Definition 2.2.**

Let  $\mathcal{C} \subseteq \mathbb{R}^n$  be a non-empty and convex set. A function  $f : \mathcal{C} \rightarrow \mathbb{R}$  is *convex*, if for all  $\mathbf{x}, \mathbf{y} \in \mathcal{C}$  and  $\lambda \in [0, 1]$

$$f(\lambda \mathbf{x} + (1 - \lambda) \mathbf{y}) \leq \lambda f(\mathbf{x}) + (1 - \lambda) f(\mathbf{y}).$$

If the inequality above is satisfied with strict inequality ( $<$ ) for all  $\mathbf{x}, \mathbf{y} \in \mathcal{C}$ ,  $\mathbf{x} \neq \mathbf{y}$  and  $\lambda \in (0, 1)$ , we say that  $f$  is *strictly convex*. The function is called *(strictly) concave*, if  $-f$  is (strictly) convex.

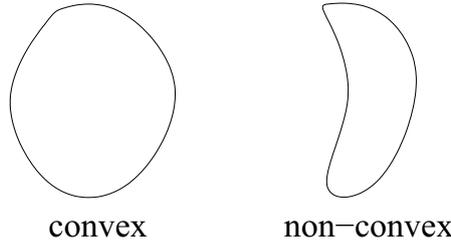


Figure 2.1: Convex and non-convex sets.

**Theorem 2.1.** (*First-order condition*)

Let  $f : \mathcal{C} \rightarrow \mathbb{R}$  be differentiable and  $\mathcal{C} \subseteq \mathbb{R}^n$  be non-empty and convex. Then,  $f$  is convex if and only if

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{C} : f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})'(\mathbf{y} - \mathbf{x}).$$

**Theorem 2.2.** (*Minimizing a convex function over a convex set*)

Let  $f : \mathcal{C} \rightarrow \mathbb{R}$  be differentiable and  $\mathcal{C} \subseteq \mathbb{R}^n$  be non-empty and convex. Then, the following three statements are equivalent

1.  $\mathbf{x}^*$  is a global minimum.
2.  $\mathbf{x}^*$  is a local minimum.
3.  $\mathbf{x}^*$  is a critical point, i.e.,  $\nabla f(\mathbf{x}^*) = 0$ . This holds only if  $\mathbf{x}^*$  is an interior point, otherwise not necessarily.

Convexity of a function can be established by the definition of convex functions or by showing that this function is from simple convex functions by operations that preserve convexity, e.g., non-negative weighted sum, composition, point-wise maximum and supremum and perspective.

## 2.1.2 Duality theory

In the following, duality theory is summarized. We call the following problem an *optimization problem in standard form*

$$\begin{aligned} & \text{minimize} && f(\mathbf{x}) && (2.1) \\ & \text{subject to} && g_i(\mathbf{x}) \leq 0, && i = 1, \dots, s \\ & && h_j(\mathbf{x}) = 0, && j = 1, \dots, m \end{aligned}$$

where

- $\mathbf{x} \in \mathbb{R}^n$  is the *optimization variable*,
- $f : \mathcal{S} \rightarrow \mathbb{R}, \mathcal{S} \subseteq \mathbb{R}^n$  is the *objective function*,
- $g_i(\mathbf{x}) \leq 0, i = 1, \dots, s$ , are the *inequality constraints*,
- $g_i : \mathbb{R}^n \rightarrow \mathbb{R}, i = 1, \dots, s$ , are the *inequality constraint functions*,
- $h_j(\mathbf{x}) = 0, j = 1, \dots, m$ , are the *equality constraints*,
- $h_j : \mathbb{R}^n \rightarrow \mathbb{R}, j = 1, \dots, m$ , are the *equality constraint functions*.

The *optimal value* of (2.1) is defined as

$$p^* = \inf\{f(\mathbf{x}) \mid g_i(\mathbf{x}) \leq 0, i = 1, \dots, s, h_j(\mathbf{x}) = 0, j = 1, \dots, m\}.$$

A *convex optimization problem in standard form* is

$$\begin{aligned} & \text{minimize} && f(\mathbf{x}) \\ & \text{subject to} && g_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, s \\ & && \mathbf{a}'_j \mathbf{x} = b_j, \quad j = 1, \dots, m \end{aligned}$$

with domain  $\mathcal{D} = \text{dom } f(\mathbf{x}) \cap \bigcap_{i=1}^s \text{dom } g_i(\mathbf{x}) \cap \bigcap_{j=1}^m \text{dom } h_j(\mathbf{x})$ , where  $f, g_1, \dots, g_s$  are convex and  $\mathbf{a}'_j \mathbf{x} = b_j, \mathbf{a}_j \in \mathbb{R}^n, j = 1, \dots, m$ , are affine. An important property of a convex optimization problem is that the feasible set is convex.

Associated with (2.1), the *Lagrangian*  $L : \mathcal{D} \times \mathbb{R}^s \times \mathbb{R}^m \rightarrow \mathbb{R}$  is defined as

$$L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f(\mathbf{x}) + \sum_{i=1}^s \lambda_i g_i(\mathbf{x}) + \sum_{j=1}^m \mu_j h_j(\mathbf{x})$$

where  $\lambda_i$  is the Lagrange multiplier associated with  $g_i(\mathbf{x}), i = 1, \dots, s$ , and  $\mu_j$  is the Lagrange multiplier associated with  $h_j(\mathbf{x}), j = 1, \dots, m$ . The vectors  $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_s)'$  and  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_m)'$  are called the *dual variables* or *Lagrange multiplier vectors*.

The *Lagrange dual function*  $L_D : \mathbb{R}^s \times \mathbb{R}^m \rightarrow \mathbb{R}$  is defined as the infimum of the Lagrangian with respect to  $\mathbf{x}$ :

$$L_D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\mathbf{x} \in \mathcal{D}} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\mathbf{x} \in \mathcal{D}} \left( f(\mathbf{x}) + \sum_{i=1}^s \lambda_i g_i(\mathbf{x}) + \sum_{j=1}^m \mu_j h_j(\mathbf{x}) \right).$$

If problem (2.1) is convex, the Lagrange dual function is concave. It gives important lower bounds on the optimal value  $p^*$  of (2.1). It holds that

$$\forall \boldsymbol{\lambda} \succeq \mathbf{0}_{s \times 1}, \boldsymbol{\mu} \in \mathbb{R}^m : \quad L_D(\boldsymbol{\lambda}, \boldsymbol{\mu}) \leq p^*.$$

Problem (2.1) is called the primal problem. The *Lagrange dual problem* associated with it is defined as

$$\begin{aligned} & \text{maximize} && L_D(\boldsymbol{\lambda}, \boldsymbol{\mu}) \\ & \text{subject to} && \boldsymbol{\lambda} \succeq \mathbf{0}_{s \times 1}. \end{aligned} \tag{2.2}$$

It is a convex optimization problem, since the objective function is concave in the dual variables and the constraint is convex. We refer to  $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$  as the *optimal dual variables* or the *optimal Lagrange multipliers*, if they are optimal for (2.2). We denote by  $d^*$  the optimal value of (2.2).

**Theorem 2.3.** (*Weak and strong duality*)

*The weak duality holds, if it holds that*

$$d^* \leq p^*.$$

*If the equality holds, we say that the strong duality holds. We refer to  $p^* - d^*$  as the optimal duality gap.*

**Definition 2.3.**

Consider the optimization problem (2.1) with differentiable  $f, g_1, \dots, g_s, h_1, \dots, h_m$ . The following four conditions are called the Karush-Kuhn-Tucker (KKT) conditions.

1. Primal constraints:  $g_i(\mathbf{x}) \leq 0, i = 1, \dots, s, h_j(\mathbf{x}) = 0, j = 1, \dots, m$ .
2. Dual constraints:  $\boldsymbol{\lambda} \succeq \mathbf{0}_{s \times 1}$ .
3. Complementary slackness:  $\lambda_i g_i(\mathbf{x}) = 0, i = 1, \dots, s$ .
4. Gradient of Lagrangian with respect to  $\mathbf{x}$ :

$$\nabla f(\mathbf{x}) + \sum_{i=1}^s \lambda_i \nabla g_i(\mathbf{x}) + \sum_{j=1}^m \mu_j \nabla h_j(\mathbf{x}) = 0.$$

As above, let  $x^*$  and  $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$  be any primal and dual optimal points with zero duality gap. Then, they must satisfy the KKT conditions.

## 2.2 Channel model

The environment for wireless communications is very complicated and is always time-varying. In this section, the behavior of wireless channels is modeled by applying some simplifications and approximations. Resource allocation methods suggested in the following chapters are not affected by these simplifications and approximations.

### 2.2.1 Multipath Rayleigh fading

In Figure 2.2 a discrete model for multipath Rayleigh fading is depicted. The longer the signal travels, the larger the tap delay is. Empirical evidence from experimental field studies suggests that at large distance received power decays exponentially with distance, see [76]. This condition is satisfied in the considered scenario. Thus, we assume that the channel response length is  $Z$ , which means that the paths beyond the  $(Z - 1)$ st tap have negligible impact on transmission. We denote by  $h_z[m]$  the complex channel coefficient of the  $z$ th tap at time  $m$ . According to the work in [76],  $h_z[m]$  can be modeled as a circular symmetric complex Gaussian random variable, distributed as  $h_z[m] \sim \text{CN}(0, \sigma_{h_z}^2)$ . The power of taps  $(\sigma_{h_0}^2, \dots, \sigma_{h_{Z-1}}^2)$  is assumed to be an exponentially decaying profile with respect to the tap index  $z$ . The magnitude  $|h_z[m]|$  of the  $z$ th tap is a *Rayleigh* random variable with density

$$f_{|h_z[m]|}(x) = \frac{x}{\sigma_{h_z}^2} \exp\left(\frac{-x^2}{2\sigma_{h_z}^2}\right) \mathbb{I}_{[0, \infty)}(x).$$

The squared magnitude  $|h_z[m]|^2$  is exponentially distributed with density

$$f_{|h_z[m]|^2}(x) = \frac{1}{\sigma_{h_z}^2} \exp\left(\frac{-x}{\sigma_{h_z}^2}\right) \mathbb{I}_{[0, \infty)}(x).$$

The delay spread is defined as the difference between the largest and the smallest tap delays. If it is comparable to or even greater than the symbol duration,  $Z > 1$

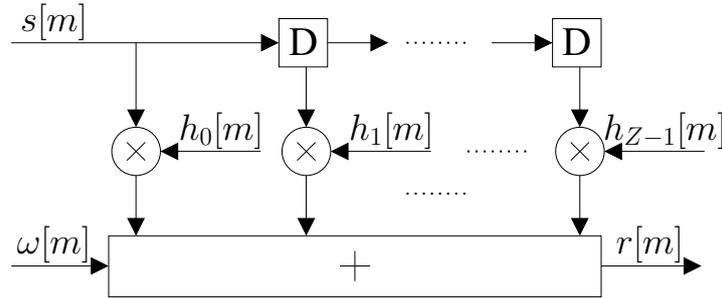


Figure 2.2: Discrete multipath Rayleigh fading channel

must hold. The channel input and output are  $s[m]$  and  $r[m]$  at time  $m$ , respectively. The additive white Gaussian noise, denoted by  $\omega[m]$ , is circular symmetric complex Gaussian distributed as  $\omega[m] \sim \text{CN}(0, N_0)$ . In mathematical terms, the channel output is expressed as

$$r[m] = \sum_{z=0}^{Z-1} h_z[m] s[m-z] + \omega[m] \quad (2.3)$$

where the symbol transmitted at time  $m$  is interfered by the symbols transmitted at previous time instances  $m-1, \dots, m-Z+1$ . This effect is called inter-symbol interference (ISI). The amplitude of the Fourier transform of the channel impulse response varies in frequency, so called frequency selective fading. Its counterpart is so called flat fading when the delay spread is much smaller than the symbol duration. The corresponding channel impulse response is approximated to a Dirac function in the time domain, while it has a constant amplitude over frequency in the frequency domain.

### 2.2.2 Time-varying channel

In the subsection above, each tap  $h_z[m]$  is modeled as a circular symmetric complex Gaussian random variable at time  $m$ . These quantities may vary in time, since the locations of the receiver, the transmitter and/or the scatterers around them are not always fixed. A statistical quantity that models this relation is known as the tap gain auto-correlation function, defined as

$$R_z(t) := \mathbb{E}\{h_z^*[m] h_z[m+t]\}.$$

Since each tap is assumed to be stationary, this auto-correlation function is independent of time. The coefficient  $R_z[0]$  is proportional to the energy received in the  $z$ th tap.

A popular statistical model to describe the temporal behavior of each channel tap is Jakes' model [35]. We assume that the receiver moves at a certain velocity  $x$  relative to the transmitter. The induced Doppler frequency is  $f_d = f_c x/c$ , where  $f_c$  is the carrier frequency and  $c$  is the speed of light. Moreover, it is assumed that a large number

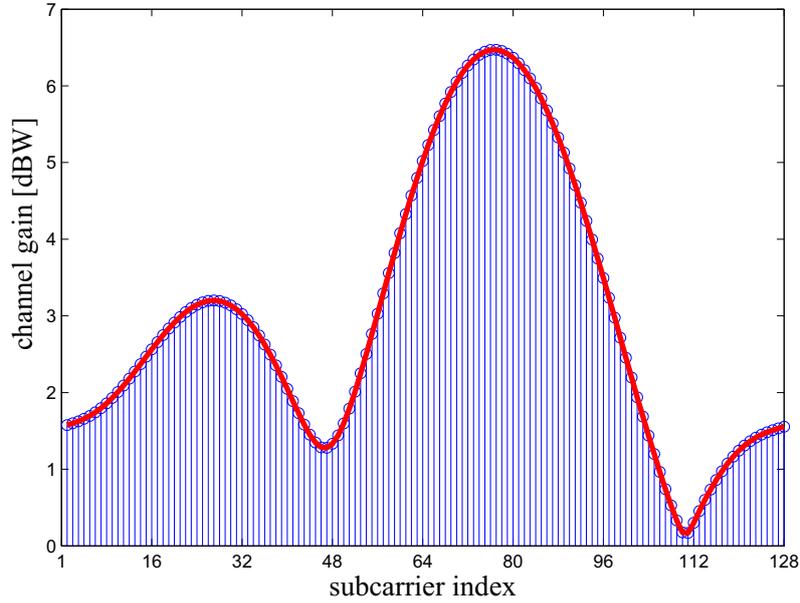


Figure 2.3: An example for subcarriers with different channel gains.

of scatterers is uniformly distributed around the transmitter and the receiver. These scatterers affect the received signal independently. Then, the auto-correlation function is written as

$$R_z(t) = J_0(2\pi f_d t)$$

where  $J_0(\cdot)$  is the 0th-order Bessel function of the first kind. Its Fourier transform is the power spectral density, written as

$$S(f) = \begin{cases} (\pi f_d \sqrt{1 - (f/f_d)^2})^{-1} & |f| < f_d \\ 0 & |f| \geq f_d \end{cases}.$$

## 2.3 System model

In this section, the OFDM system is introduced. To overcome frequency selective fading caused by multipath channels, OFDM has been proposed for the high-speed wireless data applications due to the relatively simple structure of its receivers, specified in [47]. It has been used in many present digital communication systems, such as DVB-T [22], WLAN [66] and WiMAX [2]. In OFDM, the transmission band is divided into subcarriers of equal bandwidth. Each is subject to flat fading, if the bandwidth of each is narrow enough, which means the number of subcarriers, denoted by  $N$ , is sufficiently large. An example for such a division is shown in Figure 2.3, where the transmission band with frequency selective fading is divided into 128 subcarriers. The simplified block diagram of OFDM systems is depicted in Figure 2.4 and Figure 2.5.

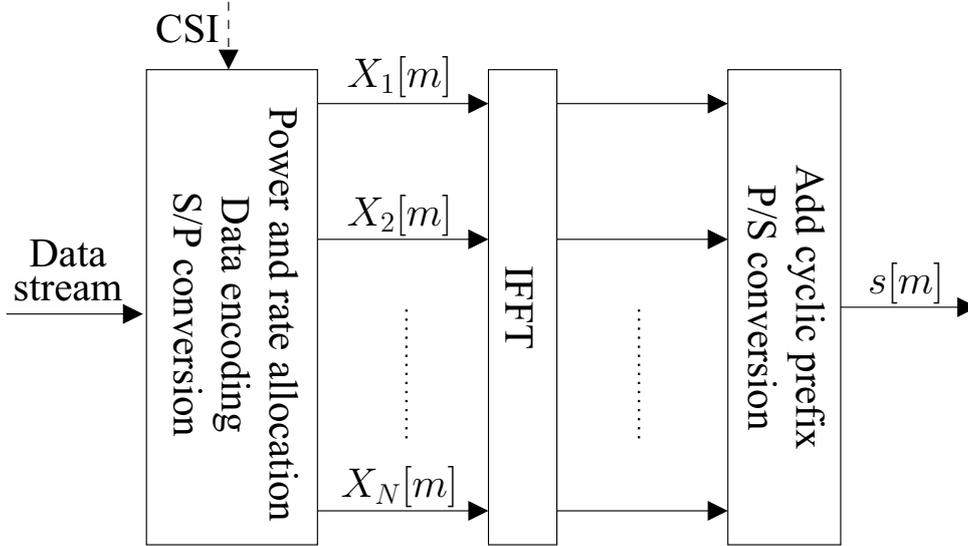


Figure 2.4: Discrete-time OFDM transmitter.

At the transmitter in Figure 2.4, before transmission the power and rate assigned to each subcarrier are determined depending on channel conditions in order to enhance system performance. Such power and rate allocation is called a resource allocation scheme. In many practical systems, CSI is only available at the receiver and must be fed back to the transmitter. According to the determined allocation scheme, the binary data stream is encoded and parallelized to subcarriers so called serial-to-parallel (S/P) conversion. At most  $N$  subcarriers may be employed. The parallelized data streams are mapped onto a complex constellation alphabet.

Let  $X_n[m]$  denote the symbol transmitted on subcarrier  $n$  at time  $m$ . We call the vector  $(X_1[m], \dots, X_N[m])'$  the  $m$ th OFDM symbol. The inverse fast Fourier transform (IFFT) allows for transmitting different data streams over subcarriers. The cyclic prefix is added to combat the ISI effect. After that the parallel-to-serial (P/S) conversion serially outputs data symbols denoted by  $s[m]$ .

The output of the multipath Rayleigh fading channel in Figure 2.2 is the received signal  $r[m]$  in the time domain. We assume perfect synchronization such that subcarriers are still orthogonal at the receiver. After synchronization, the cyclic prefix is removed and data symbols are parallelized to the input of the fast Fourier transform (FFT), where the FFT size is  $N$ . Then, the received symbols over subcarrier  $n$  in the frequency domain can be written as

$$Y_n[m] = H_n[m]X_n[m] + \Omega_n[m] \quad (2.4)$$

where  $H_n[m]$  is the channel coefficient of the  $n$ th subcarrier at time  $m$ . Since the additive white Gaussian noise follows the circular symmetric complex Gaussian distribution, this property remains after the FFT. Thus, the noise sample  $\Omega_n[m]$  is still subject to a circular symmetric complex Gaussian distribution as  $\Omega[m] \sim \text{CN}(0, N_0)$ .

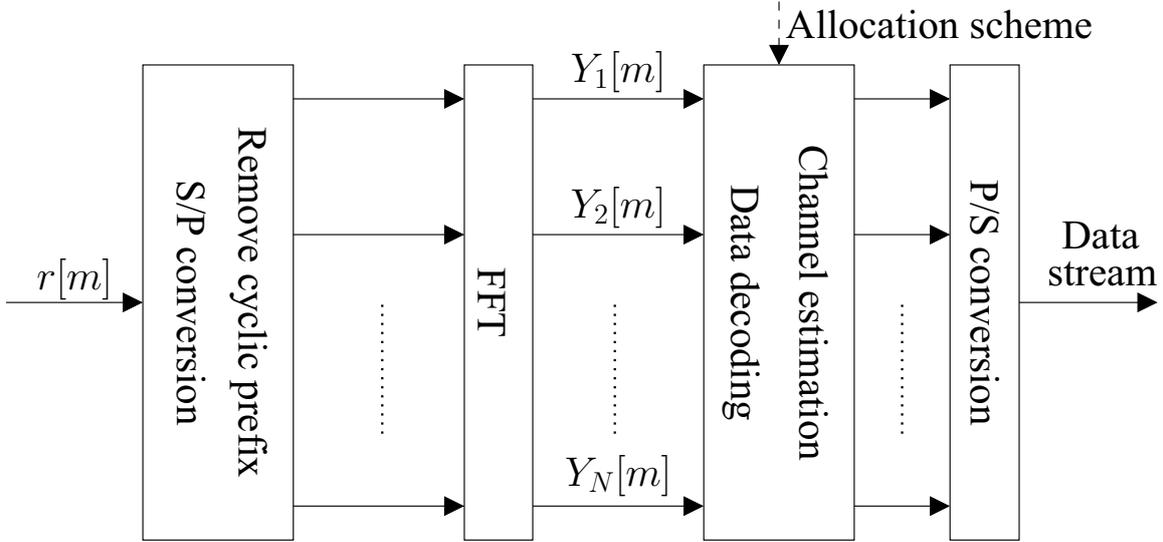


Figure 2.5: Discrete-time OFDM receiver.

Channel estimation and data decoding are performed according to the employed resource allocation scheme.

## 2.4 Problem formulation

The previous sections have summarized some important concepts of convex optimization and basic models for wireless channels and OFDM systems. OFDM employs multiple subcarriers with different channel conditions to combat ISI. This offers the flexibility of allocating different powers and rates to subcarriers to improve system performance.

### 2.4.1 Single-cell heterogeneous resource allocation

Let us first consider heterogeneous unicasting from a single BS to  $K + Q$  users over  $N$  subcarriers, where the first  $K$  users require non-real time transmission with the constraints on minimum required rates, i.e., RA users, and the other  $Q$  users demand real time transmission with fixed rates required, i.e., MA users [96]. The transmission from the BS to different users is subject to independent frequency selective fading. Each frame consists of  $L$  OFDM symbols. We assume that the resource allocation scheme is updated for each frame subject to channel variation in time. Multiuser diversity, caused by independent fading of user channels, can be exploited to further improve system performance. The transmission power at the BS is limited to  $P$ .

For heterogeneous unicasting, we aim at maximizing the weighted sum data rate for the  $K$  RA users,  $k = 1, \dots, K$ . As fairness control, their data rates are weighted by a given vector  $\mathbf{w} = (w_1, \dots, w_K)' \in \mathbb{R}_{++}^K$ , which is normalized to  $\sum_{k=1}^K w_k = 1$ . The

data rate for each user  $k = 1, \dots, K + Q$  is lower bounded by  $R_k$ . This optimization problem is stated as

$$\begin{aligned}
& \text{maximize} && R = \sum_{k=1}^K w_k \sum_{n=1}^N r_{k,n} && (2.5) \\
& \text{subject to} && r_{k,n} = \log_2(1 + p_{k,n} g_{k,n}), \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\
& && r_{k,n} \geq 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\
& && \sum_{n=1}^N r_{k,n} \geq R_k, \quad k = 1, \dots, K + Q \\
& && \sum_{k=1}^{K+Q} \sum_{n=1}^N p_{k,n} \leq P \\
& && \sum_{n=1}^N r_{k,n} r_{l,n} = 0, \quad k, l = 1, \dots, K + Q, \quad k \neq l
\end{aligned}$$

where  $p_{k,n} \in \mathbb{R}_+$  and  $r_{k,n} \in \mathbb{R}_+$  denote the power and the rate allocated to the  $n$ th subcarrier for user  $k$ , respectively. They are related by the first equality constraint function from [47], i.e., the power-rate function, where  $g_{k,n}$  is the CNR of the  $n$ th subcarrier of user  $k$ . This relation is valid for uncoded quadrature amplitude modulation. The subcarrier assignment for user  $k$  is denoted by a set  $\mathcal{S}_k$ . It contains the  $s_k = |\mathcal{S}_k|$  subcarriers assigned to user  $k$ . To avoid interference among different users, each subcarrier is assigned to at most one user at any specific period of time in the considered system as illustrated by the last constraint [36]. Thus, it follows that  $\mathcal{S}_k \cap \mathcal{S}_l = \emptyset$  for all  $k \neq l$  and  $\bigcup_{k=1}^{K+Q} \mathcal{S}_k \subseteq \{1, \dots, N\}$  always hold. By this (2.5) becomes a combinatorial problem. The transmission power for user  $k$  is referred to as  $P_k = \sum_{n \in \mathcal{S}_k} p_{k,n}$ . Let the set  $\mathcal{K}$  contain RA users and the set  $\mathcal{Q}$  comprise MA users. The resource allocation problem above is studied for heterogeneous unicasting by a single BS in Chapter 3 and Chapter 4.

## 2.4.2 Multicell heterogeneous resource allocation

The formulation above is extended for unicasting by multiple BSs. BSs, located within a large area, are clustered according to characteristics of the unicasting coverage, like topography, limits of the transmission power at BSs, the density of users and transmission demands. Within one cluster, BSs share the same transmission band by OFDM and transmission data is reachable for each BS via the fibre connection to the control center as shown in Figure 2.6. The resulting interference among different clusters is treated as noise.

Let us focus on one cluster of  $C$  BSs. Resource allocation can be jointly considered for unicasting from multiple BSs to multiple RA and MA users within one cluster, called multicell heterogeneous resource allocation. The primal single-cell problem (2.5)



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users nor among BSs. The last constraint in the single-cell case (2.5) changes to that one subcarrier is assigned to at most one pair of user and BS, formulated by the last constraint in the new problem. One user may be covered by multiple BSs over different subcarriers. Let  $\mathcal{S}_{k,c}$  denote the set of the  $s_{k,c}$  subcarriers assigned to user  $k$  at BS  $c$ . Following the last constraint,  $\mathcal{S}_{k,c} \cap \mathcal{S}_{l,v} = \emptyset$  always holds for any  $(k,c) \neq (l,v)$ . The sum power for user  $k$  at BS  $c$  is referred to as  $P_{k,c}$ . The upper limit of transmission power to each subcarrier may be additionally considered like [44] to avoid significant interference to other clusters where the same transmission band is reused. This additional constraint is omitted here, while the generality of the proposed methods is not impaired. Chapter 5 investigates multicell heterogeneous resource allocation. Note that the notation in (2.5) is used in Chapter 3 and Chapter 4, while the notation in (2.6) is used in Chapter 5.



# 3 Heterogeneous Resource Allocation

This chapter suggests primal optimal, dual optimal and heuristic solutions for the single-cell heterogeneous resource allocation problem (2.5). Since the combinatorial problem (2.5) is not convex, it is prohibitive to calculate the primal optimum of the considered problem when the numbers of users and subcarriers are large. First, the primal problem is reformulated to adapt the dual method to the heterogeneous resource allocation. A dual optimum is given for assessing heuristic methods. At the dual optimum, some RA users are treated as MA users, for whom only the minimum required rates are reached. The rates achieved for other RA users are strictly greater than the minimum required rates. This brings more challenges for the heuristic design, since these two kinds of RA users must be distinguished. The primal optimum of (2.5) is then explained.

Based on the primal and the dual optima, we obtain efficient approaches for updating the weighted sum rate for RA users after changing the subcarrier assignment. These approaches are employed to accelerate the following procedure. To adapt the proposed heuristic method to other resource allocation problems and other multi-carrier systems, a simple procedure is employed, where the subcarrier assignment is successively and iteratively adjusted with respect to subcarriers. Each subcarrier is reassigned to different users to see whether the weighted sum rate can be improved. The derived heuristic method gives a solution that is close to the primal optimum on average. However, either its computing time is long, or its performance loss is large. To improve its performance and computational efficiency, additional techniques are developed, namely sorting subcarriers and controlling iterations. By integrating these two additional techniques, the heuristic solution is closer to the primal optimum, while the average computing time is significantly reduced and increases approximately linearly in the number of users and subcarriers. The notation in (2.5) is used. Note that we only consider the feasible heterogeneous resource allocation, i.e., the minimum transmission power for reaching all minimum required rates is less than or equal to the limit of transmission power at the BS.

## 3.1 Dual optimum

Duality theory has been summarized in Chapter 2. It has been introduced in [62, 89] and widely used for resource allocation problems in OFDM and multiple-input and multiple-output systems, e.g., [32, 65, 85, 91]. It is utilized to give a dual optimum for (2.5) in the following.

### 3.1.1 Dual method

As one of the KKT conditions, the complementary slackness condition states that a constraint is met with equality if and only if the dual variable associated with the inequality is strictly greater than zero. In other words, either the constraint is met at the boundary, or the associated dual variable is zero. Hence, the constraints on the minimum rates required by the  $Q$  MA users  $k = K + 1, \dots, K + Q$  are met with equality, see [57, 72, 85, 89]. However, obviously unlike MA users, the rates achieved for the  $K$  RA users may be strictly greater than the minimum required rates at the primal optimum of (2.5).

#### Adapting to KKT conditions

To apply the dual method, each rate  $r_{k,n}$  is linearly divided into

$$r_{k,n} = a_{k,n} + b_{k,n}, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N$$

with  $a_{k,n}, b_{k,n} \in \mathbb{R}_+$ . The value  $b_{k,n}$  renders the associated rate constraint fulfilled with equality, while  $a_{k,n}$  only contributes to the objective in (2.5). Obviously, for MA users  $a_{k,n} = 0$  holds for all  $k = K + 1, \dots, K + Q, n = 1, \dots, N$ . Additionally, we set the weights  $w_k = 0$  for all  $k = K + 1, \dots, K + Q$ . Then, (2.5) is equivalent to

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{K+Q} w_k \sum_{n=1}^N a_{k,n} & (3.1) \\ \text{subject to} \quad & p_{k,n} = \frac{1}{g_{k,n}} (2^{a_{k,n} + b_{k,n}} - 1), \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\ & \sum_{n=1}^N b_{k,n} \geq R_k, \quad k = 1, \dots, K + Q \\ & \sum_{k=1}^{K+Q} \sum_{n=1}^N p_{k,n} \leq P, \\ & \sum_{n=1}^N p_{k,n} p_{l,n} = 0, \quad k, l = 1, \dots, K + Q, k \neq l \end{aligned}$$

where the power-rate function is converted to the rate-power function as the first equality constraint function. The power and rate constraints are considered while applying the KKT conditions. The constraint that subcarriers are not shared among users will be taken into account later.

For notational brevity, we define two rate matrices  $\mathbf{A} = (a_{k,n})_{1 \leq k \leq K+Q, 1 \leq n \leq N}$  and  $\mathbf{B} = (b_{k,n})_{1 \leq k \leq K+Q, 1 \leq n \leq N}$ . By applying the KKT conditions to the equivalent problem (3.1), the Lagrangian is

$$L(\boldsymbol{\lambda}, \beta, \mathbf{A}, \mathbf{B}) = - \sum_{k=1}^{K+Q} \sum_{n=1}^N w_k a_{k,n} + \sum_{k=1}^{K+Q} \lambda_k (R_k - \sum_{n=1}^N b_{k,n}) + \beta (\sum_{k=1}^{K+Q} \sum_{n=1}^N p_{k,n} - P) \quad (3.2)$$

where  $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_{K+Q})' \in \mathbb{R}_+^{K+Q}$  and  $\beta \in \mathbb{R}_+$  are non-negative dual variables. If the dual variable  $\lambda_k$ ,  $k \in \{1, \dots, K+Q\}$ , is positive, the associated rate constraint is met with equality.

### Lagrange dual problem

The Lagrange dual problem is formed as

$$\begin{aligned} & \text{maximize} && L_D(\boldsymbol{\lambda}, \beta) \\ & \text{subject to} && \boldsymbol{\lambda} \succeq \mathbf{0}_{(K+Q) \times 1} \\ & && \beta \geq 0 \end{aligned}$$

where the Lagrange dual function  $L_D$  is the unconstrained infimum of the Lagrangian (3.2) with respect to  $\mathbf{A}$  and  $\mathbf{B}$  as

$$\begin{aligned} L_D(\boldsymbol{\lambda}, \beta) &= \inf_{\mathbf{A} \in \mathbb{R}_+^{(K+Q) \times N}, \mathbf{B} \in \mathbb{R}_+^{(K+Q) \times N}} L(\boldsymbol{\lambda}, \beta, \mathbf{A}, \mathbf{B}) \\ &= \sum_{k=1}^{K+Q} \lambda_k R_k - \beta P + \inf_{\mathbf{A} \in \mathbb{R}_+^{(K+Q) \times N}, \mathbf{B} \in \mathbb{R}_+^{(K+Q) \times N}} \sum_{k=1}^{K+Q} \sum_{n=1}^N (\beta p_{k,n} - w_k a_{k,n} - \lambda_k b_{k,n}). \end{aligned}$$

We employ the substitution

$$f_{k,n} = \beta p_{k,n} - w_k a_{k,n} - \lambda_k b_{k,n}, \quad k = 1, \dots, K+Q, \quad n = 1, \dots, N$$

to simplify the following expression.

Subcarriers are separable given the dual variables, since the power allocated to a subcarrier for one user does not impact the power allocation to other subcarriers. The derivative  $\partial f_{k,n} / \partial p_{k,n}$  of the objective of the infimum in  $L_D$  with respect to the allocated power is only related to  $p_{k,n}$ . It follows that

$$L_D(\boldsymbol{\lambda}, \beta) = \sum_{k=1}^{K+Q} \lambda_k R_k - \beta P + \sum_{n=1}^N \inf_{(a_{k,n}, b_{k,n})_{1 \leq k \leq K+Q} \in \mathbb{R}_+^{2(K+Q)}} \sum_{k=1}^{K+Q} f_{k,n}.$$

Furthermore, the last constraint in (3.1) is considered so that the equation above turns to

$$L_D(\boldsymbol{\lambda}, \beta) = \sum_{k=1}^{K+Q} \lambda_k R_k - \beta P + \sum_{n=1}^N \inf_{k=1, \dots, K+Q} \left( \inf_{(a_{k,n}, b_{k,n}) \in \mathbb{R}_+^2} f_{k,n} \right). \quad (3.3)$$

Only one of the  $K+Q$  rates allocated to the  $n$ th subcarrier for the  $K+Q$  users is positive, while the others are zero. The rate allocated to subcarrier  $n \in \{1, \dots, N\}$  is positive only for the user who achieves the least infimum of  $f_{k,n}$  over  $k = 1, \dots, K+Q$  with the current dual variables.

After taking the rate-power function, the objective function of the inner infimum in (3.3) is rewritten as

$$f_{k,n} = \beta(2^{a_{k,n} + b_{k,n}} - 1) / g_{k,n} - w_k a_{k,n} - \lambda_k b_{k,n}, \quad k = 1, \dots, K+Q, \quad n = 1, \dots, N$$

which is convex in  $a_{k,n}$  and  $b_{k,n}$  given the dual variables  $\lambda_k$  and  $\beta$ . To obtain the extremum of  $f_{k,n}$ , the gradients below with respect to  $a_{k,n}$  and  $b_{k,n}$  must be zero as

$$\begin{aligned}\frac{\partial f_{k,n}}{\partial a_{k,n}} &= 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\ \frac{\partial f_{k,n}}{\partial b_{k,n}} &= 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N.\end{aligned}$$

After calculating the derivatives, we get

$$\begin{aligned}\frac{\beta \ln(2)}{g_{k,n}} 2^{a_{k,n} + b_{k,n}} - w_k &= 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\ \frac{\beta \ln(2)}{g_{k,n}} 2^{a_{k,n} + b_{k,n}} - \lambda_k &= 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N.\end{aligned}$$

When  $\lambda_k = w_k$  holds, the two equations above are identical. While  $\lambda_k \neq w_k$  holds, it is obvious that these two equations cannot hold simultaneously. In other words, either the derivative with respect to  $a_{k,n}$  or the derivative with respect to  $b_{k,n}$  is equal to zero, while the minimizer of the objective function  $f_{k,n}$  must lie on the boundary of  $\mathbb{R}_+^2$ .

### Critical points

For MA users,  $\lambda_k \geq w_k$  must hold for all  $k = K + 1, \dots, K + Q$  due to the weights  $w_k = 0, k = K + 1, \dots, K + Q$ . If the dual variable is greater than or equal to the weight for RA user  $k \in \{1, \dots, K\}$ , as  $\lambda_k \geq w_k$ , the rates and powers allocated for RA user  $k$  are

$$r_{k,n} = a_{k,n} + b_{k,n} = \max\left(\log_2\left(\frac{\lambda_k g_{k,n}}{\beta \ln(2)}\right), 0\right), \quad n = 1, \dots, N \quad (3.4)$$

$$p_{k,n} = \max\left(\frac{\lambda_k}{\beta \ln(2)} - \frac{1}{g_{k,n}}, 0\right), \quad n = 1, \dots, N. \quad (3.5)$$

The minimum of  $f_{k,n}$  is reached on the boundary  $a_{k,n} = 0$  due to convexity of  $f_{k,n}$  in  $a_{k,n}$ . The rate achieved for user  $k$  is  $R_k$ , i.e., the associated rate constraint is satisfied with equality. Such an RA user  $k$  is viewed as an MA user. The corresponding water level is denoted by

$$\mu_k = \nu \lambda_k$$

with  $\nu = (\beta \ln(2))^{-1}$ . Note that  $\mu_1, \dots, \mu_{K+Q}$  refer to the water levels for reaching the minimum required rates  $R_1, \dots, R_{K+Q}$ .

Alternatively, if  $\lambda_k < w_k$  holds,  $f_{k,n}$  reaches the minimum at

$$r_{k,n} = a_{k,n} + b_{k,n} = \max\left(\log_2\left(\frac{w_k g_{k,n}}{\beta \ln(2)}\right), 0\right), \quad n = 1, \dots, N \quad (3.6)$$

$$p_{k,n} = \max\left(\frac{w_k}{\beta \ln(2)} - \frac{1}{g_{k,n}}, 0\right), \quad n = 1, \dots, N \quad (3.7)$$

similarly to the previous case. Since  $f_{k,n}$  is also convex in  $b_{k,n}$ , the minimizer of  $f_{k,n}$  lies on the boundary where  $b_{k,n} = 0$  holds. The associated rate constraint is not of interest to (2.5), since the achieved rate must be strictly greater than the minimum required rate  $R_k$ . We must force the optimal dual variable  $\lambda_k = 0$  to assure that the KKT conditions hold. The associated water level becomes  $\nu w_k$  with  $\nu = (\beta \ln(2))^{-1}$ .

By comparing the dual variables to the corresponding weights, RA users are divided into two groups. Only the minimum required rates are reached for the RA users in one group, while the rates achieved for the RA users in the other group are strictly greater than the minimum required rates. Given the dual variables, the minima of  $f_{k,n}$ ,  $k = 1, \dots, K + Q$ ,  $n = 1, \dots, N$ , are determined. After taking these minima back to the Lagrange dual function (3.3), the subcarrier assignment is explicitly determined.

### 3.1.2 Updating dual variables

The subsection above has explained that the subcarrier assignment for (2.5) can be determined by the given dual variables  $\boldsymbol{\lambda}$  and  $\beta$ . To maximize the dual objective function, the optimal dual variables  $\boldsymbol{\lambda}^*$  and  $\beta^*$  can be found by the ellipsoid method, which is a cutting-plane method explained in [5] and is also used for other resource allocation problems in [57, 72, 85, 89]. First, it localizes the optimal dual variables  $\boldsymbol{\lambda}^*$  and  $\beta^*$  in a closed and bounded region. Then, it iteratively performs evaluation of the gradient or the subgradient of  $L_D(\boldsymbol{\lambda}, \beta)$  and shrinking the region at the center of the updated region. The region shrinks till it converges to  $\boldsymbol{\lambda}^*$  and  $\beta^*$ .

#### Subgradients of the Lagrange dual function

To apply the ellipsoid method, a subgradient of (3.2) is given in the following. The subgradients for updating  $\boldsymbol{\lambda}$  are

$$d_k = \sum_{n=1}^N b_{k,n} - R_k, \quad k = 1, \dots, K + Q \quad (3.8)$$

and the subgradient for updating  $\beta$  is

$$d_{K+Q+1} = P - \sum_{k=1}^K \sum_{n=1}^N p_{k,n} \quad (3.9)$$

where the subgradient vector is denoted by  $\mathbf{d} = (d_1, \dots, d_{K+Q+1})'$  proved as follows.

*Proof.* Assume that  $\mathbf{A}^*$  and  $\mathbf{B}^*$  are the minimizers of the Lagrangian (3.2) as  $L_D(\boldsymbol{\lambda}, \beta) = L(\boldsymbol{\lambda}, \beta, \mathbf{A}^*, \mathbf{B}^*)$ . For all  $\hat{\boldsymbol{\lambda}} \succeq \boldsymbol{\lambda}$ , it follows that

$$\begin{aligned} L_D(\boldsymbol{\lambda}, \beta) &\leq L(\hat{\boldsymbol{\lambda}}, \beta, \mathbf{A}^*, \mathbf{B}^*) \\ &= L(\boldsymbol{\lambda}, \beta, \mathbf{A}^*, \mathbf{B}^*) + (\hat{\boldsymbol{\lambda}} - \boldsymbol{\lambda})(\mathbf{R} - \mathbf{B}^* \mathbf{1}_{N \times 1}) \end{aligned}$$

**Algorithm 1** Ellipsoid method

- 
- 1:  $\mathbf{S}^{(0)} \leftarrow$  the initial ellipsoid
  - 2:  $(\boldsymbol{\lambda}^{(0)'}, \beta^{(0)'}) \leftarrow$  the center of  $\mathbf{S}$
  - 3: **repeat**
  - 4:    $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q} \leftarrow \boldsymbol{\lambda}^{(i)}$  and  $\beta^{(i)}$
  - 5:    $\mathbf{d}^{(i)} \leftarrow (d_1^{(i)}, \dots, d_{K+Q+1}^{(i)})'$  from  $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q}$
  - 6:    $\tilde{\mathbf{d}} \leftarrow \mathbf{d}^{(i)} / \sqrt{\mathbf{d}^{(i)'} \mathbf{S}^{(i)} \mathbf{d}^{(i)}}$
  - 7:    $(\boldsymbol{\lambda}^{(i+1)'}, \beta^{(i+1)'}) \leftarrow \max \left( (\boldsymbol{\lambda}^{(i)'}, \beta^{(i)'}) - \frac{\mathbf{S}^{(i)} \tilde{\mathbf{d}}}{K+Q+2}, (\mathbf{w}' - \epsilon, 0)' \right)$
  - 8:    $\mathbf{S}^{(i+1)} \leftarrow \frac{(K+Q+1)^2}{(K+Q+1)^2 - 1} (\mathbf{S}^{(i)} - \frac{2}{K+Q+2} \mathbf{S}^{(i)} \tilde{\mathbf{d}} \tilde{\mathbf{d}}' \mathbf{S}^{(i)})$
  - 9: **until**  $\sqrt{\mathbf{d}^{(i)'} \mathbf{S}^{(i)} \mathbf{d}^{(i)}} \leq \epsilon$
- 

where  $\mathbf{R} = (R_1, \dots, R_{K+Q})'$ . Similarly, for all  $\hat{\beta} \geq 0$ , it also holds that

$$\begin{aligned} L_D(\boldsymbol{\lambda}, \beta) &\leq L(\boldsymbol{\lambda}, \hat{\beta}, \mathbf{A}^*, \mathbf{B}^*) \\ &= L(\boldsymbol{\lambda}, \beta, \mathbf{A}^*, \mathbf{B}^*) + (\hat{\beta} - \beta) \left( \sum_{k=1}^K \sum_{n=1}^N p_{k,n} - P \right). \end{aligned}$$

Because of concavity of the objective function of (3.1) in the dual variables, the subgradients (3.8) and (3.9) follow.  $\square$

**Ellipsoid method**

Only the direction of gradients affects the sliced ellipsoid but not their lengths. Thus, subgradients may substitute gradients. The result from the previous subsection has shown that the dual variable  $\lambda_k$  affects the power and rate allocation for user  $k$  if and only if  $\lambda_k \geq w_k$  holds. Therefore, when the minimum rate constraint of user  $k$  is satisfied as  $w_k > \lambda_k$ , the associated subgradient is forced to zero and the dual variable  $\lambda_k$  is not updated. The deep-cut ellipsoid method from [5] can be utilized to prevent the dual variables from being out of the bound during updating. Alternatively, we may use the conventional ellipsoid method in Algorithm 1, see line 8, where  $\beta$  and  $\lambda_k$  are bounded by 0 and  $w_k - \epsilon$  with accuracy  $\epsilon$ , respectively.

The KKT conditions may be directly applied to (2.5) without dividing an allocated rate into two portions. The Lagrange dual function is

$$L_D(\boldsymbol{\alpha}, \beta) = \sum_{k=1}^{K+Q} \alpha_k R_k - \beta P + \sum_{n=1}^N \min_{k=1, \dots, K+Q} \left( \min_{r_{k,n} \in \mathbb{R}_+} (\beta p_{k,n} - w_k r_{k,n} - \alpha_k r_{k,n}) \right)$$

where  $\alpha_k$  is the dual variable and is equivalent to the difference  $\lambda_k - w_k$  for all  $k = 1, \dots, K+Q$  in the Lagrange dual function (3.3). The ellipsoid method is still used to find the optimal dual variables. In each iteration, the subcarrier assignment

is determined by the water levels  $\{w_k(\beta \ln(2))^{-1} | k = 1, \dots, K\}$  for RA users and  $\{\alpha_k(\beta \ln(2))^{-1} | k = K + 1, \dots, K + Q\}$  for MA users. If the rates achieved for RA users are greater than or equal to the minimum required rates, the subgradients are determined and the dual variables  $(\alpha_{K+1}, \dots, \alpha_{K+Q})'$  and  $\beta$  are updated. Otherwise, if the minimum required rate  $R_k$  by RA user  $k \in \{1, \dots, K\}$  is not reached, the water level  $\alpha_k(\beta \ln(2))^{-1}$  is used to recalculate the power and rate allocation for user  $k$ . After that the subgradient is determined and the dual variable  $\alpha_k$  is additionally updated. Different from the previous case,  $(\alpha_1, \dots, \alpha_{K+Q})$  must be lower bounded by  $\mathbf{0}_{1 \times (K+Q)}$  in the ellipsoid method.

The duality gap is the difference between the primal optimum and the dual optimum. It is always non-negative. In [89] it has been proved that the duality gap tends to zero as the number of subcarriers increases. This will become evident by the following simulation result. The number of iterations needed by the ellipsoid method behaves as  $\mathcal{O}((K + Q + 1)^2)$ , see [5]. In each iteration, the power and the rate allocated to each subcarrier for each user are calculated by (3.4), (3.5), (3.6) and (3.7) with the present dual variables. Thus, the complexity of determining the dual optimum is  $\mathcal{O}(N(K + Q)(K + Q + 1)^2)$ , which is equal to  $\mathcal{O}(N(K + Q)^3)$  according to the definition of the big Omicron notation [16]. The expected computing time for the dual optimum is linearly increasing in the number of subcarriers, while it increases with the 3rd power of the number of users. In each iteration, the logarithm must be computed  $(K + Q)N$  times. Therefore, the dual optimum is only achievable for systems of a small number of subcarriers and users. Instead of the computationally intractable primal optimum of (2.5), the dual optimum may be utilized as a reference for evaluating heuristic solutions due to its negligible performance loss compared to the primal optimum in the case of a large number of subcarriers.

## 3.2 Primal optimum

In the section above, the primal combinatorial problem has been transformed into a dual problem and a dual optimum has been obtained. In this method, the subcarrier assignment is determined by the Lagrange dual function (3.3) according to the present dual variables. On the contrary to this procedure, water levels, i.e., the dual variables, can be determined when the subcarrier assignment is fixed. This inverse procedure is used to derive the primal optimum of (2.5) in the following. Verified by simulations the duality gap is small.

### 3.2.1 Determining the primal optimum

The set  $\mathcal{S}_k$  denotes the subcarrier assignment for user  $k = 1, \dots, K + Q$ . It contains  $s_k = |\mathcal{S}_k|$  subcarriers assigned to user  $k$ . Because subcarriers are not sharable among users,  $\mathcal{S}_k \cap \mathcal{S}_l = \emptyset$  for any  $k \neq l$  and  $\bigcup_{k=1}^{K+Q} \mathcal{S}_k \subseteq \{1, \dots, N\}$  hold. According to the water-filling solution, if  $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q}$  are fixed, water levels are obtained and then the power and rate allocation is determined by (3.4), (3.5), (3.6) and (3.7). All water levels for different users include the same term  $\nu = (\beta \ln(2))^{-1}$ . Note that  $\mu_k = \nu \lambda_k$

always denotes the water level for reaching the minimum required rate  $R_k$  for all  $k = 1, \dots, K + Q$ , while  $\nu w_k$ ,  $k \in \{1, \dots, K\}$ , is the water level for the case that the achieved rate is strictly greater than the minimum required rate  $R_k$ . The sets  $\mathcal{K}$  and  $\mathcal{Q}$  contain RA users and MA users, respectively. In the following we assume that the subcarrier assignment is fixed. Given the subcarrier assignment, the optimal power and rate allocation for (2.5) is determined by the procedure below.

### Reaching minimum required rates

From the previous section, the rate constraints for MA users must be satisfied with equality due to the complementary slackness condition, while the rates achieved for RA users may be strictly greater than the minimum required rates. First, all users, even RA users, are treated as MA users. Only the minimum required rates are reached for all users. Given the subcarrier assignment  $\mathcal{S}_k$  for user  $k$  that may not be optimal, the power and rate allocation problem is independently solved as a single-user MA resource allocation problem

$$\begin{aligned} & \text{minimize} && \sum_{n \in \mathcal{S}_k} p_{k,n} \\ & \text{subject to} && \sum_{n \in \mathcal{S}_k} r_{k,n} = R_k \end{aligned}$$

for all  $k = 1, \dots, K + Q$ . The power and rate allocated to each subcarrier in the present subcarrier assignment are determined as

$$\begin{aligned} p_{k,n} &= \mu_k - \frac{1}{g_{k,n}}, \quad n \in \mathcal{S}_k \\ r_{k,n} &= \log_2(\mu_k g_{k,n}), \quad n \in \mathcal{S}_k \end{aligned}$$

where the water level  $\mu_k$  can be obtained by the equation below. According to the complementary slackness condition, the equality holds in the rate constraint as

$$\sum_{n \in \mathcal{S}_k} \log_2(\mu_k g_{k,n}) = R_k.$$

The water level  $\mu_k$  is derived as

$$\mu_k = 2^{\frac{R_k}{s_k}} \left( \prod_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}} \right).$$

Obviously, the power and rate allocated to subcarrier  $n \in \mathcal{S}_k$  may not be positive. In this case, subcarrier  $n$  should not be used even though it is assigned to user  $k$ . To identify such subcarriers, the approaches in [7,47] or the following procedure from [51] is used. Given the subcarrier assignment  $\mathcal{S}_k$ , the water level and the power and rate allocation are determined by the equations above. The subcarriers with non-positive power allocated are removed from  $\mathcal{S}_k$  and then the water level is recalculated. Calculating for the water level and removing subcarriers are iteratively performed till all subcarriers in  $\mathcal{S}_k$  are allocated with positive power. The minimum transmission power is then derived as  $P_k$  for reaching the minimum required rate  $R_k$ .

### Allocating remaining power

Given the subcarrier assignment,  $K + Q$  single-user MA resource allocation problems are solved independently so that all rate constraints are satisfied with equality. The water levels are obtained as  $(\mu_1, \dots, \mu_{K+Q})$  to reach the minimum required rates  $R_1, \dots, R_{K+Q}$ . The corresponding sum of transmission power for all users is  $\sum_{k=1}^{K+Q} P_k$ . The problem (2.5) is feasible for the given subcarrier assignment, if the remaining power  $\tilde{P} = P - \sum_{k=1}^{K+Q} P_k$  is non-negative. The remaining power must be allocated only to RA users. After that the rates achieved for some RA users are strictly greater than the minimum required rates.

Note that  $\boldsymbol{\mu} = (\mu_1, \dots, \mu_K)'$  denotes the water levels for RA users while reaching the minimum required rates  $R_1, \dots, R_K$ , but are not related to the weights  $\boldsymbol{w}$  and the limit of transmission power  $P$ . After distributing the remaining power  $\tilde{P}$ , the power allocated to subcarrier  $n$  may increase, if the  $n$ th subcarrier is assigned to RA users. The power increment is the increment of the associated water level due to the linear relation in (3.5) and (3.7) and is expressed by

$$\begin{aligned} \tilde{p}_{k,n} &= \max\left(\nu w_k - \frac{1}{g_{k,n}}\right) - \max\left(\mu_k - \frac{1}{g_{k,n}}, 0\right) \\ &= \max\left(\nu w_k - \underbrace{\max\left(\mu_k, \frac{1}{g_{k,n}}\right)}_{=1/\tilde{g}_{k,n}}, 0\right) \\ &= \max\left(\nu w_k - \frac{1}{\tilde{g}_{k,n}}, 0\right) \end{aligned} \quad (3.10)$$

for  $n \in \mathcal{S}_k$ ,  $k \in \{1, \dots, K\}$ . Here, we define  $1/\tilde{g}_{k,n} = \max(\mu_k, 1/g_{k,n})$ . The inequality  $\mu_k < 1/g_{k,n}$  means that no power is allocated to subcarrier  $n$  for reaching the minimum required rate  $R_k$ . If  $\tilde{p}_{k,n} = \nu w_k - \mu_k$  or  $\tilde{p}_{k,n} = \nu w_k - 1/g_{k,n}$  holds, some of the remaining power is assigned to subcarrier  $n$  with the present water level  $\nu w_k$ . The equality  $\tilde{p}_{k,n} = 0$  holds when one of the following two cases occurs. When  $\nu w_k \leq \mu_k$  and  $\mu_k > 1/g_{k,n}$  hold for RA user  $k$ , only the minimum required rate  $R_k$  is reached. The remaining power will not be assigned to RA user  $k$ . It may also occur that  $\nu w_k \leq 1/g_{k,n}$  and  $\mu_k \leq 1/g_{k,n}$  hold. This means that subcarrier  $n$  cannot be used for RA user  $k$ .

The remaining power must be allocated to different RA users and their subcarriers. This two-dimensional allocation to RA users and subcarriers is equivalent to the weighted sum rate maximization problem [72], stated as

$$\begin{aligned} &\text{maximize} && \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} w_k \log_2(1 + \tilde{p}_{k,n} \tilde{g}_{k,n}) + \sum_{k=1}^K w_k R_k \\ &\text{subject to} && \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \tilde{p}_{k,n} = \tilde{P}. \end{aligned} \quad (3.11)$$

The term  $\sum_{k=1}^K w_k R_k$  is constant, since the minimum required rates have been reached by the previous process where all users are treated as MA users. The power constraint

**Algorithm 2** Two-dimensional water-filling (TDWF)

---

$(P_k, \mu_k) \leftarrow$  MA water-filling over  $\{g_{k,n} \mid n \in \mathcal{S}_k\}$  to achieve  $R_k$ ,  $k = 1, \dots, K + Q$   
 $\tilde{P} \leftarrow P - \sum_{k \in \mathcal{K}} P_k$  the remaining power  
 $\tilde{g}_{k,n} \leftarrow \min(g_{k,n}, 1/\mu_k)$ ,  $n \in \mathcal{S}_k$ ,  $k \in \mathcal{K}$   
**repeat**  
 $\nu \leftarrow (3.12)$  .  
 $\mathcal{Q}, \mathcal{K} \leftarrow$  comparing  $\mu_k$  and  $\nu w_k$ ,  $k = 1, \dots, K$   
 $\tilde{p}_{k,n} \leftarrow \nu w_k - 1/\tilde{g}_{k,n}$ ,  $k \in \mathcal{K}, n \in \bigcup_{k \in \mathcal{K}} \mathcal{S}_k$   
 excluding subcarriers with non-positive power increments from  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_k$   
**until** all power increments are positive

---

must hold with equality again due to the complementary slackness condition. The CNRs for allocating the remaining power are denoted by  $\tilde{g}_{k,n}$ , see (3.10),

$$\tilde{g}_{k,n} = \begin{cases} \min(\mu_k^{-1}, g_{k,n}), & n \in \mathcal{S}_k \\ 0, & n \notin \mathcal{S}_k \end{cases}, \quad k = 1, \dots, K.$$

For an arbitrary subcarrier  $n \in \mathcal{S}_k, k \in \{1, \dots, K\}$ , when  $\nu w_k > \mu_k > 1/g_{k,n}$  holds in (3.10), it follows that

$$\begin{aligned} \log_2(1 + \tilde{p}_{k,n} \tilde{g}_{k,n}) &= \log_2(1 + (\nu w_k - \mu_k) \mu_k^{-1}) \\ &= \log_2(\nu w_k g_{k,n}) - \log_2(\mu_k g_{k,n}), \end{aligned}$$

which is the rate increment to subcarrier  $n$  for user  $k$  after the water level increases from  $\mu_k$  to  $\nu w_k$ . The rate achieved for user  $k$  must be strictly greater than the minimum required rate  $R_k$ . If  $\nu w_k > 1/g_{k,n} > \mu_k$  holds in (3.10), the equation above becomes

$$\begin{aligned} \log_2(1 + \tilde{p}_{k,n} \tilde{g}_{k,n}) &= \log_2\left(1 + (\nu w_k - \frac{1}{g_{k,n}}) g_{k,n}\right) \\ &= \log_2(\nu w_k g_{k,n}). \end{aligned}$$

It is the rate allocated to subcarrier  $n$  for user  $k$  with the new water level  $\nu w_k$ , while no rate is allocated with the original water level  $\mu_k$ .

**Two-dimensional water-filling**

The new maximization problem (3.11) can be solved by a procedure similar to the one solving the single-user RA resource allocation problem in [47] specified as follows, since the power and rate allocation is only subject to the limit of transmission power. The transmission power limit is reached as

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \nu w_k - \frac{1}{\tilde{g}_{k,n}} = \tilde{P},$$

according to which  $\nu$  is derived as

$$\nu = \frac{\tilde{P} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} 1/\tilde{g}_{k,n}}{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} w_k}. \quad (3.12)$$

The set  $\mathcal{K}$  is initialized to  $\mathcal{K} = \{1, \dots, K\}$ . First, by comparing  $\boldsymbol{\mu}$  and  $\nu\boldsymbol{w}$ , we find the RA users treated as MA users, for whom only the minimum required rates are reached. If  $\mu_k \geq \nu w_k, k \in \mathcal{K}$ , holds, RA user  $k$  is excluded from  $\mathcal{K}$ . It is treated as an MA user and its achieved rate is equal to the lower bound  $R_k$ . Otherwise, a larger value is achieved and  $k$  remains in  $\mathcal{K}$ . The water levels for the RA users in  $\mathcal{K}$  are  $\{\nu w_k | k \in \mathcal{K}\}$ . After that we determine which subcarriers cannot carry the remaining power. The power increments to the subcarriers in  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_k$  are obtained. The subcarriers with non-positive power increments are excluded from  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_k$ . After that,  $\nu$  is recalculated. This procedure repeats till the power increments to all subcarriers of the RA users in  $\mathcal{K}$  are positive.

The two-dimensional water-filling (TDWF) is described in Algorithm 2. Since the complexity for calculating the water-filling solutions is increasing linearly in the number of subcarriers [47], TDWF has complexity  $\mathcal{O}(N)$  as the subcarrier assignment is given. To find the optimal subcarrier assignment by exhaustive search,  $(K + Q)^N$  searches must be performed, since each subcarrier may be assigned to one of  $K + Q$  users. Hence, the primal optimum of (2.5) is achieved with complexity  $\mathcal{O}(N(K+Q)^N)$ . It makes determining the primal optimum prohibitive for systems with a large number of subcarriers and users.

### 3.2.2 Comparison between primal and dual optima

Because of the high complexity of determining the primal optimum, it is computationally tractable neither in practice nor in simulations with a large number of users and subcarriers. Thus, it cannot be used as a reference for evaluating heuristic methods. On the other hand, the complexity of obtaining the dual optimum is linearly increasing in  $N$ . Hence, when there are only a few users, like WLAN, or when channels are subject to relatively slow variation in time, the dual optimum is computationally tractable. Furthermore, it provides a reliable benchmark to quantify the performance loss of heuristic methods in simulations.

#### Rate region

For a fixed channel realization, Figure 3.1 plots the achieved sum rate against  $R_1$ , i.e., the minimum rate required by user 1, while there are only two RA users. The minimum required rate  $R_2$  for user 2 is always zero. Let us consider the curves in increasing  $R_1$ . In (3.4), (3.5), (3.6) and (3.7), the powers and rates allocated to subcarriers are subject to  $\nu = (\beta \ln(2))^{-1}$ , which is separately scaled by  $\boldsymbol{\lambda}$  and  $\boldsymbol{w}$ . By comparing the dual variables  $(\lambda_1, \dots, \lambda_K)'$  and the weights  $\boldsymbol{w}$ , RA users are divided into two groups. In one group, the achieved rates are strictly greater than the lower bounds and  $\nu = (\beta \ln(2))^{-1}$  is scaled by the associated weights. The two users are in this group in the example, when  $R_1$  is small. In this region, the weighted sum rate is constant due to the fixed  $\boldsymbol{w}$ , even though the minimum required rate  $R_1$  by user 1 grows.

Different from the first group, only the minimum required rates are reached for the RA users in the other group that are viewed as additional MA users, while the

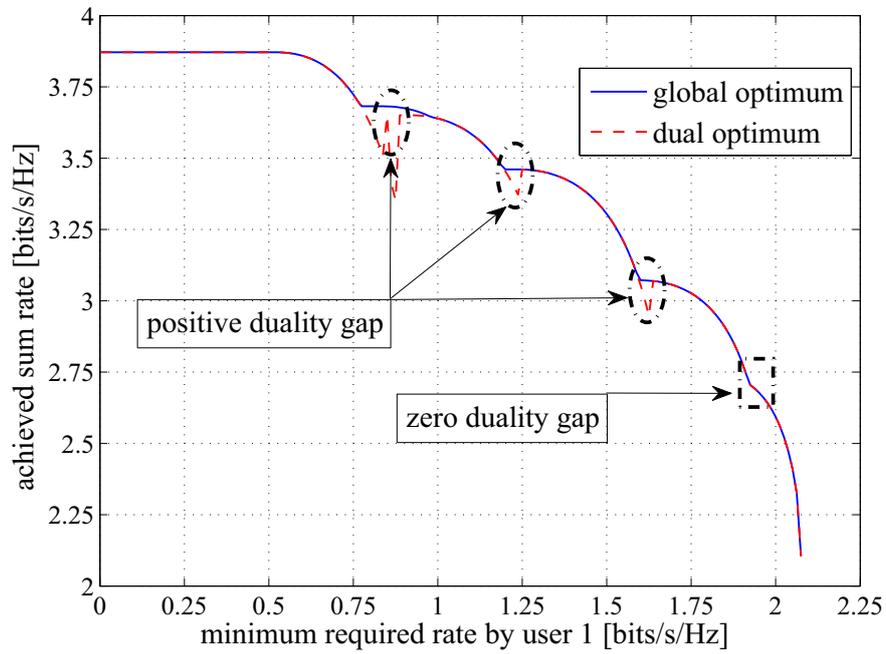


Figure 3.1: Achieved sum rate region with  $K = 2$ ,  $Q = 0$ ,  $N = 8$ ,  $P = 15$  dBW,  $\mathbf{w} = (1, 1)'$  and  $R_2 = 0$ .

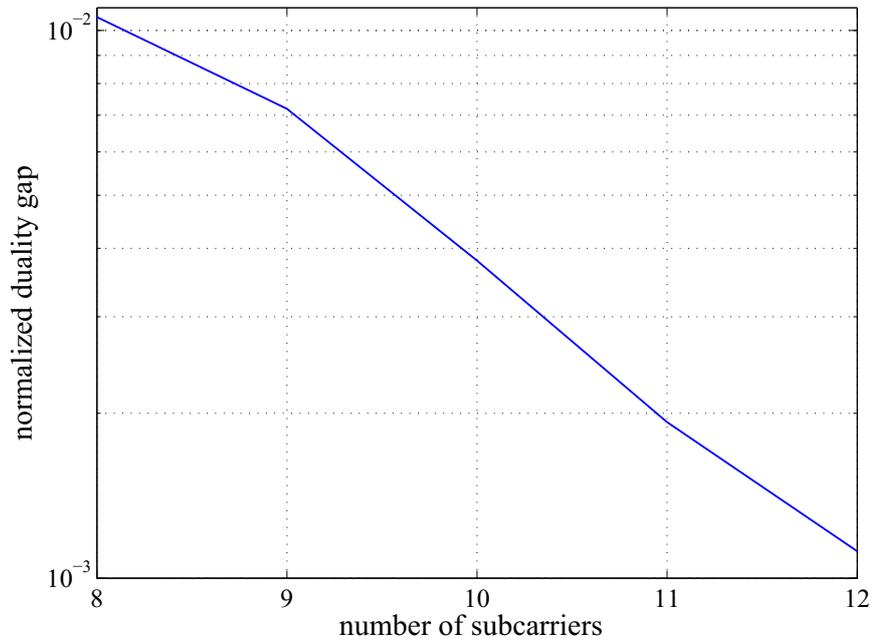


Figure 3.2: Duality gap vs. number of subcarriers with  $K = 2$ ,  $Q = 0$ ,  $P = 15$  dBW,  $\mathbf{w} = (0.5, 0.5)'$ .

associated weights do not have any impact. As  $R_1$  increases, user 1 becomes an MA user. The rate constraint for user 1 is met with equality by scaling  $\nu = (\beta \ln(2))^{-1}$  with  $\lambda_1$ . The minimum required rate  $R_1$  increases till all subcarriers are assigned to user 1. After that the resource allocation problem becomes infeasible. Obviously, this example curve is not convex in  $R_1$ . When the optimal subcarrier assignment does not change as  $R_1$  increases, the achieved sum rate varies continuously in  $R_1$  within this convex sub-domain and the duality gap is zero. Since subcarriers are not sharable, the optimal subcarrier assignment changes as  $R_1$  increases. This may lead to a discrete decrement of the weighted sum rate. The duality gap may be strictly greater than zero marked by the ellipses and the square in Figure 3.1. This implies the weak duality of (2.5) that the objective function may not be continuous at the optimal dual variables, see [89].

### Duality gap

The duality gap becomes negligible as the number of subcarriers  $N$  increases. In this subsection, the duality gap between the dual optimum and the primal optimum is obtained for a small simulation system, which is limited by the high complexity of determining the primal optimum. The frequency selective channels are modeled as consisting of 8 independently Rayleigh fading paths with an exponentially decaying profile. The expected CNR of each subcarrier is set to 5 dB. The total transmission power is limited to 15 dBW. There are  $K = 2$  users in the simulation. The weights are equal to 0.5. The minimum required rates are independently uniformly distributed within [10, 20] bits per OFDM symbol. Figure 3.2 shows that the duality gap is an approximately exponential function of the number of subcarriers. Even though the duality gap becomes larger as the number of users increases, it is still very small since the number of users is generally much less than the number of subcarriers. Hence, the dual optimum can be used as a reference to assess heuristic solutions of (2.5).

## 3.3 Heuristic solutions

The dual optimum and the primal optimum of (2.5) have been given in the previous two sections. The duality gap becomes negligible in the case of a large number of subcarriers. However, although determining the dual optimum is of much smaller complexity than determining the primal optimum, it is still computationally complex for fast time-varying environments, if the number of users is large. To further reduce complexity, a heuristic method is designed in the following.

To determine the primal optimum, the water levels, i.e., the dual variables, are determined as the subcarrier assignment is given, while the optimal subcarrier assignment is obtained by exhaustive search. Heuristics for resource allocation in multiuser OFDM systems aim at substituting exhaustive search so that a good balance is achieved between performance and complexity. In previous works [3, 10, 11, 23, 25, 41, 42, 55, 86, 90], the subcarrier assignment is updated in a heuristic way till some stopping criterion is satisfied.

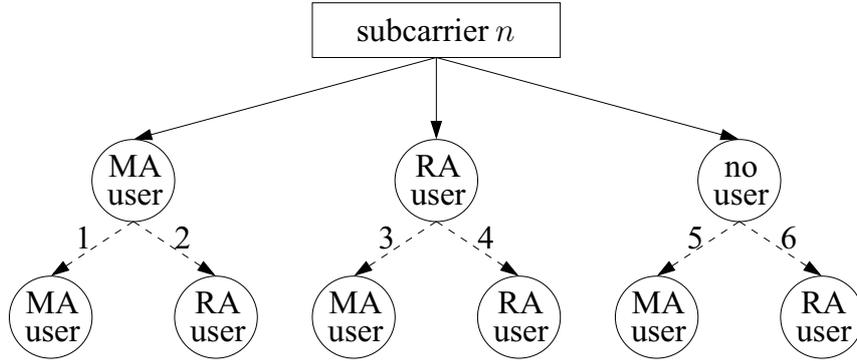


Figure 3.3: Assigning map for subcarrier  $n$ .

While adjusting the subcarrier assignment for (2.5), each subcarrier can be reassigned to some user and then the weighted sum rate for RA users may be improved. For example in Figure 3.3, subcarrier  $n$  is assigned to an MA user or an RA user, or to no user as described by the solid lines. After excluding subcarrier  $n$  from the subcarrier assignment for this MA user (RA user), the transmission power for this MA user (the weighted sum rate achieved for RA users) increases (decreases). By reassigning subcarrier  $n$  to an alternative MA (RA) user as shown by the dashed lines in Figure 3.3, the transmission power for this alternative MA user (the weighted sum rate for RA users) may decrease (increase). These increments and decrements of power and rate can be obtained by TDWF in Algorithm 2. If this reassignment has a positive overall effect on the objective of (2.5), it is actually executed. Otherwise, subcarrier  $n$  is still assigned to the original user.

The subcarrier assignment can be adjusted in the following way. Each subcarrier is reassigned to different users to investigate whether the objective function can be improved. While the reassignment of one subcarrier is investigated, the assignment of others is fixed. This successive procedure repeats till no improvement can be made, e.g., [23, 71]. For the considered heterogeneous resource allocation problem, TDWF with complexity  $\mathcal{O}(N)$  may be used to optimally quantify the variation of the weighted sum rate after changing the subcarrier assignment. Then, this method has complexity  $\mathcal{O}((K + Q)N^2)$ . Since the number of subcarriers in the present or future OFDM system is very large [2, 73], it may cause a large delay. To avoid this problem, we propose the following efficient but non-optimal approaches to replace TDWF.

### 3.3.1 Genetic water-filling

In Figure 3.3, one subcarrier may be excluded from the subcarrier assignment for one user and then reassigned to another user. In general, changes of the subcarrier assignment are classified into two cases. One is excluding a subcarrier from the subcarrier assignment, while the other is including an additional subcarrier in the subcarrier assignment. After such a change, the weighted sum rate achieved for RA users may vary. Under certain conditions, such variations can be efficiently determined. We still

denote by  $\mathcal{K}$  and  $\mathcal{Q}$  the sets of RA and MA users, respectively. It always holds that  $\mathcal{K} \cap \mathcal{Q} = \emptyset$  and  $\mathcal{K} \cup \mathcal{Q} = \{1, \dots, K + Q\}$ . The water levels for reaching the minimum required rates are referred to as  $\mu_1, \dots, \mu_{K+Q}$ .

From TDWF, given the subcarrier assignment  $\mathcal{S}_k$  for user  $k \in \{1, \dots, K + Q\}$ , if positive power is assumed for all subcarriers in  $\mathcal{S}_k$ , the water level for user  $k$  is

$$\mu_k = 2^{\frac{R_k}{s_k}} \left( \prod_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}} \right)^{\frac{1}{s_k}} \quad (3.13)$$

in order to reach the minimum required rate  $R_k$ . The transmission power for user  $k$  is

$$P_k = s_k \mu_k - \sum_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}}. \quad (3.14)$$

Assume that the transmission power for the RA users in  $\mathcal{K}$  is limited to  $P^{(\text{RA})}$  and the subcarrier assignment for RA users is fixed. Without considering the minimum rate requirement, the primal problem (2.5) is relaxed to maximizing the weighted sum rate only subject to the power limit, formulated as

$$\begin{aligned} & \text{maximize} && R = \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} r_{k,n} \\ & \text{subject to} && r_{k,n} = \log_2(1 + p_{k,n} g_{k,n}), \quad k \in \mathcal{K}, n \in \mathcal{S}_k \\ & && \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} p_{k,n} \leq P^{(\text{RA})}. \end{aligned}$$

If all subcarriers in  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_k$  are allocated with positive power, the term  $\nu = (\beta \ln(2))^{-1}$  is derived as

$$\nu = \frac{P^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n}}{\sum_{k \in \mathcal{K}} s_k w_k} \quad (3.15)$$

which explicitly gives the water levels  $\nu w_k, k \in \mathcal{K}$ . Then, the powers and rates allocated to subcarriers are determined. The weighted sum rate is achieved as

$$R = \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu w_k g_{k,n}). \quad (3.16)$$

### Excluding a subcarrier

As explained before, there are two basic changes of the subcarrier assignment, i.e., excluding and including a subcarrier. In the following, the details of these changes are investigated. After excluding subcarrier  $m$  from  $\mathcal{S}_l$  of MA user  $l \in \mathcal{Q}$ , the rate

originally allocated to subcarrier  $m$  must be distributed to the remaining subcarriers in  $\mathcal{S}_l$  to reach the fixed data rate  $R_l$ . Therefore, the water level  $\mu_l$  must increase to

$$\begin{aligned}
\mu_l^{(\text{ex})}(m) &= 2^{\frac{R_l}{s_l-1}} \left( \prod_{n \in \mathcal{S}_l \setminus \{m\}} \frac{1}{g_{l,n}} \right)^{\frac{1}{s_l-1}} \\
&= \left( \underbrace{\left( 2^{\frac{R_l}{s_l}} \left( \prod_{n \in \mathcal{S}_l} \frac{1}{g_{l,n}} \right)^{\frac{1}{s_l}} \right)^{s_l}}_{=\mu_l} g_{l,m} \right)^{\frac{1}{s_l-1}} \\
&= \mu_l (\mu_l g_{l,m})^{\frac{1}{s_l-1}}, \quad l \in \mathcal{Q},
\end{aligned} \tag{3.17}$$

since  $\mu_l g_{l,m} > 1$  and  $1/(s_l - 1) \in (0, 1)$  hold because of (3.4) and (3.5). The set  $\mathcal{S}_l$  denotes the primal subcarrier assignment before excluding subcarrier  $m$ . The upper index (ex) indicates the operation of excluding. The original water level is  $\mu_l$ . Removing the  $m$ th subcarrier from  $\mathcal{S}_l$  may be viewed as assigning less resource to user  $l$ . Intuitively, the transmission power must increase to maintain the required data rate. The induced power increment is

$$\begin{aligned}
\Delta P_l^{(\text{ex})}(m) &= ((s_l - 1)\mu_l^{(\text{ex})}(m) - \sum_{n \in \mathcal{S}_l \setminus \{m\}} \frac{1}{g_{l,n}}) - (s_l \mu_l - \sum_{n \in \mathcal{S}_l} \frac{1}{g_{l,n}}) \\
&= s_l (\mu_l^{(\text{ex})}(m) - \mu_l) - (\mu_l^{(\text{ex})}(m) - 1/g_{l,m}), \quad l \in \mathcal{Q}.
\end{aligned} \tag{3.18}$$

There are two terms on the right-hand side of the equation above. The first term  $s_l (\mu_l^{(\text{ex})}(m) - \mu_l)$  illustrates that the power increments on the  $s_l$  subcarriers are the same as the increment  $\mu_l^{(\text{ex})}(m) - \mu_l$  of the water level. The second term  $\mu_l^{(\text{ex})}(m) - 1/g_{l,m}$  is the power decrement by not allocating any power to the  $m$ th subcarrier for MA user  $l$ .

If subcarrier  $m$  is excluded from the subcarrier assignment  $\mathcal{S}_l$  of RA user  $l \in \mathcal{K}$ , the power allocated to the  $m$ th subcarrier for user  $l$  is distributed to the remaining subcarriers assigned to the RA users in  $\mathcal{K}$  in order to make the power constraint satisfied with equality. Hence, the sharing term  $\nu$  in the water levels increases to

$$\begin{aligned}
\nu^{(\text{ex})}(m) &= \frac{P^{(\text{RA})} + \sum_{k \in \mathcal{K} \setminus \{l\}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n} + \sum_{n \in \mathcal{S}_l \setminus \{m\}} 1/g_{l,n}}{\sum_{k \in \mathcal{K} \setminus \{l\}} s_k w_k + (s_l - 1)w_l} \\
&= \frac{P^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n}}{\sum_{k \in \mathcal{K}} s_k w_k} \sum_{k \in \mathcal{K}} s_k w_k - 1/g_{l,m} \\
&= \frac{= \nu}{\sum_{k \in \mathcal{K}} w_k s_k - w_l} \\
&= \frac{\nu \sum_{k \in \mathcal{K}} w_k s_k - 1/g_{l,m}}{\sum_{k \in \mathcal{K}} w_k s_k - w_l}
\end{aligned} \tag{3.19}$$

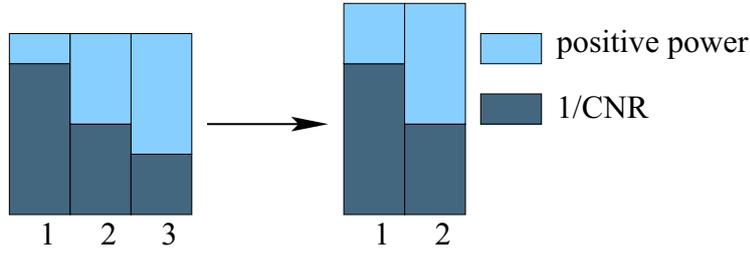


Figure 3.4: Example for excluding a subcarrier from the subcarrier assignment.

where  $\mathcal{S}_l$  including the  $m$ th subcarrier is the subcarrier assignment for user  $l$  before removing subcarrier  $m$  and  $\nu$  is the old sharing term. This resource reduction by excluding subcarrier  $m$  leads to the decrement of the weighted sum rate, given as

$$\begin{aligned}
 \Delta R^{(\text{ex})}(m) &= \sum_{k \in \mathcal{K} \setminus \{l\}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu^{(\text{ex})}(m) w_k g_{k,n}) + w_l \sum_{n \in \mathcal{S}_l \setminus \{m\}} \log_2(\nu^{(\text{ex})}(m) w_l g_{l,n}) \\
 &\quad - \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu w_k g_{k,n}) \\
 &= \log_2\left(\frac{\nu^{(\text{ex})}(m)}{\nu}\right) \sum_{k \in \mathcal{K}} w_k s_k - w_l \log_2(\nu^{(\text{ex})}(m) w_l g_{l,m}) \tag{3.20}
 \end{aligned}$$

where the rate increments on the remaining subcarriers of the RA users in  $\mathcal{K}$  are the same as  $\log_2(\nu^{(\text{ex})}(m)/\nu)$  in the first term  $\log_2(\nu^{(\text{ex})}(m)/\nu) \sum_{k \in \mathcal{K}} w_k s_k$  on the right-hand side. The excluded subcarrier does not carry any bit for the RA users in  $\mathcal{K}$ . It is used to load  $\log_2(\nu w_l g_{l,m})$  bits per OFDM symbol, which is weighted by  $w_l$  and subtracted from the weighted sum rate, shown by the second term  $w_l \log_2(\nu^{(\text{ex})}(m) w_l g_{l,m})$ .

After excluding a subcarrier from the subcarrier assignment for an MA or RA user, the associated water level increases, see for instance Figure 3.4. This ensures that the equations above give the optimal power increment and rate decrement. Compared to the water-filling solution, only one exponential operation or two logarithmic operations are necessary to determine the optimal variation of the weighted sum rate after removing one subcarrier, while others are simple operations, like additions, subtractions and multiplications.

### Including a subcarrier

Alternatively, a subcarrier may be included in the subcarrier assignment, shown by the dashed lines in Figure 3.3. Including subcarrier  $m$  may increase the weighted sum rate, if  $1/g_{l,m}$  is less than the present water level  $\mu_l, l \in \mathcal{Q}$ , or  $\nu w_l, l \in \mathcal{K}$ . Otherwise, it cannot be used for user  $l$ , see (3.5) and (3.7).

After adding the  $m$ th subcarrier to  $\mathcal{S}_l$  of MA user  $l \in \mathcal{Q}$ , the water level for the single-user MA resource allocation problem decreases, since some portions of the rates originally allocated to the subcarriers in  $\mathcal{S}_l$  transfer to the subcarrier added newly in

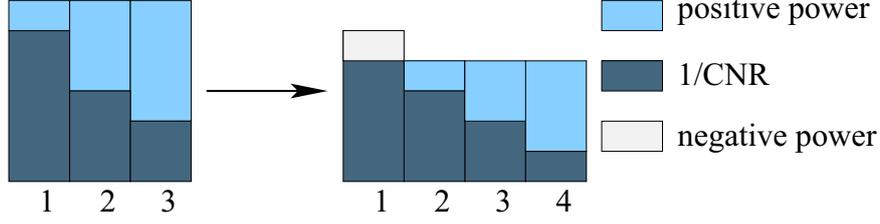


Figure 3.5: Example for including a subcarrier in the subcarrier assignment.

order to keep the achieved rate fixed as  $R_l$ . The inequality  $\mu_l g_{l,m} > 1$  from (3.4) and (3.5) leads to that the water level  $\mu_l$  reduces to

$$\begin{aligned}
 \mu_l^{(\text{in})}(m) &= 2^{\frac{R_k}{s_k+1}} \left( \prod_{n \in \mathcal{S}_k \cup \{m\}} \frac{1}{g_{k,n}} \right)^{\frac{1}{s_k+1}} \\
 &= \left( \underbrace{\left( 2^{\frac{R_k}{s_k}} \left( \prod_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}} \right)^{\frac{1}{s_k}} \right)^{s_k}}_{=\mu_l} \frac{1}{g_{l,m}} \right)^{\frac{1}{s_k+1}} \\
 &= \mu_l (\mu_l g_{l,m})^{-\frac{1}{s_l+1}}
 \end{aligned} \tag{3.21}$$

where  $\mathcal{S}_l$  is the original subcarrier assignment before including subcarrier  $m$  and where (in) indicates the operation of including. Including a subcarrier may be viewed as assigning more resource to this MA user. The transmission power therefore reduces. The induced power decrement is

$$\begin{aligned}
 \Delta P_l^{(\text{in})}(m) &= ((s_l + 1)\mu_l^{(\text{in})}(m) - \sum_{n \in \mathcal{S}_l \cup \{m\}} \frac{1}{g_{l,n}}) - (s_l \mu_l - \sum_{n \in \mathcal{S}_l} \frac{1}{g_{l,n}}) \\
 &= s_l (\mu_l^{(\text{in})}(m) - \mu_l) + (\mu_l^{(\text{in})}(m) - 1/g_{l,m}).
 \end{aligned} \tag{3.22}$$

The powers allocated to the  $s_l$  subcarriers in  $\mathcal{S}_l$  equally reduce by  $(\mu_l^{(\text{in})}(m) - \mu_l)$  on the right-hand side. The power allocated to the  $m$ th subcarrier with the new water level is  $\mu_l^{(\text{in})} - 1/g_{l,m}$ . As explained before, the transmission power should decrease. However, the output may not be optimal, when negative power is allocated to some subcarriers in the previous subcarrier assignment  $\mathcal{S}_l$  due to the decrement  $\mu_l^{(\text{in})}(m) - \mu_l$  of the water level as illustrated in Figure 3.5.

When user  $l$  is an RA user in  $\mathcal{K}$ , the sharing term  $\nu$  in the water levels decreases to

$$\begin{aligned}
\nu^{(\text{in})}(m) &= \frac{P^{(\text{RA})} + \sum_{k \in \mathcal{K} \setminus \{l\}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n} + \sum_{n \in \mathcal{S}_l \cup \{m\}} 1/g_{l,n}}{\sum_{k \in \mathcal{K} \setminus \{l\}} s_k w_k + (s_l + 1)w_l} \\
&= \frac{\frac{P^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n}}{\sum_{k \in \mathcal{K}} s_k w_k} \sum_{k \in \mathcal{K}} s_k w_k + 1/g_{l,m}}{\sum_{k \in \mathcal{K}} w_k s_k + w_l} \\
&= \frac{\nu \sum_{k \in \mathcal{K}} s_k w_k + 1/g_{l,m}}{\sum_{k \in \mathcal{K}} s_k w_k + w_l} \tag{3.23}
\end{aligned}$$

where  $\mathcal{S}_l$  contains the subcarriers originally assigned to RA user  $l$  before including subcarrier  $l$ . Subcarrier  $l$  must be allocated with positive power because only the case of  $\nu w_l > 1/g_{l,m}$  is considered as explained earlier. Since more resource is assigned to the RA users in  $\mathcal{K}$ , the achieved weighted sum rate grows. This rate increment is

$$\begin{aligned}
\Delta R^{(\text{in})}(m) &= \sum_{k \in \mathcal{K} \setminus \{l\}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu^{(\text{ex})}(m) w_k g_{k,n}) + w_l \sum_{n \in \mathcal{S}_l \cup \{m\}} \log_2(\nu^{(\text{ex})}(m) w_l g_{l,n}) \\
&\quad - \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu w_k g_{k,n}) \\
&= \log_2\left(\frac{\nu^{(\text{in})}(m)}{\nu}\right) \sum_{k \in \mathcal{K}} s_k w_k + w_l \log_2(\nu^{(\text{in})}(m) w_l g_{l,m}). \tag{3.24}
\end{aligned}$$

The rate  $\log_2(\nu^{(\text{in})}(m) w_l g_{l,m})$  is allocated to subcarrier  $m$  with the new water level  $\nu^{(\text{in})}(m) w_l$ . It is weighted by  $w_l$ , see the second term  $w_l \log_2(\nu^{(\text{in})}(m) w_l g_{l,m})$  on the right-hand side of the equation above. The rates allocated to the  $s_l$  subcarriers in the previous subcarrier assignment  $\mathcal{S}_l$  decrease by  $\log_2(\nu^{(\text{in})}(m)/\nu)$ . These rate decrements are weighted and then subtracted from the weighted sum rate, shown by the first term  $\log_2(\nu^{(\text{in})}(m)/\nu) \sum_{k \in \mathcal{K}} s_k w_k$ . The derived variation of the weighted sum rate may not be optimal again because some subcarriers in  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_k$  may load negative power with the new water levels. Similar to excluding a subcarrier, only a small number of complex operations, i.e., exponential and logarithmic operations, is necessary after including a subcarrier.

### Variation of transmission power

When the subcarrier assignment for the RA users in  $\mathcal{K}$  is fixed, the weighted sum rate still varies after changing the transmission power  $P^{(\text{RA})}$  for them by  $\Delta P^{(\text{RA})}$ . As the transmission power for the RA users in  $\mathcal{K}$  grows, i.e.,  $\Delta P^{(\text{RA})} > 0$ , the water levels for the RA users in  $\mathcal{K}$  linearly increase due to the linear relation in (3.15). When  $\Delta P^{(\text{RA})}$

is negative, the water levels become lower. After changing the transmission power for the RA users in  $\mathcal{K}$ , the new sharing term is

$$\begin{aligned}
\nu^{(p)}(\Delta P^{(RA)}) &= \frac{P^{(RA)} + \Delta P^{(RA)} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n}}{\sum_{k \in \mathcal{K}} s_k w_k} \\
&= \underbrace{\frac{P^{(RA)} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} 1/g_{k,n}}{\sum_{k \in \mathcal{K}} s_k w_k}}_{=\nu} + \frac{\Delta P^{(RA)}}{\sum_{k \in \mathcal{K}} s_k w_k} \\
&= \nu + \frac{\Delta P^{(RA)}}{\sum_{k \in \mathcal{K}} s_k w_k} \tag{3.25}
\end{aligned}$$

where (p) indicates the operation of changing the transmission power for RA users. The variation of the weighted sum rate is

$$\begin{aligned}
\Delta R^{(p)}(\Delta P^{(RA)}) &= \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu^{(p)}(\Delta P^{(RA)}) w_k g_{k,n}) - \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} \log_2(\nu w_k g_{k,n}) \\
&= \log_2\left(\frac{\nu^{(p)}(\Delta P^{(RA)})}{\nu}\right) \sum_{k \in \mathcal{K}} s_k w_k. \tag{3.26}
\end{aligned}$$

It may not be optimal either because of the decrement of water levels when  $\Delta P^{(RA)}$  is negative. The variation of the rate to each subcarrier in  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_k$  is equal to  $\log_2(\nu^{(p)}(\Delta P^{(RA)})/\nu)$ .

In the equations above, the water levels  $\{\nu w_k | k \in \mathcal{K}\}$ ,  $\{\mu_k | k \in \mathcal{Q}\}$  and the term  $\sum_{k \in \mathcal{K}} s_k w_k$  are always needed. They can be buffered and only updated as they actually change. The only complex operations used in these equations are exponential and logarithmic operations. The number of such complex operations is limited to two for each case.

### Efficient updating approaches

In the following, two examples are given for updating the weighted sum rate by utilizing the equations above for branch 1 and 2 in Figure 3.3. After subcarrier  $m$  is removed from the subcarrier assignment for an MA user, the transmission power for this MA user increases and this increment is derived by (3.17) and (3.18). For branch 1, if subcarrier  $m$  is reassigned to an MA user, the transmission power for this MA user decreases and this decrement is available via (3.21) and (3.22). The variation of the transmission power for all MA users is then acquired. The additional transmission power is reallocated to the RA users in  $\mathcal{K}$  and the increment of the weighted sum rate is obtained via (3.25) and (3.26).

For branch 2, if subcarrier  $m$  is reassigned to an RA user, the resulting power increment for the MA users in  $\mathcal{Q}$  must be taken from the transmission power for the RA users in  $\mathcal{K}$ . First, the weighted sum rate reduces as the transmission power for RA users decreases, see (3.25) and (3.26). The sharing term  $\nu$  then further decreases, while the weighted sum rate grows by including subcarrier  $m$  in the subcarrier assignment

for RA users, illustrated by (3.23) and (3.24). Similar to branch 1 and 2, the variation of the weighted sum rate is derived for branch 3, 4, 5 and 6 by applying the equations above in different orders, listed below.

- For branch 1, (3.17) and (3.18), (3.21) and (3.22), (3.25) and (3.26).
- For branch 2, (3.17) and (3.18), (3.23) and (3.24), (3.25) and (3.26).
- For branch 3, (3.19) and (3.20), (3.21) and (3.22), (3.25) and (3.26).
- For branch 4, (3.19) and (3.20), (3.23) and (3.24).
- For branch 5, (3.21) and (3.22), (3.25) and (3.26).
- For branch 6, (3.23) and (3.24).

Such investigation is performed for all users who presently do not use subcarrier  $m$  but may potentially use it. Finally, if there indeed exists an improvement to the weighted sum rate by reassigning subcarrier  $m$ , then subcarrier  $m$  is actually reassigned to the user associated to the greatest positive variation of the weighted sum rate.

In summary, the power or rate variation induced by reassigning subcarriers can be efficiently but possibly not optimally obtained by inheriting the previous result. We call these efficient updating approaches genetic water-filling (GWF). It can substitute TDWF to reduce the computing time in previous works, e.g., [23, 25, 71, 90]. Since the output of GWF may not be optimal, a good starting point and some controls are required to suppress the negative side effect of GWF. The heuristic method employing GWF is described in the following.

### 3.3.2 Initialization

GWF can efficiently update the weighted sum rate after changing the subcarrier assignment by inheriting the previous solution. The derived result may not be optimal after including a subcarrier in the subcarrier assignment. The output of GWF is probably not optimal, when the water level for one user decreases too much relatively to the CNRs of the subcarriers originally assigned to this user. This large decrement of the water level may be caused by two cases. One is that the number of the subcarriers assigned to this user is very small. After one subcarrier is additionally assigned to this user, the water level decreases significantly. The other is that the subcarrier included newly has a much higher CNR than the subcarriers previously assigned to this user. Negative power may likely appear in these two cases.

A good initialization for subcarrier assignment can mitigate the side effect of GWF. The initialization from [41] first evaluates the number of subcarriers assigned to each user. According to this evaluation, specific subcarriers are then assigned to users. By using this basic idea, the following initialization is devised for our problem and is composed of two steps. To keep complexity low, the primal problem (2.5) is relaxed without the rate constraints for RA users. Compared to the initialization in [41], additional conditions are considered to evaluate the number of subcarriers assigned

to each user here so that a number of subcarriers are assured for each user regarding its transmission demand and channel quality. An iterative process is introduced into the second step to guarantee that all users can get subcarriers with relatively high CNRs. In doing so, the output of GWF is only occasionally not optimal in the later processing.

### Cardinality evaluation

Algorithm 3 returns the approximate number of subcarriers assigned to each user according to average channel conditions, transmission demands and the limit of transmission power. All cardinalities  $s_1, \dots, s_{K+Q}$  are initially set to 1. They will increase via two parts. The computational efficiency of the initialization is more important than its performance, since the following complex processing will effectively improve performance. We therefore assume that all subcarriers have the same CNR for each user  $k$ , which is the arithmetic average CNR over subcarriers given by

$$g_k = \frac{1}{N} \sum_{n=1}^N g_{k,n}, \quad k = 1, \dots, K + Q.$$

Other averages may be employed, like the geometric or harmonic average, while the resulting difference is limited. In part 1, the limit of transmission power is not considered and we aim at solving the problem below

$$\begin{aligned} & \text{minimize} && \sum_{k=1}^{K+Q} P_k \\ & \text{subject to} && s_k \log_2(1 + P_k/s_k g_k) = R_k, \quad k = 1, \dots, K + Q \\ & && \sum_{k=1}^{K+Q} s_k \leq N \end{aligned}$$

where  $P_k$  is the transmission power for user  $k$  and the cardinalities  $s_1, \dots, s_{K+Q}$  must be determined.

In each iteration of part 1, we only increase that cardinality by 1 which causes the greatest power decrement, which is obtained by

$$\Delta P_k = \frac{s_k}{g_k} (2^{\frac{R_k}{s_k}} - 1) - \frac{s_k + 1}{g_k} (2^{\frac{R_k}{s_k+1}} - 1), \quad k = 1, \dots, K + Q \quad (3.27)$$

where the first term is the transmission power assigned to user  $k$  with the present cardinality  $s_k$  as

$$P_k = \frac{s_k}{g_k} (2^{\frac{R_k}{s_k}} - 1).$$

The iteration finishes when the power for reaching all minimum required rates is less than the power limit or the sum of cardinalities is equal to  $N$ .

**Algorithm 3** Cardinality evaluation

---

$s_k \leftarrow 1, \quad k = 1, \dots, K+Q$   
**Part 1:**  
**repeat**  
 $\tilde{k} \leftarrow \operatorname{argmax}_{k=1, \dots, K+Q} \Delta P_k$   
 $s_{\tilde{k}} \leftarrow s_{\tilde{k}} + 1$   
**until**  $\sum_{k=1}^{K+Q} P_k < P$  or  $\sum_{k=1}^{K+Q} s_k = N$   
**Part 2:**  
**if**  $\sum_{k=1}^{K+Q} s_k < N$  **then**  
**repeat**  
 $\tilde{k} \leftarrow \operatorname{argmax}_{k=K+1, \dots, K+Q} \Delta P_k$   
 $R^{(\text{MA})} \leftarrow (3.28)$   
 $\hat{k} \leftarrow \operatorname{argmax}_{l=1, \dots, K} R^{(\text{RA})}(l)$   
**if**  $R^{(\text{MA})} > R^{(\text{RA})}$  **then**  
 $s_{\tilde{k}} \leftarrow s_{\tilde{k}} + 1$   
**else**  
 $s_{\hat{k}} \leftarrow s_{\hat{k}} + 1$   
**end if**  
**until**  $\sum_{k=1}^{K+Q} s_k = N$   
**end if**

---

Those cardinalities proceed increasing in part 2, if their sum from part 1 is less than  $N$ . In part 2, without considering the constraints on the minimum required rates, the primal problem (2.5) is relaxed to

$$\begin{aligned}
 & \text{maximize} && \sum_{k=1}^K w_k s_k r_k \\
 & \text{subject to} && r_k = \log_2 \left( w_k g_k \frac{P - \sum_{i=K+1}^{K+Q} P_i + \sum_{i=1}^K s_i / g_i}{\sum_{i=1}^K s_i w_i} \right), \quad k = 1, \dots, K \\
 & && \sum_{k=1}^{K+Q} s_k = N
 \end{aligned}$$

where  $r_k$  is the rate allocated to each subcarrier with the same CNR  $g_k$  of user  $k$ . There are two possible ways of increasing one cardinality in each iteration of part 2. One is that one of the cardinalities  $s_{K+1}, \dots, s_{K+Q}$  for MA users increases by one so that more transmission power is allocated to RA users. The maximum power decrement is  $\Delta P_{\tilde{k}}$ , obtained by (3.27) as

$$\tilde{k} = \operatorname{argmin}_{k=K+1, \dots, K+Q} \Delta P_k.$$

The resulting weighted sum rate is obtained as

$$R^{(\text{MA})} = \sum_{k=1}^K w_k s_k \log_2 \left( w_k g_k \frac{P - \sum_{i=K+1}^{K+Q} P_i + \Delta P_{\hat{k}} + \sum_{i=1}^K s_i / g_i}{\sum_{i=1}^K s_i w_i} \right) \quad (3.28)$$

from (3.25) and (3.26). In the other way, one of the cardinalities  $s_1, \dots, s_K$  for RA users grows by one. The induced weighted sum rate is derived as

$$\begin{aligned} R^{(\text{RA})}(l) = & \sum_{k=1, k \neq l}^K w_k s_k \log_2 \left( w_k g_k \frac{P - \sum_{i=K+1}^{K+Q} P_i + \sum_{i=1}^K s_i / g_i}{\sum_{i=1}^K s_i w_i} \right) \\ & + w_l (s_l + 1) \log_2 \left( w_l g_l \frac{P - \sum_{i=K+1}^{K+Q} P_i + \sum_{i=1}^K s_i / g_i + 1 / g_l}{\sum_{i=1}^K s_i w_i + w_l} \right) \end{aligned}$$

via (3.23) and (3.24) for all RA users  $l = 1, \dots, K$ . Among them, the greatest weighted sum rate is denoted by  $R^{(\text{RA})}$  after adding one to  $s_{\hat{k}}$  for RA user  $\hat{k}$ . It is compared to the weighted sum rate  $R^{(\text{MA})}$  (3.28). Only one of these two cardinalities  $s_{\hat{k}}$  and  $s_{\hat{k}}$  indeed increases by one.

The original idea from [41] is extended and improved for the heterogeneous resource allocation problem by additionally considering the constraints on minimum required rates and transmission power. The number of subcarriers assigned to each user is evaluated through two serial parts. First, one aims at reducing the total transmission power to the value less than the limit of transmission power. After that the other part increases cardinalities to maximize the weighted sum rate without considering the constraints on minimum required rates. Every cardinality must be evaluated and only one of them increases by one in each iteration. The total number of iterations in part 1 and part 2 must be  $N$  to let the sum of cardinalities be  $N$ . Thus, the complexity of Algorithm 3 is  $\mathcal{O}((K + Q)N)$ .

### Initial subcarrier allocation

According to the evaluated cardinalities returned by Algorithm 3, specific subcarriers are assigned to users by an iterative process in Algorithm 4. In [41], each user  $k$  takes the  $s_k$  available subcarriers with the greatest CNRs sequentially. It may occur that some user has only a few subcarriers to choose due to its bad channel quality, while it has to select subcarriers at last. In other words, the subcarriers that are supposed to be assigned to this user are taken by other users due to this user-sequential procedure. However, it is very difficult to decide which users should choose subcarriers first. To avoid this problem, an iterative procedure in Algorithm 4 is devised, such that every user may have the opportunity of selecting subcarriers with relatively high CNRs.

At first, all subcarriers are included in  $\mathcal{N}$ . The sets  $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q}$  are empty. After Algorithm 4,  $|\mathcal{S}_k| = s_k$  will be made for all  $k = 1, \dots, K + Q$ . Thus,  $|\mathcal{S}_k| \neq s_k$  may hold within the iteration. In each iteration, each user  $k$  only takes the  $\tilde{s}_k$  available subcarriers with the greatest CNRs in  $\mathcal{N}$ , where  $\tilde{s}_k$  is related to the geometric average of the evaluated cardinalities. Here, the geometric average is empirically chosen. The

**Algorithm 4** Initial subcarrier allocation

---


$$\mathcal{N} \leftarrow \{1, \dots, N\}$$

$$\mathcal{S}_k \leftarrow \emptyset, k = 1, \dots, K + Q$$

$$s \leftarrow \left( \prod_{k=1}^{K+Q} s_k \right)^{\frac{1}{K+Q}}$$

$$\tilde{s}_k \leftarrow \lceil s_k / s \rceil, \quad k = 1, \dots, K + Q$$

**repeat**

**for** each  $k = 1, \dots, K + Q$  **do**

**if**  $|\mathcal{S}_k| < s_k$  **then**

$\hat{s}_k \leftarrow \min(s_k - |\mathcal{S}_k|, \tilde{s}_k)$

$\mathcal{M} \leftarrow \{\hat{s}_k \text{ subcarriers with the greatest CNRs, } n \in \mathcal{N}\}$

$\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \mathcal{M}$

$\mathcal{N} \leftarrow \mathcal{N} \setminus \mathcal{M}$

**end if**

**end for**

**until**  $\mathcal{N} = \emptyset$

$$(\mathcal{K}, \mathcal{Q}, \nu, \mu_1, \dots, \mu_{K+Q}, P_1, \dots, P_{K+Q}, \mathcal{S}_1, \dots, \mathcal{S}_{K+Q}) \leftarrow \text{TDWF Algorithm 2}$$


---

larger evaluated cardinalities users have, the more subcarriers they occupy in one iteration. It may happen that the remaining number of subcarriers  $s_k - |\mathcal{S}_k|$  needed by user  $k$  is smaller than  $\tilde{s}_k$ . Then, only  $s_k - |\mathcal{S}_k|$  subcarriers are selected from  $\mathcal{N}$ . The iteration stops when  $\mathcal{N}$  is empty.

Finally, TDWF in Algorithm 2 is executed over the subcarrier assignment from the iteration above to output the parameters for using GWF. Note that it may occur that  $\mathcal{K} \neq \{1, \dots, K\}$  holds. Only the RA users, whose achieved rates are strictly greater than the minimum required rates, are included in  $\mathcal{K}$ . The other RA users, for whom only the minimum required rates are reached, are included in  $\mathcal{Q}$  as well as all MA users. Algorithm 4 iteratively assigns  $N$  subcarriers to  $K + Q$  users according to the evaluated cardinalities. Thus, it has complexity  $\mathcal{O}((K + Q)N)$ . It allows that subcarriers with high CNRs for several users are dispersed to these users so that a good starting point is given for the following adjustment of subcarrier assignment.

### 3.3.3 Subcarrier adjustment

In the previous two subsections, GWF and the initialization for subcarrier assignment have been given. GWF can efficiently update the weighted sum rate after changing the subcarrier assignment, while its output may not be optimal. The initialization for subcarrier assignment allows for that every user has the opportunity of getting subcarriers with relatively high CNRs. In this way, the side effect of GWF is mitigated. In the following, the subcarrier assignment from the initialization is adjusted iteratively and successively with respect to subcarriers, while each subcarrier is reassigned to different users to enhance the rate achievement. Two additional techniques, i.e., sorting

**Algorithm 5** Iterative successive subcarrier adjustment (ISSA)

---

```

for  $i = 1, \dots, I$  do
  for each  $n = 1, \dots, N$  do
    if  $\sum_{k=1}^{K+Q} P_k > P$  then
      minimize  $\sum_{k=1}^{K+Q} P_k \leftarrow$  adjusting  $n$  among users with GWF
    else
      maximize  $\sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} r_{k,n} \leftarrow$  adjusting  $n$  among users with GWF
    end if
  end for
   $(\mathcal{K}, \mathcal{Q}, \nu, \mu_1, \dots, \mu_{K+Q}, P_1, \dots, P_{K+Q}, \mathcal{S}_1, \dots, \mathcal{S}_{K+Q}) \leftarrow$  TDWF Algorithm 2
end for

```

---

subcarriers and controlling iterations, are suggested to improve the performance of that procedure.

**Iterative successive subcarrier adjustment (ISSA)**

Before explaining Algorithm 5, let us clarify that  $\mathcal{Q}$  contains the users whose rate constraints are satisfied with equality and  $\mathcal{K}$  contains the users whose achieved rates are strictly greater than the minimum required rates given the subcarrier assignment. Some primal RA users may be in  $\mathcal{Q}$ . The water levels for reaching the minimum required rates are  $\mu_1, \dots, \mu_{K+Q}$  and the associated transmission powers are  $P_1, \dots, P_{K+Q}$ . As the term  $\nu$  varies, only the water levels  $\{\nu w_k | k \in \mathcal{K}\}$  for the RA users in  $\mathcal{K}$  change accordingly. It may occur that one subcarrier is not used by any user after the initialization, since this subcarrier cannot be used by the user to whom it is assigned after TDWF, shown in Figure 3.3.

In Algorithm 5, subcarrier adjustment is performed along subcarriers successively in the inner loop and iteratively in the outer loop. In the successive procedure of the inner loop, each subcarrier is reassigned to different users to see if the weighted sum rate can be improved. When the reassignment of one subcarrier is investigated, the assignment of others remains fixed. Within the inner loop, the present subcarrier assignment may make (2.5) infeasible, i.e.,  $\sum_{k=1}^{K+Q} P_k > P$ . If this occurs, the objective changes to minimizing the sum power for reaching all minimum required rates. Otherwise, the weighted sum rate is maximized. GWF is utilized to determine to whom one subcarrier should be reassigned. This successive procedure repeats  $I$  times in the outer loop. The remarks below are listed for implementing Algorithm 5.

- Some RA users are contained in  $\mathcal{K}$  and others are included in  $\mathcal{Q}$ . All MA users are included in  $\mathcal{Q}$ . These two sets are only changed by TDWF. They are fixed within the inner loop. All users in  $\mathcal{Q}$  are treated as MA users by GWF, even though some of them are primal RA users in (2.5). Only the RA users in  $\mathcal{K}$  are treated as RA users by GWF.

- If only subcarrier  $n$  is assigned to user  $k$  in the current subcarrier assignment, subcarrier  $n$  should not be reassigned to any other user.
- It may occur that the minimum required rates are not reached for some RA users in  $\mathcal{K}$  after reassigning subcarrier  $n$ . In such a case, adjusting subcarrier  $n$  should be skipped.

The successive adjustment repeats  $I$  times. The case of  $I = \infty$  is equivalent to adjusting subcarriers till no improvement can be made. Finally, a heuristic solution is returned by Algorithm 5 for (2.5). As explained before, the output of GWF may not be optimal. If such an error occurs, it may propagate to adjusting the latter subcarriers. Hence, TDWF is used at the end of each iteration to terminate this propagation. In the processing above, each subcarrier is adjusted among at most  $K + Q$  users. Thus, Algorithm 5 has linear complexity  $\mathcal{O}((K + Q)N)$ . Since the number of iterations  $I$  is not a system parameter, it is not included in the complexity. To combat the error propagation and to let  $I$  be adaptive to channel conditions and transmission demands, iterative successive subcarrier adjustment (ISSA) is modified as follows.

### Sorted subcarrier adjustment

For ISSA, the subcarrier assignment is successively adjusted from the 1st to the  $N$ th subcarrier in the inner loop. The successive adjustment repeats  $I$  times in the outer loop. The structure of ISSA is simple. Thus, ISSA can be easily extended to other resource allocation problems. However, there is a trade-off between its performance and its computing time regarding the number of iterations  $I$ . Intuitively, as the iteration counter  $i$  increases, the achieved weighted sum rate increases, while its computing time grows. In the following, insights into ISSA are revealed to balance its performance and its computing time.

As mentioned earlier, the error propagation may degrade ISSA within the successive procedure, since adjusting one subcarrier highly depends on the earlier subcarrier adjustment. Even if only one subcarrier is assigned to an inappropriate user in the previous subcarrier adjustment, the degradation may significantly increase in the following adjustment. To suppress this error propagation, the earlier subcarrier adjustment must be more effective within the inner loop. Let us consider the extreme case that only one user can access one subcarrier and others have too low CNRs on this subcarrier to utilize it. If this subcarrier is adjusted, improperly reassigning this subcarrier rarely occurs. Thus, such subcarriers should be adjusted earlier. On the other hand, if many users attempt to use one subcarrier, this subcarrier is likely assigned or reassigned to an inappropriate user. The induced impairment of the weighted sum rate may be enlarged in the following adjustment. Hence, such subcarriers should be adjusted later.

We can investigate characteristics of subcarriers to take advantage of the property above. The CNR of one subcarrier varies for different users. This variation can be employed to quantify the characteristic of this subcarrier. The bigger the variation is, the fewer users have relatively high CNR on this subcarrier and the earlier this subcarrier should be adjusted. Furthermore, the characteristics of subcarriers must

be additionally related to transmission requirements. Here, we exploit the variation of potential rates to each subcarrier for different users as defined below. First, the subcarrier index for users in the different sets  $\mathcal{K}$  and  $\mathcal{Q}$  is defined as

$$\begin{aligned}\varphi_{k,n} &= \begin{cases} 1, & \nu w_k \geq 1/g_{k,n} \\ 0, & \nu w_k < 1/g_{k,n} \end{cases}, \quad k \in \mathcal{K}, n = 1, \dots, N \\ \varphi_{k,n} &= \begin{cases} 1, & \mu_k \geq 1/g_{k,n} \\ 0, & \mu_k < 1/g_{k,n} \end{cases}, \quad k \in \mathcal{Q}, n = 1, \dots, N.\end{aligned}$$

Certainly, the inequality  $\sum_{k=1}^{K+Q} \varphi_{k,n} \leq K + Q$  holds. With the present water levels  $\{\nu w_k | k \in \mathcal{K}\}$  and  $\{\mu_k | k \in \mathcal{Q}\}$ , the rates that may be potentially assigned to the  $n$ th subcarrier are  $\{\varphi_{k,n} \log_2(\nu w_k g_{k,n}) | k \in \mathcal{K}\}$  and  $\{\varphi_{k,n} \log_2(\mu_k g_{k,n}) | k \in \mathcal{Q}\}$ . We define the variation of potential rates allocated to subcarrier  $n = 1, \dots, N$  with respect to different users as

$$\sigma_{\bar{r}}[n] = \frac{1}{\sum_{k=1}^{K+Q} \varphi_{k,n}} \left( \sum_{k \in \mathcal{K}} \varphi_{k,n} |\log_2(\nu w_k g_{k,n}) - \bar{r}_n| + \sum_{k \in \mathcal{Q}} \varphi_{k,n} |\log_2(\mu_k g_{k,n}) - \bar{r}_n| \right) \quad (3.29)$$

where  $\bar{r}_n$  is the average of potential rates to the  $n$ th subcarrier, expressed as

$$\bar{r}_n = \frac{1}{\sum_{k=1}^{K+Q} \varphi_{k,n}} \left( \sum_{k \in \mathcal{K}} \varphi_{k,n} \log_2(\nu w_k g_{k,n}) + \sum_{k \in \mathcal{Q}} \varphi_{k,n} \log_2(\mu_k g_{k,n}) \right).$$

The variation (3.29) is used as the metric for sorting subcarriers before each iteration of the successive subcarrier adjustment. Subcarriers are sorted in a descending order of the variations of potential rates before each iteration in Algorithm 5 as

$$\{o_1, \dots, o_N\} \leftarrow \text{a descending order of } \{\sigma_{\bar{r}}[1], \dots, \sigma_{\bar{r}}[N]\}. \quad (3.30)$$

The order is related to the water levels that are updated by GWF. If the order were updated within the inner loop, the order of subcarriers might worsen due to the side effect of GWF and the complexity would significantly increase.

Note that the variations (3.29) of potential rates allocated to subcarriers are not updated within the successive procedure. In other words, the subcarrier order (3.30) is fixed within the inner loop in Algorithm 5. In doing so, the computing time only increases slightly. The complexity of sorting  $N$  subcarriers is  $\mathcal{O}(N \log(N))$ . Subcarriers are adjusted following the descending order. Then, the complexity of sorted ISSA is  $\mathcal{O}((K + Q)N \log(N))$ . Although the complexity becomes higher, the performance is improved, which is verified by simulations.

### Iteration control

The criterion has been designed for sorting subcarriers to suppress the error propagation in Algorithm 5 so that its performance can be improved. In the following, let us focus on the outer loop. For some channel realizations, only a small number of iterations is necessary to achieve a small performance loss. In other cases, a large

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**Algorithm 6** Iterative successive subcarrier adjustment with sorting and iteration control (ISSA-SIC)

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```

1: for  $i = 1, \dots, I$  do
2:    $\{o_1, \dots, o_N\} \leftarrow$  descending order of  $\{\sigma_{\bar{\tau}}[1], \dots, \sigma_{\bar{\tau}}[N]\}$  from (3.30)
3:   for  $j = 1, \dots, N/2$  do
4:     if  $\sum_{k=1}^{K+Q} P_k > P$  then
5:       minimize  $\sum_{k=1}^{K+Q} P_k \leftarrow$  adjusting  $o_j$  among users with GWF
6:     else
7:       maximize  $\sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} r_{k,n} \leftarrow$  adjusting  $o_j$  among users with GWF
8:     end if
9:   end for
10:   $(\hat{R}^{(i)}, \mathcal{K}, \mathcal{Q}, \nu, \mu_1, \dots, \mu_{K+Q}, P_1, \dots, P_{K+Q}, \mathcal{S}_1, \dots, \mathcal{S}_{K+Q}) \leftarrow$  TDWF

11:  for  $j = N/2 + 1, \dots, N$  do
12:    if  $\sum_{k=1}^{K+Q} P_k > P$  then
13:      minimize  $\sum_{k=1}^{K+Q} P_k \leftarrow$  adjusting  $o_j$  among users with GWF
14:    else
15:      maximize  $\sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} r_{k,n} \leftarrow$  adjusting  $o_j$  among users with GWF
16:    end if
17:  end for
18:   $(R^{(i)}, \mathcal{K}, \mathcal{Q}, \nu, \mu_1, \dots, \mu_{K+Q}, P_1, \dots, P_{K+Q}, \mathcal{S}_1, \dots, \mathcal{S}_{K+Q}) \leftarrow$  TDWF
19:  if  $\frac{|\hat{R}^{(i)} - R^{(i)}|}{\hat{R}^{(i)}} \leq \rho$  then
20:    BREAK
21:  end if
22: end for

```

---

number of iterations is required. To let the number of iterations  $I$  be adaptive to channel conditions and the constraints on transmission power and data rates, the simplest stopping criterion for the outer loop in Algorithm 5 is comparison between the outputs of two successive iterations, expressed as

$$\frac{|R^{(i)} - R^{(i-1)}|}{R^{(i-1)}} \leq \rho$$

where  $R^{(i)}$  is the weighted sum rate achieved in the  $i$ th iteration and  $\rho \in (0, 1)$  is the control factor given by the transmission system.

However, one additional iteration has to be executed to determine whether improvement is insignificant or not when the stopping criterion above is adopted. Alternatively, in [74], the iteration control for the turbo decoding is to compare the outputs of the two inner decoders in each iteration. If the two outputs are similar, iterations ter-

minate. By using this idea, the stopping criterion below is devised to avoid the last unnecessary iteration, expressed as

$$\frac{|\hat{R}^{(i)} - R^{(i)}|}{\hat{R}^{(i)}} \leq \rho. \quad (3.31)$$

Subcarriers are equally divided into two groups according to the order of subcarriers (3.30). The first  $N/2$  subcarriers are adjusted at first. After that the weighted sum rate  $\hat{R}^{(i)}$  is achieved in the  $i$ th iteration. Adjustment then proceeds over the latter  $N/2$  subcarriers. The weighted sum rate changes to  $R^{(i)}$ . When the improvement, i.e., the term on the left-hand side of the inequality above, is smaller than the given factor for iteration control  $\rho \in (0, 1)$ , subcarrier adjustment terminates and the present power and rate allocation is returned.

The two techniques, i.e., sorting subcarriers and controlling iterations, are integrated into ISSA. The modified heuristic method is rearranged in Algorithm 6, denoted by ISSA-SIC. Subcarriers are sorted in line 2. They are divided into two groups in line 3 and 11. The iteration control is used in line 19. Obviously, one additional TDWF is added for the iteration control, while the complexity is still  $\mathcal{O}((K + Q)N \log(N))$ . The complexities of determining the primal optimum, the dual optimum and the proposed heuristic solutions are compared in Table 3.1.

Table 3.1: Complexity comparison for single-cell resource allocation

primal optimum	dual optimum	ISSA	ISSA-SIC
$\mathcal{O}(N(K + Q)^N)$	$\mathcal{O}((K + Q)^3 N)$	$\mathcal{O}((K + Q)N)$	$\mathcal{O}((K + Q)N \log(N))$

### 3.4 Simulation results

In this section, the proposed heuristic solution is compared to the dual optimum by simulations. As illustrated before, the dual optimum of (2.5) is qualified to be a reference for assessing heuristic solutions. The simulation system consists of at most  $N = 128$  subcarriers. The transmission power is limited to  $P = 20$  dBW. The number of RA users  $K$  varies from 1 to 6 and  $Q = K$  always holds, which means up to 12 users appear in the simulation system at the same time. Each required rate is independently and uniformly distributed within  $[10, 20]$  bits per OFDM symbol. Each weight is distributed independently and uniformly within  $[1, 10]$ . Their sum is normalized to one. The channel is modeled as consisting of  $N/8$  independently Rayleigh fading paths with an exponentially decaying profile. The expected CNR of each subcarrier is 5 dB. Problem (2.5) often becomes infeasible in the case of  $K > 6$  and  $K = Q$  in our simulation, which means that the transmission power for reaching all minimum required rates is strictly greater than the power limit  $P$ .

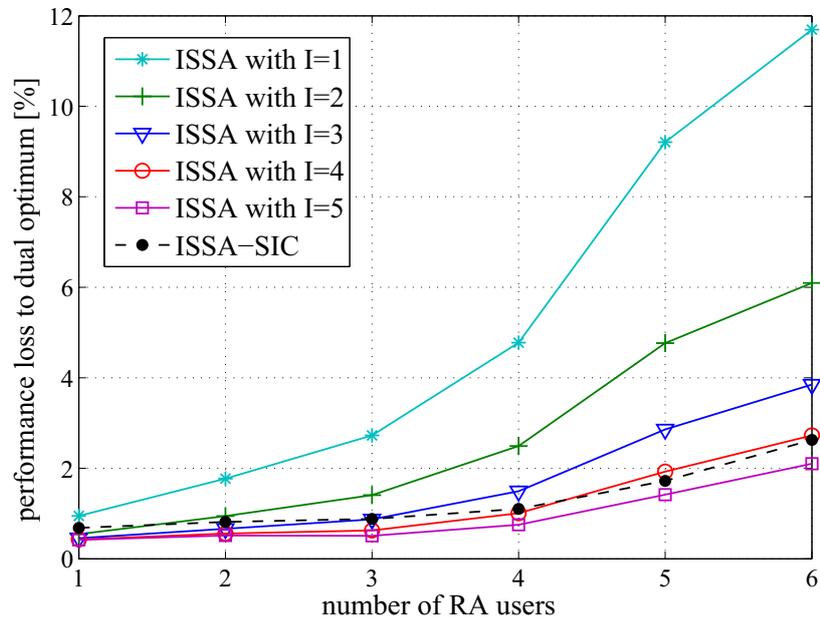


Figure 3.6: Performance loss of heuristic methods in percent compared to the dual optimum vs. number of RA users with  $N = 128$ ,  $K = Q$ ,  $P = 20$  dBW and  $\rho = 0.01$ .

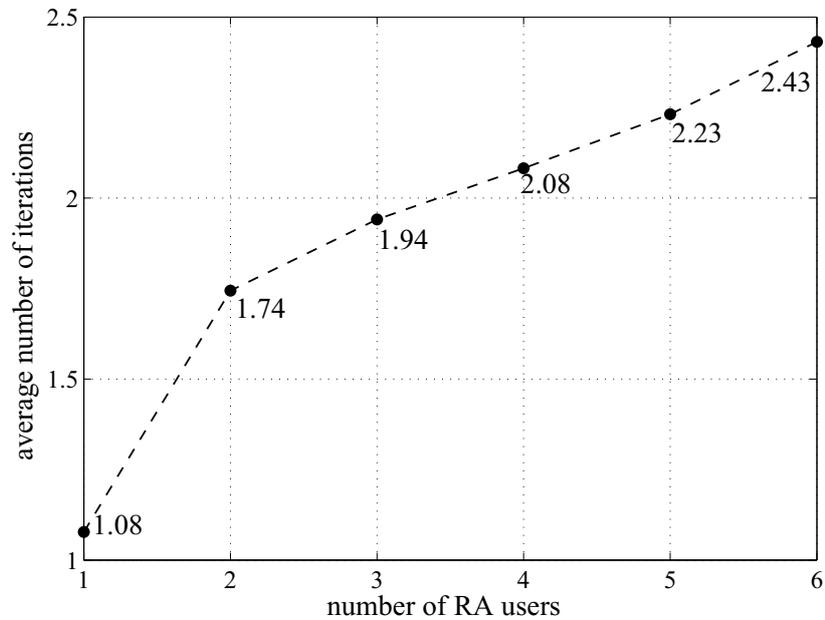


Figure 3.7: Average number of iterations in ISSA-SIC vs. number of RA users with  $N = 128$ ,  $K = Q$ ,  $P = 20$  dBW and  $\rho = 0.01$ .

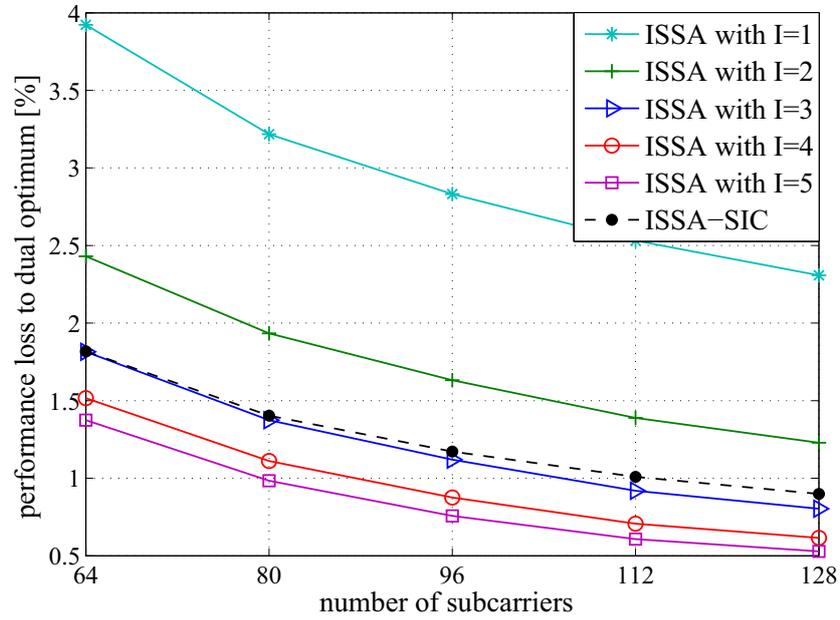


Figure 3.8: Performance loss of heuristic methods in percent compared to the dual optimum vs. number of subcarriers with  $K = Q = 3$ ,  $P = 20$  dBW and  $\rho = 0.01$ .

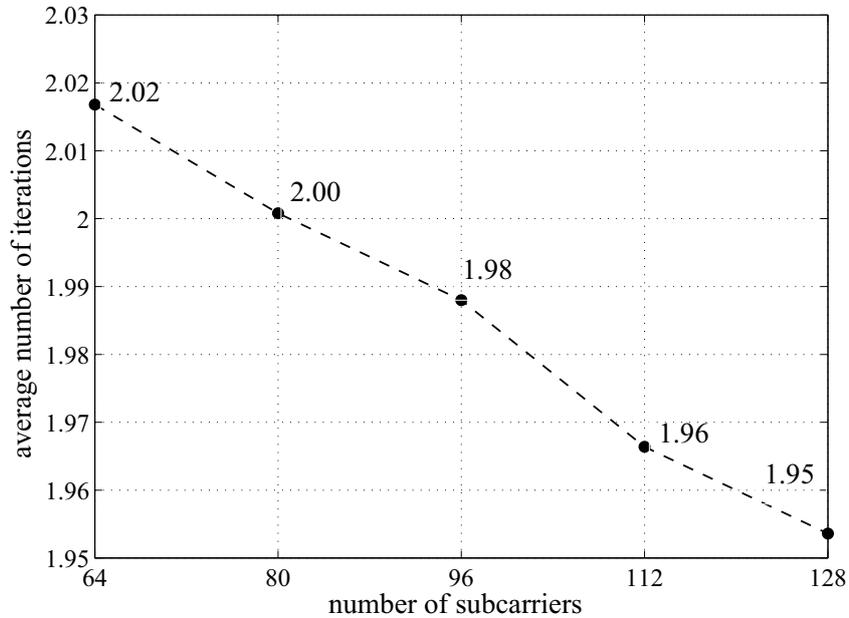


Figure 3.9: Average number of iterations in ISSA-SIC vs. number of subcarriers with  $K = Q = 3$ ,  $P = 20$  dBW and  $\rho = 0.01$ .

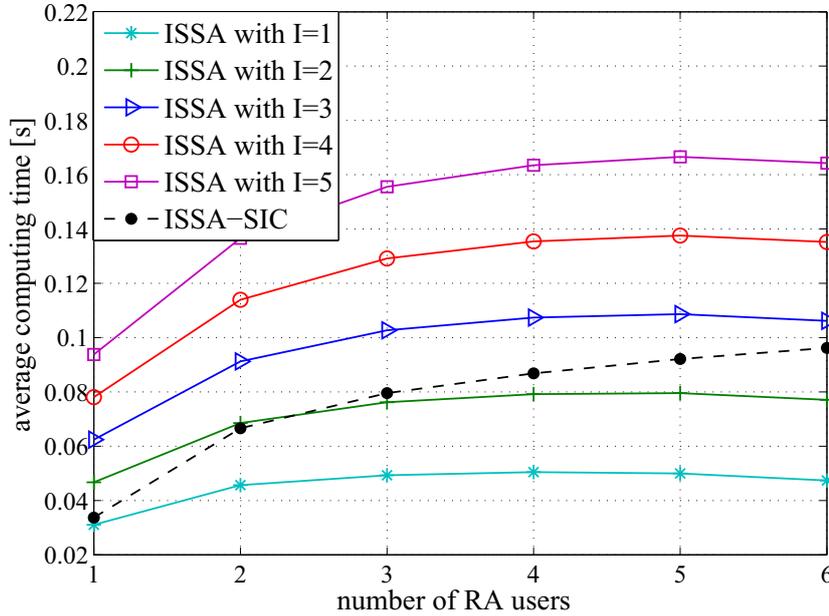


Figure 3.10: Average computing time in seconds vs. number of RA users  $N = 128, K = Q, P = 20$  dBW and  $\rho = 0.01$ .

Figure 3.6 plots the performance loss of the heuristic method compared to the dual optimum against different numbers of users. The performance loss is expressed as

$$\frac{R^{(\text{dual})} - R^{(\text{sub})}}{R^{(\text{dual})}} \times 100\%$$

where  $R^{(\text{dual})}$  denotes the dual optimum of (2.5) and  $R^{(\text{sub})}$  represents the suboptimum given by the proposed heuristic method. As the number of iterations  $I$  grows, the performance loss reduces and the improvement becomes smaller between two successive iterations. When the number of iterations is small, for example  $I = 1$  or  $I = 2$ , the performance loss increases dramatically as the number of users grows. If the number of iterations is large, the performance loss becomes insensitive to the number of users. When  $I > 5$  holds, the performance cannot be improved significantly as  $I$  grows and the associated performance loss is omitted here.

The performance loss of the heuristic method with subcarrier sorting and iteration control integrated is expressed by the dashed line. The associated average number of iterations is given in Figure 3.7, which illustrates that sorting subcarriers makes subcarrier adjustment more effective. For instance, the average number of iterations for  $K = 5$  is 2.23, while the performance is better than ISSA with  $I = 4$ . Computing time is reduced by nearly 50% on average. The average number of iterations increases approximately linearly in the number of users.

In Figure 3.8, the performance of the proposed method is compared to the dual optimum for different numbers of subcarriers. The performance loss reduces as  $N$

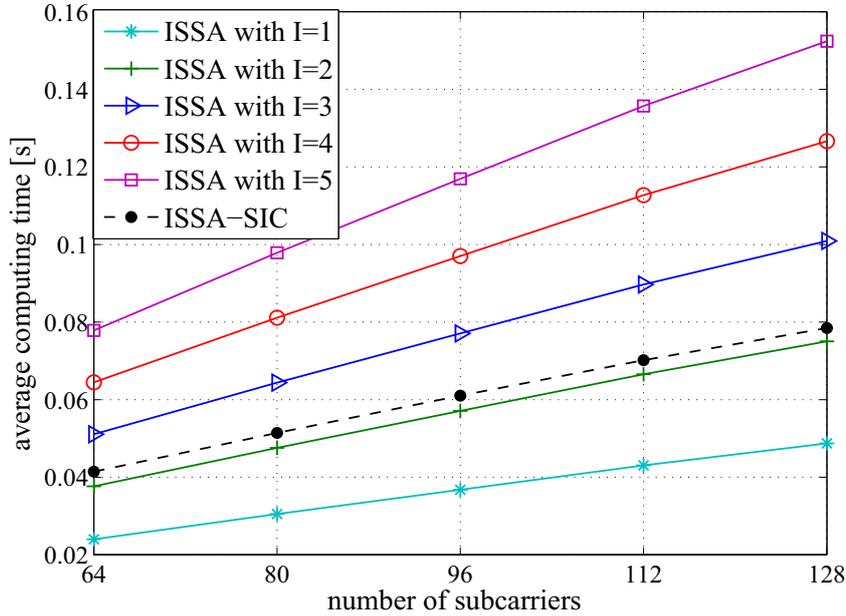


Figure 3.11: Average computing time in seconds vs. number of subcarriers with  $K = Q = 3$ ,  $P = 20$  dBW and  $\rho = 0.01$ .

increases. When the number of iterations  $I$  increases, the performance loss becomes smaller. ISSA-SIC has performance similar to ISSA with  $I = 3$ . Its average number of iterations while employing subcarrier sorting and iteration control is given in Figure 3.9. The average number of iterations decreases slightly as the number of subcarriers increases. Over 30% computing time is saved.

The computing time for the proposed method is recorded in Figure 3.10 and Figure 3.11 for different numbers of users and subcarriers. It is measured by the pair of commands (tic, toc) in MATLAB. This pair measures elapsed time and is recommended by MATLAB. Simulations run on PCs equipped with AMD Athlon(tm) XP 2600+, 2.0 GHz and cache 512 KB. The computing time almost remains constant as the number of users varies, while it increases approximately linearly in the number of subcarriers. The gap of computing time between any two successive iterations is almost constant. The computing time increases slightly after integrating subcarrier sorting and iteration control into ISSA.

### 3.5 Conclusions

In this chapter, resource allocation has been investigated for heterogeneous OFDM unicasting by a single BS. Non-convexity of the considered problem leads to the prohibition for obtaining the primal optimum in the case of a large number of users and subcarriers. Within the framework of convex optimization, duality theory has been

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used to derive a dual optimum, whose performance loss compared to the primal optimum is decreasing approximately exponentially in the number of subcarriers. The performance loss is negligible when the number of subcarriers is large. Thus, the dual optimum can be used as a reference for evaluating heuristic methods. After that a heuristic solution has been proposed. Its generality has been maintained by employing a simple procedure, where the subcarrier assignment is adjusted iteratively and successively along subcarriers. Besides, the criteria for sorting subcarriers and controlling iterations have been developed to make this subcarrier adjustment more effective in order to improve performance and reduce computing time. The derived heuristic solution is near to the dual optimum as demonstrated by simulations. By using subcarrier sorting and iteration control, the average computing time is still increasing approximately linearly in the number of users and subcarriers, while the balance between performance and computing time is significantly improved.



## 4 Equal Rate per User Resource Allocation

In Chapter 3, the primal optimum, the dual optimum and the heuristic solution have been given for the heterogeneous resource allocation problem (2.5), where the water-filling strategy is employed. In a water-filling solution, multiple subcarriers are allocated with different powers and rates according to their channel qualities. However, the water-filling strategy may degrade system performance in fast time-varying environments. The faster a channel varies, the more frequently the resource allocation scheme changes. The time variation of channels depends on the velocity of receivers relative to the transmitter. Additionally, it is sensitive to the scatterers around the transmitter and receivers, which may move fast during transmission. For multiuser resource allocation, even if the channel changes for only one user, the resource allocation scheme has to be updated. Hence, the resource allocation scheme must be updated frequently in wireless communication systems.

In many existing transmission systems, channels are measured at receivers, while resource allocation is performed at the transmitter, see Figure 2.4 and Figure 2.5. To make the employed resource allocation scheme available at receivers, one approach is to forward all CSI to receivers after the transmitter collects CSI from all receivers. All receivers then perform the same resource allocation method. The induced energy consumption however shortens the life time of the portable terminals due to the capacity limit of batteries. Alternatively, the resource allocation scheme is sent via the signalling overhead. Since different modulation schemes may be assigned to subcarriers, the signalling overhead is significantly large, see [28]. The resulting delay further worsens performance.

To solve the problem above, in [88, 95] the same power is assigned to subcarriers. Employed modulation schemes must be identified for different subcarriers at receivers. Thus, the signalling overhead still remains large. To reduce the signalling overhead, [61] suggests to cluster subcarriers into blocks. If many subcarriers are in one block and CSI variation with respect to subcarriers in one block is large, performance deteriorates severely. Otherwise, the signalling overhead is not significantly reduced. The same power and rate are statically allocated to a fixed number of subcarriers, which have greater CNRs in [19]. This fixed number is known to the receiver. This implies that resource allocation does not adapt to channel conditions. In [68], the same signal-to-noise ratios (SNRs) are obtained over all subcarriers. However, the resulting performance loss may be very large when the frequency selectivity of channels is strong [14].

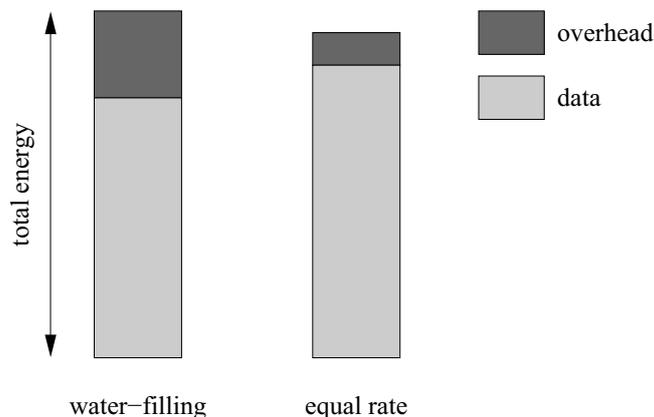


Figure 4.1: New balance of energy consumption on data and signalling.

Different from previous works, in this chapter, we propose that a dynamic and equal rate is allocated to subcarriers assigned to one user so that the same modulation scheme is employed over these subcarriers, while the rates allocated to subcarriers of different users differ. In doing so, the signalling overhead is significantly reduced such that the energy consumption for signalling and data transmission is balanced and the overall energy efficiency is improved for heterogeneous unicasting. This idea is depicted in Figure 4.1. Furthermore, the power and rate allocation can be efficiently updated after changing the subcarrier assignment while applying the proposed equal rate resource allocation strategy. Consequently, an easily implementable heuristic method is designed. It has a small performance loss compared to the dual optimum of (2.5) presented in Chapter 3. Moreover, in present applications, a constant rate is allocated to subcarriers, e.g., WLAN. Thus, the proposed strategy can be applied in the current protocols with simple modifications.

The remainder of this chapter first reformulates the heterogeneous resource allocation problem while additionally considering the signalling overhead. The equal rate resource allocation is then investigated for single-user OFDM systems. We give the asymptotic limit for instantaneous per-symbol performance loss by the proposed strategy. After that a heuristic method is designed for heterogeneous resource allocation by taking advantage of the simplicity of the proposed strategy. Finally, the suggested method is compared to the dual optimum from Chapter 3.

## 4.1 Problem reformulation

The faster channels vary in time, the smaller is the amount of data that each resource allocation scheme is effective for. Thus, the water-filling strategy becomes less energy efficient when the employed resource allocation scheme is frequently renewed via the signalling overhead. In this section, the heterogeneous resource allocation problem (2.5) is reformulated with the load of the signalling overhead included.

### 4.1.1 Water-filling with signalling overhead

We still consider heterogeneous unicasting by a single BS. The transmitter executes resource allocation, while receivers are notified of the employed resource allocation scheme via signalling overhead. We assume that the resource allocation scheme is updated for each frame consisting of  $L$  OFDM symbols due to time-varying channels. In other words, each resource allocation scheme is effective for  $L$  OFDM symbols. Each modulation scheme is identified by  $M$  bits. According to the investigation in [28], if the water-filling strategy is used, in which different powers and rates are allocated to subcarriers, the amount of bits required to express one resource allocation scheme is

$$\underbrace{\lceil \log_2(K) \rceil N}_{\text{user identification}} + \underbrace{NM}_{\text{rate identification}}.$$

The first  $\lceil \log_2(K) \rceil$  bits are used to identify the user assigned with each subcarrier and are named user identification. The BS must do the best effort broadcasting for the first  $\lceil \log_2(K) \rceil N$  bits to let receivers know which subcarriers are assigned to them. The associated energy consumption is fixed given the number of subcarriers and users. After receiving the user identification, each receiver knows which subcarriers are assigned to it. The other  $MN$  bits can be sent separately over subcarriers so that each receiver knows which modulation schemes are employed over its subcarriers. This part of the signalling overhead is called rate identification and may be treated as data bits. After that data symbols follow and receivers perform data detection according to resource allocation schemes.

Based on the setting above, the energy efficiency for heterogeneous unicasting is related to the energy for transmitting the rate identification and the energy for transmitting data symbols, while the energy for the user identification is fixed. The amount of data bits in  $L$  OFDM symbols over subcarrier  $n$  is  $Lr_{k,n} - M$ , in which  $r_{k,n}$  is the rate allocated to subcarrier  $n$  for user  $k$ . Then, the heterogeneous resource allocation problem is reformulated to

$$\begin{aligned} \text{maximize} \quad & R = \sum_{k=1}^K w_k \sum_{n=1}^N r_{k,n} & (4.1) \\ \text{subject to} \quad & r_{k,n} = \log_2(1 + p_{k,n} g_{k,n}) + \frac{M}{L}, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\ & r_{k,n} \geq 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\ & \sum_{n=1}^N r_{k,n} \geq R_k, \quad k = 1, \dots, K + Q \\ & \sum_{k=1}^{K+Q} \sum_{n=1}^N p_{k,n} \leq P \\ & \sum_{n=1}^N r_{k,n} r_{l,n} = 0, \quad k, l = 1, \dots, K + Q, \quad k \neq l \end{aligned}$$

where only the first equality constraint function, i.e., the power-rate function, is changed compared to the primal heterogeneous resource allocation problem (2.5).

Once a subcarrier is allocated with positive power,  $M$  bits must be transmitted for rate identification. Note that the partial signalling overhead for rate identification cannot be treated as data bits in practice. They must be transmitted with the modulation schemes determined preliminarily. Otherwise, the receiver cannot detect the rate identification. In this way, the energy efficiency becomes even worse. For the convenience of theoretical investigation, the bits expressing the rate identification are simply treated as data bits.

The dual optimum of (4.1) can be obtained in a similar way to the previous chapter due to the fixed shift  $M/L$ . Given the dual variables with  $\lambda_k \geq w_k$ , the rate (3.4) and the power (3.5) allocated to subcarriers change to

$$\begin{aligned} r_{k,n} &= \max\left(\log_2\left(\frac{\lambda_k g_{k,n}}{\beta \ln(2)}\right) - \frac{M}{L}, 0\right), \quad n = 1, \dots, N \\ p_{k,n} &= \max\left(\frac{\lambda_k}{\beta \ln(2)} - \frac{1}{g_{k,n}}, 0\right), \quad n = 1, \dots, N \end{aligned}$$

where  $\lambda_k$  is the dual variable associated to the rate  $R_k$  required by user  $k$  and  $\beta$  is the dual variable associated to the limit of transmission power  $P$  at the BS. The allocated rate (3.6) and power (3.7) with  $\lambda_k < w_k$  become

$$\begin{aligned} r_{k,n} &= \max\left(\log_2\left(\frac{w_k g_{k,n}}{\beta \ln(2)}\right) - \frac{M}{L}, 0\right), \quad n = 1, \dots, N \\ p_{k,n} &= \max\left(\frac{w_k}{\beta \ln(2)} - \frac{1}{g_{k,n}}, 0\right), \quad n = 1, \dots, N. \end{aligned}$$

By taking the derived power and rate allocation back to the Lagrange dual function (3.3), the subcarrier assignment is determined provided the dual variables. The ellipsoid method can be used to find the optimal dual variables. The complexity of determining the dual optimum of (4.1) is still  $\mathcal{O}(N(K+Q)^3)$ .

### 4.1.2 Equal rate resource allocation

To reduce the signalling overhead, an adaptive and equal rate, i.e., the same modulation scheme, is allocated to subcarriers assigned to one user. The rates to subcarriers belonging to different users may not be the same. The user identification does not change, while the rate identification becomes much smaller. The amount of bits for expressing one equal rate resource allocation scheme is

$$\underbrace{\lceil \log_2(K) \rceil N}_{\text{user identification}} + \underbrace{KM}_{\text{rate identification}}.$$

Based on this setting, the resource allocation problem is reformulated for heterogeneous unicasting as

$$\begin{aligned}
& \text{maximize} && \bar{R} = \sum_{k=1}^K w_k s_k r_k && (4.2) \\
& \text{subject to} && r_k = \log_2(1 + p_{k,n} g_{k,n}), \quad k = 1, \dots, K + Q, \quad n \in \mathcal{S}_k \\
& && r_{k,n} \geq 0, \quad k = 1, \dots, K + Q, \quad n = 1, \dots, N \\
& && s_k r_k \geq R_k + \frac{M}{L}, \quad k = 1, \dots, K + Q \\
& && \sum_{k=1}^{K+Q} \sum_{n \in \mathcal{S}_k} p_{k,n} \leq P \\
& && \mathcal{S}_k \cap \mathcal{S}_l = \emptyset, \quad k, l = 1, \dots, K + Q, k \neq l.
\end{aligned}$$

The set  $\mathcal{S}_k$  still denotes the subcarrier assignment for user  $k$  for all  $k = 1, \dots, K+Q$ . Its cardinality is  $s_k$ . Different from the water-filling strategy, the same rate  $r_k$  is allocated to all subcarriers in the set  $\mathcal{S}_k$ . Only  $M$  bits are necessary for rate identification for each user. As  $K$  grows to  $N$ , the size of this signalling overhead approaches the previous one, while (4.2) tends to the problem (4.1). In other words (4.2) is equivalent to (4.1) when  $K = N$  holds.

## 4.2 Single-user equal rate resource allocation

Before solving the multiuser resource allocation problem (4.2), let us first study the single-user equal rate resource allocation. The following single-user MA and RA problems are extracted from (4.2), while the signalling overhead is not considered in order to quantify the instantaneous per-symbol performance loss of the proposed strategy compared to the water-filling solution.

### 4.2.1 Single-user MA resource allocation

Due to the complementary slackness condition, the single-user equal rate MA resource allocation problem for user  $k$  is stated as

$$\begin{aligned}
& \text{minimize} && P_k^{(\text{ER})} = \sum_{n \in \mathcal{S}_k} p_{k,n} && (4.3) \\
& \text{subject to} && r_k = \log_2(1 + p_{k,n} g_{k,n}), \quad n \in \mathcal{S}_k \\
& && s_k r_k = R_k
\end{aligned}$$

for all  $k = 1, \dots, K + Q$ . The transmission power for user  $k$  is minimized subject to the fixed required rate  $R_k$ . If the subcarrier assignment  $\mathcal{S}_k$  is fixed for user  $k$ , the solution of (4.3) is simple as

$$r_k = \frac{R_k}{s_k}, \quad n \in \mathcal{S}_k,$$

while the power allocation is explicitly given by the power-rate function in (4.3) as

$$p_{k,n} = \frac{1}{g_{k,n}}(2^{r_k} - 1), \quad n \in \mathcal{S}_k.$$

The transmission power for user  $k$  given the subcarrier assignment  $\mathcal{S}_k$  is

$$P_k^{(\text{ER})} = \frac{S_k}{H_k}(2^{r_k} - 1) \quad (4.4)$$

where  $H_k$  is the harmonic average of the CNRs  $\{g_{k,n} \mid n \in \mathcal{S}_k\}$ , defined as

$$\frac{1}{H_k} = \frac{1}{s_k} \sum_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}}.$$

The equal rate resource allocation above may be interpreted as a water-filling solution over  $s_k$  subcarriers that have the same CNR  $H_k$ . When the subcarrier assignment varies, the harmonic average  $H_k$  changes accordingly and the power and rate allocation can be efficiently updated.

From (3.14), the transmission power for reaching the fixed required rate while applying the water-filling strategy is

$$\begin{aligned} P_k &= \sum_{n \in \mathcal{S}_k} \left( \frac{2^{R_k/s_k}}{G_k} - \frac{1}{g_{k,n}} \right) \\ &= s_k \left( \frac{2^{R_k/s_k}}{G_k} - \frac{1}{H_k} \right) \end{aligned}$$

where  $G_k$  is the geometric average of the CNRs  $\{g_{k,n} \mid n \in \mathcal{S}_k\}$  as

$$G_k = \left( \prod_{n \in \mathcal{S}_k} g_{k,n} \right)^{\frac{1}{s_k}}.$$

By comparing  $P_k^{(\text{ER})}$  to  $P_k$ , we obtain the instantaneous per-symbol performance loss of the proposed strategy compared to the water-filling solution for user  $k$ , expressed as

$$\begin{aligned} \frac{P_k^{(\text{ER})} - P_k}{P_k} &= \frac{\frac{s_k}{H_k}(2^{R_k/s_k} - 1) - s_k \frac{2^{R_k/s_k}}{G_k} + \frac{s_k}{H_k}}{s_k \frac{2^{R_k/s_k}}{G_k} - \frac{s_k}{H_k}} \\ &= \frac{G_k - H_k}{H_k - G_k 2^{-\frac{R_k}{s_k}}} \end{aligned} \quad (4.5)$$

where  $H_k \leq G_k$  always holds, see [8]. The equality  $H_k = G_k$  holds and there is no performance loss, when transmission experiences flat fading. When  $R_k$  goes to infinity, the performance loss (4.5) is

$$\lim_{R_k/s_k \rightarrow \infty} \frac{G_k - H_k}{H_k - G_k 2^{-\frac{R_k}{s_k}}} = \frac{G_k}{H_k} - 1. \quad (4.6)$$

The equation above implies that the performance loss is limited, when at least one of the required rate and the number of users is large. This may be satisfied in large scale systems, where many users are served and each of them demands a large rate.

### 4.2.2 Single-user RA resource allocation

The other problem extracted from (4.2) is the equal rate RA resource allocation problem, while the signalling overhead is still not considered here. It is formulated as

$$\begin{aligned} & \text{maximize} && R_k^{(\text{ER})} = s_k r_k && (4.7) \\ & \text{subject to} && r_k = \log_2(1 + p_{k,n} g_{k,n}), \quad n \in \mathcal{S}_k \\ & && \sum_{n \in \mathcal{S}_k} p_{k,n} = P_k \end{aligned}$$

for  $k = 1, \dots, K$ . The transmission power for user  $k$  is limited to  $P_k$ . Given the subcarrier assignment  $\mathcal{S}_k$  for user  $k$ , the solution of (4.7) is obtained from the equation

$$\sum_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}} (2^{r_k} - 1) = P_k.$$

Then, the equal rate is derived as

$$r_k = \log_2 \left( 1 + \frac{P_k H_k}{s_k} \right). \quad (4.8)$$

The water level with the proposed strategy applied is

$$\mu_k^{(\text{ER})} = \frac{P_k}{s_k} + \frac{1}{H_k}$$

which is the same as the water level while applying the water-filling strategy. The achieved rate for user  $k$  is

$$\sum_{n \in \mathcal{S}_k} \log_2(\mu_k^{(\text{ER})} g_{k,n}) = s_k \log_2(\mu_k^{(\text{ER})} G_k).$$

As before, the equal rate RA problem may also be interpreted as a water-filling solution over  $s_k$  subcarriers that have the same CNR  $H_k$ . The instantaneous per-symbol performance loss of the proposed strategy compared to the water-filling strategy is given as

$$\begin{aligned} \frac{s_k \log_2(\mu_k^{(\text{ER})} G_k) - s_k \log_2(\mu_k^{(\text{ER})} H_k)}{s_k \log_2(\mu_k^{(\text{ER})} G_k)} &= \frac{\log_2(\mu_k^{(\text{ER})} G_k) - \log_2(\mu_k^{(\text{ER})} H_k)}{\log_2(\mu_k^{(\text{ER})} G_k)} \\ &= \frac{\log_2(G_k/H_k)}{\log_2(G_k P_k/s_k + G_k/H_k)}. \end{aligned} \quad (4.9)$$

The performance loss (4.9) tends to

$$\lim_{P_k/s_k \rightarrow \infty} \frac{\log_2(G_k/H_k)}{\log_2(G_k P_k/s_k + G_k/H_k)} = 0. \quad (4.10)$$

Similar to the previous asymptotic limit of performance loss for the single-user MA problem, the performance loss for the single-user RA problem is limited when a small number of subcarriers is assigned to user  $k$  or the transmission power is large.

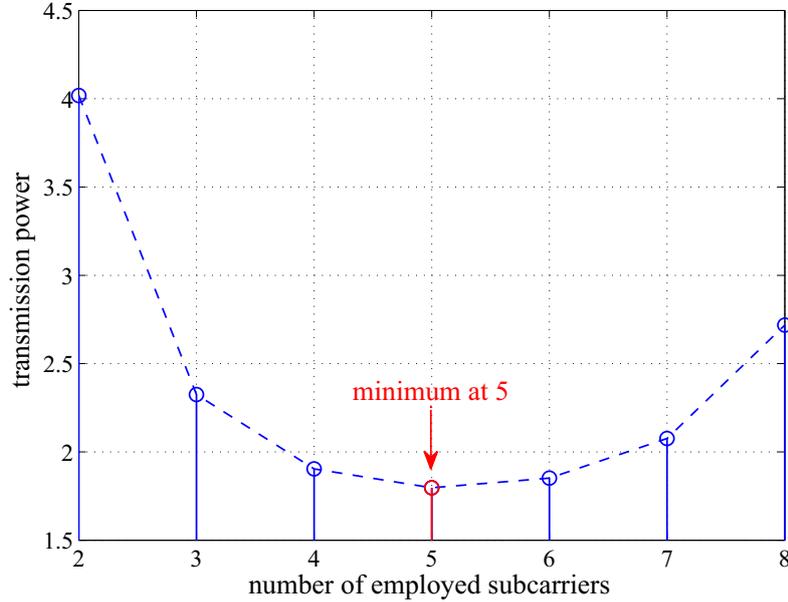


Figure 4.2: Example of transmission power vs. number of employed subcarriers with  $s_k = 8$ ,  $g_{k,n} = 9 - n$ ,  $n = 1, \dots, s_k$  and  $R_k = 8$  bits per OFDM symbol.

### 4.2.3 Single-user subcarrier allocation

Given the subcarrier assignment, the solutions of (4.3) and (4.7) can be viewed as water-filling solutions over subcarriers having the same CNR. When equal rate resource allocation is applied, the power and rate allocation for a single user  $k$  simplifies to the power-rate function

$$R_k^{(\text{ER})} = s_k \log_2 \left( 1 + \frac{P_k^{(\text{ER})}}{s_k} H_k \right) \quad (4.11)$$

from (4.4) and (4.8) given the subcarrier assignment  $\mathcal{S}_k$  with the cardinality  $s_k$ . The transmission power for user  $k$  is  $P_k^{(\text{ER})}$ , while  $R_k^{(\text{ER})}$  is the achieved rate. We denote by  $H_k$  the harmonic average of the CNRs  $\{g_{k,n} | n \in \mathcal{S}_k\}$ . It shows that (4.3) and (4.7) are dual to each other. When the subcarrier assignment changes, the achieved rate is simply obtained after updating the harmonic average. In the following, the subcarrier assignment is derived for the single-user equal rate resource allocation.

#### Unimodality

When the water-filling strategy is adopted for resource allocation and the subcarrier assignment  $\mathcal{S}_k$  is not optimal for user  $k$ , there exist some subcarriers with non-positive power allocated, see (3.4), (3.5), (3.6) and (3.7). In this case, these subcarriers are not usable for user  $k$  and should be excluded from  $\mathcal{S}_k$ . Thereafter, the new power and rate allocation is computed. This iterative procedure finishes when all subcarriers in

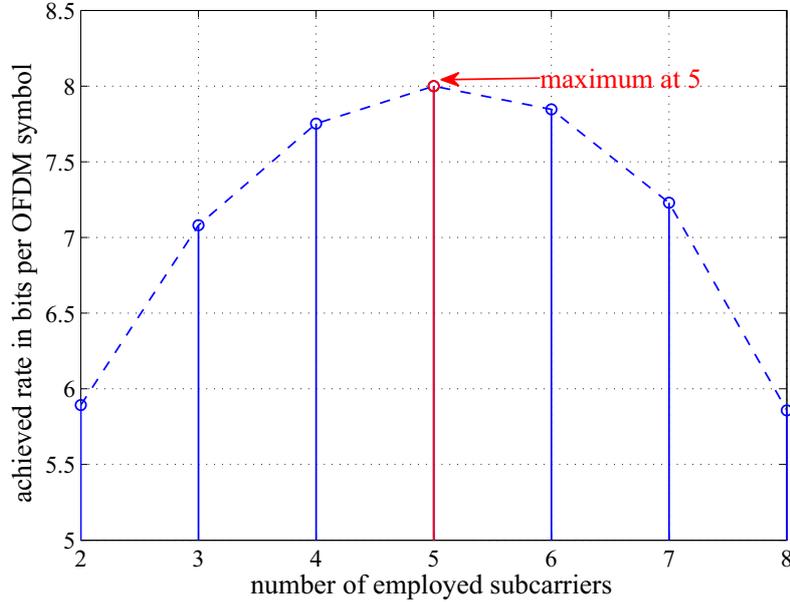


Figure 4.3: Example of achieved rate vs. number of employed subcarriers with  $s_k = 8$ ,  $g_{k,n} = 9 - n$ ,  $n = 1, \dots, s_k$  and  $P_k = 2.55$  dBW.

$\mathcal{S}_k$  are allocated with positive power. However, this procedure of excluding useless subcarriers is not suitable for the proposed strategy. When the proposed equal rate resource allocation is applied, positive power is allocated to all subcarriers in  $\mathcal{S}_k$ , even though it is not optimal to use some of them.

Obviously, exhaustive search for excluding useless subcarriers cannot work for the proposed strategy in practice due to its high complexity. To efficiently determine the subcarrier assignment, we find the following properties of the proposed strategy in examples. Without loss of generality, it is assumed that the first  $s_k$  subcarriers  $\{1, \dots, s_k\}$  are assigned to user  $k$  and their CNRs are subject to  $g_{k,1} \geq \dots \geq g_{k,s_k}$ . There may be some useless subcarriers for the equal rate resource allocation. Intuitively, they must be the subcarriers with the smallest CNRs among the  $s_k$  subcarriers. For example,  $s_k = 8$  subcarriers are assigned to user  $k$  for the single-user MA problem (4.3) in Figure 4.2. The transmission power for reaching the required rate  $R_k = 8$  bits per OFDM symbol is plotted against different numbers of employed subcarriers with the largest CNRs among the  $s_k = 8$  subcarriers. In this example, the optimum number of employed subcarriers is 5. On the left-hand side of the minimum, the transmission power is decreasing monotonically in the number of employed subcarriers. On the right-hand side, it is monotonically increasing. The relation between the transmission power and the number of employed subcarriers can be approximated by an inverse unimodal function. In Figure 4.3, an example for the single-user RA problem is given, where the achieved rate is a unimodal function of the number of employed subcarriers.

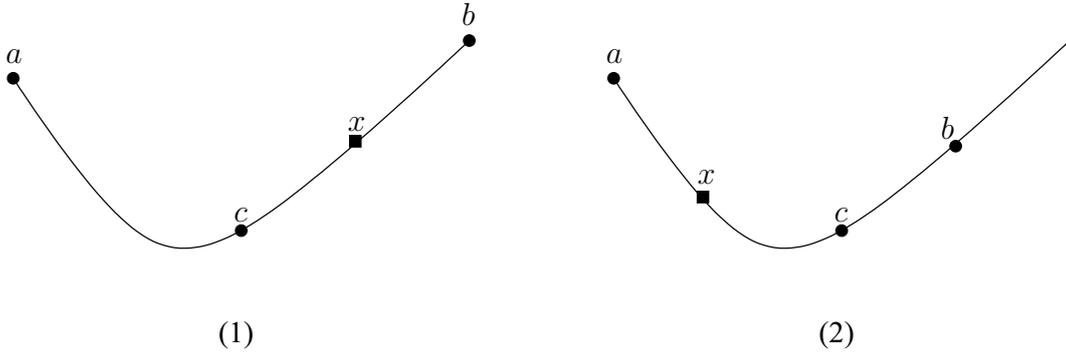


Figure 4.4: Example for the golden section search.

### Upgraded bisection search

To take advantage of the properties above, an upgraded bisection search is designed by using the idea of the golden section search. The conventional bisection search is used only for the monotonic relation between the objective and the argument. The golden section search is used for continuous functions, which is described in Figure 4.4. In each iteration of the golden section search, the searching interval  $[a, b]$  is divided into two sections by the third point  $c$  with  $a < c < b$ . A new point  $x$  is chosen either between  $a$  and  $c$  or between  $b$  and  $c$ . By taking the latter choice, shown in the left part of Figure 4.4, if the objective at  $c$  is smaller than the objective at  $x$ , the new bracketing triplet of points is  $a < c < x$ . Otherwise, the new bracketing triplet is  $c < x < b$ , shown in the right part of Figure 4.4. This iteration finishes until  $|b - a| < \epsilon$ , where  $\epsilon$  is the calculation accuracy.

The golden section search is modified as an upgraded bisection method in Algorithm 7 to fit for the single-user MA problem (4.3) applying the proposed equal rate resource allocation. The subcarriers assigned to this single user are first sorted following a descending order of their CNRs. The ceiling and floor functions in Algorithm 7 are used to render the number of employed subcarriers always integer-valued. When the number of employed subcarriers  $x$  varies, the harmonic average and the transmission power accordingly change to

$$H_k(x) = \frac{x}{\sum_{n=1}^x 1/g_{k,n}}$$

$$P_k^{(\text{ER})}(x) = \frac{x(2^{R_k/x} - 1)}{H_k(x)},$$

respectively. At last, the three total transmission powers  $P_k^{(\text{ER})}(a)$ ,  $P_k^{(\text{ER})}(c)$ ,  $P_k^{(\text{ER})}(b)$  at different numbers of used subcarriers are compared and the lowest transmission power is determined. For the single-user MA problem (4.3), the relation of the objective and the number of employed subcarriers is inverse unimodal. For the single-user RA problem (4.7), the objective is a unimodal function. Thus, Algorithm 7 can be

**Algorithm 7** Upgraded bisection search

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```

 $a \leftarrow 1$ 
 $b \leftarrow s_k$ 
 $c \leftarrow \lceil 0.5s_k \rceil$ 
repeat
  if  $c - a \leq b - c$  then
     $x \leftarrow \lceil 0.5(c + b) \rceil$ 
  else
     $x \leftarrow \lfloor 0.5(a + c) \rfloor$ 
  end if
  if  $c - a \leq b - c$  then
    if  $P_k^{(\text{ER})}(c) \leq P_k^{(\text{ER})}(x)$  then
       $b \leftarrow x$ 
    else
       $a \leftarrow c$ 
       $c \leftarrow x$ 
    end if
  else
    if  $P_k^{(\text{ER})}(c) \leq P_k^{(\text{ER})}(x)$  then
       $a \leftarrow x$ 
    else
       $b \leftarrow c$ 
       $c \leftarrow x$ 
    end if
  end if
until  $b - a \leq 2$ 
 $P^{(\text{ER})} \leftarrow \min(P_k^{(\text{ER})}(a), P_k^{(\text{ER})}(c), P_k^{(\text{ER})}(b))$ 

```

---

used to determine the subcarrier assignment for (4.7) with a simple modification for the unimodal relation.

**Numerical examples**

Without considering the signalling overhead, Figure 4.5 shows the instantaneous per-symbol performance loss by the proposed strategy compared to water-filling as

$$\frac{P_k^{(\text{ER})} - P_k}{P_k} \times 100\%$$

where  $P_k^{(\text{ER})}$  is the transmission power with the equal rate resource allocation applied and  $P_k$  is the transmission power while using the water-filling strategy. The performance loss tends to the limit (4.6) as the required rate increases.

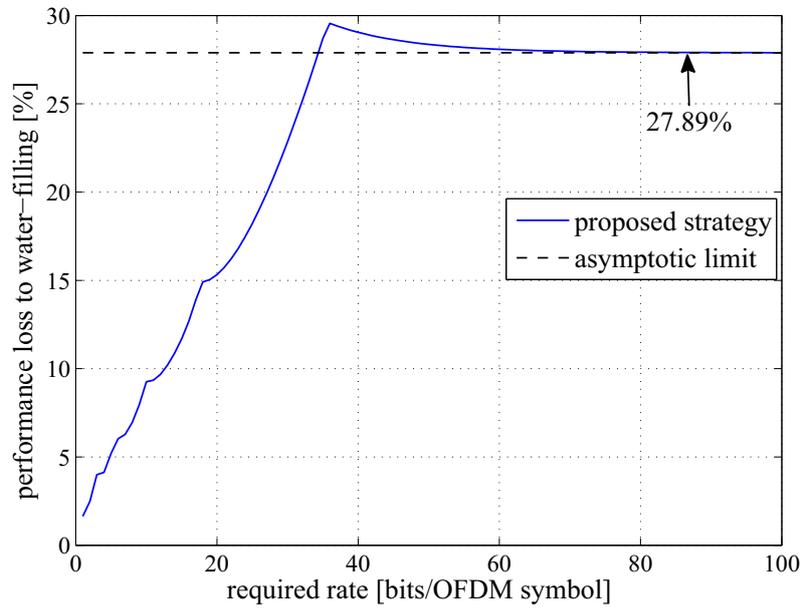


Figure 4.5: Performance loss of the proposed strategy compared to water-filling for MA user  $k$  with  $N = 8$  subcarriers  $g_{k,n} = 9 - n, n = 1, \dots, 8$ .

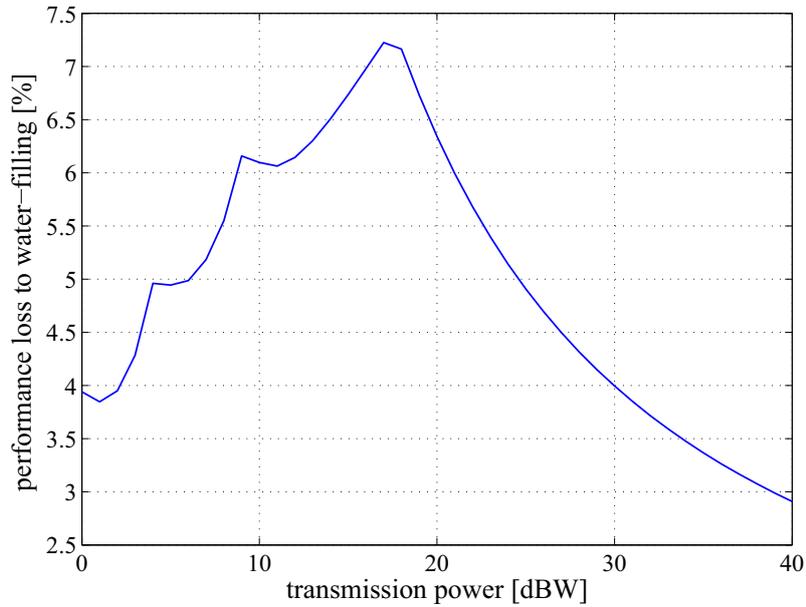


Figure 4.6: Performance loss of the proposed strategy compared to water-filling for RA user  $k$  with  $N = 8$  subcarriers  $g_{k,n} = 9 - n, n = 1, \dots, 8$ .

For the single-user RA problem, the performance loss by the proposed strategy is shown in Figure 4.6, defined as

$$\frac{R_k^{(\text{ER})} - R_k}{R_k} \times 100\%$$

where  $R_k^{(\text{ER})}$  is the rate achieved with the equal rate resource allocation applied and  $R_k$  is the one while using the water-filling strategy. The performance loss asymptotically reduces to zero (4.10). The maximum loss appears, when these two strategies employ different numbers of subcarriers. Note that the asymptotic limit is 27.89% in Figure 4.5, while the loss goes up to 7.4% in Figure 4.6, since CSI variation is very large with respect to the example subcarriers. Verified by simulations, the average performance loss of the proposed strategy is low.

## 4.3 Multiuser equal rate resource allocation

The previous section has solved the single-user equal rate resource allocation problems, while the signalling overhead is not considered. In this section, we focus on the multiuser equal rate resource allocation problem (4.2) with the signalling overhead considered. Given the subcarrier assignment, the single-user equal rate resource allocation simplifies to the power-rate function (4.11), which turns to

$$R_k^{(\text{ER})} = \max \left( s_k \log_2 \left( 1 + \frac{P_k^{(\text{ER})}}{s_k} H_k \right) - \frac{M}{L}, 0 \right), \quad (4.12)$$

for all  $k = 1, \dots, K + Q$ . The problem (4.2) can be separated in two problems, i.e., a multiuser MA resource allocation problem and a multiuser RA resource allocation problem. Note that the same rate is allocated to subcarriers assigned to one user, while the equal rates for users may differ.

### 4.3.1 Multiuser MA resource allocation

When only MA users appear in OFDM unicasting, the multiuser MA resource allocation problem is stated as

$$\begin{aligned} & \text{minimize} && \sum_{k \in \mathcal{Q}} P_k^{(\text{ER})} && (4.13) \\ & \text{subject to} && \frac{1}{H_k} = \frac{1}{s_k} \sum_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}}, \quad k \in \mathcal{Q} \\ & && P_k^{(\text{ER})} = \frac{s_k (2^{(R_k + M/L)/s_k} - 1)}{H_k}, \quad k \in \mathcal{Q} \\ & && \mathcal{S}_k \cap \mathcal{S}_l = \emptyset, \quad k, l \in \mathcal{Q}, \quad k \neq l. \end{aligned}$$

MA users are contained in a set  $\mathcal{Q}$ . When the subcarrier assignment  $\{\mathcal{S}_k | k \in \mathcal{Q}\}$  is fixed, (4.13) is solved by (4.12) independently for different users. To determine

**Algorithm 8** Multiuser equal rate subcarrier adjustment

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```

 $\mathcal{N} \leftarrow \{1, \dots, N\}$ 
for  $i = 1, \dots, I$  do
  for  $n = 1, \dots, N$  do
     $k \leftarrow \{j \in \mathcal{Q} \mid \mathcal{S}_k \cap \{n\} \neq \emptyset\}$ 
    for  $m \in \mathcal{N} \setminus \mathcal{S}_k$  do
       $l \leftarrow \{j \in \mathcal{Q} \mid \mathcal{S}_k \cap \{m\} \neq \emptyset\}$ 
      if  $Z_k(\frac{1}{g_{k,m}} - \frac{1}{g_{k,n}}) + Z_l(\frac{1}{g_{l,n}} - \frac{1}{g_{l,m}}) < 0$  then
         $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \{m\} \setminus \{n\}$ 
         $\mathcal{S}_l \leftarrow \mathcal{S}_l \cup \{n\} \setminus \{m\}$ 
      end if
    end for
  end for
end for

```

---

the subcarrier assignment, the following feature of the proposed strategy offers convenience for the heuristic design. Assume that subcarrier  $n$  is assigned to user  $k$  and subcarrier  $m$  is assigned to user  $l$  in the present subcarrier assignment. If these two subcarriers are swapped between user  $k$  and user  $l$  that means subcarrier  $n$  is reassigned to user  $l$  while subcarrier  $m$  is reassigned to user  $k$ , the induced power variation of the sum power is

$$\begin{aligned} \Delta_m^n P(k, l) &= \underbrace{\left(2^{\frac{R_k+M/L}{s_k}} - 1\right)}_{=Z_k} \left( \sum_{i \in \mathcal{S}_k} \frac{1}{g_{k,i}} + \frac{1}{g_{k,m}} - \sum_{i \in \mathcal{S}_k} \frac{1}{g_{k,i}} - \frac{1}{g_{k,n}} \right) \\ &\quad + \underbrace{\left(2^{\frac{R_l+M/L}{s_l}} - 1\right)}_{=Z_l} \left( \sum_{i \in \mathcal{S}_l} \frac{1}{g_{l,i}} + \frac{1}{g_{l,n}} - \sum_{i \in \mathcal{S}_l} \frac{1}{g_{l,i}} - \frac{1}{g_{l,m}} \right). \end{aligned}$$

where  $\mathcal{S}_k$  and  $\mathcal{S}_l$  denote the previous subcarrier assignments for user  $k$  and user  $l$  before swapping, respectively. After the substitutions  $Z_k = 2^{\frac{R_k}{s_k}} - 1$  and  $Z_l = 2^{\frac{R_l}{s_l}} - 1$ , it simplifies to

$$\Delta_m^n P(k, l) = Z_k \left( \frac{1}{g_{k,m}} - \frac{1}{g_{k,n}} \right) + Z_l \left( \frac{1}{g_{l,n}} - \frac{1}{g_{l,m}} \right)$$

Since  $Z_k$  is only determined by the cardinality of  $\mathcal{S}_k$ , it remains constant after swapping. If  $\Delta_m^n P(k, l) < 0$  holds, we actually perform this swapping. Then, the criterion for this adjustment is simplified to

$$Z_k \left( \frac{1}{g_{k,m}} - \frac{1}{g_{k,n}} \right) + Z_l \left( \frac{1}{g_{l,n}} - \frac{1}{g_{l,m}} \right) < 0. \quad (4.14)$$

It can be used as long as fixing the number of subcarriers assigned to each user.

After some initialization for subcarrier assignment, the cardinalities are fixed and  $\{Z_k \mid k \in \mathcal{Q}\}$  remains constant. The criterion (4.14) is utilized in Algorithm 8 to adjust

the subcarrier assignment. For each subcarrier  $n$  assigned to user  $k$ , all subcarriers  $\{m \in \mathcal{N} \setminus \mathcal{S}_k\}$  that may be swapped are checked. If the associated swapping improves the performance, the swapping is actually executed. This successive procedure repeats  $I$  times. Its complexity is  $\mathcal{O}(N^2)$  and is not related to the number of users. Then, a heuristic solution is obtained for (4.13).

### 4.3.2 Multiuser RA resource allocation

Maximizing the weighted sum rate for RA users in a set  $\mathcal{K}$  is equivalent to relaxing the original problem (4.2) by not considering the minimum required rates, stated as

$$\begin{aligned} & \text{maximize} && \sum_{k \in \mathcal{K}} w_k \max \left( s_k \log_2 \left( 1 + \frac{P_k^{(\text{ER})}}{s_k} H_k \right) - \frac{M}{L}, 0 \right) && (4.15) \\ & \text{subject to} && \frac{1}{H_k} = \frac{1}{s_k} \sum_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}}, \quad k \in \mathcal{K} \\ & && \sum_{k \in \mathcal{K}} P_k^{(\text{ER})} \leq P - \sum_{k \in \mathcal{Q}} P_k^{(\text{ER})} \\ & && \mathcal{S}_k \cap \mathcal{S}_l = \emptyset, \quad k, l \in \mathcal{K}, k \neq l \end{aligned}$$

where  $P - \sum_{k \in \mathcal{Q}} P_k^{(\text{ER})}$  is the remaining power for the RA users in  $\mathcal{K}$  after reaching the minimum rates required by the users in  $\mathcal{Q}$ . Given the subcarrier assignment  $\{\mathcal{S}_k \mid k \in \mathcal{K}\}$  for the RA users in  $\mathcal{K}$ , the problem above is solved with the KKT conditions in the similar procedure to (3.15) as

$$P_k^{(\text{ER})} = s_k \max \left( \nu^{(\text{ER})} w_k - \frac{1}{H_k}, 0 \right), \quad k \in \mathcal{K} \quad (4.16)$$

$$R^{(\text{ER})} = \max \left( s_k \log_2 \left( \nu^{(\text{ER})} w_k H_k \right) - \frac{M}{L}, 0 \right) \quad (4.17)$$

where the sharing term  $\nu^{(\text{ER})}$  is

$$\nu^{(\text{ER})} = \frac{P - \sum_{k \in \mathcal{Q}} P_k^{(\text{ER})} + \sum_{k \in \mathcal{K}} s_k / H_k}{\sum_{k \in \mathcal{K}} w_k s_k} \quad (4.18)$$

and  $\nu^{(\text{ER})} w_k$  is the water level for user  $k \in \mathcal{K}$ . However, from (4.16) and (4.17), it can be seen that the rate allocated to user  $k$  may be zero. If this occurs on user  $k$ , it is treated as an MA user. User  $k$  should be excluded from  $\mathcal{K}$  and included in  $\mathcal{Q}$ . After that  $\nu^{(\text{ER})}$  is recalculated. This iterative procedure repeats till all users in  $\mathcal{K}$  are assigned with positive transmission power. This solution can be viewed as a water-filling solution over different users  $k \in \mathcal{K}$ . Hence, its complexity is  $\mathcal{O}(K)$ .

### 4.3.3 Heterogeneous resource allocation

For the single-user equal rate resource allocation in Section 4.2, the transmission power (4.4) for MA user  $k$  is treated as an inverse unimodal function of the number of employed subcarriers in  $\mathcal{S}_k$ , where subcarriers are sorted in a descending order

**Algorithm 9** ISSA with equal rate strategy applied

---

```

for  $i = 1, \dots, I$  do
  for each  $n \in \{1, \dots, N\}$  do
    if  $\sum_{k=1}^{K+Q} P_k > P$  then
       $\min \sum_{k=1}^{K+Q} P_k \leftarrow$  adjusting  $n$  among users via harmonic averages
    else
       $\max \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_k} r_{k,n} \leftarrow$  adjusting  $n$  among users via harmonic averages
    end if
  end for
end for

```

---

of the CNRs  $\{g_{k,n} | n \in \mathcal{S}_k\}$ . The achieved rate (4.8) for RA user  $k$  is viewed as a unimodal function of the number of employed subcarriers in  $\mathcal{S}_k$ . The upgraded bisection search is used to exclude the useless subcarriers for user  $k$  while using the proposed strategy.

At the optimum of (4.2), only the minimum required rate  $R_k$  is reached for each MA user  $k = K + 1, \dots, K + Q$  due to the complementary slackness condition. The achieved rates may be strictly greater than the minimum required rates for RA users. The set  $\mathcal{Q}$  contains the users, for whom only the minimum required rates are reached. Other users are contained in  $\mathcal{K}$ , for whom the achieved rates are strictly greater than the corresponding minimum required rates. Note that  $\mathcal{Q}$  includes all MA users as well as some primal RA users. In the following, first, the solution is determined for (4.2) as long as fixing the subcarrier assignment. The heuristic method used in Chapter 3 is then utilized to determine the subcarrier assignment.

Given the subcarrier assignments  $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q}$  for all users, (4.2) can be solved as follows. The sets  $\mathcal{K}$  and  $\mathcal{Q}$  are initialized as  $\{1, \dots, K\}$  and  $\{K + 1, \dots, K + Q\}$ , respectively. First, the transmission power is obtained for each user for reaching the minimum required rate by independently solving the single-user MA problem (4.3) as  $P_k^{(\text{ER})}$ ,  $k = 1, \dots, K + Q$ . With the present subcarrier assignments,  $\nu^{(\text{ER})}$  is then obtained by (4.18) for (4.15). With the derived  $\nu^{(\text{ER})}$ , we obtain the transmission power and rate for each user in  $\mathcal{K}$  by (4.16) and (4.17). If the minimum required rate cannot be satisfied for user  $k \in \mathcal{K}$ ,  $k$  is moved from  $\mathcal{K}$  to  $\mathcal{Q}$ . After that the transmission power and rate are computed for each user in the new  $\mathcal{K}$ . This procedure repeats till satisfying all constraints on the minimum required rates. Finally, the multiuser equal rate resource allocation is determined as the subcarrier assignment is given.

The heuristic method ISSA suggested in Chapter 3 for (2.5) can be used to determine the subcarrier assignment for (4.2) with small modifications. First, the initialization in Chapter 3 remains for initializing the subcarrier assignment. The obtained subcarrier assignment is then adjusted iteratively and successively along subcarriers as ISSA.

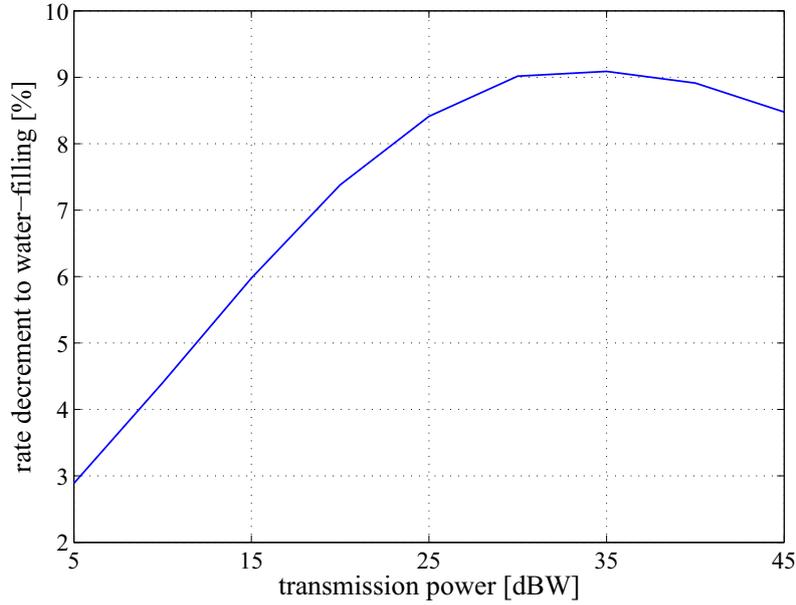


Figure 4.7: Decrement of instantaneous per-symbol rate achievement by the single-user equal rate resource allocation compared to water-filling vs. increasing power limit.

Each subcarrier is reassigned to different users to see whether the weighted sum rate can be improved. While adjusting one subcarrier, the assignment of others is fixed. The modified method is arranged in Algorithm 9, which consists of two loops. In the inner loop, the subcarrier assignment is successively adjusted from the first to the  $N$ th subcarrier. The inner loop repeats  $I$  times within the outer loop. GWF presented in Chapter 3 is not necessary, since updating the harmonic average of CNRs is sufficient for obtaining the resulting weighted sum rate while applying the equal rate resource allocation. Finally, the upgraded bisection search of Algorithm 7 is performed over the output subcarrier assignment from Algorithm 9 separately for different users. The only change to ISSA in Chapter 3 is that the weighted sum rate is renewed via updating the harmonic averages instead of GWF. The structure of ISSA does not change. Hence, the complexity of this heuristic method is still  $\mathcal{O}((K + Q)N)$ .

## 4.4 Simulation results

In this section, simulations are performed to assess the proposed strategy. The simulation system is built with the parameters of WiMAX from [2]. It consists of 128 subcarriers. The frequency selective channel is modeled as consisting of  $N/8$  independently Rayleigh distributed multiple paths with an exponentially decaying profile. The expected CNR of each subcarrier is normalized to 5 dB.

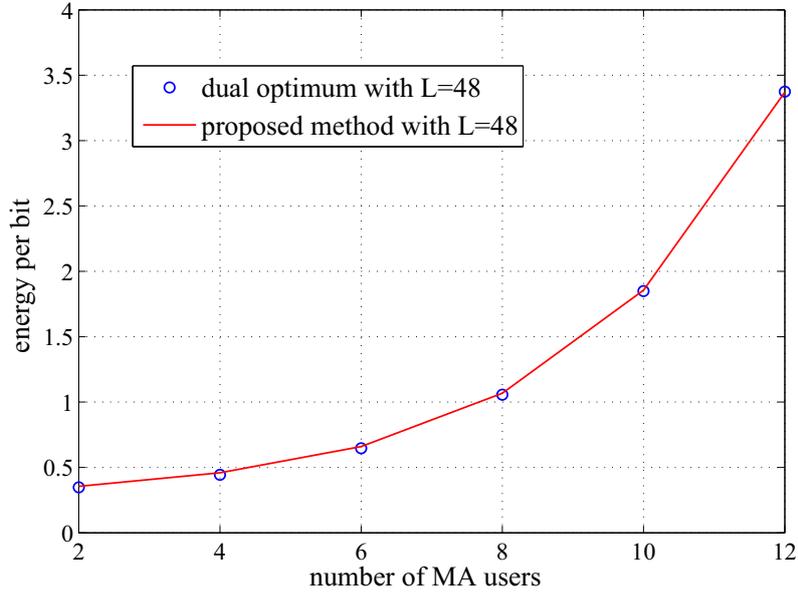


Figure 4.8: Energy efficiency vs. number of users with  $L = 48$ .

### Single user

For single-user RA resource allocation (4.7) in Figure 4.7, the decrement of data rate caused by the proposed strategy compared to the water-filling solution is

$$\frac{R^{(\text{WF})} - R^{(\text{ER})}}{R^{(\text{WF})}} \times 100\%$$

where  $R^{(\text{WF})}$  is the rate achieved by the water-filling solution with different rates allocated to subcarriers and  $R^{(\text{ER})}$  is the rate achieved by the proposed strategy with an equal rate allocated to subcarriers. This decrement is subject to an upper limit, which is determined by CNR variation with respect to subcarriers. On the left-hand side of the maximum, the higher the power limit becomes, the more subcarriers are employed. On the right-hand side, when all subcarriers are in use, CNR variation with respect to subcarriers becomes negligible as the transmission power increases, see the asymptotic limit (4.10).

### Multiple MA users

In WiMAX, a frame is composed of 48 OFDM symbols, while the time duration of one OFDM symbol is  $102.9 \mu\text{s}$ . When there are only MA users, Algorithm 8 is used. Its performance is compared to the dual optimum applying the water-filling strategy [72], while one resource allocation scheme is only effective for one frame containing  $L$  OFDM symbols. The data rate demanded by each MA user is distributed uniformly within  $[50, 100]$  bits per OFDM symbols, i.e.,  $[455, 910]$  kbits/s. Each rate is expressed by

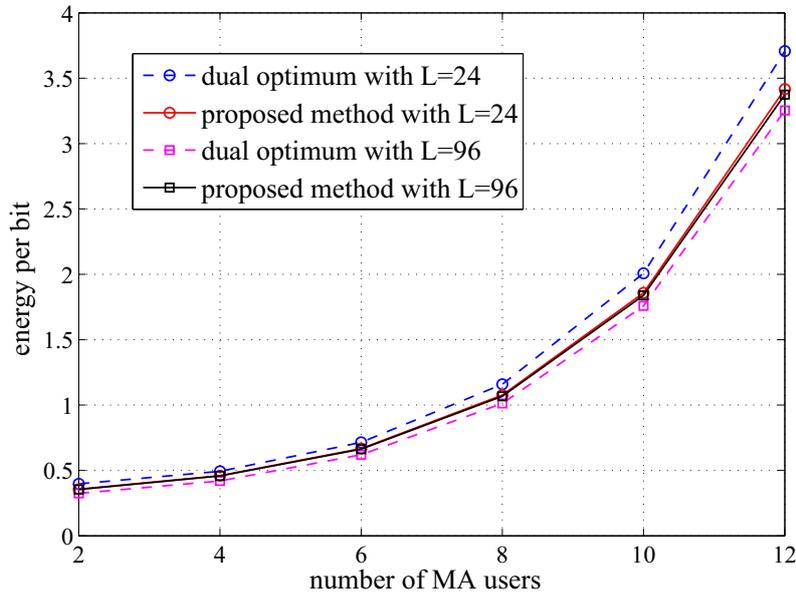


Figure 4.9: Energy efficiency vs. number of users with  $L = 24$  or  $L = 96$ .

$M = 6$  bits to distinguish different modulation schemes. Figure 4.8 draws the average energy per bit by the proposed heuristic method of Algorithm 8 with  $I = 4$  and the one by the dual optimal multiuser water-filling against different numbers of users. They have similar performance. In this simulation, each resource allocation scheme is effective for  $L = 48$  OFDM symbols, which is the frame length in WiMAX. As explained before, the average computing time for our method is much shorter than that for the dual optimum, see Table 4.1. The smaller  $L$  becomes, the faster the channel varies in time so that the power and rate allocation scheme must be updated more frequently and vice versa. Shown in Figure 4.9, when the primal  $L$  doubles to 96, our method still gives a solution close to the dual optimum, whereas it has better performance than the dual optimum as  $L$  reduces to 24.

Table 4.1: Average computing time [s]

$K$	4	6	8	10	12
dual optimum	57.5	161.5	344.6	640.9	1055.0
proposed method	27.8	28.3	28.6	28.9	29.3

### Multiple RA and MA users

For heterogeneous resource allocation, each minimum rate is distributed independently and uniformly within  $[10, 20]$  bits per OFDM symbol. Each weight is distributed

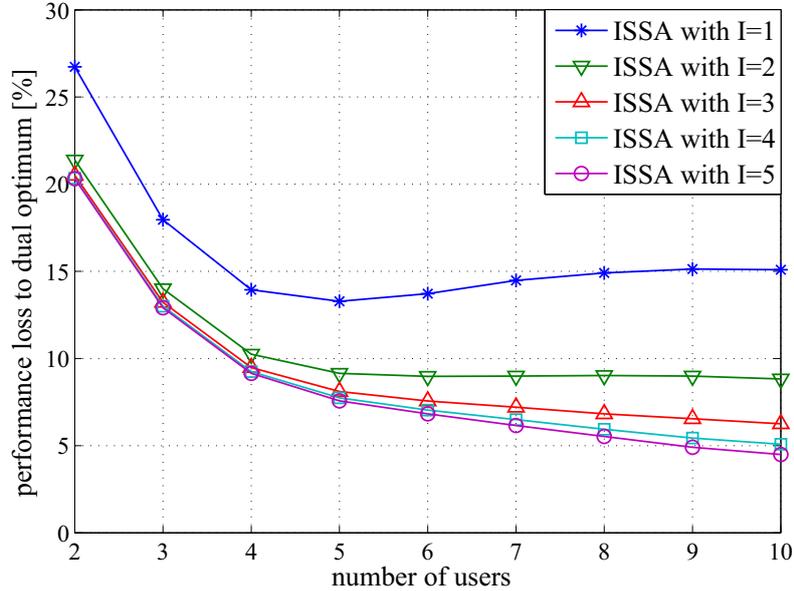


Figure 4.10: Instantaneous per-symbol performance loss of the equal rate resource allocation compared to the dual optimum vs. number of users.

independently and uniformly within  $[1, 10]$ . Their sum is normalized to one. The transmission power is limited to 20 dBW. The modified ISSA with the equal rate strategy applied is compared to the dual optimum of (2.5). First, we quantify the instantaneous per-symbol performance loss as

$$\frac{R^{(\text{dualWF})} - R^{(\text{subER})}}{R^{(\text{dualWF})}} \times 100\%$$

where  $R^{(\text{dualWF})}$  is the dual optimum of (2.5) with the water-filling solution applied and  $R^{(\text{subER})}$  is the heuristic solution proposed for (4.2) by Algorithm 9. Figure 4.10 plots this performance loss with  $K = Q$ . It is decreasing in the number of iterations  $I$  and the number of users  $K$ . It converges at around  $I = 5$  in our simulation. The previous criteria for sorting subcarriers and controlling iterations in Chapter 3 can also be applied, while the associated performance is omitted here.

As explained earlier, the faster channels change, the smaller is the number of OFDM symbols that one resource allocation scheme is effective for. One resource allocation scheme is only effective for  $L$  OFDM symbols. We denote by  $R$  the dual optimum of (4.1) and by  $\bar{R}$  the heuristic solution of (4.2) from Algorithm 9. According to [28],  $N \lceil \log(K) \rceil + NM$  bits represent one resource allocation scheme of the water-filling solution, while there are  $LR - (N \lceil \log(K) \rceil + NM)$  data bits within  $L$  OFDM symbols. For the proposed strategy,  $N \lceil \log(K) \rceil + KM$  bits are adequate for expressing one resource allocation scheme, while  $L\bar{R} - (N \lceil \log(K) \rceil + NM)$  data bits are contained

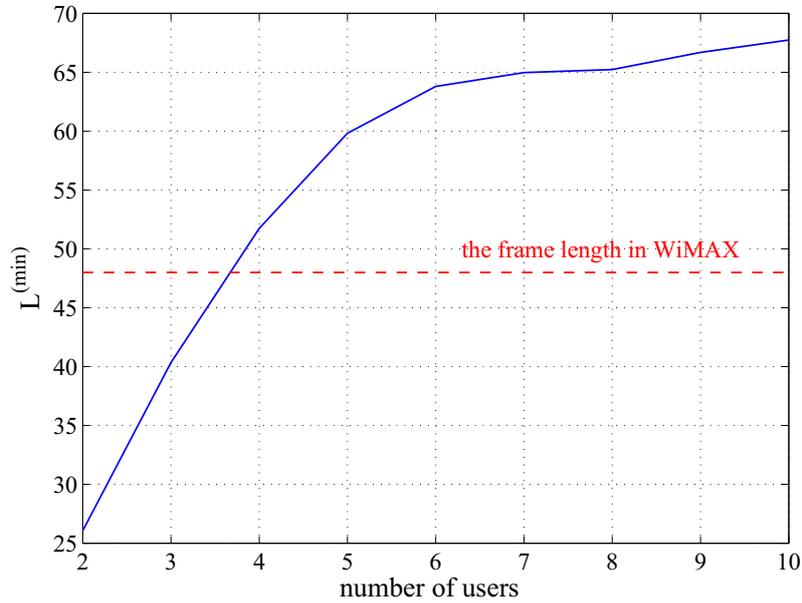


Figure 4.11: Minimum number of OFDM symbols for the case that water-filling has better performance vs. number of users.

in  $L$  OFDM symbols. With the signalling overhead considered, if the water-filling solution has better performance, the following inequality must hold as

$$LR - (N\lceil\log(K)\rceil + NM) > L\bar{R} - (N\lceil\log(K)\rceil + KM),$$

which is equivalent to

$$L > \frac{M(N - K)}{R - \bar{R}}.$$

The resource allocation scheme is updated for every  $L$  OFDM symbols. We define that  $L^{(\min)} = M(N - K)/(R - \bar{R})$  is the minimum frame length for which the water-filling strategy has better performance than the proposed strategy. For comparison, we use  $\forall k : w_k = 1$  and  $M = 6$ . The inequality above does not hold below the solid curve in Figure 4.11, which is much greater than 48, i.e., the frame length in WiMAX [2]. This implies that our strategy performs better in the region below the solid curve. In practice, its performance is expected even better, since the signalling overhead must be transmitted at a rate much lower than the data rate to assure that every user receives it correctly, as explained while formulating (4.2).

TDWF presented in Chapter 3 can be performed over the subcarrier assignment given by the proposed method of Algorithm 9. A heuristic solution is then obtained for (2.5). Its instantaneous per-symbol performance loss compared to the dual optimum is quantified as

$$\frac{R^{(\text{dual})} - R^{(\text{sub})}}{R^{(\text{dual})}} \times 100\%$$

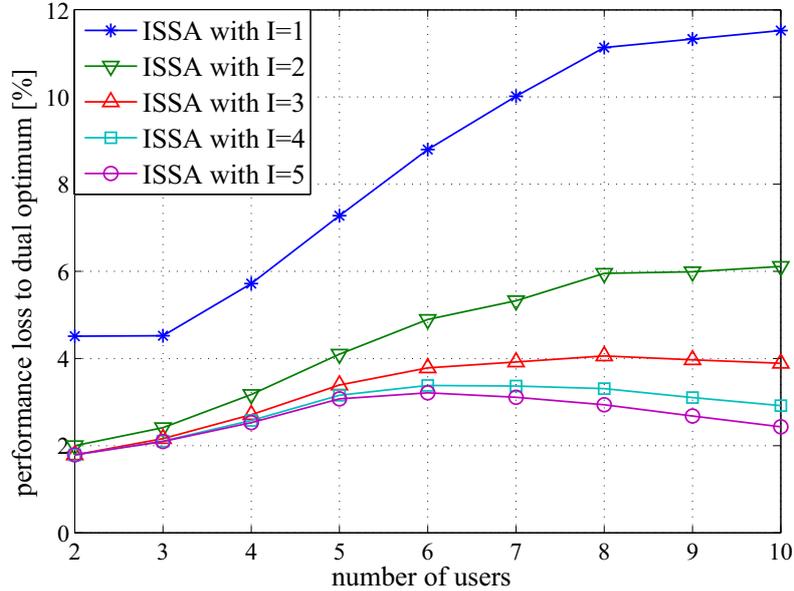


Figure 4.12: Instantaneous per-symbol performance loss of water-filling over the subcarrier assignment given by the proposed method compared to the dual optimum vs. number of users.

where  $R^{(\text{dual})}$  denotes the dual optimum of (2.5) and  $R^{(\text{sub})}$  represents the suboptimum of (2.5) by TDWF over the subcarrier assignment from Algorithm 9. The performance loss in Figure 4.12 is small. It decreases as  $K$  grows. Hence, Algorithm 9 can also offer a near-optimum solution for the heterogeneous resource allocation problem (2.5).

## 4.5 Conclusions

Different from the conventional resource allocation strategy, we have proposed that the same rate is allocated to subcarriers assigned to one user. This leads to a small signalling overhead, low complexity and easy implementation. Based on this strategy, a heuristic solution has been derived for heterogeneous resource allocation by simply revising the heuristic method presented in Chapter 3. Additionally, a heuristic solution has been developed specially for multiuser MA resource allocation by taking advantage of the feature of the proposed strategy. The proposed strategy and heuristic methods have been thoroughly evaluated by simulations. The simulation results demonstrate that the proposed strategy has better performance w.r.t. energy consumption than the water-filling strategy in fast time-varying environments when the signalling overhead is taken into account.

## 5 Multicell Resource Allocation

In the previous two chapters, resource allocation has been studied for heterogeneous unicasting from a single BS to multiple RA and MA users. This chapter investigates resource allocation for heterogeneous unicasting by multiple BSs. BSs in one cluster share the transmission band via OFDM. Unicasting data is available for all BSs through the fibre connection to the control center, see Figure 2.6. Resource allocation is performed by the control center, where we assume that perfect CSI is available. Each subcarrier is assigned to at most one pair of BS and user.

When signals from different BSs are synchronized, each receiver can receive different data streams via subcarriers from multiple BSs at the same time as illustrated by problem (2.6) in Chapter 2. To solve (2.6), the dual optimum and the heuristic solution offered for single-cell resource allocation are extended to the multicell case. Alternatively, when synchronization of signals from different BSs cannot be achieved, we stipulate that each receiver receives data streams from only one BS at any specific period of time. For this case, an additional constraint must be added to (2.6). It follows that a specific BS must be selected for each user, called BS selection. A dual optimum and a heuristic solution are given for the new problem.

With the BS dimension added, the notation used in the earlier two chapters changes to the subcarrier assignment  $\mathcal{S}_{k,c}$  for user  $k$  at BS  $c$ , the rate  $r_{k,n,c}$  and the power  $p_{k,n,c}$  allocated to the  $n$ th subcarrier for user  $k$  at BS  $c$  and the limit  $P_c$  of transmission power at BS  $c$ . It always holds that  $\mathcal{S}_{k,c} \cap \mathcal{S}_{l,v} = \emptyset$  for any  $(k, c) \neq (l, v)$ .

### 5.1 Dual optimum

Duality theory can also be applied to solve the multicell heterogeneous resource allocation problem (2.6), whose Lagrangian is given as

$$\begin{aligned}
 L(\boldsymbol{\lambda}, \boldsymbol{\beta}, \mathbf{A}, \mathbf{B}) &= - \sum_{k=1}^{K+Q} \sum_{n=1}^N \sum_{c=1}^C w_k a_{k,n,c} + \sum_{k=1}^{K+Q} \lambda_k (R_k - \sum_{n=1}^N \sum_{c=1}^C b_{k,n,c}) \\
 &\quad + \sum_{c=1}^C \beta_c \left( \sum_{k=1}^{K+Q} \sum_{n=1}^N p_{k,n,c} - P_c \right) \\
 &= \sum_{k=1}^{K+Q} \lambda_k R_k - \sum_{c=1}^C \beta_c P_c + \sum_{k=1}^{K+Q} \sum_{n=1}^N \sum_{c=1}^C (\beta_c p_{k,n,c} - w_k a_{k,n,c} - \lambda_k b_{k,n,c})
 \end{aligned}$$

where the dual variables, denoted by  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_C)'$ , are associated to the limits of transmission power at  $C$  BSs and where  $\boldsymbol{\lambda}$  is still related to the constraints

on the minimum required rates. For notational brevity, we redefine the matrices  $\mathbf{A} = (\mathbf{A}_1, \dots, \mathbf{A}_C)$  with  $\mathbf{A}_c = (a_{k,n,c})_{1 \leq k \leq K+Q, 1 \leq n \leq N}$ ,  $c = 1, \dots, C$ , for the objective and  $\mathbf{B} = (\mathbf{B}_1, \dots, \mathbf{B}_C)$  with  $\mathbf{B}_c = (b_{k,n,c})_{1 \leq k \leq K+Q, 1 \leq n \leq N}$ ,  $c = 1, \dots, C$ , for the rate constraints, while the linear relation  $r_{k,n,c} = a_{k,n,c} + b_{k,n,c}$  holds. We again set  $(w_{K+1}, \dots, w_{K+Q})' = \mathbf{0}_{Q \times 1}$  as obtaining the dual optimum of (2.5) in Chapter 3.

Then, the Lagrange dual problem is

$$\begin{aligned} & \text{maximize} && L_D(\boldsymbol{\lambda}, \boldsymbol{\beta}) \\ & \text{subject to} && \boldsymbol{\lambda} \succeq \mathbf{0}_{(K+Q) \times 1} \\ & && \boldsymbol{\beta} \succeq \mathbf{0}_{C \times 1} \end{aligned}$$

where the Lagrange dual function  $L_D$  is the unconstrained infimum of the Lagrangian above as

$$\begin{aligned} L_D(\boldsymbol{\lambda}, \boldsymbol{\beta}) &= \inf_{\mathbf{A} \in \mathbb{R}_+^{C(K+Q) \times N}, \mathbf{B} \in \mathbb{R}_+^{C(K+Q) \times N}} L(\boldsymbol{\lambda}, \boldsymbol{\beta}, \mathbf{A}, \mathbf{B}) \\ &= \sum_{k=1}^{K+Q} \lambda_k R_k - \sum_{c=1}^C \beta_c P_c \\ &\quad + \sum_{n=1}^N \inf_{c=1, \dots, C, k=1, \dots, K} \inf_{a_{k,n,c}, b_{k,n,c} \in \mathbb{R}_+^2} (\beta_c p_{k,n,c} - w_k a_{k,n,c} - \lambda_k b_{k,n,c}). \end{aligned} \quad (5.1)$$

The Lagrange dual function is written in the form of (5.1) due to the last constraint in (2.6) that each subcarrier is assigned to at most one pair of user and BS. The function

$$f_{k,n,c} = \beta_c p_{k,n,c} - w_k a_{k,n,c} - \lambda_k b_{k,n,c}$$

is convex in  $a_{k,n,c}$  and  $b_{k,n,c}$  given the dual variables  $\boldsymbol{\lambda}$  and  $\boldsymbol{\beta}$  due to concavity of the first equality constraint in (2.6). The extremum of  $f_{k,n,c}$  is obtained by setting the derivative to zero, while the derivative of  $f_{k,n,c}$  with respect to  $a_{k,n,c}$  and the derivative of  $f_{k,n,c}$  with respect to  $b_{k,n,c}$  cannot be zero simultaneously, explained in Chapter 3. Thus, the extremum must be reached at the boundary of  $\mathbb{R}_+^2$ .

For MA users,  $\lambda_k \geq w_k$  must hold for all  $k = K+1, \dots, K+Q$  due to  $\lambda_k \geq 0$  and  $w_k = 0$ . If the dual variable is greater than the weight for RA user  $k$  as  $\lambda_k \geq w_k$ ,  $k \in \{1, \dots, K\}$ , the rates and powers allocated to RA user  $k$  are

$$r_{k,n,c} = \max \left( \log_2 \left( \frac{\lambda_k g_{k,n,c}}{\beta_c \ln(2)} \right), 0 \right), \quad n = 1, \dots, N \quad (5.2)$$

$$p_{k,n,c} = \max \left( \frac{\lambda_k}{\beta_c \ln(2)} - \frac{1}{g_{k,n,c}}, 0 \right), \quad n = 1, \dots, N. \quad (5.3)$$

The minimum of  $f_{k,n,c}$  is reached at the boundary  $a_{k,n,c} = 0$  because of convexity of  $f_{k,n,c}$  in  $a_{k,n,c}$ . We define  $\boldsymbol{\nu} = (\boldsymbol{\beta} \ln(2))^{-1}$ , where the multiplicative inverse is componentwise as  $\boldsymbol{\beta}^{-1} = (\beta_1^{-1}, \dots, \beta_C^{-1})'$ . The water level  $\mu_{k,c} = \lambda_k \nu_c$  is related to the minimum required rate via  $\lambda_k$  and the power limit via  $\beta_c$ . The associated achieved rate is  $R_k$ , which implies that the rate constraint is satisfied with equality.

Alternatively, if  $\lambda_k < w_k$  holds for RA user  $k \in \{1, \dots, K\}$ ,  $f_{k,n}$  reaches the minimum at

$$r_{k,n,c} = \max \left( \log_2 \left( \frac{w_k g_{k,n,c}}{\beta_c \ln(2)} \right), 0 \right), \quad n = 1, \dots, N \quad (5.4)$$

$$p_{k,n,c} = \max \left( \frac{w_k}{\beta_c \ln(2)} - \frac{1}{g_{k,n,c}}, 0 \right), \quad n = 1, \dots, N. \quad (5.5)$$

The associated rate constraint is not of interest to the primal problem, since the rate achieved for RA user  $k$  is strictly greater than the minimum required rate  $R_k$ . The water level becomes  $w_k \nu_c$  that is only related to the power limit  $P_c$ . In this case, we must force the dual variable  $\lambda_k = 0$  to let the KKT conditions hold.

After taking the minima derived above into the dual objective function (5.1), the subcarrier assignment is determined explicitly by the given dual variables  $\boldsymbol{\lambda}$  and  $\boldsymbol{\beta}$ . The ellipsoid method is still employed to search the  $K + Q + C$  dual variables  $\boldsymbol{\lambda}$  and  $\boldsymbol{\beta}$ . As explained before, the number of iterations in the ellipsoid method is proportional to  $(K + Q + C)^2$  and in each iteration (5.2), (5.3), (5.4) and (5.5) must be calculated for all subcarriers and users. Thus, the computational complexity for determining the dual optimum is  $\mathcal{O}(NC(K + Q)(K + Q + C)^2)$ . The subgradients are the same as the ones in Section 3.1 for updating dual variables.

From (5.2), (5.3), (5.4) and (5.5), some insights to (2.6) are obtained regarding the user dimension and the BS dimension. On one hand, for different users, the transmission power  $P_c$  is allocated to subcarriers at BS  $c \in \{1, \dots, C\}$ . This power allocation is subject to the dual variable  $\beta_c$  and is independent of dual variables  $\{\beta_v | v \neq c\}$  related to other BSs. On the other hand, for an arbitrary MA user  $k$ , its fixed data rate must be satisfied by the  $C$  BSs. The rate allocation to the  $C$  BSs is subject to  $\lambda_k$ . In other words, the sum rate for user  $k$  must be equal to  $R_k$  covered by up to  $C$  BSs, while each rate is related to the dual variable  $\lambda_k$ . The heuristic design will benefit from this property.

## 5.2 Heuristic solutions

Similar to single-cell resource allocation, at the optimum of (2.6), the rate constraints are satisfied with equality for some RA users, while the rates achieved for other RA users are strictly greater than the minimum required rates. In Chapter 3, the heuristic method has been suggested for single-cell heterogeneous resource allocation. It generally consists of two steps. First, the subcarrier assignment is initialized. It is then adjusted successively and iteratively along subcarriers. Within the subcarrier adjustment, each subcarrier is reassigned to different users to improve the weighted sum rate, see Figure 3.3, while the efficient approach GWF is utilized to accelerate this procedure.

In the multicell scenario, the BS dimension is additionally considered. As before, the set  $\mathcal{K}$  contains the RA users, for whom the achieved rates are strictly greater than the minimum required rates. The set  $\mathcal{Q}$  includes all MA users and some primal RA users, for whom only the minimum required rates are reached. A subcarrier may

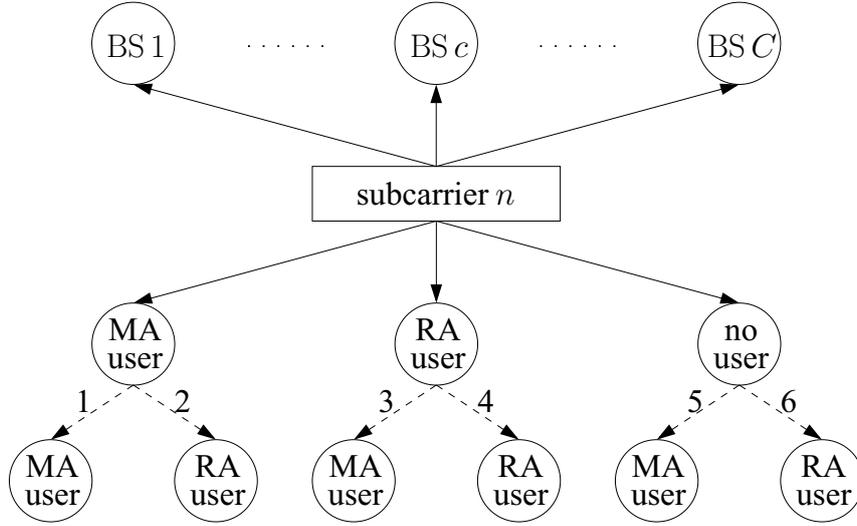


Figure 5.1: Assigning map for subcarrier  $n$ .

be assigned to different users while employed by different BSs, see the upper part of Figure 5.1. Even though it is fixed that subcarrier  $n$  is assigned to a user, it can be adjusted among different BSs to improve the weighted sum rate. The proposed heuristic method for single-cell resource allocation is extended to the multicell resource allocation problem (2.6) in the following.

### 5.2.1 Multicell genetic water-filling

For single-cell resource allocation in Chapter 3, changes on the subcarrier assignment have been divided into two groups. In one group, a subcarrier is excluded from the subcarrier assignment. In the other group, a subcarrier is included in the subcarrier assignment. Efficient approaches have been developed to update the weighted sum rate after such a change by inheriting the previous water-filling solution, so called genetic water-filling (GWF). It can be extended to multicell genetic water-filling (MGWF) for multicell resource allocation.

#### Changing subcarrier assignment for MA users

In single-cell resource allocation, if the subcarrier assignment is fixed, the power and rate allocation for each MA user is formulated as a typical single-user MA resource allocation problem [47]. However, in multicell resource allocation, even though the subcarrier assignment is fixed, the single-user MA resource allocation problem is not typical. As shown by (5.2) and (5.3), the power and rate allocation for MA user  $k$  is subject to the dual variables  $\lambda_k$  and  $\beta$ , since subcarriers assigned to MA user  $k$  may be employed by different BSs. The rate allocation to different BSs is determined by the dual variable  $\lambda_k$  for MA user  $k$ . The power allocation of BS  $c$  is subject to the dual variable  $\beta_c$  for MA user  $k$ . Therefore, if GWF is extended to MGWF for each MA

user, the dual variables associated to the power limits must be fixed while updating the dual variable associated to the minimum required rate. Otherwise, MGWF cannot be efficient.

Given the subcarrier assignment  $\mathcal{S}_{k,1}, \dots, \mathcal{S}_{k,C}$  for MA user  $k \in \mathcal{Q}$  at different BSs, the multicell single-user MA resource allocation problem is extracted from (2.6) as

$$\begin{aligned} & \text{minimize} && \sum_{c=1}^C \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} \\ & \text{subject to} && \sum_{c=1}^C \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} = R_k. \end{aligned} \quad (5.6)$$

The equality in the constraint holds because of the complementary slackness condition. If  $\nu = (\beta \ln(2))^{-1}$  is fixed and the assigned subcarriers in  $\bigcup_{c=1}^C \mathcal{S}_{k,c}$  are allocated with positive power, the dual variable  $\lambda_k$  is obtained as

$$\lambda_k = \left( 2^{R_k} \prod_{c=1}^C \prod_{n \in \mathcal{S}_{k,c}} \frac{1}{\nu_c g_{k,n,c}} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c}}} \quad (5.7)$$

where the term  $\prod_{c=1}^C 1/\nu_c$  is fixed. It is equivalent to that each CNR  $g_{k,n,c}$  is normalized by  $\nu_c$  to  $\nu_c g_{k,n,c}$ .

If subcarrier  $m$  is excluded from the set  $\mathcal{S}_{k,v}$ ,  $v \in \{1, \dots, C\}$ , the corresponding dual variable  $\lambda_k$  becomes

$$\begin{aligned} \lambda_k^{(\text{ex})}(m, v) &= \left( 2^{R_k} \left( \prod_{c=1, c \neq v}^C \prod_{n \in \mathcal{S}_{k,c}} \frac{1}{\nu_c g_{k,n,c}} \right) \prod_{n \in \mathcal{S}_{k,v} \setminus \{m\}} \frac{1}{\nu_v g_{k,n,v}} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c} - 1}} \\ &= \left( \underbrace{\left[ \left( 2^{R_k} \prod_{c=1}^C \prod_{n \in \mathcal{S}_{k,c}} \frac{1}{\nu_c g_{k,n,c}} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c}}} \right]^{\sum_{c=1}^C s_{k,c}}}_{=\lambda_k} \nu_v g_{k,m,v} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c} - 1}} \\ &= \lambda_k (\lambda_k \nu_v g_{k,m,v})^{\frac{1}{\sum_{c=1}^C s_{k,c} - 1}}, \end{aligned} \quad (5.8)$$

while  $\mathcal{S}_{k,v} \setminus \{m\} \neq \emptyset$  holds. The operation of excluding subcarrier  $m$  is still denoted by (ex). The set  $\mathcal{S}_{k,v}$  is the subcarrier assignment before removing subcarrier  $m$ . Since subcarrier  $m$  is allocated with positive power for user  $k$  as  $\lambda_k \nu_v > 1/g_{k,m,v}$  from (5.2) and (5.3),  $(\lambda_k \nu_v g_{k,m,v})^{\frac{1}{\sum_{c=1}^C s_{k,c} - 1}}$  must be greater than 1. Thus, the dual variable  $\lambda_k$  increases after removing subcarrier  $m$  from  $\mathcal{S}_{k,v}$ . The transmission power for MA user  $k$  at BS  $v$  varies by

$$\begin{aligned} \Delta P_{k,v}^{(\text{ex})}(m) &= (s_{k,v} - 1) \left( \lambda_k^{(\text{ex})}(m, v) \nu_v - \sum_{n \in \mathcal{S}_{k,v} \setminus \{m\}} \frac{1}{g_{k,n,v}} \right) - s_{k,v} \left( \lambda_k \nu_v - \sum_{n \in \mathcal{S}_{k,v}} \frac{1}{g_{k,n,v}} \right) \\ &= \left( \lambda_k^{(\text{ex})}(m, v) - \lambda_k \right) \nu_v s_{k,v} - \left( \lambda_k^{(\text{ex})}(m, v) \nu_v - \frac{1}{g_{k,m,v}} \right), \end{aligned} \quad (5.9)$$

while the transmission power for MA user  $k$  at BS  $c \neq v$  varies by

$$\begin{aligned} \forall c \neq v : \Delta P_{k,c}^{(\text{ex})}(m) &= s_{k,c}(\lambda_k^{(\text{ex})}(m,v)\nu_c - \sum_{n \in \mathcal{S}_{k,c}} \frac{1}{g_{k,n,c}}) - s_{k,c}(\lambda_k \nu_c - \sum_{n \in \mathcal{S}_{k,c}} \frac{1}{g_{k,n,c}}) \\ &= (\lambda_k^{(\text{ex})}(m,v) - \lambda_k)\nu_c s_{k,c}. \end{aligned} \quad (5.10)$$

The transmission power for other MA users  $l \neq k$  does not change at each BS. Because of the increased dual variable (5.8), more power is allocated to each of the remaining subcarriers assigned to user  $k$  at all BSs in order to reach the fixed required rate  $R_k$ . The resulting power increments are proportional to the increment of the dual variable  $\lambda_k^{(\text{ex})}(m,v) - \lambda_k$ . A lower transmission rate is achieved for user  $k$  at BS  $v$ , while other BSs  $c \neq v$  transmit faster to MA user  $k$ , seen from (5.9) and (5.10).

If subcarrier  $m$  is included in  $\mathcal{S}_{k,v}$ , i.e., the subcarrier assignment for user  $k$  at BS  $v$ , the dual variable  $\lambda_k$  changes to

$$\begin{aligned} \lambda_k^{(\text{in})}(m,v) &= \left( 2^{R_k} \left( \prod_{c=1, c \neq v}^C \prod_{n \in \mathcal{S}_{k,c}} \frac{1}{\nu_c g_{k,n,c}} \right) \prod_{n \in \mathcal{S}_{k,v} \cup \{m\}} \frac{1}{\nu_v g_{k,n,v}} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c} + 1}} \\ &= \left( \underbrace{\left[ \left( 2^{R_k} \prod_{c=1}^C \prod_{n \in \mathcal{S}_{k,c}} \frac{1}{\nu_c g_{k,n,c}} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c}}} \right]}_{=\lambda_k} \frac{1}{\nu_v g_{k,m,v}} \right)^{\frac{1}{\sum_{c=1}^C s_{k,c} + 1}} \\ &= \lambda_k (\lambda_k \nu_v g_{k,m,v})^{-\frac{1}{\sum_{c=1}^C s_{k,c} + 1}}. \end{aligned} \quad (5.11)$$

Because of (5.2) and (5.3),  $(\lambda_k \nu_v g_{k,m,v})^{-\frac{1}{\sum_{c=1}^C s_{k,c} + 1}}$  in (5.11) must be positive and less than 1. The dual variable  $\lambda_k$  decreases after adding subcarrier  $m$  to  $\mathcal{S}_{k,v}$ . The transmission power for user  $k$  at BS  $v$  changes by

$$\begin{aligned} \Delta P_{k,v}^{(\text{in})}(m) &= (s_{k,v} + 1)(\lambda_k^{(\text{in})}(m,v)\nu_v - \sum_{n \in \mathcal{S}_{k,v} \cup \{m\}} \frac{1}{g_{k,n,v}}) - s_{k,v}(\lambda_k \nu_v - \sum_{n \in \mathcal{S}_{k,v}} \frac{1}{g_{k,n,v}}) \\ &= (\lambda_k^{(\text{in})}(m,v) - \lambda_k)\nu_v s_{k,v} + (\lambda_k^{(\text{in})}\nu_v - \frac{1}{g_{k,m,v}}). \end{aligned} \quad (5.12)$$

The power decrements on subcarriers in  $\mathcal{S}_{k,v}$  are the same as  $(\lambda_k^{(\text{in})}(m,v) - \lambda_k)\nu_v$ . The transmission power for user  $k$  at other BS  $c \neq v$  changes by

$$\begin{aligned} \forall c \neq v : \Delta P_{k,c}^{(\text{in})}(m) &= s_{k,c}(\lambda_k^{(\text{in})}(m,v)\nu_c - \sum_{n \in \mathcal{S}_{k,c}} \frac{1}{g_{k,n,c}}) - s_{k,c}(\lambda_k \nu_c - \sum_{n \in \mathcal{S}_{k,c}} \frac{1}{g_{k,n,c}}) \\ &= (\lambda_k^{(\text{in})}(m,v) - \lambda_k)\nu_c s_{k,c}. \end{aligned} \quad (5.13)$$

The set  $\mathcal{S}_{k,v}$  is the subcarrier assignment before including subcarrier  $m$  and (in) refers to the operation of including a subcarrier. Due to the decrement of the dual variable  $\lambda_k^{(\text{in})}(m,v) - \lambda_k$ , less transmission power is allocated for user  $k$  at other BSs  $c \neq v$ . The power decrements on the subcarriers originally assigned to user  $k$  at all BSs are proportional to the term  $\lambda_k^{(\text{in})}(m,v) - \lambda_k$  in (5.12) and (5.13).

### Changing subcarrier assignment for RA users

The equations above extend GWF to MGWF for the non-typical single-user MA resource allocation problem (5.6). For RA users in  $\mathcal{K}$ , without considering the constraints on minimum required rates, the primal optimization problem is relaxed to

$$\begin{aligned} & \text{maximize} && \sum_{k \in \mathcal{K}} w_k \sum_{c=1}^C \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} && (5.14) \\ & \text{subject to} && \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} = P_c^{(\text{RA})}, \quad c = 1, \dots, C \end{aligned}$$

where  $P_c^{(\text{RA})}$  is the transmission power for the RA users in  $\mathcal{K}$  at BS  $c$ . If the subcarrier assignment is fixed, (5.14) is simply solved by (5.4) and (5.5). The sharing term  $\nu_c = (\beta_c \ln(2))^{-1}$  is obtained via the equality constraint as

$$\nu_c = \frac{P_c^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c}}, \quad c = 1, \dots, C, \quad (5.15)$$

which is similar to (3.15). It shows that  $\nu_1, \dots, \nu_C$  are independent once the subcarrier assignment is fixed. This implies that the change of  $\nu_c$  does not impact other terms  $\nu_l, l \neq c$ . The problem (5.14) is separated into  $C$  independent single-cell RA resource allocation problems as

$$\begin{aligned} & \text{maximize} && \hat{R}_c = \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} && (5.16) \\ & \text{subject to} && \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} = P_c^{(\text{RA})} \end{aligned}$$

for  $c = 1, \dots, C$ . The total achieved weighted sum rate is  $R = \sum_{c=1}^C \hat{R}_c$ , where the weighted sum rate achieved by BS  $c$  is

$$\hat{R}_c = \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c g_{k,n,c}),$$

if all subcarriers in  $\bigcup_{k \in \mathcal{K}} \mathcal{S}_{k,c}$  are allocated with positive power. It is the same as (3.16).

If subcarrier  $m$  is excluded from  $\mathcal{S}_{l,c}, l \in \mathcal{K}, c \in \{1, \dots, C\}$ , only the sharing term  $\nu_c$  changes. It increases to

$$\begin{aligned} \nu_c^{(\text{ex})}(m, l) &= \frac{P_c^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c} - 1/g_{l,m,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c} - w_l} \\ &= \frac{\underbrace{P_c^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c}}_{=\nu_c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c} - w_l} \sum_{k \in \mathcal{K}} w_k s_{k,c} - 1/g_{l,m,c}} \\ &= \frac{\nu_c \sum_{k \in \mathcal{K}} w_k s_{k,c} - 1/g_{l,m,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c} - w_l}. \end{aligned} \quad (5.17)$$

Correspondingly, the weighted sum rate achieved by BS  $c$  decreases by

$$\begin{aligned}
\Delta \hat{R}_c^{(\text{ex})}(m, l) &= \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c^{(\text{ex})}(m, l) g_{k,n,c}) - w_l \log_2(w_l \nu_c^{(\text{ex})}(m, l) g_{l,m,c}) \\
&\quad - \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c g_{k,n,c}) \\
&= \log_2 \left( \frac{\nu_c^{(\text{ex})}(m, l)}{\nu_c} \right) \sum_{k \in \mathcal{K}} w_k s_{k,c} - w_l \log_2(w_l \nu_c^{(\text{ex})}(m, l) g_{l,m,c}), \quad (5.18)
\end{aligned}$$

while the weighted sum rates achieved by other BSs  $v \neq c$  do not change. The rate decrements on the remaining subcarriers are the same as  $\log_2(\nu_c^{(\text{ex})}(m, l)/\nu_c)$ . The rate  $\log_2(w_l \nu_c^{(\text{ex})}(m, l) g_{l,m,c})$  used to be allocated to subcarrier  $m$  for user  $l$  at BS  $c$ .

After including subcarrier  $m$  in  $\mathcal{S}_{l,c}$ , the resulting sharing factor is

$$\begin{aligned}
\nu_c^{(\text{in})}(m, l) &= \frac{P_c^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c} + 1/g_{l,m,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c} + w_l} \\
&= \frac{\frac{P_c^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c}} \sum_{k \in \mathcal{K}} w_k s_{k,c} + 1/g_{l,m,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c} + w_l} \\
&= \frac{\nu_c \sum_{k \in \mathcal{K}} w_k s_{k,c} + 1/g_{l,m,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c} + w_l}. \quad (5.19)
\end{aligned}$$

Consequently, the weighted sum rate provided by BS  $c$  decreases by

$$\begin{aligned}
\Delta \hat{R}_c^{(\text{in})}(m, l) &= \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c^{(\text{ex})}(m, l) g_{k,n,c}) + w_l \log_2(w_l \nu_c^{(\text{ex})}(m, l) g_{l,m,c}) \\
&\quad - \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c g_{k,n,c}) \\
&= \log_2 \left( \frac{\nu_c^{(\text{in})}(m, l)}{\nu_c} \right) \sum_{k \in \mathcal{K}} w_k s_{k,c} + w_l \log_2(w_l \nu_c^{(\text{ex})}(m, l) g_{l,m,c}). \quad (5.20)
\end{aligned}$$

When the subcarrier assignment for an MA user varies, the power and rate allocation for this MA user changes at all BSs. Changing the subcarrier assignment for an RA user only affects the power and rate allocation at that BS, at which the change occurs. As mentioned before, the limit of transmission power at one BS is independent of others and it is the sum power over users and subcarriers. The minimum required rate for one user is the sum rate over subcarriers assigned to it at all BSs.

### Changing transmission power for RA users

If the transmission power for the RA users in  $\mathcal{K}$  varies by  $\Delta P_c^{(\text{RA})}$  at BS  $c$ , the induced sharing factor and variation of the weighted sum rate are

$$\begin{aligned} \nu_c^{(\text{p})}(\Delta P_c^{(\text{RA})}) &= \frac{P_c^{(\text{RA})} + \Delta P_c^{(\text{RA})} + \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c}}{\sum_{k \in \mathcal{K}} w_k s_{k,c}} \\ &= \nu_c + \frac{\Delta P_c^{(\text{RA})}}{\sum_{k \in \mathcal{K}} w_k s_{k,c}} \end{aligned} \quad (5.21)$$

$$\begin{aligned} \Delta \hat{R}_c^{(\text{p})}(\Delta P_c^{(\text{RA})}) &= \sum_{k \in \mathcal{K}} w_k \left( \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c^{(\text{p})}(\Delta P_c^{(\text{RA})}) g_{k,n,c}) - \sum_{n \in \mathcal{S}_{k,c}} \log_2(w_k \nu_c g_{k,n,c}) \right) \\ &= \log_2 \left( \frac{\nu_c^{(\text{p})}(\Delta P_c^{(\text{RA})})}{\nu_c} \right) \sum_{k \in \mathcal{K}} w_k s_{k,c}, \end{aligned} \quad (5.22)$$

respectively. In single-cell resource allocation, GWF is used to update the objective while adjusting a subcarrier among users. In multicell resource allocation, this updating is obtained by using the equations (5.17), (5.18), (5.19), (5.20), (5.21) and (5.22), called MGWF. Additionally, one subcarrier is only employed by at most one BS. MGWF is also utilized for reassigning subcarriers to different BSs.

#### 5.2.2 Three-dimensional water-filling

Given the subcarrier assignment for single-cell resource allocation in Chapter 3, power is allocated to different users and subcarriers. This two-dimensional power and rate allocation is returned by TDWF. It consists of two serial steps. First,  $K+Q$  single-user MA resource allocation problems are independently solved to obtain the transmission power for reaching all minimum required rates. The remaining power is then distributed to RA users  $k = 1, \dots, K$ . However, for multicell resource allocation, the BS dimension must be additionally considered. Even though the subcarrier assignment is fixed, power must be allocated over three dimensions, i.e., different users, subcarriers and BSs. As explained before, the power and rate allocated to each subcarrier are subject to at most two dual variables, see (5.2), (5.3), (5.4) and (5.5). The optimal dual variables cannot be derived by a procedure similar to TDWF. In the following, the ellipsoid method is employed to determine the optimal dual variables in  $\beta^*$  associated to the limits of transmission power at  $C$  BSs, when the subcarrier assignment is fixed.

From the analysis of the dual optimum and MGWF, the power and rate allocation is subject to the dual variable  $\beta_c$  associated with the power limit at BS  $c$ . If the subcarrier assignment is fixed and  $\nu$  is given, the dual variables  $\lambda$  are explicitly determined via (5.7). For an RA user  $k$ , if  $\lambda_k \geq w_k$  holds for the derived  $\lambda_k$ , the water level of user  $k$  is  $\mu_{k,c} = \lambda_k \nu_c$  at BS  $c$ . Otherwise, the water level is  $w_k \nu_c$ . Through this comparison, users are divided into two sets  $\mathcal{K}$  and  $\mathcal{Q}$ . Only the minimum required rates are reached for users in  $\mathcal{Q}$ , while the rates achieved for users in  $\mathcal{K}$  are strictly greater than the minimum required rates.

Given the subcarrier assignment and  $\nu$ , the transmission power required at BS  $c$  is

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} \left( w_k \nu_c - \frac{1}{g_{k,n,c}} \right) + \sum_{k \in \mathcal{Q}} \sum_{n \in \mathcal{S}_{k,c}} \left( \lambda_k \nu_c - \frac{1}{g_{k,n,c}} \right)$$

in order to satisfy the constraints on the minimum required rates. According to the complementary slackness condition, the power constraint is met with equality. The subgradient for updating  $\beta_c$  is

$$P_c - \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_{k,c}} \left( w_k \nu_c - \frac{1}{g_{k,n,c}} \right) - \sum_{k \in \mathcal{Q}} \sum_{n \in \mathcal{S}_{k,c}} \left( \lambda_k \nu_c - \frac{1}{g_{k,n,c}} \right)$$

for  $c = 1, \dots, C$ . Then,  $\nu$  can be updated by the ellipsoid method till all power constraints are satisfied with equality. With the derived optimal  $\nu^*$ , TDWF is finally utilized to determine the power and rate allocation over the given subcarrier assignment at each BS. The  $C$  optimal dual variables must be determined by the ellipsoid method. Thus, three-dimensional water-filling has complexity  $\mathcal{O}(NC^2)$ . Exhaustive search including  $((K + Q)C)^N$  searches is performed to find the optimal subcarrier assignment. Hence, the complexity for determining the optimal power and rate allocation is  $\mathcal{O}(NC^2((K + Q)C)^N)$ . This implies that the optimal multicell heterogeneous resource allocation is computationally intractable even for simulations.

### 5.2.3 Heuristic extension

In the previous subsection, MGWF and three-dimensional water-filling have been obtained for multicell heterogeneous resource allocation. They will be used to provide a heuristic solution for (2.6). The heuristic method for single-cell heterogeneous resource allocation is extended in the following.

#### Initialization

In Chapter 3, the subcarrier assignment is initialized in two steps. First, the number of subcarriers assigned to each user is evaluated by Algorithm 3. According to these evaluated numbers, specific subcarriers are then assigned to users in Algorithm 4. To directly use these two algorithms, the multicell resource allocation problem is reduced to a single-cell resource allocation problem by using the substitutions below. In this virtual single cell, users have CNRs

$$g_{k,n} = \max_{c=1, \dots, C} g_{k,n,c}, \quad n = 1, \dots, N, \quad k = 1, \dots, K + Q.$$

The transmission power is limited to  $P = \sum_{c=1}^C P_c$  in the virtual single cell. With these two substitutions, Algorithm 3 can be directly used to evaluate the number of subcarriers  $s_k$  assigned to each user  $k = 1, \dots, K + Q$ , which is the sum  $s_k = \sum_{c=1}^C s_{k,c}$ .

According to the evaluated numbers of subcarriers  $s_1, \dots, s_{K+Q}$ , specific subcarriers are allocated to users in Algorithm 4. Before using Algorithm 4, let us consider the

**Algorithm 10** Initialization for multicell subcarrier assignments

- 
- 1: **Part 1:** assignment of  $C$  subcarriers
  - 2:  $\mathcal{N} \leftarrow \{1, \dots, N\}$
  - 3:  $\mathcal{S}_{k,c} \leftarrow \emptyset, \quad c = 1, \dots, C, \quad k = 1, \dots, K + Q$
  - 4: **for**  $c = 1, \dots, C$  **do**
  - 5:    $(l, m) \leftarrow \operatorname{argmax}_{k=1, \dots, K, n \in \mathcal{N}} g_{k,n,c}$
  - 6:    $\mathcal{S}_{l,c} \leftarrow \{m\}$
  - 7:    $\mathcal{N} \leftarrow \mathcal{N} \setminus \{m\}$
  - 8: **end for**
  
  - 9: **Part 2:** assignment of the remaining  $N - C$  subcarriers
  - 10:  $(\mathcal{S}_1, \dots, \mathcal{S}_{K+Q}) \leftarrow$  Algorithm 4
  - 11: **for**  $k = 1, \dots, K + Q$  **do**
  - 12:   **for**  $n \in \mathcal{S}_k$  **do**
  - 13:      $v \leftarrow \operatorname{argmax}_{c=1, \dots, C} g_{k,n,c}$
  - 14:      $\mathcal{S}_{k,v} \leftarrow \mathcal{S}_{k,v} \cup \{n\}$
  - 15:   **end for**
  - 16: **end for**
  - 17:  $(\mathcal{K}, \mathcal{Q}, \nu_1, \dots, \nu_C, \lambda_1, \dots, \lambda_{K+Q}) \leftarrow$  three-dimensional water-filling
- 

following problem. If BS  $c$  does not cover any RA user in  $\mathcal{K}$ , the term  $\sum_{k \in \mathcal{K}} w_k s_{k,c}$  is equal to zero in (5.17) and (5.19). This leads to inconvenience for utilizing MGWF. Hence, we stipulate that at least one subcarrier is assigned to RA users at each BS. With this stipulation, Algorithm 4 is revised to Algorithm 10. After deriving the evaluated numbers of subcarriers assigned to users from Algorithm 3,  $C$  subcarriers separately from  $C$  BSs are first assigned to RA users in part 1. At each BS, the available subcarrier with the greatest CNR is assigned to the corresponding RA user, see line 5. After that Algorithm 4 is employed to allocate subcarriers to users according to the evaluated numbers of subcarriers, see line 10. Till now, subcarriers are specifically assigned to different users in the virtual single cell. Finally, each subcarrier is only employed by the BS, at which it has the greatest CNR, see line 13 and 14. Then, the initial subcarrier assignment  $\{\mathcal{S}_{k,c} | k = 1, \dots, K + Q, c = 1, \dots, C\}$  is obtained. Obviously, the complexity of Algorithm 10 is still linear in the number of users, subcarriers and BSs as  $\mathcal{O}((K + Q)NC)$ .

With the initialized subcarrier assignment, three-dimensional water-filling is executed to give initial values of  $\boldsymbol{\lambda}$  and  $\boldsymbol{\nu}$  for using MGWF. Alternatively, three-dimensional water-filling can be replaced by the following two steps. First, the primal heterogeneous resource allocation problem (2.6) is relaxed to the  $C$  individual RA resource allocation problems as the problem (5.14) without considering the rate constraints in order to initialize  $\boldsymbol{\nu}$ . With the initialized  $\boldsymbol{\nu}$ ,  $\boldsymbol{\lambda}$  is then given by independently solving the  $K + Q$  non-typical MA resource allocation problems as (5.6). The sets  $\mathcal{K}$  and  $\mathcal{Q}$  are consequently obtained.

### Subcarrier adjustment

In the heuristic method for single-cell resource allocation in Chapter 3, subcarriers are iteratively and successively adjusted among users in Algorithm 6, where GWF is used. Within this subcarrier adjustment, each subcarrier is reassigned to different users to investigate whether the weighted sum rate can be improved. When one subcarrier is reassigned, the assignment of others is fixed. Additionally, two techniques, i.e., sorting subcarriers and controlling iterations, have been used to improve performance while reducing computing time. Algorithm 6 is extended to multicell resource allocation by utilizing MGWF as follows.

Compared to unicasting by a single BS, one subcarrier may be also adjusted among different BSs even for one user in unicasting by multiple BSs, see Figure 5.1. The modified heuristic method is given in Algorithm 11. Subcarriers are adjusted among users and BSs with MGWF iteratively (see line 1) and successively (see lines 3 and 11). Different from unicasting by a single BS, the two remarks below must be additionally taken in account while implementing Algorithm 11.

- The denominators in (5.17), (5.19) and (5.21) must remain positive for utilizing MGWF. Thus, if only one subcarrier is assigned to RA users in  $\mathcal{K}$  at one BS, adjusting this subcarrier is skipped.
- While adjusting subcarriers at one BS, the limits of transmission power at some BSs may be violated. Hence, if such a case occurs, the associated adjustment is also skipped.

In the heuristic method for single-cell resource allocation, two additional techniques are embedded into the iterative successive subcarrier adjustment to improve performance and reduce computing time. In Chapter 3, subcarriers are adjusted following the order determined by (3.29) and (3.30) to make subcarrier assignment more effective. This technique can be also used here. Subcarriers are adjusted following the descending order of variations of potential rates allocated to subcarriers, see line 2. The variation of potential rates allocated to the  $n$ th subcarrier at BS  $c$  is defined below. First, the subcarrier index  $\varphi_{k,n,c}$  is redefined as

$$\varphi_{k,n,c} = \begin{cases} 1, & \nu_c w_k \geq 1/g_{k,n,c} \\ 0, & \nu_c w_k < 1/g_{k,n,c} \end{cases}, \quad k \in \mathcal{K}, n = 1, \dots, N$$

$$\varphi_{k,n,c} = \begin{cases} 1, & \lambda_k \nu_c \geq 1/g_{k,n,c} \\ 0, & \lambda_k \nu_c < 1/g_{k,n,c} \end{cases}, \quad k \in \mathcal{Q}, n = 1, \dots, N.$$

for  $c = 1, \dots, C$ . The variation of potential rates is then obtained with respect to users and BSs as

$$\sigma_{\bar{r}}[n] = \frac{1}{\sum_{c=1}^C \sum_{k=1}^{K+Q} \varphi_{k,n,c}} \left( \sum_{c=1}^C \sum_{k \in \mathcal{K}} \varphi_{k,n,c} |\log_2(\nu_c w_k g_{k,n,c}) - \bar{r}_n| \right. \\ \left. + \sum_{c=1}^C \sum_{k \in \mathcal{Q}} \varphi_{k,n,c} |\log_2(\nu_k \lambda_k g_{k,n,c}) - \bar{r}_n| \right), \quad n = 1, \dots, N \quad (5.23)$$

**Algorithm 11** Multicell ISSA-SIC

---

```

1: for  $i = 1, \dots, I$  do
2:    $\{o_1, \dots, o_N\} \leftarrow$  descending order of  $\{\sigma_{\bar{r}}[1], \dots, \sigma_{\bar{r}}[N]\}$  from (3.30)
3:   for  $j = 1, \dots, N/2$  do
4:     if  $\sum_{k=1}^{K+Q} P_{k,c} > P_c, c = 1, \dots, C$  then
5:       minimize  $\sum_{k=1}^{K+Q} P_{k,c} \leftarrow$  adjusting  $o_j$  among users and BSs with MGWF
6:     else
7:       maximize  $\sum_{c=1}^C \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \leftarrow$  adjusting  $o_j$  among users and
          BSs with MGWF
8:     end if
9:   end for
10:   $(\hat{R}^{(i)}, \mathcal{K}, \mathcal{Q}, \boldsymbol{\nu}, \boldsymbol{\lambda}) \leftarrow$  TDWF with current  $\boldsymbol{\nu}$  and  $\boldsymbol{\lambda}$ 

11:  for  $j = N/2 + 1, \dots, N$  do
12:    if  $\sum_{k=1}^{K+Q} P_{k,c} > P_c, c = 1, \dots, C$  then
13:      minimize  $\sum_{k=1}^{K+Q} P_{k,c} \leftarrow$  adjusting  $o_j$  among users and BSs with MGWF
14:    else
15:      maximize  $\sum_{c=1}^C \sum_{k \in \mathcal{K}} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \leftarrow$  adjusting  $o_j$  among users and
          BSs with MGWF
16:    end if
17:  end for
18:   $(R^{(i)}, \mathcal{K}, \mathcal{Q}, \boldsymbol{\nu}, \boldsymbol{\lambda}) \leftarrow$  TDWF with current  $\boldsymbol{\nu}$  and  $\boldsymbol{\lambda}$ 
19:  if  $\frac{|\hat{R}^{(i)} - R^{(i)}|}{\hat{R}^{(i)}} \leq \rho$  then
20:    break
21:  end if
22: end for

```

---

where the average potential rate is

$$\bar{r}_n = \frac{1}{\sum_{k=1}^{K+Q} \varphi_{k,n,c}} \left( \sum_{k \in \mathcal{K}} \varphi_{k,n,c} \log_2(\nu_c w_k g_{k,n,c}) + \sum_{k \in \mathcal{Q}} \varphi_{k,n,c} \log_2(\lambda_k \nu_c g_{k,n,c}) \right).$$

Note that  $\boldsymbol{\nu}$  is only updated as the subcarrier assignment for RA users in  $\mathcal{K}$  changes.

The other technique for reducing computing time is the iteration control, see lines 10, 18 and 19. In the  $i$ th iteration, first,  $N/2$  subcarriers are adjusted and the resulting weighted sum rate is denoted by  $\hat{R}^{(i)}$ . The latter  $N/2$  subcarriers are then adjusted and the new weighted sum rate is  $R^{(i)}$ . The subcarrier adjustment terminates when

$$\frac{|\hat{R}^{(i)} - R^{(i)}|}{\hat{R}^{(i)}} \leq \rho$$

is satisfied, where  $\rho \in (0, 1)$  is given by the transmission system.

Without these two additional techniques, multicell iterative and successive subcarrier adjustment (ISSA) has complexity  $\mathcal{O}((K+Q)NC)$ , called multicell ISSA, since



Compared to the primal multicell resource allocation problem (2.6), the last constraint in (5.24) is additionally considered for the new scenario. The set  $\mathcal{S}_{k,c}$  contains the  $s_{k,c}$  subcarriers assigned to user  $k$  at BS  $c$ . One subcarrier is assigned to only one pair of BS and user, explained by the second last constraint. It can also be expressed as  $\mathcal{S}_{k,c} \cap \mathcal{S}_{l,v} = \emptyset$  for any  $(k,c)' \neq (l,v)'$ . Additionally, one user is covered by only one BS at any specific period of time, expressed by the last constraint. In other words, only one of  $\mathcal{S}_{k,1}, \dots, \mathcal{S}_{k,C}$  is not empty for user  $k = 1, \dots, K + Q$ . The set  $\mathcal{Q}$  contains the users, for whom only the minimum required rates are reached, while other users are in  $\mathcal{K}$ . Users covered by BS  $c$  are in the set  $\mathcal{U}_c$ . The equalities  $\mathcal{U}_c \cap \mathcal{U}_v = \emptyset$  for any  $c \neq v$  and  $\bigcup_{c=1}^C \mathcal{U}_c = \{1, \dots, K + Q\}$  hold.

The combinatorial problem above can be viewed as consisting of two problems. On one hand, users are covered by different BSs separately, i.e., BS selection. On the other hand, different subcarriers are separately assigned to BSs, i.e., subcarrier assignment. Obviously, these two problems affect each other. With the KKT conditions, the Lagrangian of the new resource allocation problem is

$$L_D(\boldsymbol{\lambda}, \boldsymbol{\beta}) = \sum_{k=1}^{K+Q} \lambda_k R_k - \sum_{c=1}^C \beta_c P_c + \inf_{\mathbf{A} \in \mathbb{R}_+^{N \times C(K+Q)}, \mathbf{B} \in \mathbb{R}_+^{N \times C(K+Q)}} \sum_{k=1}^{K+Q} \sum_{n=1}^N \sum_{c=1}^C (\beta_c p_{k,n,c} - w_k a_{k,n,c} - \lambda_k b_{k,n,c}).$$

If the BS selection  $\mathcal{U}_1, \dots, \mathcal{U}_C$  are determined, subcarriers are still separable as before (5.1) and the Lagrange dual function above is further written as

$$L_D(\boldsymbol{\lambda}, \boldsymbol{\beta}) = \sum_{k=1}^{K+Q} \lambda_k R_k - \sum_{c=1}^C \beta_c P_c + \sum_{n=1}^N \inf_{c=1, \dots, C, k \in \mathcal{U}_c} \inf_{a_{k,n,c}, b_{k,n,c} \in \mathbb{R}_+^2} (\beta_c p_{k,n,c} - w_k a_{k,n,c} - \lambda_k b_{k,n,c}).$$

As explained in Section 5.1, the subcarrier assignment can be determined by the ellipsoid method. Thus, given the BS selection, the complexity of determining a dual optimum is  $\mathcal{O}(N(K+Q)(K+Q+C)^2)$ . The optimal BS section must be determined by exhaustive search, where  $C^{K+Q}$  searches must be performed, since one user is served by one of  $C$  BSs. Thus, the complexity of determining the dual optimum of (5.24) is  $\mathcal{O}(N(K+Q)(K+Q+C)^2 C^{K+Q})$ . Obviously, it is computationally intractable for OFDM systems with a large number of users and BSs. Hence, heuristic solutions are required to achieve a good balance between performance and complexity.

The essential difference between (5.24) and (2.6) is that users are separately served by BSs. Given the subcarrier assignment, three-dimensional water-filling must be performed to determine the power and rate allocation for (2.6). However, for the new problem (5.24), the power and rate allocation can be efficiently determined by TDWF when the subcarrier assignment and the BS selection are fixed, explained as follows.

RA users and MA users served by BS  $c$  are contained in  $\mathcal{K} \cap \mathcal{U}_c$  and  $\mathcal{Q} \cap \mathcal{U}_c$ , respectively. Because of  $\mathcal{K} \cup \mathcal{Q} = \{1, \dots, K + Q\}$ ,  $(\mathcal{K} \cap \mathcal{U}_c) \cup (\mathcal{Q} \cap \mathcal{U}_c) = \mathcal{U}_c$  always holds.

One user is only served by one BS, as  $\mathcal{U}_c \cap \mathcal{U}_v = \emptyset$  for any  $c \neq v$ . When the subcarrier assignment and the BS selection are fixed, the power and rate allocation at BS  $c$  is not related to that at other BSs and (5.24) is therefore separated into  $C$  single-cell heterogeneous resource allocation problems as

$$\begin{aligned} & \text{maximize} && \hat{R}_c = \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} && (5.25) \\ & \text{subject to} && \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \geq R_k, \quad \forall k \in \mathcal{U}_c \\ & && \sum_{k \in \mathcal{U}_c} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} \leq P_c \end{aligned}$$

for  $c = 1, \dots, C$ . It follows that GWF for single-cell resource allocation can be directly used here. For the heuristic design, note that each BS must serve at least one RA user in  $\mathcal{K}$ , as  $\mathcal{K} \cap \mathcal{U}_c \neq \emptyset$  for all  $c = 1, \dots, C$ . Otherwise, the remaining power cannot be allocated to any RA user after reaching the minimum required rates.

### 5.3.1 Revised genetic water-filling

To extend the heuristic method from Chapter 3 to (5.24), GWF is simply revised in the following. Given the non-empty subcarrier assignment  $\mathcal{S}_{k,c}$  for MA user  $k \in \mathcal{Q} \cap \mathcal{U}_c$  served by BS  $c$ , the typical single-user MA problem is extracted from (5.24) as

$$\begin{aligned} & \text{minimize} && \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} \\ & \text{subject to} && \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} = R_k. \end{aligned}$$

The equality constraint must hold due to the complementary slackness condition. If all subcarriers are allocated with positive power, the minimum transmission power is

$$P_{k,c} = \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} = s_{k,c} \mu_k - \sum_{n \in \mathcal{S}_{k,c}} \frac{1}{g_{k,n,c}}$$

where the water level  $\mu_k$  is obtained as

$$\mu_k = 2^{\frac{R_k}{s_{k,c}}} \left( \prod_{n \in \mathcal{S}_{k,c}} \frac{1}{g_{k,n,c}} \right)^{\frac{1}{s_{k,c}}}.$$

After excluding subcarrier  $m$  from  $\mathcal{S}_{k,c}$  or including subcarrier  $m$  in  $\mathcal{S}_{k,c}$ , the new water levels and the resulting power variations are

$$\begin{aligned} \mu_{k,c}^{(\text{ex})}(m) &= \mu_k (\mu_k g_{k,m,c})^{\frac{1}{s_{k,c}-1}} \\ \Delta P_{k,c}^{(\text{ex})}(m) &= s_{k,c} (\mu_k^{(\text{ex})}(m) - \mu_k) - (\mu_k^{(\text{ex})}(m) - 1/g_{k,m,c}) \end{aligned}$$

and

$$\begin{aligned}\mu_{k,c}^{(\text{in})}(m) &= \mu_k(\mu_k g_{k,m,c})^{-\frac{1}{s_{k,c}+1}} \\ \Delta P_{k,c}^{(\text{in})}(m) &= s_{k,c} (\mu_k^{(\text{in})}(m) - \mu_k) + (\mu_k^{(\text{in})}(m) - 1/g_{k,m,c}),\end{aligned}$$

respectively. In the equations above, (ex) and (in) still indicate the operations of excluding and including a subcarrier, respectively.

By omitting the constraints on the minimum required rates, (5.25) is relaxed to the single-cell multiuser RA problem as

$$\begin{aligned}\text{maximize} \quad & \hat{R}_c = \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_k} r_{k,n,c} \\ \text{subject to} \quad & \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} \sum_{n \in \mathcal{S}_k} p_{k,n,c} = P_c^{(\text{RA})}\end{aligned}$$

given  $\mathcal{S}_{k,c}$  and  $\mathcal{U}_c$  for  $c = 1, \dots, C$ . The power constraint above must be met with equality due to the complementary slackness condition, where  $P_c^{(\text{RA})} = P_c - \sum_{k \in \mathcal{Q} \cap \mathcal{U}_c} P_{k,c}$  is the transmission power for RA users in  $\mathcal{K}$  at BS  $c$ . It is solved as

$$\hat{R}_c = \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_{k,c}} \log_2(\nu_c w_k g_{k,n,c})$$

where  $\nu_c w_k$  is the water level and is determined by the power limit, given as

$$\nu_c = \frac{P_c^{(\text{RA})} + \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} \sum_{n \in \mathcal{S}_{k,c}} 1/g_{k,n,c}}{\sum_{k \in \mathcal{K} \cap \mathcal{U}_c} s_{k,c} w_k}.$$

The weighted sum rate achieved by all BSs is  $\sum_{c=1}^C \hat{R}_c$ . Given the BS selection and the subcarrier assignment,  $\hat{R}_1, \dots, \hat{R}_C$  are separable.

After excluding subcarrier  $m$  from  $\mathcal{S}_{k,c}$  for RA user  $k \in \mathcal{K} \cap \mathcal{U}_c$ ,  $\nu_c$  becomes

$$\nu_c^{(\text{ex})}(m, k) = \frac{\nu_c \sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c} - 1/g_{k,m,c}}{\sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c} - w_k}.$$

The resulting decrement of the weighted sum rate is

$$\Delta \hat{R}_c^{(\text{ex})}(m, k) = \log_2 \left( \frac{\nu_c^{(\text{ex})}(m, k)}{\nu_c} \right) \sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c} - w_k \log_2 (\nu_c^{(\text{ex})}(m, k) w_k g_{k,m,c}).$$

Alternatively, after including subcarrier  $m$  in  $\mathcal{S}_{k,c}$  for RA user  $k$  in  $\mathcal{K} \cap \mathcal{U}_c$ ,  $\nu_c$  turns to

$$\nu_c^{(\text{in})}(m, k) = \frac{\nu_c \sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c} + 1/g_{k,m,c}}{\sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c} + w_k}.$$

The variation of the weighted sum rate is derived as

$$\Delta \hat{R}_c^{(\text{in})}(m, k) = \log_2 \left( \frac{\nu_c^{(\text{in})}(m, k)}{\nu_c} \right) \sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c} + w_k \log_2 (\nu_c^{(\text{in})}(m, k) w_k g_{k,m,c}).$$

**Algorithm 12** Initialization for BS selection

- 
- 1: **Step 1:**
  - 2:  $\bar{g}_{k,c} \leftarrow$  average of  $N/2$  greatest CNRs of user  $k$  by BS  $c, k = 1, \dots, K, c = 1, \dots, C$
  - 3: **for**  $c = 1, \dots, C$  **do**
  - 4:   the non-assigned RA user with the greatest  $\bar{g}_{k,c}$  is served by BS  $c$
  - 5: **end for**
  
  - 6: **Step 2:**
  - 7:  $\bar{g}_{k,c} \leftarrow$  average of the  $N/(K + Q)$  greatest CNRs of user  $k$   
by BS  $c, k = 1 + K, \dots, K + Q, c = 1, \dots, C$
  - 8: each non-assigned user is served by the BS, where it has the greatest  $\bar{g}_{k,c}$
- 

When the transmission power for RA users at BS  $c$  varies by  $\Delta P^{(\text{RA})}$ , the resulting variation of the weighted sum rate is

$$\Delta \hat{R}_c^{(\text{p})}(\Delta P^{(\text{RA})}) = \log_2 \left( \frac{\nu_c^{(\text{p})}(\Delta P_c^{(\text{RA})})}{\nu_c} \right) \sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c}$$

where (p) indicates the operation of changing the transmission power for RA users and where the sharing term becomes

$$\nu_c^{(\text{p})}(\Delta P_c^{(\text{RA})}) = \nu_c + \frac{\Delta P^{(\text{RA})}}{\sum_{l \in \mathcal{K} \cap \mathcal{U}_c} w_l s_{l,c}}.$$

With the revised GWF, Algorithm 11 can be easily modified to solve (5.24).

### 5.3.2 Joint BS selection and subcarrier assignment

BS selection must be considered in the heuristic design for the new resource allocation problem (5.24). To achieve a good balance between BS selection and subcarrier assignment, they must be jointly performed. In general, a joint signal processing is achieved by some iterative procedure in order to keep its output reliable and its complexity low. For example, joint channel estimation and data detection is realized by the iteration between channel estimation and data detection, e.g., [82]. Similarly, joint BS selection and subcarrier assignment can also be provided for (5.24) by the following iterative process. The revised GWF is used to update the weighted sum rate after changing the subcarrier assignment. Before that, the BS selection and the subcarrier assignment must be initialized.

#### Initialization for BS selection

Different from the initialization for solving (2.6), the BS selection must be additionally initialized. The starting point of BS selection and subcarrier assignment is provided by Algorithm 12 and Algorithm 13. After that the BS selection and the subcarrier assignment are iteratively adjusted.

**Algorithm 13** Initialization for subcarrier assignment

- 
- 1: **Step 1:**
  - 2:  $s_1, \dots, s_{K+Q} \leftarrow$  Algorithm 3
  
  - 3: **Step 2:**
  - 4:  $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q} \leftarrow$  Algorithm 4
  - 5:  $\mathcal{S}_{k,c} \leftarrow \mathcal{S}_k, \quad k \in \mathcal{U}_c$
- 

The BS selection for (5.24) is initialized by two sequential steps in Algorithm 12. Seen from the  $C$  parallel problems (5.25), if no RA user is served by one BS, the rate for RA users cannot be maximized at this BS, even though the remaining power is positive after satisfying the constraints on the minimum required rates. Thus, each BS should serve at least one RA user as  $\mathcal{K} \cap \mathcal{U}_c \neq \emptyset$  for all  $c = 1, \dots, C$ . To ensure this, one RA user is assigned to each BS in the first step, see line 4 in Algorithm 12. Among the RA users that are not covered by any BS, the one who has the greatest average CNR at BS  $c$  is included in  $\mathcal{U}_c$ . In the second step, each remaining user is covered by the BS, from which it has the best channel condition, see lines 7 and 8.

In previous works [41], the average CNR of  $N$  subcarriers is used to assess channel quality for each user at each BS. Here, only some subcarriers are used for each user. On one hand, when the channel of one user is very frequency selective that means CNR variation is large with respect to subcarriers, the subcarriers with low CNRs are not used for this user in general due to the user diversity. Thus, for evaluating channel qualities from different BSs, we empirically employ the average of the greatest  $N/2$  CNRs and the average of the  $N/(K+Q)$  greatest CNRs, instead of all  $N$  CNRs, for each RA and MA user, respectively. The impact of low CNRs is suppressed. On the other hand, if the frequency selectivity is small for this user, the average of the  $N/2$  or  $N/(K+Q)$  greatest CNRs is similar to the average of  $N$  CNRs. Finally, the BS selection  $\mathcal{U}_1, \dots, \mathcal{U}_C$  is initialized in Algorithm 12.

**Initialization for subcarrier assignment**

Since users are separated to different BSs in Algorithm 12,  $\mathcal{U}_c \cap \mathcal{U}_v = \emptyset$  always holds for any  $c \neq v$ . We denote by  $s_k$  the number of subcarriers assigned to user  $k$  without regarding BSs for all  $k = 1, \dots, K+Q$ . It follows that

$$s_k = \sum_{c=1}^C s_{k,c},$$

while only one of  $s_{k,1}, \dots, s_{k,C}$  is positive and others are zero, since one user is only served by one BS. As explained before, the multicell resource allocation problem (5.24) can be viewed as a single-cell resource allocation when the BS selection is fixed. In other words, given the BS selection, multicell subcarrier assignment is reduced to a single-cell case. Thus, previous algorithms of the initialization for single-cell resource allocation are used in Algorithm 13.

**Algorithm 14** Joint BS selection and subcarrier adjustment

---

```

1: for  $i = 1, \dots, I$  do
2:    $\{o_1, \dots, o_N\} \leftarrow$  descending order of  $\{\sigma_{\bar{r}}[1], \dots, \sigma_{\bar{r}}[N]\}$  from (3.30)
3:    $(\mathcal{K}, \mathcal{Q}) \leftarrow$  independent water-filling with  $\mathcal{S}_{k,c}, \mathcal{U}_c$ 

4:   for each  $j = 1, \dots, N/2$  do
5:     if  $\sum_{k \in \mathcal{K} \cap \mathcal{U}_c} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} > P_c$  then
6:       minimize  $\sum_{k \in \mathcal{K} \cap \mathcal{U}_c} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} \leftarrow$  adjusting  $o_j$  among users by GWF
7:     else
8:       maximize  $\sum_{c=1}^C \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \leftarrow$  adjusting  $o_j$  among users by GWF
9:     end if
10:  end for

11:  for each  $k = 1, \dots, K + Q$  do
12:    maximize  $\hat{R}^{(i)} = \sum_{c=1}^C \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \leftarrow$  adjusting  $k$  among BSs by TDWF
13:  end for

14:  for each  $j = N/2 + 1, \dots, N$  do
15:    if  $\sum_{k \in \mathcal{K} \cap \mathcal{U}_c} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} > P_c$  then
16:      minimize  $\sum_{k \in \mathcal{K} \cap \mathcal{U}_c} \sum_{n \in \mathcal{S}_{k,c}} p_{k,n,c} \leftarrow$  adjusting  $o_j$  among users by GWF
17:    else
18:      maximize  $\sum_{c=1}^C \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \leftarrow$  adjusting  $o_j$  among users by GWF
19:    end if
20:  end for

21:  for each  $k = 1, \dots, K + Q$  do
22:    maximize  $R^{(i)} = \sum_{c=1}^C \sum_{k \in \mathcal{K} \cap \mathcal{U}_c} w_k \sum_{n \in \mathcal{S}_{k,c}} r_{k,n,c} \leftarrow$  adjusting  $k$  among BSs by TDWF
23:  end for

24:  if  $(R^{(i)} - \hat{R}^{(i)}) / \hat{R}^{(i)} \leq \rho$  then
25:    break
26:  end if
27: end for

```

---

In the first step, the transmission power is limited to  $P = \sum_{c=1}^C P_c$  in the virtual single cell. We assume that all subcarriers of each user have the same CNR as the average of their CNRs at the BS that this user is assigned to. For example, if  $k \in \mathcal{U}_c$  holds, the average CNR for user  $k$  is

$$g_k = \frac{1}{N} \sum_{n=1}^N g_{k,n,c}.$$

Algorithm 3 without any modification returns  $s_1, \dots, s_{K+Q}$ , see line 2 in Algorithm 13.

In the second step, CNRs in the virtual single cell are  $g_{k,n} = g_{k,n,c}, n = 1, \dots, N$ , if  $k \in \mathcal{U}_c$  holds. With the evaluated cardinalities  $s_1, \dots, s_{K+Q}$  from the first step, specific subcarriers are simply assigned to users in the second step in the same way as Algorithm 4. The subcarrier assignments  $\mathcal{S}_1, \dots, \mathcal{S}_{K+Q}$  are obtained, see line 4.

Since the BS selection is fixed, only one of  $\mathcal{S}_{k,1}, \dots, \mathcal{S}_{k,C}$  is not empty for each user  $k$ . Thus, the derived subcarrier assignment from Algorithm 4 is the initialized subcarrier assignment for (5.24), as  $\mathcal{S}_{k,c} = \mathcal{S}_k$  if  $k \in \mathcal{U}_c$  and otherwise  $\mathcal{S}_{k,c} = \emptyset$ , see line 5.

### Joint BS and subcarrier adjustment

After the initialization above, TDWF from Chapter 3 is independently performed at BSs. The sets  $\mathcal{K}$  and  $\mathcal{Q}$  are then determined. Note that  $\mathcal{Q}$  may include some primal RA users as before. These two sets remain constant within the inner loop in Algorithm 14.

First, subcarriers are successively adjusted following the descending order of  $\sigma_{\bar{r}}[1], \dots, \sigma_{\bar{r}}[N]$ , which is redefined by (5.23) for unicasting by multiple BSs, see lines 2, 4 and 16. Each subcarrier is reassigned to different users to see if the weighted sum rate can be improved. In this successive procedure, if the power limits are violated, the sum power for reaching all minimum required rates must be minimized by adjusting subcarriers among users and by adjusting users among BSs, see lines 6 and 18. Otherwise, the weighted sum rate is maximized, see lines 8 and 20. While adjusting one subcarrier among users, the revised GWF is utilized to accelerate this adjustment.

We then investigate if the weighted sum rate can be improved by selecting different BSs for each user, while the subcarrier assignment is fixed. TDWF is used to assess this user adjustment, see lines 14 and 27. These two serial procedures repeat  $I$  times. If the previous subcarrier sorting and iteration control are applied, we call Algorithm 14 joint BS selection and subcarrier assignment with sorting and iteration control (JBSA-SIC). Otherwise, JBSA with  $I$  iterations is given. Obviously, Algorithm 14 has the same complexity as Algorithm 11 as  $\mathcal{O}((K + Q)N \log(N)C)$ .

## 5.4 Simulation results

In this section, the solutions offered for (2.6) and (5.24) are compared by simulations. The transmission power is limited to 20 dBW at each BS. Each required rate is distributed independently uniformly within [10, 20] bits per OFDM symbol. Each weight is distributed independently uniformly within [1, 10]. Their sum is normalized to one. The multipath Rayleigh fading channel is simulated with  $N/8$  independent paths with an exponentially decaying profile. In simulations,  $C$  BSs are equally located on the circle with radius  $D$ . In polar coordinates, user's location is expressed by the pair of the angular coordinate  $\theta$  and the radial coordinate  $\gamma$ , while  $\theta$  and  $\gamma$  are distributed uniformly within  $[0^\circ, 360^\circ)$  and within  $[0, 1.5D]$ , respectively. The transmission fading between one BS and one user is related to their distance via free-space path loss.

### 5.4.1 Primal multicell resource allocation

In the nine figures below, the dual optimum and the proposed heuristic solution of (2.6) are compared. The performance loss by the proposed heuristic method compared to the dual optimum is plotted against the number of users, subcarriers and BSs in Figure 5.2, Figure 5.3 and Figure 5.4, respectively. Multicell ISSA-SIC represents Algorithm 11.

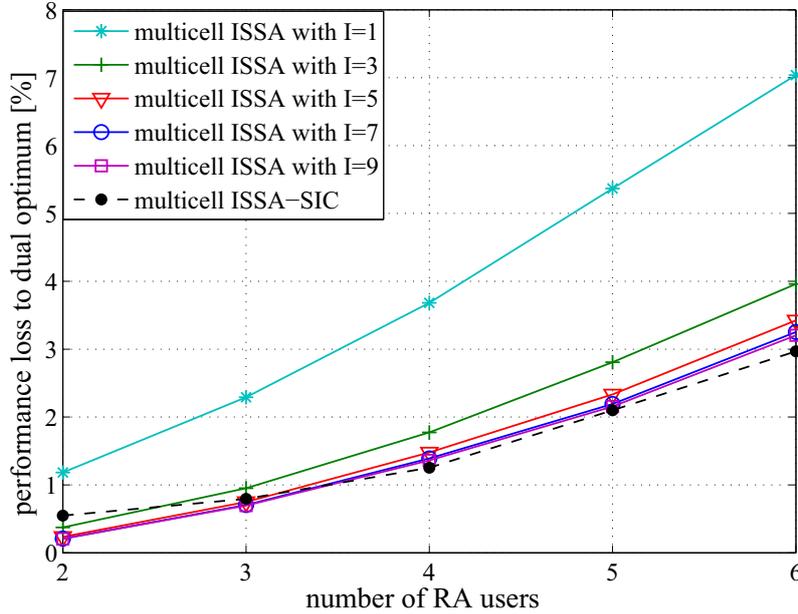


Figure 5.2: Performance loss of the suboptimum in percent compared to the dual optimum vs. number of RA users with  $K = Q$ ,  $N = 128$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

It becomes multicell ISSA after removing the two additional techniques, i.e., sorting subcarrier and controlling iteration. The performance loss is denoted as

$$\frac{R^{(\text{dual})} - R^{(\text{sub})}}{R^{(\text{dual})}} \times 100\%$$

where  $R^{(\text{dual})}$  denotes the dual optimum of (2.6) and  $R^{(\text{sub})}$  denotes the suboptimum by the proposed methods. It is increasing in the number of users and decreasing in the number of subcarriers. It is not sensitive to the number of BSs. As the number of iterations  $I$  grows, the performance loss becomes negligible and the gap between two successive iterations reduces significantly, which is similar to the results in Chapter 3.

The corresponding average numbers of iterations required by multicell ISSA-SIC are given in Figure 5.5, Figure 5.6 and Figure 5.7. The extended criteria for sorting subcarriers and controlling iterations are even more effective for multicell heterogeneous resource allocation. The associated performance, expressed by the dashed lines, is similar to or even better than the performance of multicell ISSA with  $I = 9$ , while computing time is reduced by up to 60%. For example, multicell ISSA-SIC has better performance than multicell ISSA with  $I = 9$  in Figure 5.2, while it only needs on average  $I = 4.23$  iterations when the number of RA users is 5. Computing time is reduced by over  $(1 - 4.23/9) \times 100\% \approx 53\%$ . The corresponding average computing time is recorded in Figure 5.8, Figure 5.9 and Figure 5.10. It is approximately linearly increasing in the number of users, subcarriers and BSs.

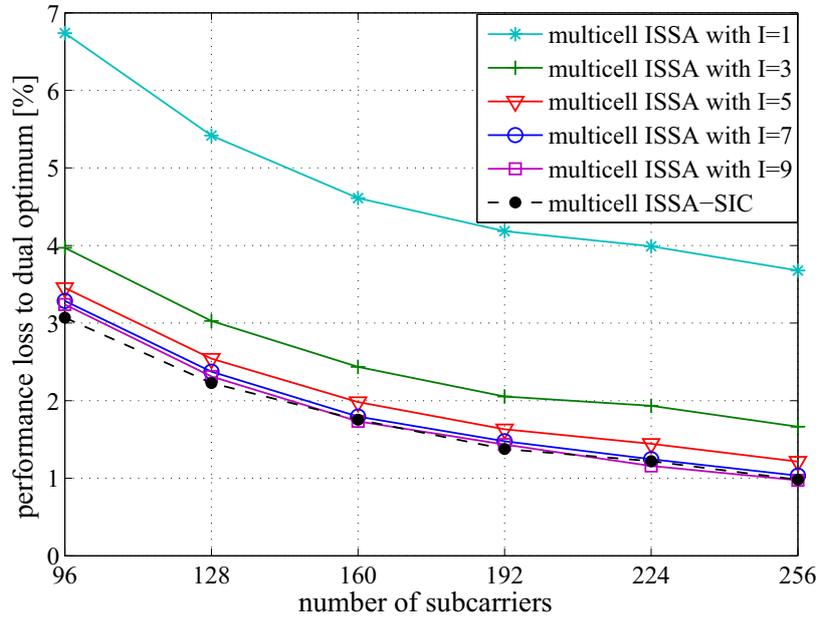


Figure 5.3: Performance loss of the suboptimum in percent compared to the dual optimum vs. number of subcarriers with  $K = Q = 4$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

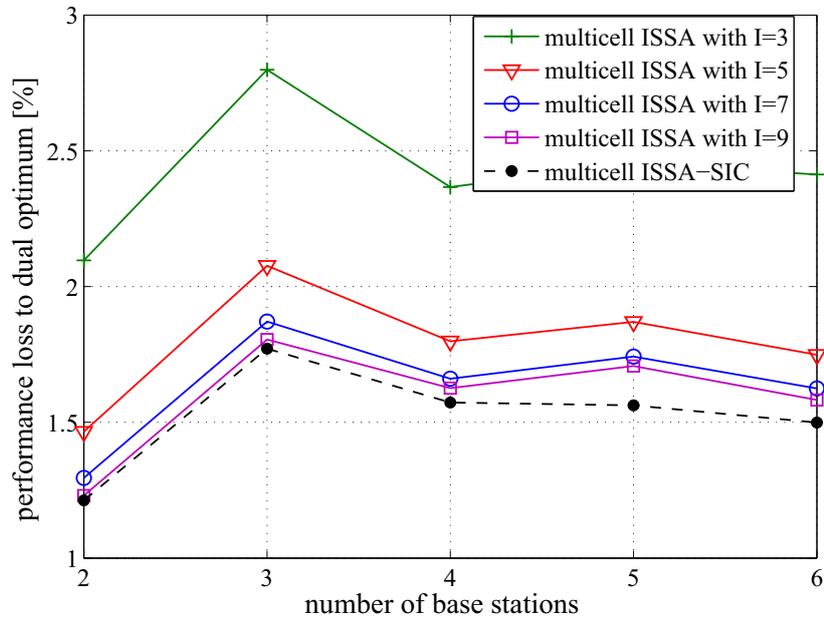


Figure 5.4: Performance loss of the suboptimum in percent compared to the dual optimum vs. number of BSs with  $K = Q = 4$ ,  $N = 128$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

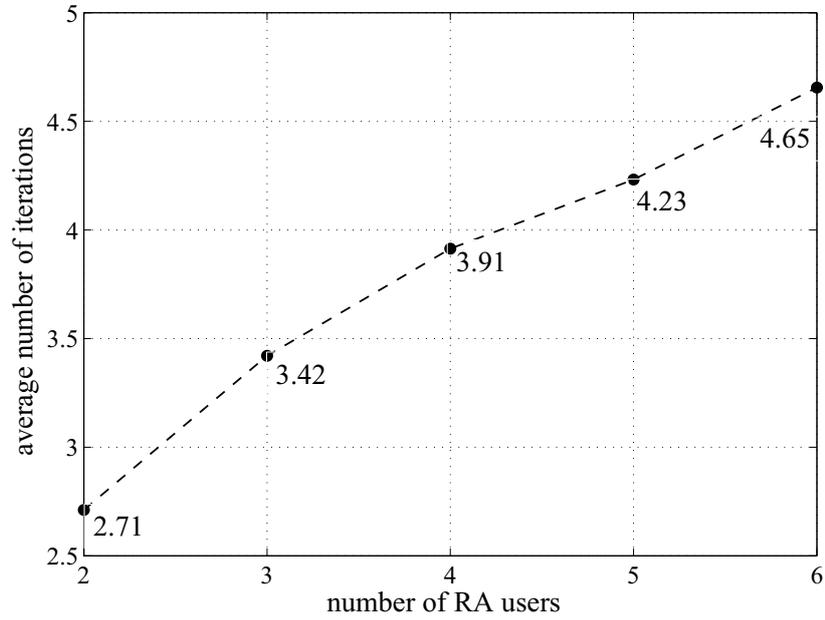


Figure 5.5: Average number of iterations required by multicell ISSA-SIC vs. number of RA users with  $K = Q$ ,  $N = 128$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

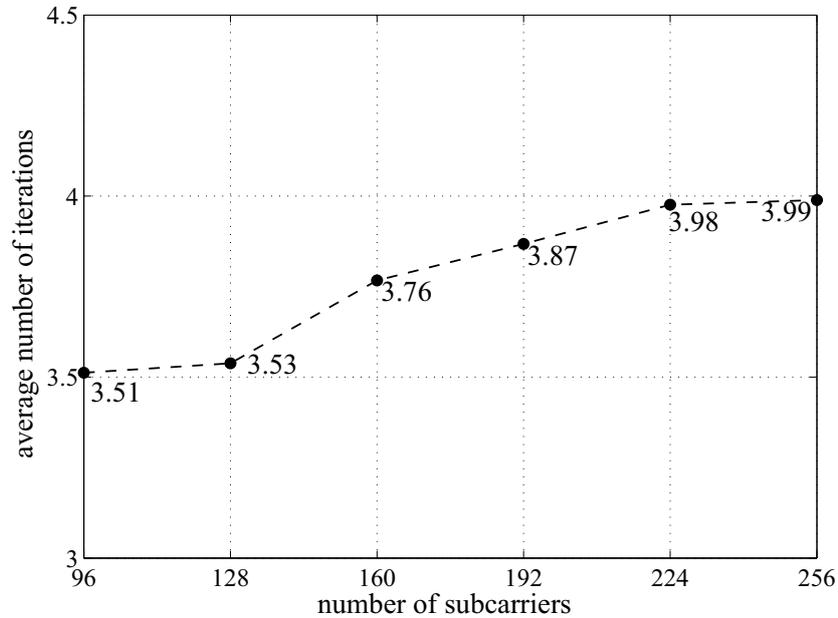


Figure 5.6: Average number of iterations required by multicell ISSA-SIC vs. number of subcarriers with  $K = Q = 4$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

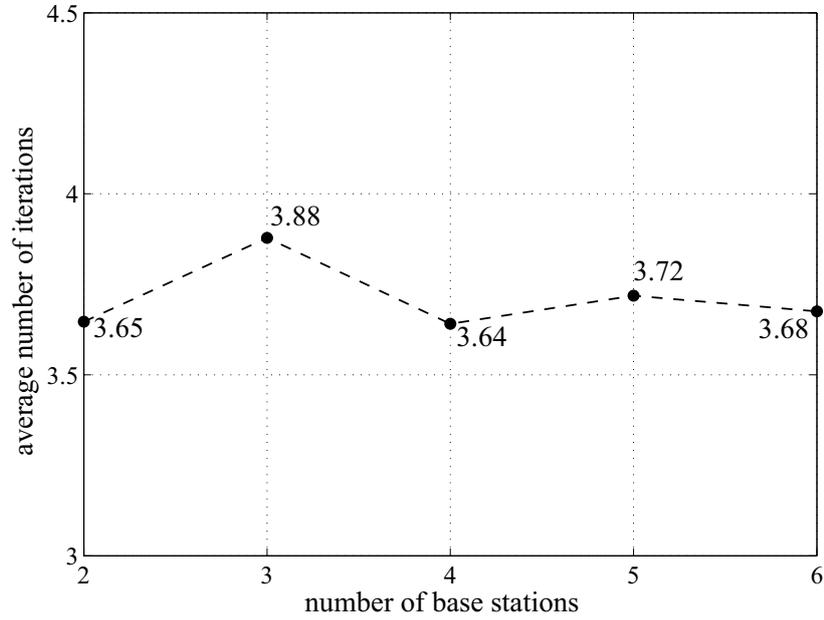


Figure 5.7: Average number of iterations required by multicell ISSA-SIC vs. number of BSs with  $K = Q = 4$ ,  $N = 128$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

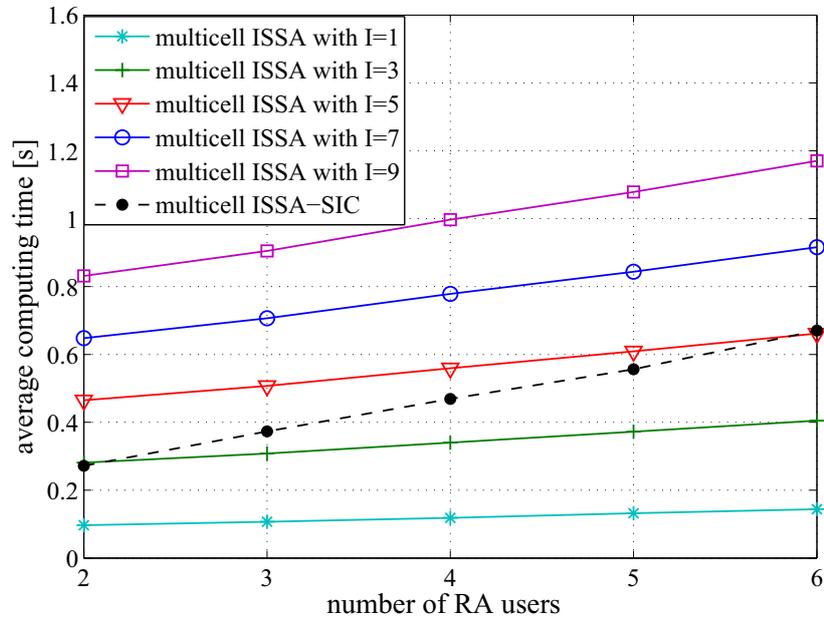


Figure 5.8: Average computing time in seconds vs. number of RA users  $K = Q$ ,  $N = 128$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

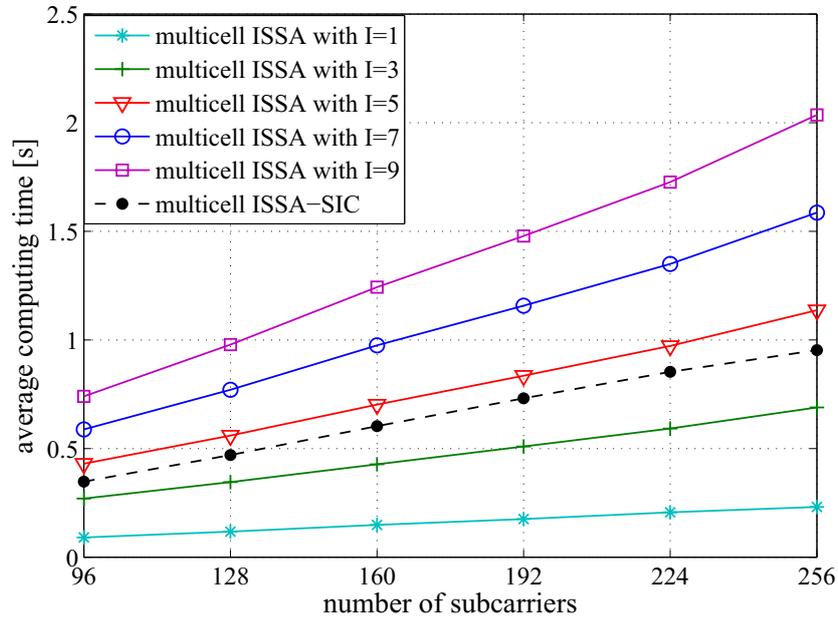


Figure 5.9: Average computing time in seconds vs. number of subcarriers  $K = Q = 4$ ,  $C = 3$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

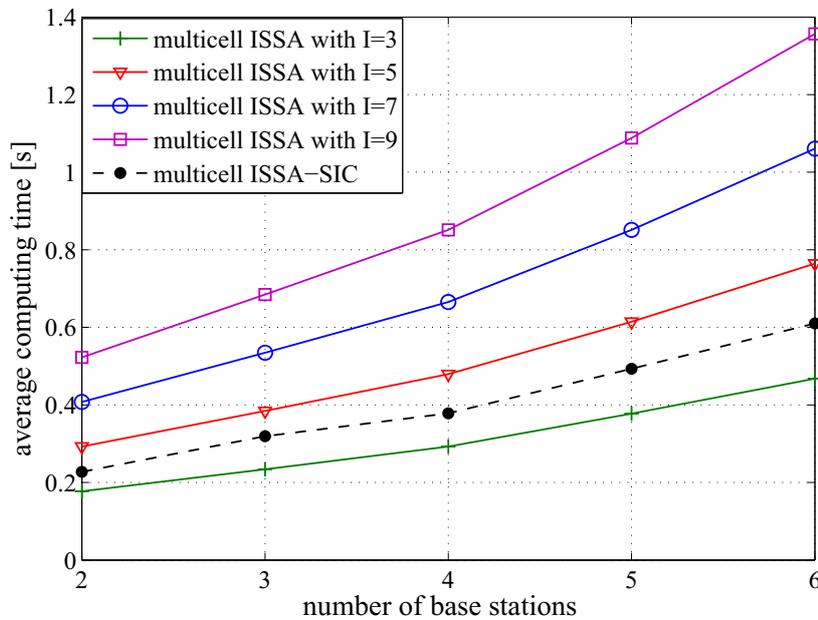


Figure 5.10: Average computing time in seconds vs. number of BSs  $K = Q = 4$ ,  $N = 128$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

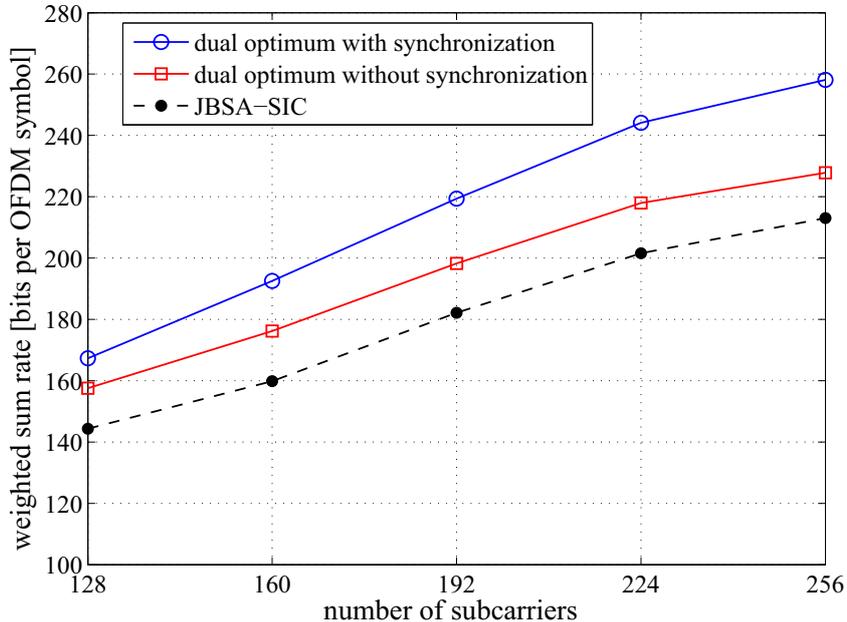


Figure 5.11: Comparison between the dual optimum when BSs are synchronized and the dual optimum when BSs are not synchronized with  $K = Q = 3$ ,  $P_c = 20$  dBW,  $C = 2$  and  $\rho = 0.01/(C(K + Q))$ .

### 5.4.2 Joint BS selection and subcarrier assignment

The heuristic solution proposed for (5.24) is evaluated below. The simulation setting does not change, while one user is only served by one BS. In Figure 5.11, the dual optima of (2.6) and (5.24) are given. Since the dual optimum of (5.24) must be determined by exhaustive  $K^C$  searches, the simulation system is limited to two BSs and three users. When BSs are not synchronized, BSs cannot transmit to one user simultaneously. The user and BS diversities are not fully used. Thus, the gap between these two dual optima is large. It is increasing in the number of subcarriers. Compared to this gap, the gap between the suboptimum by JBSA-SIC and the dual optimum of (5.24) is small. When even only one user is covered by an inappropriate BS, the resulting subcarrier assignment significantly differs from the dual optimal one and the induced performance loss is large.

Without using the two additional techniques, i.e., subcarrier sorting and iteration control, the proposed method simplifies to JBSA. The achieved weighted sum rate is given in Figure 5.12, Figure 5.13 and Figure 5.14, respectively. The average number of iterations required by JBSA-SIC is recorded in Figure 5.15, Figure 5.16 and Figure 5.17. The average computing time is shown in Figure 5.18, Figure 5.19 and Figure 5.20. They have the same properties as the earlier results. The performance converges as the number of iterations grows. The criteria for sorting subcarriers and

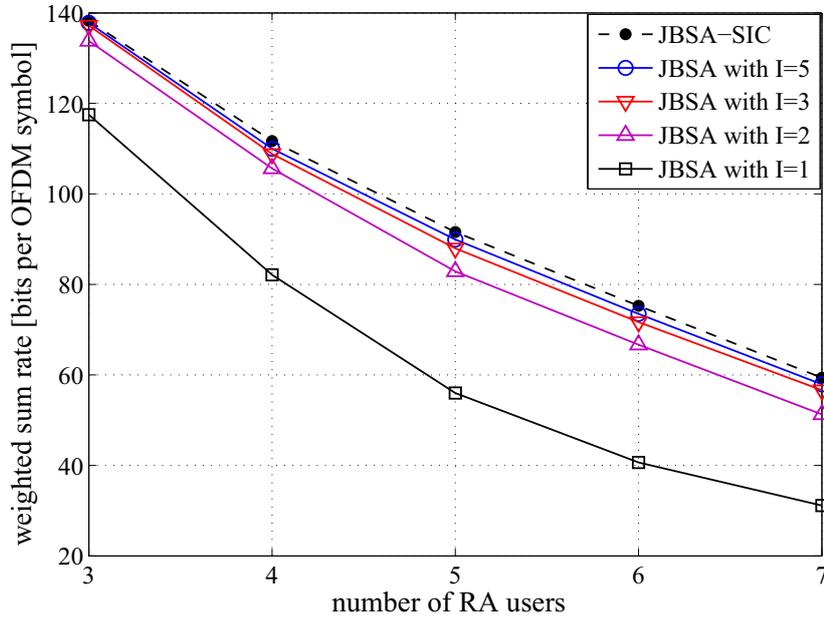


Figure 5.12: Rate achievement of the suboptimum in percent compared to the dual optimum vs. number of RA users with  $K = Q$ ,  $N = 128$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

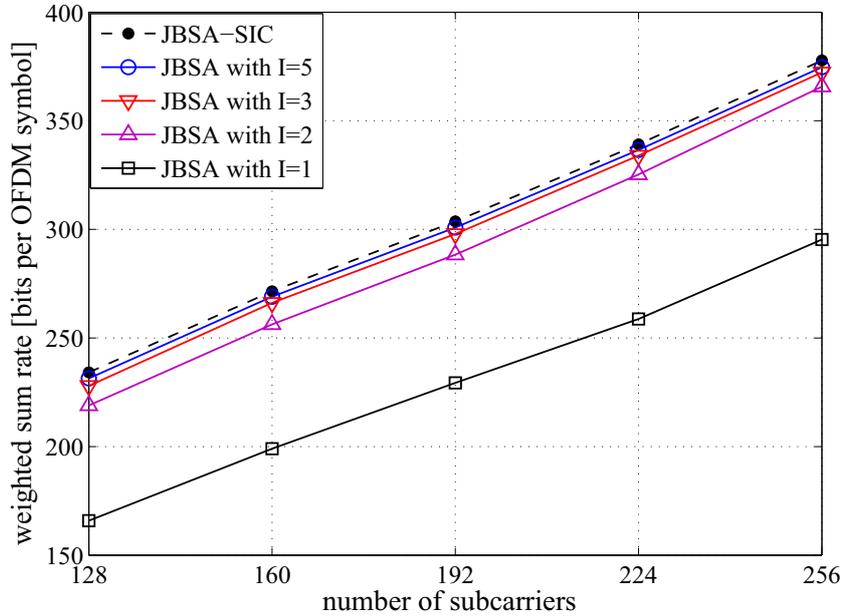


Figure 5.13: Rate achievement of the suboptimum in percent compared to the dual optimum vs. number of subcarriers with  $K = Q = 4$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

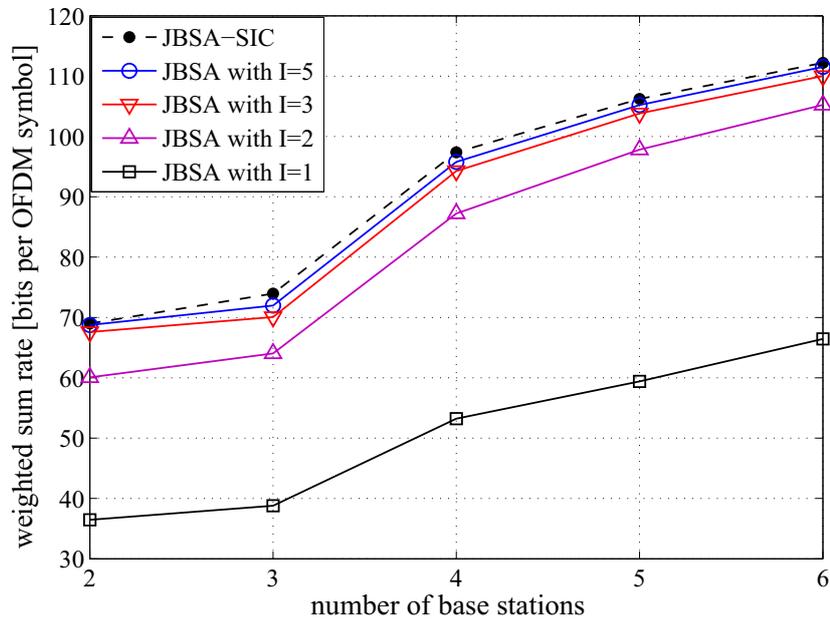


Figure 5.14: Rate achievement of the suboptimum in percent compared to the dual optimum vs. number of BSs with  $K = Q = 4$ ,  $N = 128$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

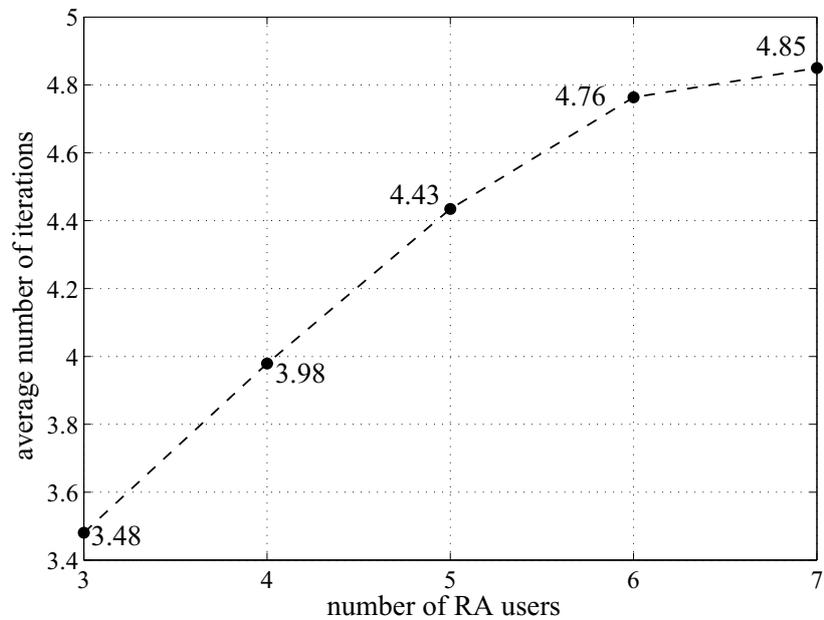


Figure 5.15: Average number of iterations required by JBSA-SIC vs. number of RA users with  $K = Q$ ,  $N = 128$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

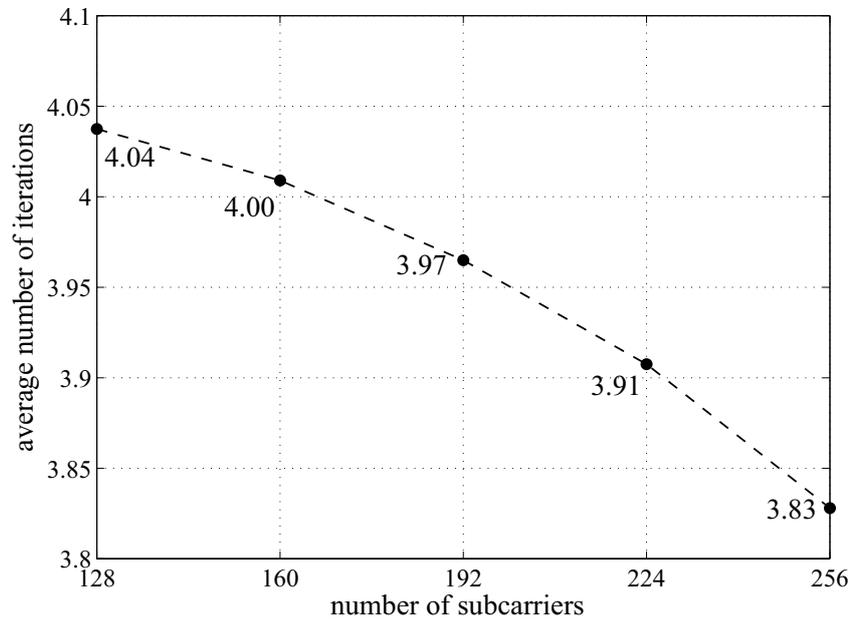


Figure 5.16: Average number of iterations required by JBSA-SIC vs. number of subcarriers with  $K = Q = 4$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

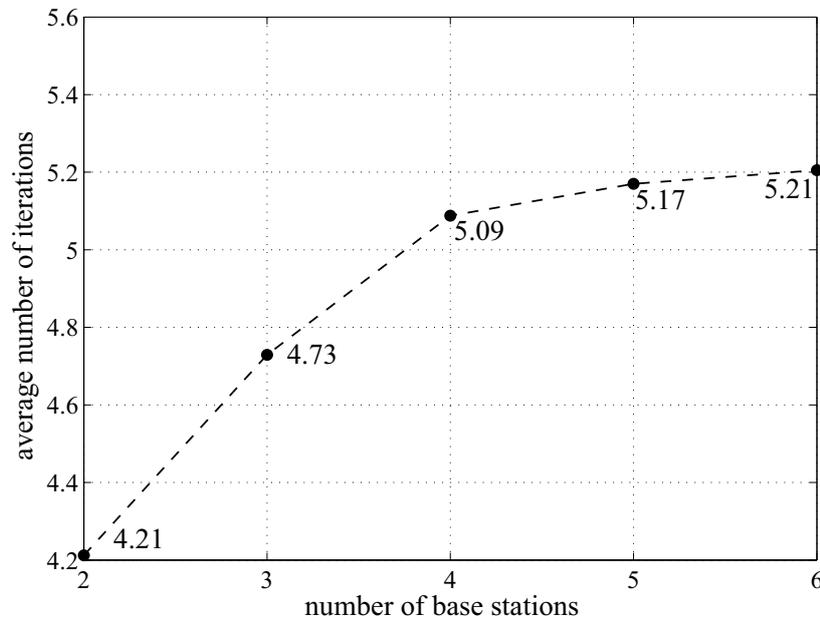


Figure 5.17: Average number of iterations required by JBSA-SIC vs. number of BSs with  $K = Q = 4$ ,  $N = 128$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

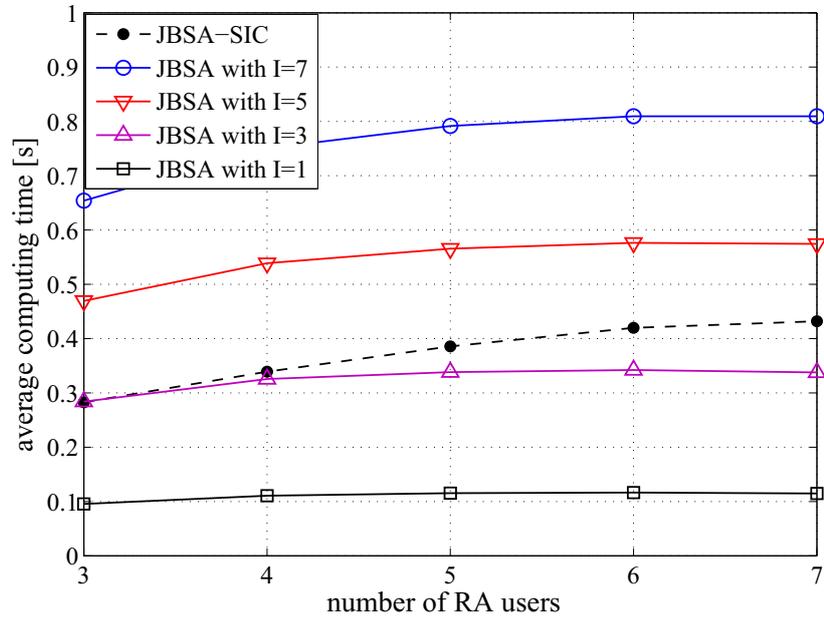


Figure 5.18: Average computing time in seconds vs. number of RA users  $K = Q$ ,  $N = 128$ ,  $P_c = 20$  dBW,  $C = 3$  and  $\rho = 0.01/(C(K + Q))$ .

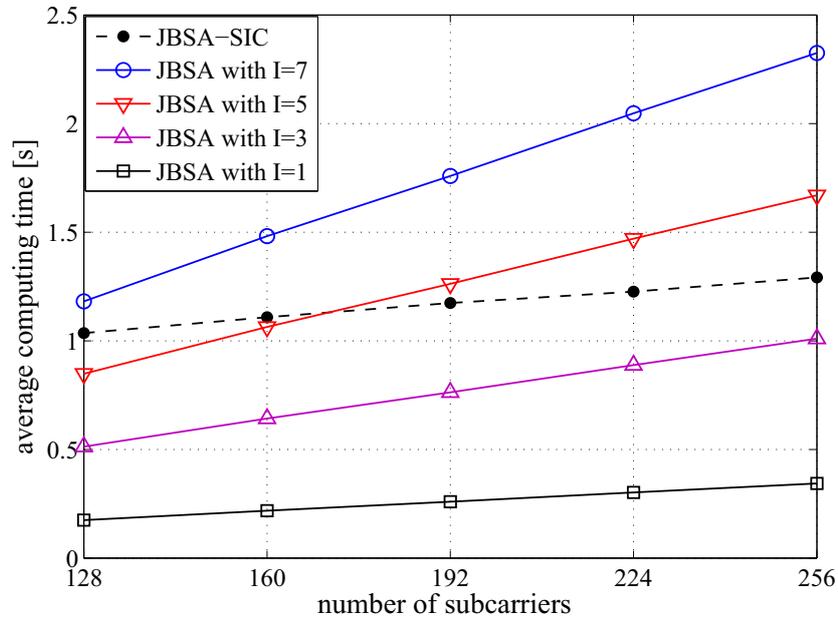


Figure 5.19: Average computing time in seconds vs. number of subcarriers  $K = Q = 4$ ,  $C = 3$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

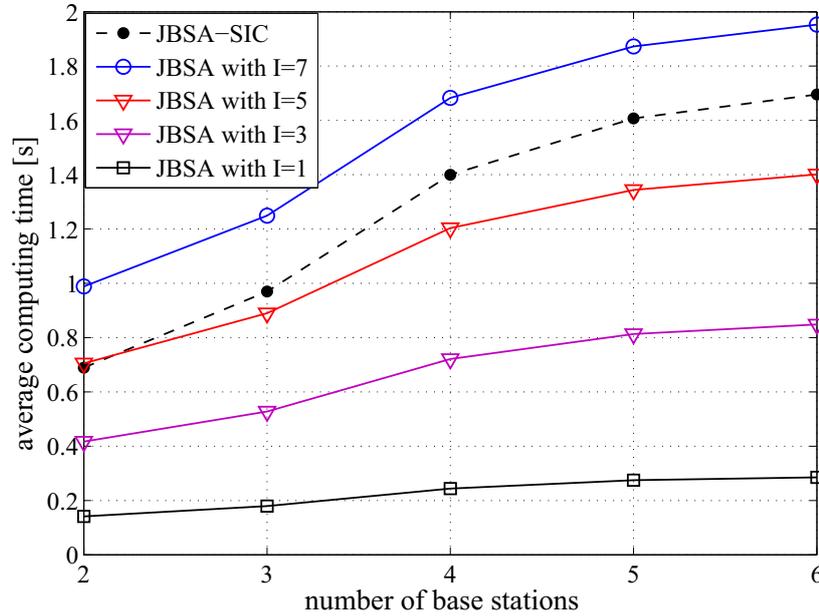


Figure 5.20: Average computing time in seconds vs. number of BSs  $K = Q = 4$ ,  $N = 128$ ,  $P_c = 20$  dBW and  $\rho = 0.01/(C(K + Q))$ .

controlling iterations are still effective. The computing time is an approximately linear function of the number of users, subcarriers and BSs.

## 5.5 Conclusions

In this chapter, resource allocation methods have been proposed for heterogeneous unicasting by multiple BSs. When signals from multiple BSs can be synchronized, BSs can transmit to one user at the same time. For this multicell heterogeneous resource allocation, a dual optimum and a heuristic solution have been obtained by extending the earlier methods for single-cell resource allocation. However, if synchronization of signals received from different BSs cannot be achieved, one user is only served by one BS at any specific period of time. This additional constraint leads to an extra consideration, i.e., BS selection, where only one BS is selected for each user. A heuristic method has been given for simultaneous BS selection and subcarrier assignment. Finally, the heuristic solutions of the multicell heterogeneous resource allocation problems have been assessed. Simulations demonstrate that the provided heuristic methods have small performance loss. The criteria for sorting subcarriers and controlling iterations are still effective in multicell scenarios.

# 6 Imperfect CSI and Rate Quantization

Resource allocation has been studied for heterogeneous unicasting in previous chapters. Perfect channel knowledge has been assumed at BSs. However, it is impossible to obtain perfect CSI in realistic scenarios due to noisy channel estimation and channel variation during the unavoidable feedback delay between the transmitter and the receiver. Only imperfect channel knowledge is available at BSs in practice. The resulting performance degradation has been studied in [43, 75]. In this chapter, we quantify the CSI imperfection induced by noisy channel estimation and feedback delay and investigate its impact on resource allocation.

Furthermore, when rates are constrained to be discrete, the water-filling solution must be modified. Methods have been suggested to solve this bit loading problem in [13, 24, 61]. They have complexity  $\mathcal{O}(N \log(N))$ . The method in [97] reduces the complexity but cannot give the optimal solution. Here, a non-iterative procedure is suggested for optimally quantizing the continuous rates of a water-filling solution. Its complexity is linearly increasing in the number of subcarriers.

Note that in this chapter we focus on single-user OFDM systems and thus the user and BS indices are omitted. Yet, the derived methods can be carried over to multiuser resource allocation.

## 6.1 Imperfect channel knowledge

In this section, the imperfection of feedback CSI is quantified. As introduced in Chapter 2, each transmission frame is composed of  $L$  OFDM symbols. It is assumed that one resource allocation scheme is only effective for one frame. The time duration of each frame is  $T$ . Channels remain constant within one frame. This is usually assumed in works on channel estimation for OFDM systems, e.g., [12, 18, 48]. In the time domain, the considered channel consists of  $Z$  independent paths, see Chapter 2. The channel coefficient of path  $z$  while transmitting the  $m$ th frame is denoted by  $h_z[m]$  with zero mean and variance  $\sigma_{h_z}^2$ . The corresponding channel coefficient of subcarrier  $n$  in the frequency domain is derived by the discrete Fourier transform (DFT) as

$$H_n[m] = \frac{1}{\sqrt{N}} \sum_{z=0}^{Z-1} h_z[m] e^{-j \frac{2\pi n}{N} z}. \quad (6.1)$$

The channel coefficients of different subcarriers are identically but not independently distributed as  $H_n[m] \sim \text{CN}(0, \sigma_H^2)$ , where the variance is

$$\sigma_H^2 = \frac{1}{N} \sum_{z=0}^{Z-1} \sigma_{h_z}^2 \quad (6.2)$$

due to independence of multiple paths.

According to Clark's model introduced in Chapter 2, the temporal auto-correlation function of a fading process on the path  $z$  is given by

$$\mathbb{E}\{h_z[m]^* h_z[m+j]\} = \alpha_j^2 \sigma_{h_z}^2$$

where  $\alpha_j^2$  is defined by the 0th-order Bessel function of the first kind  $J_0(2\pi f_D jT)$  [33] and  $f_D$  is the Doppler frequency. Note that  $m$  indicates the frame index but not the time index. It follows that

$$h_z[m+j] = \alpha_j h_z[m] + \sqrt{1 - \alpha_j^2} v_z[m] \quad (6.3)$$

where the channel variation  $v_z[m]$  is a random variable with zero mean and variance  $\sigma_{h_z}^2$  and is independent of the channel coefficient  $h_z[m]$ . In the frequency domain, the  $m$ th received vector via the  $n$ th subcarrier is referred to as

$$\mathbf{Y}_n[m] = H_n[m] \mathbf{X}_n[m] + \boldsymbol{\Omega}_n[m],$$

see (2.4). The noise  $\boldsymbol{\Omega}_n[m]$  is independently complex Gaussian distributed with zero mean and variance  $\sigma_{\Omega}^2$  and  $\mathbf{X}_n[m]$  is the transmitted vector.

### 6.1.1 Noisy channel estimation

After receiving the  $m$ th frame, the receiver performs channel estimation. The channel coefficient of subcarrier  $n$  while transmitting the  $m$ th frame is obtained as

$$H_n[m] = \hat{H}_n[m] + E_n[m] \quad (6.4)$$

where  $E_n[m]$  is the channel estimation error and  $\hat{H}_n[m]$  denotes the estimated channel coefficient of the  $n$ th subcarrier in the frequency domain. To measure the channel estimation error, we employ the mean square error (MSE) of channel estimation over subcarriers, which is defined as

$$\text{MSE} = \mathbb{E}\left\{\frac{1}{N} \sum_{n=1}^N |H_n[m] - \hat{H}_n[m]|^2\right\}.$$

If multipath components experience independent fading as assumed before, it follows that all subcarriers undergo identical fading [87]. Then, the MSE is given as

$$\text{MSE} = \mathbb{E}\{|E_n[m]|^2\} = \mathbb{E}\{|H_n[m] - \hat{H}_n[m]|^2\}.$$

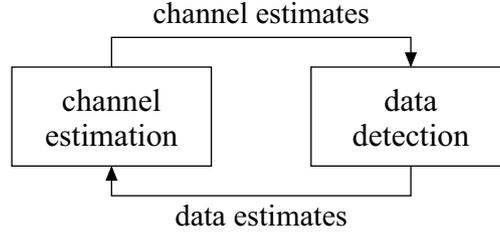
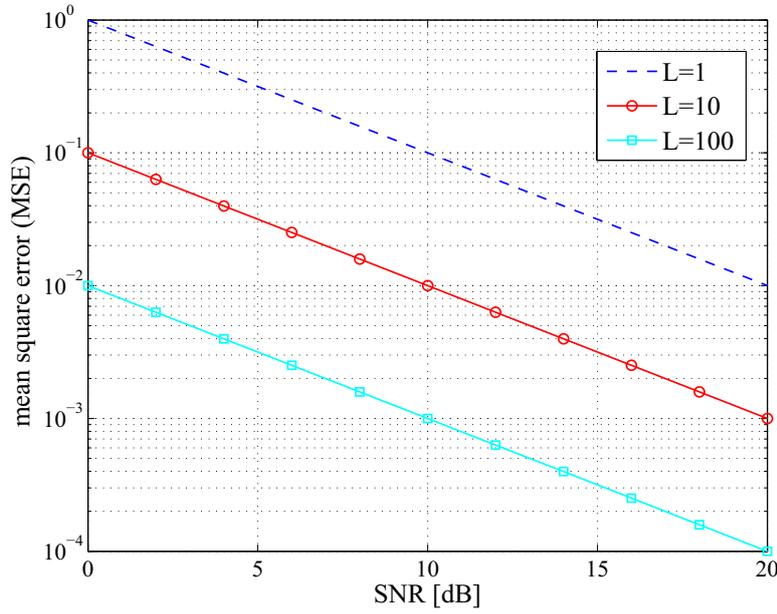


Figure 6.1: Joint channel estimation and data detection.

Figure 6.2: The Cramer-Rao lower bound of the mean square error of channel estimation for  $L = 1$ ,  $L = 10$  and  $L = 100$ .

According to the Cramer-Rao lower bound (CRLB) from [12], we obtain

$$\text{MSE} = \mathbb{E}\{|E_n[m]|^2\} \geq \frac{Z\sigma_\Omega^2}{LP[m]}$$

where  $P[m]$  is the transmission power for the  $m$ th frame. Channel estimation errors  $E_1[m], \dots, E_N[m]$  have zero mean and the same variance  $\sigma_E^2[m] = Z\sigma_\Omega^2/(LP[m])$ .

The variance  $\sigma_E^2[m]$  above is obtained, when channel estimation and data detection are jointly performed. This joint processing is implemented by an iterative procedure as shown in Figure 6.1, e.g., [12, 18, 48]. In Figure 6.2, the CRLB is plotted against SNR for different frame lengths with  $Z = N/8$ .

### 6.1.2 Time-varying channel

Resource allocation is performed at the transmitter. In many present wireless applications, channels are measured at the receiver. After channel estimation, channel estimates are fed back to the transmitter. The resulting delay cannot be avoided. Thus, a power and rate allocation scheme is only available for the  $(m + j)$ th frame at the transmitter, where the feedback delay  $j > 0$  is assumed.

From (6.1), the channel coefficient of subcarrier  $n$  in the frequency domain while transmitting the  $(m + j)$ th frame is written as

$$H_n[m + j] = \frac{1}{\sqrt{N}} \sum_{z=0}^{Z-1} h_z[m + j] e^{-j \frac{2\pi n}{N} z}.$$

By taking (6.3) to the equation above, it is derived that

$$\begin{aligned} H_n[m + j] &= \frac{1}{\sqrt{N}} \sum_{z=0}^{Z-1} (\alpha_j h_z[m] + \sqrt{1 - \alpha_j^2} v_z[m]) e^{-j \frac{2\pi n}{N} z} \\ &= \alpha_j \underbrace{\frac{1}{\sqrt{N}} \sum_{z=0}^{Z-1} h_z[m] e^{-j \frac{2\pi n}{N} z}}_{=H_n[m]} + \sqrt{1 - \alpha_j^2} \frac{1}{\sqrt{N}} \sum_{z=0}^{Z-1} v_z[m] e^{-j \frac{2\pi n}{N} z} \\ &= \alpha_j H_n[m] + \sqrt{1 - \alpha_j^2} V_n[m]. \end{aligned} \quad (6.5)$$

The channel variation in the frequency domain is referred to as  $V_n[m]$ . It is identically distributed for different subcarriers with zero mean and variance

$$\sigma_V^2 = \frac{1}{N} \sum_{z=0}^{Z-1} \sigma_{h_z}^2,$$

which is equal to  $\sigma_H^2$  in (6.2), see (6.3).

Since the time variation  $v_z[m]$  of channels is independent of the additive white Gaussian noise, by taking (6.4) and (6.5), the channel for the  $(m + j)$ th frame is predicted by the channel estimate from the  $m$ th frame as

$$\begin{aligned} H_n[m + j] &= \alpha_j \hat{H}_n[m] + \underbrace{\alpha_j E_n[m] + \sqrt{1 - \alpha_j^2} V_n[m]}_{=\eta_n[m+j]} \\ &= \alpha_j \hat{H}_n[m] + \eta_n[m + j]. \end{aligned} \quad (6.6)$$

The channel coefficient of subcarrier  $n$  for the frame  $m + j$  is viewed as a random variable with mean  $\alpha_j \hat{H}_n[m]$  and variance  $\sigma_\eta^2[m + j]$  as

$$\sigma_\eta^2[m + j] = \alpha_j^2 \sigma_E^2[m] + (1 - \alpha_j^2) \sigma_V^2.$$

The pair  $(\alpha_j \hat{H}_n[m], \sigma_\eta^2[m + j])$  is called the *soft channel prediction* of subcarrier  $n$  for the  $(k + m)$ th frame, where  $\alpha_j \hat{H}_n[m]$  is the predicted channel coefficient of subcarrier  $n$  for the  $(m + j)$ th frame. The term  $\sigma_\eta^2[m + j]$  may be interpreted as the reliability of the channel prediction.

## 6.2 Resource allocation with imperfect CSI

In the previous section, CSI imperfection is quantified, while feedback delay and noisy channel estimation are taken into account. In the following, resource allocation is studied in the presence of imperfect CSI.

After receiving the channel estimates for the  $m$ th frame, channel prediction can only be applied for the  $(m + j)$ th frame due to the feedback delay  $j$ . If perfect CSI is available, the RA resource allocation problem for the  $(m + j)$ th frame reads as

$$\begin{aligned} & \text{maximize} && \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m + j] |H_n[m + j]|^2}{\sigma_\Omega^2} \right) \\ & \text{subject to} && \sum_{n=1}^N p_n[m + j] \leq P \end{aligned}$$

where  $p_n[m + j]$  is the power allocated to subcarrier  $n$  within the  $(m + j)$ th frame. However, as explained earlier, only soft channel predictions are available at the transmitter including predicted channel coefficients and their reliability. The objective of the RA resource allocation problem changes to maximizing the expected transmission rate, formulated as

$$\begin{aligned} & \text{maximize} && \mathbb{E} \left\{ \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m + j] |H_n[m + j]|^2}{\sigma_\Omega^2} \right) \right\} && (6.7) \\ & \text{subject to} && \sum_{n=1}^N p_n[m + j] \leq P. \end{aligned}$$

If  $H_n[m + j]$  is complex Gaussian distributed with zero mean and variance  $\sigma_\eta^2[m + j]$ , it follows that the optimization problem above has an analytical solution, see [20]. If the distribution of  $H_n[m + j]$  is unknown while only its mean  $\alpha_j \hat{H}_n[m]$  and variance  $\sigma_\eta^2[m + j]$  are available, it is very difficult to solve the resource allocation problem analytically. Thus, the approximation below is employed.

Using Jensen's inequality [17, 46], an upper bound for (6.7) is given as

$$\begin{aligned} R[m + j] &= \max \mathbb{E} \left\{ \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m + j] |H_n[m + j]|^2}{\sigma_\Omega^2} \right) \right\} \\ &\leq \max \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m + j] \mathbb{E} \{ |H_n[m + j]|^2 \}}{\sigma_\Omega^2} \right), \end{aligned}$$

since the logarithm is concave. With the soft prediction (6.6), the upper bound, denoted by  $R^{(1)}[m+j]$ , is written as

$$\begin{aligned} R^{(1)}[m+j] &= \max \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m+j] \mathbb{E} \{|H_n[m+j]|^2\}}{\sigma_\Omega^2} \right) \\ &= \max \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m+j] \mathbb{E} \{|\alpha_j \hat{H}_n[m] + \eta_n[m+j]|^2\}}{\sigma_\Omega^2} \right) \\ &= \max \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m+j] (\alpha_j^2 |\hat{H}_n[m]|^2 + \sigma_\eta^2[m+j])}{\sigma_\Omega^2} \right). \end{aligned}$$

As the frame length grows, the channel estimation error becomes negligible as

$$\lim_{L \rightarrow \infty} R^{(1)}[m+j] = \max \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m+j] (\alpha_j^2 |\hat{H}_n[m]|^2 + (1 - \alpha_j^2) \sigma_V^2)}{\sigma_\Omega^2} \right)$$

where exists only the impairment from feedback delay.

The primal RA resource allocation problem is reformulated as

$$\begin{aligned} &\text{maximize} && \sum_{n=1}^N \log_2 \left( 1 + \frac{p_n[m+j] (\alpha_j^2 |\hat{H}_n[m]|^2 + \sigma_\eta^2[m+j])}{\sigma_\Omega^2} \right) \\ &\text{subject to} && \sum_{n=1}^N p_n[m+j] \leq P \end{aligned}$$

The water-filling solution can be obtained to the new resource allocation problem with complexity  $\mathcal{O}(N)$ . The MA resource allocation with imperfect channel knowledge can also be derived in the same way.

The impairment of imperfect CSI on resource allocation is quantified by the following simulations. We still use the system parameters of WiMAX from [2]. The OFDM system is composed of 128 subcarriers. The duration of one OFDM symbol is  $103 \mu s$ . The carrier frequency is 2.4 GHz. The frequency selective channel is modeled as consisting of  $N/8$  independent Rayleigh distributed paths with an exponentially decaying profile and  $\sigma_H^2 = 1$ . Noise power to each subcarrier is  $-5$  dBW. For convenience, the transmission power is equally allocated to all subcarriers for the  $m$ th frame. Channel estimates are given by least-square channel estimation. With the channel estimates for the frame  $m$ , we perform resource allocation for the  $(m+j)$ th frame.

The transmission rate achieved in the presence of imperfect CSI is shown in Figure 6.3, while the transmitter and the receiver are static and CSI imperfection is only subject to noisy channel estimation. When channels remain constant within 10 OFDM symbols, the effect of channel estimation error can be ignored. In WiMAX, one frame contains 48 OFDM symbols, which is much greater than 10.

Alternatively, in Figure 6.4, perfect CSI for the  $m$ th frame is known and then CSI imperfection for the  $(m+1)$ th frame is only caused by feedback delay. The resulting performance loss is not significant. Obviously, after the receiver obtains the  $m$ th frame,

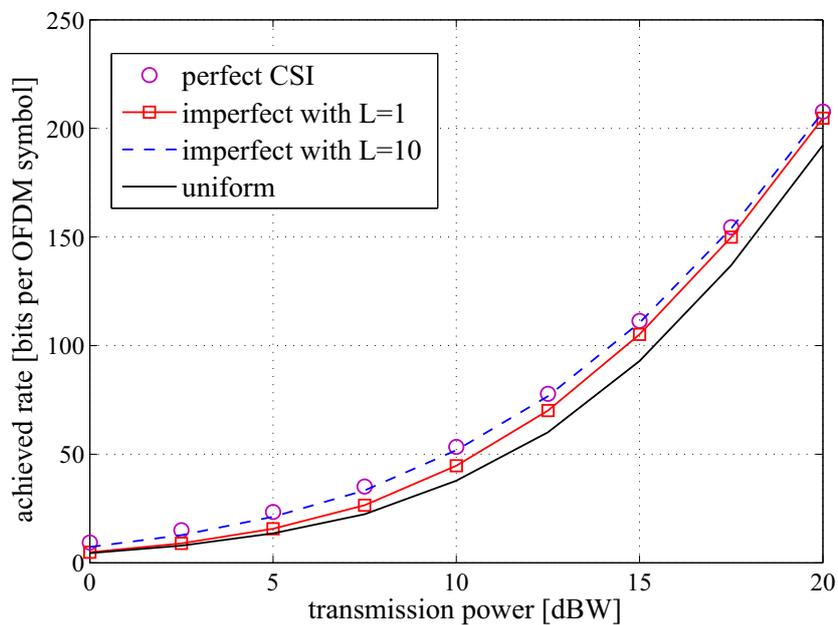


Figure 6.3: Achieved rate vs. transmission power with imperfect CSI only subject to noisy channel estimation.

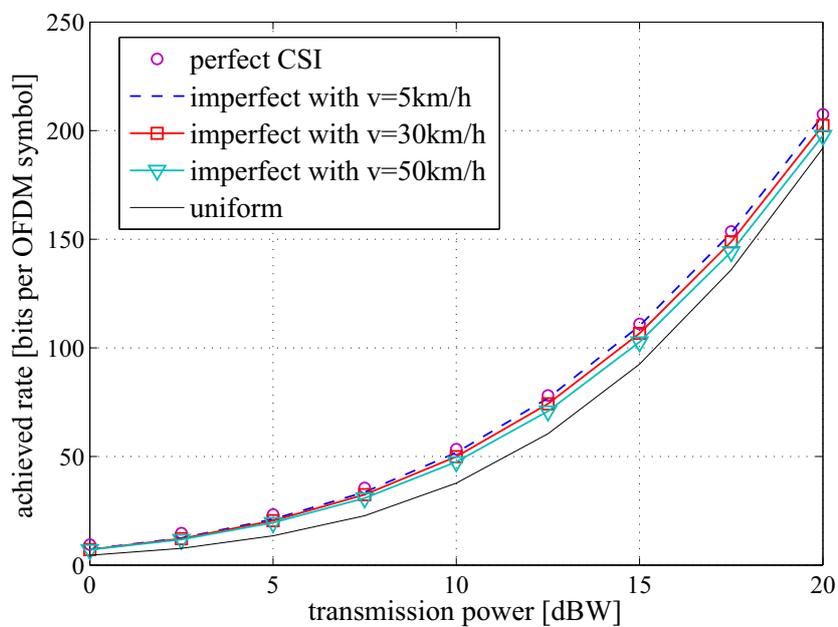


Figure 6.4: Achieved rate vs. transmission power with imperfect CSI only subject to the feedback delay of  $L = 48$  OFDM symbols, i.e.,  $j = 1$ .

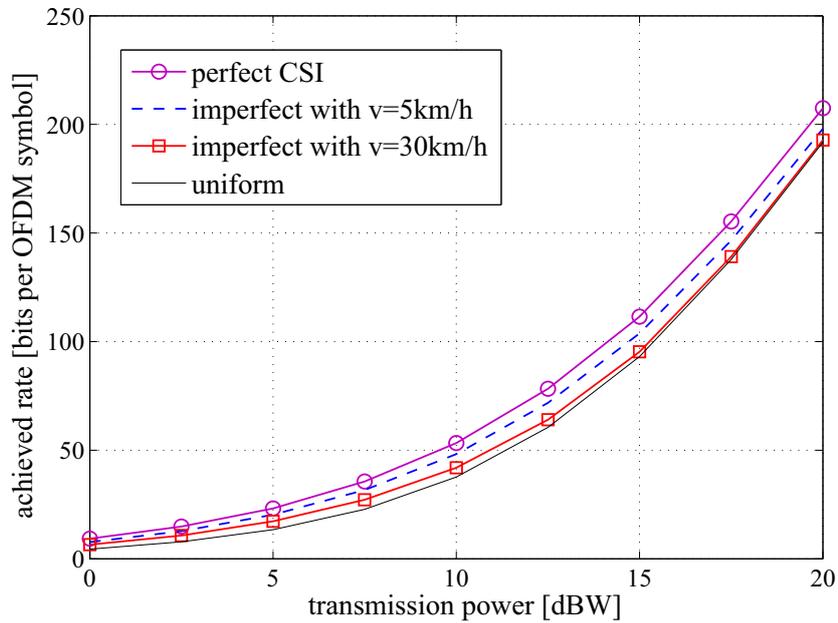


Figure 6.5: Achieved rate vs. transmission power with imperfect CSI only subject to the feedback delay of  $L = 96$  OFDM symbols, i.e.,  $j = 2$ .

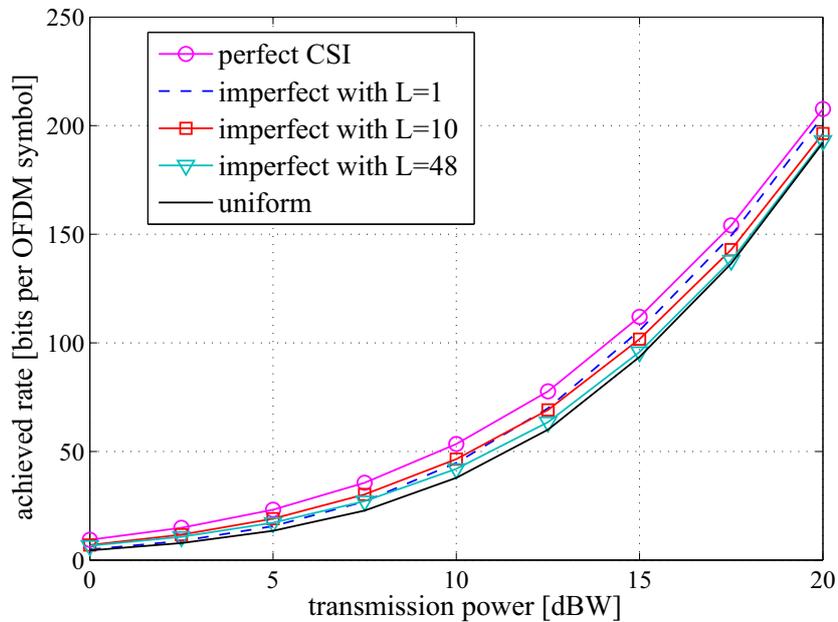


Figure 6.6: Achieved rate vs. transmission power with imperfect CSI subject to noisy channel estimation and feedback delay of  $2L$  OFDM symbols at speed of  $30\text{ km/h}$ .

resource allocation cannot be immediately performed for the  $(m + 1)$ th frame at the transmitter due to the propagation delay from the receiver to the transmitter and the delay for computing channel estimation and resource allocation. In Figure 6.5, the feedback delay time is doubled to  $j = 2$ , i.e.,  $2L = 96$  OFDM symbols. Resource allocation becomes ineffective, when the receiver moves fast or the transmission power is large.

Finally, both noisy channel estimation and feedback delay are considered in Figure 6.6, where the receiver moves at speed of 30 km/h and the feedback delay is  $2L$  OFDM symbols. There exists a trade-off for the frame length  $L$ . The larger the frame length is, the smaller the channel estimation error is, while the longer the feedback delay is. When the transmission power is low, e.g.,  $P = 5$  dBW, resource allocation with  $L = 10$  has better performance than the one with  $L = 1$ . As the transmission power grows, this relation is reversed. When the transmission power is large, e.g.,  $P = 20$  dBW, the shorter a frame is, the better performance is achieved. We may conclude that resource allocation for relatively long frames shows better performance when the transmission power is low. It has worse performance when the transmission power is high.

## 6.3 Rate quantization

In the previous two sections, CSI imperfection is embedded into resource allocation, while noisy channel estimation and feedback delay are taken into account. In practice, rates are constrained to be discrete due to a limited number of available coding and mapping schemes. Thus, continuous rates from resource allocation must be quantized to available discrete rates. In this section, a non-iterative method is proposed to optimally quantize continuous rates of a water-filling solution.

The continuous rates allocated by a water-filling solution are derived as  $r_1, \dots, r_N$  for a single user. We assume that all allocated rates are positive. Let  $\Gamma$  denote the constant granularity of available discrete rates, which implies the fixed distance between any two successive available discrete rates. If the rate  $R$  is required by a user, the achievable rate is

$$\hat{R} = \left\lceil \frac{R}{\Gamma} \right\rceil \Gamma \geq R.$$

The equality holds, when  $R/\Gamma$  is integer. Rounding up a continuous rate to the nearest available discrete rate is expressed by

$$r_n^+ = \Gamma \left\lceil \frac{r_n}{\Gamma} \right\rceil, \quad n = 1, \dots, N.$$

Rounding down a continuous rate to the nearest available discrete rate is performed as

$$r_n^- = \Gamma \left\lfloor \frac{r_n}{\Gamma} \right\rfloor, \quad n = 1, \dots, N.$$

The rate increment and decrement by rounding up and down are denoted by

$$\begin{aligned}\Delta r_n^+ &= r_n^+ - r_n \\ \Delta r_n^- &= r_n - r_n^-, \end{aligned}$$

respectively. It follows that

$$\begin{aligned}\Delta r_n^+ &\geq 0 \\ \Delta r_n^- &\geq 0 \\ \Delta r_n^+ + \Delta r_n^- &= r_n^+ - r_n^- = \Gamma. \end{aligned}$$

Hence,  $\Delta r_n^+$  and  $\Delta r_n^-$  are both in the closed interval  $[0, \Gamma]$ . Correspondingly, the power allocated to each subcarrier after rounding up and down changes to

$$\begin{aligned}p_n^+ &= \frac{1}{g_n}(2^{r_n^+} - 1), \\ p_n^- &= \frac{1}{g_n}(2^{r_n^-} - 1), \end{aligned}$$

respectively.

To achieve the sum rate  $\hat{R}$  over subcarriers, first, all continuous rates are rounded down. The following iterative procedure is then performed. In each iteration, we increase the rate by  $\Gamma$  bits to the subcarrier with the least power increment, which is defined as  $\Delta p_n = p_n^+ - p_n^-$ . This iteration finishes when  $\hat{R}$  is met. However, this iterative procedure can be avoided, which is proved as follows.

Let  $\tilde{R}$  denote the difference between the achievable sum rate  $\hat{R}$  and the achieved sum rate after rounding down all continuous rates, shown as

$$\tilde{R} = \hat{R} - \sum_{n=1}^N r_n^-$$

where the term  $\sum_{n=1}^N r_n^-$  must be an integer multiple of  $\Gamma$ . Then, if the iterative procedure above is executed,  $\tilde{R}/\Gamma$  iterations are performed and  $\tilde{R}/\Gamma < N$  always holds. The power increment  $\Delta p_n$  by additionally allocating  $\Gamma$  to subcarrier  $n$  is obtained as

$$\begin{aligned}\Delta p_n &= p_n^+ - p_n^- \\ &= \frac{1}{g_n} 2^{r_n} (2^{\Delta r_n^+} - 2^{-\Delta r_n^-}) \\ &= \frac{1}{g_n} 2^{r_n} (2^{\Delta r_n^+} - 2^{(\Delta r_n^+ - \Gamma)}) \\ &= \underbrace{\frac{1}{g_n} 2^{r_n}}_{\text{water level}} 2^{\Delta r_n^+} (1 - 2^{-\Gamma}) \end{aligned}$$

It is monotonically increasing in  $\Delta r_n^+$ . Note that  $2^{r_n}/g_n$  is the water level, where  $g_n$  is the CNR of the  $n$ th subcarrier. The equation  $2^{r_n}/g_n = 2^{r_l}/g_l$  always holds for any  $n \neq l$ . Thus, it leads to

$$\Delta r_n^+ \leq \Delta r_l^+ \Rightarrow \Delta p_n \leq \Delta p_l.$$

**Algorithm 15** Optimal rate quantization

$$\begin{aligned}
\tilde{R} &\leftarrow \left\lceil \sum_{n=1}^N \Delta r^-[n] \right\rceil \\
\mathcal{B} &\leftarrow \{n = 1, \dots, N \mid \tilde{R}/\Gamma \text{ smallest } \Delta r_n^+\} \\
r_n &\leftarrow r_n^+, \quad n \in \mathcal{B} \\
r_n &\leftarrow r_n^-, \quad n \in \{1, \dots, N\} \setminus \mathcal{B} \\
p_n &\leftarrow 1/g_n(2^{r_n} - 1), \quad n = 1, \dots, N
\end{aligned}$$

The following holds for rate quantization. If one of two continuous rates must be rounded up, the rate rounded up must be the one with a smaller rate increment.

With the insight above, a non-iterative method for optimal rate quantization is given in Algorithm 15. It is executed after deriving the single-user water-filling solution, which outputs a continuous rate allocation. The number of subcarriers with rate rounded up is  $\tilde{R}/\Gamma$ . They are determined by the associated rate increments. By using the order statistic selection algorithms from [16], this step can be efficiently implemented with complexity  $\mathcal{O}(N)$  in the worst case. The remaining continuous rates are rounded down. The achievable rate  $\hat{R}$  is then satisfied. Finally, the transmission power allocated to each subcarrier is calculated. In previous works [13, 24, 61, 67], continuous rates to subcarriers are rounded down (up) iteratively till meeting the rate requirement. Instead of power increments used in [7, 78], rate increments are used here so that the number of exponential operations is reduced to  $N$  only for obtaining powers allocated to subcarriers after rate quantization. The other operations are simple, like plus, minus and compare. Obviously, its complexity is  $\mathcal{O}(N)$ .

Algorithm 15 is designed for rate quantization for single-user MA resource allocation. It can be simply revised for the single-user RA resource allocation problem. After obtaining the RA water-filling solution, the achieved sum rate over subcarriers is  $R$ . To maintain the total transmission power below the power limit, the achievable rate after rate quantization is

$$\hat{R} = \left\lfloor \frac{R}{\Gamma} \right\rfloor \Gamma.$$

After that Algorithm 15 follows and optimal rate quantization is obtained.

## 6.4 Conclusions

To apply resource allocation in practice, two problems cannot be avoided. One is the impact of imperfect channel knowledge on resource allocation. The other is that transmission rates must be discrete. In this chapter, Jensen's inequality has been employed to approximate resource allocation in the presence of imperfect CSI. In doing so, the primal water-filling solution can still be used. A non-iterative method has been given to quantize the continuous rates of a water-filling solution. The derived discrete rates are optimal, if the granularity of available discrete rates is constant.



# 7 Conclusions

In future wireless communications, efficient methods are required for adaptive resource allocation in fast time-varying environments. Within this thesis, resource allocation has been investigated for heterogeneous unicasting by multiple BSs. The primal optimum, the dual optimum and heuristic solutions have been given. Compared to the dual optimum, the proposed heuristic solutions have been thoroughly assessed.

## 7.1 Summary

For convenience, we have called users demanding non-real time services RA users and users requiring real time services MA users. In general, RA and MA users always appear simultaneously in OFDM unicasting, called heterogeneous unicasting. We have aimed at maximizing the weighted sum rate for RA users subject to limited transmission power, while the minimum required rates are satisfied for RA and MA users. A trade-off relation exists between subcarrier assignment and power allocation. If more subcarriers are assigned to MA users, more power is kept for RA users, while they have fewer available subcarriers and may not have a large rate achievement, and vice versa. Thus, resource allocation must be considered for RA and MA users jointly.

First, we have concentrated on resource allocation for heterogeneous unicasting by a single BS. Within the framework of convex optimization, a dual optimum has been obtained. It may be used in OFDM systems consisting of a small number of users and subcarriers. Simulations have verified that the duality gap is decreasing approximately exponentially in the number of subcarriers. Thus, the dual optimum is qualified as a reference for assessing heuristic solutions. At the dual optimum, for some RA users only the minimum required rates are reached and these RA users are actually treated as MA users, while for other RA users the achieved rates are strictly greater than the minimum required rates. Distinguishing these two kinds of RA users must be taken into account to the design of heuristic methods.

To render the heuristic method suitable for different resource allocation problems and multicarrier systems, a simple procedure has been employed. In this procedure, the subcarrier assignment is adjusted successively in the inner loop and iteratively in the outer loop. In the successive adjustment of the inner loop, while reassigning one subcarrier to different users, the assignment of others is fixed. In the iterative adjustment of the outer loop, this successive procedure repeats several times. To reduce the computational complexity of this simple procedure, a class of efficient approaches has been deduced for updating the objective after changing the subcarrier assignment by inheriting the previous water-filling solution, i.e., so called genetic water-filling. It ensures the linear complexity of the proposed heuristic method. Furthermore, the

criteria for sorting subcarriers within the inner loop and controlling iterations for the outer loop have been developed in order to make the adjustment more effective. Simulations show that the derived solutions achieve a better balance between computing time and performance.

Furthermore, resource allocation has been studied for heterogeneous unicasting by multiple BSs. When synchronization of signals received from different BSs can be achieved, each user may be served by more than one BS at any specific period of time. The resource allocation methods derived above have been extended to this multicell resource allocation. In contrast, if signals from BSs cannot be synchronized at receivers, each user must be covered by only one BS. In this scenario, BS selection has been additionally considered, where a specific BS must be selected for each user. BS selection and subcarrier assignment are jointly performed by an iterative process. The criteria for sorting subcarriers and controlling iterations are still effective. The proposed heuristic methods have been thoroughly evaluated with respect to performance and computing time against the number of users, subcarriers and BSs in simulations. The simulation results demonstrate that the proposed heuristic solution is comparable to the dual optimum with a small number of iterations. The average computing time is increasing approximately linearly in the number of users, subcarriers and BSs.

Moreover, performance of resource allocation in multicarrier systems may degrade, when the employed power and rate allocation scheme must be sent via the signalling overhead from the BS to receivers for data detection. To moderate this degradation, we have proposed a new resource allocation strategy that an equal rate is allocated to subcarriers assigned to each user. In doing so, the signalling overhead reduces significantly. Theoretical asymptotic limits have been deduced for the instantaneous per-symbol performance loss by the proposed strategy. The same iterative successive procedure in the previous chapter has been used for resource allocation with the proposed strategy applied. The only modification is that genetic water-filling mentioned earlier is replaced by updating harmonic average CNRs, since the equal rate allocation is only related to the harmonic average CNR of subcarriers assigned to each user. Simulations illustrate that the proposed strategy has better performance than the water-filling strategy when channels are subject to rapid variation in time.

Finally, we have dealt with two important issues on resource allocation. On one hand, resource allocation must be executed at the transmitter, while noisy channels are measured at receivers. The feedback channel knowledge is imperfect. We have quantified this imperfection subject to noisy channel estimation and channel variation during feedback delay. It has been embedded into resource allocation problems. The water-filling solution can still be used. On the other hand, practical transmission rates must be discrete due to a limited number of available coding and mapping schemes. The continuous rates of a water-filling solution have been quantized by a non-iterative process. The derived solution is optimal, if the granularity of available transmission rates is constant.

## 7.2 Outlook

The offered dual optimum and the proposed heuristic methods are not limited to OFDM systems and the considered heterogeneous resource allocation problems. They can be extended to other resource allocation problems or to other multicarrier systems with small modifications. Some examples are given in the following.

### Energy efficiency

The increase in global average air and ocean temperatures implies that warming of the climate system is unequivocal [15]. One cause for the increasing temperatures is the abuse CO<sub>2</sub> emissions, which are related to the energy consumption by humans. Also in communications, energy efficiency becomes a stringent factor to moderate or even reverse the effects of the global warming. The consumed energy is the product of the transmission power and the time duration of transmission.

Energy efficiency can be enhanced for heterogeneous unicasting by resource allocation [30, 63, 70]. Reducing the energy consumption on real time transmission is equivalent to minimizing the transmission power, while the transmission time is fixed to communication systems. On the other hand, there exists a fixed data amount for non-real time transmission, e.g., online movie and data downloading, so that the energy efficiency is related to the transmission time, i.e., the quotient of data amount and rate. However, the transmission time cannot be explicitly minimized without long-term channel prediction, since transmission rates must change subject to time-varying channels. The energy efficiency of the transmitter can be enhanced by greedily maximizing the weighted sum rate for RA users at each specific time subject to limited transmission power while meeting the fixed rates required by MA users. The proposed methods can be directly used for this purpose.

### Cognitive radio

The proposed methods can also be used for cognitive radio [27, 31, 92] realized by multicarrier systems. Over the sharing spectrum presently used by the licensed users, there is an upper limit of the interference from unlicensed users. Thus, the power allocated to the subcarriers for unlicensed users must be less than or equal to the associated upper limit [29, 64]. While searching the optimal dual variables, the power allocated to each subcarrier is additionally upper bounded by the interference limit. A dual optimum can be derived. In heuristic methods, the upper limit must be checked during adjusting the subcarrier assignment, while the computing time does not increase significantly.

### Self-organizing networks

In this thesis, BSs within one cluster cooperatively perform resource allocation for heterogeneous unicasting. However, the clustering of BSs is subject to users' demand, topology and other impacts. Neither the transmission power for pure MA users nor the achieved sum rate for pure RA users can be used for evaluating the quality of service

due to the trade-off mentioned earlier. Thus, the considered heterogeneous resource allocation explicitly offers a metric for assessing performance of current networks. Some adjustment can be performed according to these metrics. Finally, networks can be self-organized, self-configured and self-healed [1, 21, 77].

# Acronyms

<b>3GPP</b>	Third generation partnership project
<b>BS</b>	Base station
<b>CNR</b>	Channel gain-to-noise ratio
<b>CRLB</b>	Cramer-Rao lower bound
<b>CSI</b>	Channel state information
<b>DFT</b>	Discrete Fourier transform
<b>DVB-T</b>	Digital video broadcasting-terrestrial
<b>FFT</b>	Fast Fourier transform
<b>GWF</b>	Genetic water-filling
<b>IFFT</b>	Inverse fast Fourier transform
<b>ISI</b>	Inter-symbol interference
<b>ISSA</b>	Iterative successive subcarrier adjustment
<b>JBSA</b>	Joint base station selection and subcarrier assignment
<b>KKT</b>	Karush-Kuhn-Tucker
<b>LTE</b>	Long-term evolution
<b>MA</b>	Margin-adaptive
<b>MGWF</b>	Multicell genetic water-filling
<b>MSE</b>	Mean square error
<b>OFDM</b>	Orthogonal frequency division multiplexing
<b>RA</b>	Rate-adaptive
<b>S/P</b>	Serial-to-parallel
<b>SIC</b>	Sorting and iteration control
<b>SNR</b>	Signal-to-noise ratio

<b>TDWF</b>	Two-dimensional water-filling
<b>WiMAX</b>	Worldwide interoperability for microwave access
<b>WLAN</b>	Wireless local area network

# Notations

## Symbols for general purpose

$(\cdot)'$	Transpose of a matrix.
$(\cdot)^*$	Optimal value.
$\mathbb{E}(\cdot)$	Expectation of a random variable.
$\mathbb{I}_{[0,\infty)}(x)$	For any $x \in [0, \infty)$ .
$\mathbf{0}_{m \times n}$	Matrix with $m$ rows and $n$ columns of zeros.
$\mathbf{1}_{m \times n}$	Matrix with $m$ rows and $n$ columns of ones.
$J_0(\cdot)$	The 0th-order Bessel function.
$L(\cdot)$	Lagrangian of an optimization problem.
$L_D(\cdot)$	Lagrange dual function.
$\mathcal{O}(\cdot)$	Big Omicron notation.
$\mathbb{R}$	Set of real numbers.
$\mathbb{R}_+$	Set of non-negative real numbers.
$\mathbb{R}_{++}$	Set of positive real numbers.

## Symbols for single-cell resource allocation

$H_k$	Harmonic average of CNRs of user $k$ , p. 60.
$I$	Number of iterations, p. 47.
$G_k$	Geometric average of CNRs of user $k$ , p. 60.
$K$	Number of RA users, p. 14.
$\mathcal{K}$	Set of RA users, p. 15.
$L$	A resource allocation scheme is effective for $L$ OFDM symbols, p. 14.

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$L^{(\min)}$	Minimum frame length that water-filling has better performance than the proposed equal rate resource allocation, p. 75.
$M$	Number of bits for expressing one rate, p. 57.
$N$	Number of subcarriers, p. 14.
$P$	Limit of the transmission power of a base station, p. 14.
$P_k$	Transmission power allocated to user $k$ , p. 15.
$\tilde{P}$	Remaining power after reaching all minimum required rates, p. 27.
$\Delta P_l^{(\text{ex})}(m)$	Power variation by excluding $m$ from $\mathcal{S}_l$ , p. 34.
$\Delta P_l^{(\text{in})}(m)$	Power variation by including $m$ in $\mathcal{S}_l$ , p. 36.
$\Delta P^{(\text{RA})}$	Power variation for RA users, p. 37.
$\Delta P_k$	Power variation of user $k$ , p. 40.
$P_k^{(\text{ER})}$	Power allocated to user $k$ while using the equal rate resource allocation, p. 59.
$\Delta_m^n P(k, l)$	Power variation after swapping subcarrier $m$ and subcarrier $n$ between user $k$ and user $l$ , p. 68.
$Q$	Number of MA users, p. 14.
$\mathcal{Q}$	Set of MA users, p. 15.
$R$	Achieved weighted sum rate, p. 15.
$R_k$	Minimum rate required by user $k$ , p. 15.
$\Delta R^{(\text{ex})}(m)$	Rate variation by not assigning $m$ to RA users, p. 35.
$\Delta R^{(\text{in})}(m)$	Rate variation by reassigning $m$ to RA users, p. 37.
$\Delta R^{(\text{p})}(\Delta P^{(\text{RA})})$	Rate variation by changing power for RA users by $\Delta P^{(\text{RA})}$ , p. 38.
$R^{(i)}$	Achieved weighted sum rate in the $i$ th iteration, p. 47.
$\hat{R}^{(i)}$	Achieved weighted sum rate after adjusting half subcarriers in the $i$ th iteration, p. 48.
$R^{(\text{dual})}$	Achieved weighted sum rate by the dual method, p. 51.
$R^{(\text{sub})}$	Achieved weighted sum rate by the heuristic method, p. 51.

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$\bar{R}$	Achieved weighted sum rate while using the equal rate resource allocation, p. 59.
$R_k^{(\text{ER})}$	Achieved rate for user $k$ while using the equal rate resource allocation, p. 61.
$R^{(\text{WF})}$	Achieved weighted sum rate while using the water-filling resource allocation, p. 61.
$R^{(\text{ER})}$	Achieved weighted sum rate while using the equal rate resource allocation, p. 61.
$\mathcal{S}_k$	Set of subcarriers assigned to user $k$ , p. 15.
$a_{k,n}$	Allocated rate associated to the objective, p. 20.
$b_{k,n}$	Allocated rate associated to the rate constraint, p. 20.
$g_{k,n}$	CNR of the $n$ th subcarrier for user $k$ , p. 15.
$\tilde{g}_{k,n}$	Equivalent CNR for distributing the remaining power, p. 27.
$g_k$	Average CNR of user $k$ over all subcarriers, p. 40.
$i$	Iteration counter, p. 47.
$k$	User index, p. 14.
$n$	Subcarrier index, p. 15.
$o_n$	Index of sorted subcarriers, p. 46.
$p_{k,n}$	Power allocated to the $n$ th subcarrier for user $k$ , p. 15.
$\tilde{p}_{k,n}$	Additional power after reaching minimum required rates, p. 27.
$r_k$	Rate allocated to each subcarrier assigned to user $k$ , p. 59.
$\bar{r}_n$	Average of potential rates to subcarrier $n$ over users, p. 46.
$r_{k,n}$	Rate allocated to the $n$ th subcarrier for user $k$ , p. 15.
$s_k$	Cardinality of $\mathcal{S}_k$ , p. 15.
$\mathbf{w}$	Vector of weights for achieved rates, p. 14.
$w_k$	Weight for the achieved rate for user $k$ , p. 14.
$\beta$	Dual variable associated to the power limit, p. 21.

$\epsilon$	Accuracy for the ellipsoid method, p. 24.
$\lambda_k$	Dual variable associated to the minimum rate required by user $k$ , p. 21.
$\boldsymbol{\lambda}$	Vector of dual variables $(\lambda_1, \dots, \lambda_{K+Q})'$ , p. 21.
$\lambda_k^{(\text{ex})}(m, v)$	Dual variable after excluding $m$ from $\mathcal{S}_{k,v}$ , p. 81.
$\lambda_k^{(\text{in})}(m, v)$	Dual variable after including $m$ in $\mathcal{S}_{k,v}$ , p. 82.
$\mu_k$	Water level for reaching $R_k$ , p. 22.
$\mu_l^{(\text{ex})}(m)$	Water level after excluding $m$ from $\mathcal{S}_l$ , p. 34.
$\mu_l^{(\text{in})}(m)$	Water level after including $m$ in $\mathcal{S}_l$ , p. 36.
$\mu_k^{(\text{ER})}$	Water level for user $k$ while using the equal rate resource allocation, p. 61.
$\nu^{(\text{in})}(m)$	Sharing term by reassigning $m$ to RA users, p. 37.
$\nu^{(\text{p})}(\Delta P^{(\text{RA})})$	Sharing term by changing power for RA user by $\Delta P^{(\text{RA})}$ , p. 38.
$\nu$	Normalized reciprocal $(\beta \ln(2))^{-1}$ of $\beta$ , p. 22.
$\nu^{(\text{ex})}(m)$	Sharing term by not assigning $m$ to RA users, p. 34.
$\nu^{(\text{ER})}$	Sharing term while using the equal rate resource allocation, p. 61.
$\rho$	Given factor for iteration control, p. 47.
$\varphi_{k,n}$	Subcarrier index of the $n$ th subcarrier of user $k$ , p. 46.
$\varphi_{k,n,c}$	Subcarrier index of the $n$ th subcarrier of user $k$ at BS $c$ , p. 88.
$\sigma_{\bar{r}}[n]$	Variation of potential rates to subcarrier $n$ over users, p. 46.

### Additional symbols for multi-cell resource allocation

$C$	Number of BSs in one cluster, p. 15.
$P_{k,c}$	Power allocated to user $k$ at BS $c$ , p. 17.
$\Delta P_{k,v}^{(\text{ex})}(m)$	Power variation after excluding $m$ from $\mathcal{S}_{k,v}$ at BS $v$ , p. 81.
$\Delta P_{k,v}^{(\text{in})}(m)$	Power variation after including $m$ in $\mathcal{S}_{k,v}$ at BS $v$ , p. 82.

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$P_c^{(\text{RA})}$	Power for RA users at BS $c$ , p. 83.
$\Delta P_c^{(\text{RA})}$	Change to power for RA users at BS $c$ , p. 85.
$\Delta \hat{R}_c^{(\text{ex})}(m, l)$	Variation of weighted sum rate by not assigning $m$ to RA user $l$ at BS $c$ , p. 84.
$\Delta \hat{R}_c^{(\text{in})}(m, l)$	Variation of weighted sum rate by reassigning $m$ to RA user $l$ at BS $c$ , p. 84.
$\Delta \hat{R}_c^{(\text{p})}(\Delta P_c^{(\text{RA})})$	Variation of weighted sum rate by changing power for RA users by $\Delta P_c^{(\text{RA})}$ at BS $c$ , p. 85.
$\hat{R}_c$	Achieved weighted sum rate at BS $c$ , p. 83.
$\mathcal{S}_{k,c}$	Set of subcarriers assigned to user $k$ at BS $c$ , p. 17.
$a_{k,n,c}$	Allocated rate associated to the objective, p. 77.
$b_{k,n,c}$	Allocated rate associated to the rate constraint, p. 77.
$c$	BS index, p. 16.
$g_{k,n,c}$	CNR of the $n$ th subcarrier for user $k$ at BS $c$ , p. 16.
$p_{k,n,c}$	Power allocated to the $n$ th subcarrier for user $k$ at BS $c$ , p. 16.
$r_{k,n,c}$	Rate allocated to the $n$ th subcarrier for user $k$ at BS $c$ , p. 16.
$s_{k,c}$	Cardinality of $\mathcal{S}_{k,c}$ , p. 17.
$\beta_c$	Dual variable associated to the power limit at BS $c$ , p. 77.
$\boldsymbol{\beta}$	Vector of dual variables $(\beta_1, \dots, \beta_C)'$ , p. 77.
$\mu_{k,c}$	Water level for reaching $R_k$ at BS $c$ , p. 78.
$\mu_{k,c}^{(\text{ex})}(m)$	Water level after excluding $m$ from $\mathcal{S}_{k,c}$ , p. 92.
$\mu_{k,c}^{(\text{in})}(m)$	Water level after including $m$ in $\mathcal{S}_{k,c}$ , p. 93.
$\nu_c$	Normalized reciprocal $(\beta_c \ln(2))^{-1}$ of $\beta_c$ , p. 78.
$\boldsymbol{\nu}$	Componentwise inverse $(\boldsymbol{\beta} \ln(2))^{-1}$ of $\boldsymbol{\beta}$ , p. 78.
$\nu_c^{(\text{ex})}(m, l)$	Sharing term by not assigning $m$ to RA user $l$ at BS $c$ , p. 83.
$\nu_c^{(\text{in})}(m, l)$	Sharing term by reassigning $m$ to RA user $l$ at BS $c$ , p. 84.

$\nu_c^{(p)}(\Delta P_c^{(\text{RA})})$  Sharing term by changing power for RA users by  $\Delta P_c^{(\text{RA})}$  at BS  $c$ , p. 85.

### Symbols for single-user resource allocation

$E_n[m]$	Channel estimation error for of subcarrier $n$ while transmitting frame $m$ , p. 110.
$H_n[m]$	Channel coefficient of subcarrier $n$ while transmitting frame $m$ , p. 109.
$\hat{H}_n[m]$	Estimated channel coefficient of subcarrier $n$ while transmitting frame $m$ , p. 110.
$P[m]$	Power of transmitting frame $m$ , p. 111.
$R_z(t)$	Temporal auto-correlation function, p. 11.
$R[m]$	Achieved rate within frame $m$ , p. 113.
$R^{(1)}[m]$	Approximated achieved rate within frame $m$ , p. 114.
$\hat{R}$	Achieved rate after rate quantization, p. 117.
$\tilde{R}$	Difference between the achievable sum rate and the sum rate by rounding all rates down, p. 118.
$T$	Time duration of a frame consisting of $L$ OFDM symbols, p. 109.
$\mathbf{X}_n[m]$	The $m$ th transmitted vector over the $n$ subcarrier, p. 110.
$\mathbf{Y}_n[m]$	The $m$ th received vector over the $n$ subcarrier, p. 110.
$Z$	Channel response length, p. 10.
$\boldsymbol{\Omega}_n[m]$	Noise vector in the $m$ th received vector over the $n$ subcarrier, p. 110.
$f_d$	Doppler frequency, p. 11.
$f_c$	Carrier frequency, p. 11.
$g_n$	CNR of the $n$ th subcarrier, p. 118.
$r_n^+$	Quantized rate by rounding up, p. 117.
$r_n^-$	Quantized rate by rounding down, p. 117.
$\Delta r_n^+$	Bit increment by rounding up, p. 118.
$\Delta r_n^-$	Bit decrement by rounding down, p. 118.

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$h_z[m]$	Channel coefficient of path $m$ while transmitting frame $m$ , p. 10.
$p_n[m]$	Power allocated to the $n$ th subcarrier within frame $m$ , p. 113.
$p_n^+$	Power allocated to subcarrier $n$ after rounding up, p. 118.
$p_n^-$	Power allocated to subcarrier $n$ after rounding down, p. 118.
$\Delta p_n$	Difference of allocated powers after rounding up and down, p. 118.
$v_z[m]$	Channel variation of path $z$ while transmitting frame $m$ , p. 110.
$z$	Index of channel taps, p. 10.
$\Gamma$	Granularity of available discrete rates, p. 117.
$\alpha_j^2$	The 0th order Bessel function of the first kind $J_0(2\pi f_D jT)$ , p. 110.
$\eta_n[m]$	Effective error of channel estimation and prediction for frame $m$ , p. 112.
$\sigma_\eta^2[m]$	Variance of the effective error of channel estimation and prediction for frame $m$ , p. 112.
$\sigma_E^2$	Variance of the channel estimation error, p. 111.
$\sigma_H^2$	Channel variance of a subcarrier in the frequency domain, p. 110.
$\sigma_{h_z}^2$	Variance of the channel path $z$ , p. 10.
$\sigma_V^2$	Channel variation of a subcarrier in the frequency domain, p. 112.
$\sigma_\Omega^2$	Noise variance, p. 110.



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# Curriculum Vitae

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