Power Allocation in Sensor Networks for Surveilling Security Zones

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***Abstract*—Power consumption and lifetime are essential features of sensor networks. On the one hand, the power consumption should be as low as possible to enable an energy-aware system. On the other hand, the lifetime should be as long as possible to ensure a comprehensive coverage. Especially, for surveilling security zones, e.g., the frontier between two countries or the terrain of military base stations, it is also necessary to achieve high reliability over the whole lifetime. However, these features are contrary and they must be optimized simultaneously to achieve optimal performance. In this paper, we thus aim to investigate the lifetime of bistatic radars for a required minimum reliability and any given power resources. We first describe the underlying scenario and subsequently develop a graph based optimization method to maximize the lifetime. By selected results, we finally show the performance of the new approach and discuss the power consumption of the sensor network.**

I. Introduction

Due to the rise of Internet of things (IoT) and certain applications in 5th generation wireless systems (5G), sensor networks for various applications are drastically gaining importance. Since large-scale sensor networks usually utilize cheap and weak sensor nodes (SN), a complex signal processing within each SN is not applicable. Thus, mostly distributed SNs relay their independently observed information to a centralized unit, e.g., a fusion center, which is located at a remote position. The task of the fusion center is then to perform a powerful processing with the aid of all transmitted signals in order to increase the reliability of the individual observations and combine them to a global observation. Several publications show that this reliability increases as the transmission power of each SN increases. Naturally, the question arises how to guarantee for a minimum signal quality at the fusion center and simultaneously enable an energy-aware sensor network. Especially for sensor networks with a huge number of SNs this question becomes essential, because the overall power consumption can drastically be reduced which will consequently rise the network lifetime.

In this paper, we consider a conventional sensor network, which is utilized as a distributed radar system and is used for surveillance of security zones. In the literature, such applications are also known as barrier coverage techniques, where a large barrier region is covered by huge sensor networks in order to provide a certain level of security. Figure 1 shows a frontier (belt region) between two countries in which each combination of two sensor pairs acts as a bistatic radar. The radars only observe a small area and transmit their individual observations to a remotely located fusion center. The fusion center decides by the aid of all received observations, whether trespassers are crossing the border or not. This scenario is investigated in many publications and will also serve as our framework in the present paper.

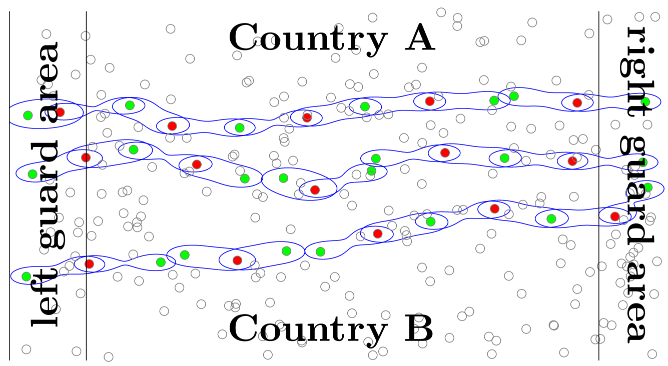

Fig. 1. A sensor network used for barrier coverage, e.g., between two countries A and B. The small circles represent the sensor nodes, which are randomly distributed over the left guard area, the barrier, and the right guard area. Activated sensor nodes are marked with red and green colors corresponding to their operating modes as transmitter and receiver, respectively. Sensor nodes in standby mode are marked by empty circles. The association of any transmitter with its corresponding receiver works as a bistatic radar. The coverage region of each bistatic radar is depicted by a Cassini oval. Three connected coverage chains, illustrated by three chains of Cassini ovals, are shown which also is known as a 3-barrier coverage situation.

The authors in [1] have investigated both the deterministic and the random deployment of SNs to achieve an n-barrier coverage. However, their approach neglects any power constraints which are important in practical applications. Later in [2], the authors have extended their previous work by a lifetime maximization analysis by exploiting the deployment of redundant SNs. Another extension has been proposed in [3], in which each SN is able to perform a local decision, instead of a global one, in order to determine the existence of local barrier coverage even when the region of deployment is arbitrarily curved. Since bistatic radars are more convenient for sensor networks than monostatic ones, and they can cover geometrically longer regions in contrast to monostatic radars, in the newer papers [4] and [5] only bistatic radars are investigated. Due to the special form of the observation region of a bistatic radar, which is outlined by a Cassini oval [6], the observation reliability can be ensured and thus new options have originated to optimize the lifetime and power consumption of sensor networks. In the present paper, we also aim to exploit this knowledge and will answer the question: 'Which lifetime is expected for a given amount of energy resources to ensure reliability and coverage simultaneously?'

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In the present paper, we first introduce the system model and formulate a lifetime maximization problem for any given amount of energy resources. Second, since the considered optimization problem is of combinatorial nature and challenging to solve, we apply a graph based method to provide more insight. We then deduce an upper bound on the maximum lifetime as well as a more practical approximation of the maximum lifetime. Afterwards, standard methods are used to maximize the lifetime numerically. Finally, selected results are visualized to show the performance of the new approaches.

II. Overview and Technical System Description

We consider a distributed sensor network, that consists of K independent SNs. The network operates cycle wise, where L denotes the maximum possible number of network cycles – equivalent to the lifetime of the considered sensor network. The SNs are randomly distributed over the left guard area, the barrier, and the right guard area as shown in Figure 1. We assume that each SN is able either to work as a radar transmitter or receiver which hereinafter will be distinguished as the specific *operating mode* of a SN. In a case, where the k^{th} SN is selected by the optimization method and activated to work as a transmitter, it sends a radar signal with constant radar power P_{radar} which will be received by an associated SN. In the other case, where a SN is selected to work as a radar receiver, it receives the radar signal of its associated transmitter and forwards the included information to the fusion center. For the transmission of information to the fusion center, we assume that each SN will use a transmission power P_{fusion} which should be adequate for a nearly error-free communication to the fusion center. Furthermore, each SN is connected to a weak power supply, e.g., a battery or an energy harvesting unit, with a total power budget P_{budget}. Note that usually most SNs are on *standby* to save energy and thus their available power budget can remain constant over a long period of network cycles. For other SNs, which are *active* either as transmitter or receiver, the available power budget is reduced either by P_{radar} or by P_{fusion}, respectively, in each network cycle. We denote the available power budget by $P_{\text{budget}}^{(k,l)}$, where k and l are the index of the SN and the current cycle of the network lifetime, respectively.

It is well-known, that the coverage area of a bistatic radar is surrounded by a Cassini oval [6]. Figure 1 shows different degenerations of coverage regions surrounded by the corresponding Cassini ovals. The degeneration of the shape emerges by variation of the distance between the position of the radar transmitter and receiver (equivalent with the distance between the foci of the oval) while the transmission power P_{radar} of the radar is kept constant. In order to avoid disconnected coverage regions, the distance between two SNs establishing a bistatic radar is limited by $d_{\max} = 2\sqrt[4]{\frac{P_{\text{radar}}}{P_{\min}}C}$. The quantity $P_{\min}$ is the minimum required power at the radar receiver to enable a proper radar operation, while C contains the radar cross section, the influence of antennas, the impact of filters, and other additional effects on the transmitted radar signal. Because of $d_{\max}$, only adjacent SNs are suitable candidates for establishing a bistatic radar. In this way, we can combine each combination of two different sensors k_1 and k_2, which have a smaller distance between their positions than $d_{\max}$, to the potential bistatic radars $R_{(k_1,k_2)}$ and $R_{(k_2,k_1)}$. Note, that although $R_{(k_1,k_2)}$ and $R_{(k_2,k_1)}$ consist of the same sensors k_1 and k_2, we distinguish them because of the two permissible operating modes of the sensors, i.e., the first index is preserved for the sensor working as transmitter while the second index identifies the corresponding receiver.

In order to achieve overlapping coverage regions for establishing long coverage chains, we have to determine all bistatic radars, e.g., $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$ with $k_1 \neq k_2$, $k_3 \neq k_4$, $k_1 \neq k_4$, $k_2 \neq k_3$, $(k_1,k_2) \neq (k_3,k_4)$ and $(k_1,k_2) \neq (k_4,k_3)$, which can have overlapping coverage regions, i.e., the Cassini oval corresponding to the sensors k_1 and k_2 has at least one intersection with the Cassini oval corresponding to the sensors k_3 and k_4. If any overlapping of the coverage areas of $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$ exists, then we point this situation out by $T_{(k_1,k_2),(k_3,k_4)} = 1$, otherwise $T_{(k_1,k_2),(k_3,k_4)} = 0$. Note, that the cases $T_{(k_1,k),(k,k_4)}$ and $T_{(k,k_2),(k_3,k)}$ for any k can never exist, since the k^{th} SN cannot operate in receive and transmit mode simultaneously. In contrast, the cases $T_{(k,k_2),(k,k_4)}$ and $T_{(k_1,k),(k_3,k)}$ are conceivable. Consider, that a connected chain of coverage regions only consists of adjacent bistatic radars, e.g., $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$ for which $T_{(k_1,k_2),(k_3,k_4)} = 1$ holds. In addition, we treat the left and the right guard area as coverage regions of two *virtual* radars R_ℓ and R_r, respectively. Both R_ℓ and R_r are treated as a normal representant of bistatic radars such that $T_{(k_1,k_2),\ell} = T_{\ell,(k_1,k_2)} = 1$ and $T_{r,(k_3,k_4)} = T_{(k_3,k_4),r} = 1$ whenever the coverage of radars $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$ overlap the left and right guard area, respectively. We denote by $\mathcal{T} = \{\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_N\}$ the family of sets over $\{(k_1,k_2) \mid 1 \leq k_1 \leq K, 1 \leq k_2 \leq K\}$ for which $T_{(k_1,k_2),(k_3,k_4)} = 1$ must hold for every two pair $(k_1,k_2) \in \mathcal{T}_i$ and $(k_3,k_4) \in \mathcal{T}_i$ with $1 \leq i \leq N$. In addition, each subset $\mathcal{T}_i$ must at least include both virtual radar R_ℓ and R_r. Furthermore, both requirements $\mathcal{T}_{i_1} \cup \mathcal{T}_{i_2} \neq \mathcal{T}_{i_3}$ and $\mathcal{T}_{i_3} \neq \emptyset$ are evident for each $i_1 \neq i_2$ and any i_3. Note, that any connected coverage chain is presented by a corresponding subset $\mathcal{T}_i$, which can also contain loops. Moreover, be aware that $\mathcal{T}$ does not necessarily contain disjoint coverage chains (paths). The number N of possible coverage chains is scenario dependent and usually very high. Hence, searching all possible combinations for the best power allocation strategy and for the optimal operating modes of all SNs will in practice be highly computation intensive.

In conclusion, each subset $\mathcal{T}_i$ includes all SNs, which can be chosen to connect the virtual radar R_ℓ with the virtual radar R_r, by a connected chain of coverage regions. In turn, each single coverage region is provided by a bistatic radar consisting of two SNs. The family of sets $\mathcal{T}$ includes all connected coverage chains, which are possible for the given scenario.

III. Lifetime Optimization

In order to investigate the lifetime of such a sensor network we can describe the maximal lifetime $L^\star$ by the optimization problem

$$\underset{L,\boldsymbol{\alpha},\mathcal{U}}{\text{maximize}} \quad L\,, \qquad \text{s. t.} \quad \mathcal{U} \subseteq \mathcal{T}\,,\ \alpha_{k,l} \in \{0,1\}\,,\ L \in \mathbb{N}\,,$$
$$\sum\nolimits_{l=1}^{L} \alpha_{k,l} P_{\text{radar}} + (1-\alpha_{k,l}) P_{\text{fusion}} \leq P_{\text{budget}}, \forall k \in \mathcal{U}_j, \forall \mathcal{U}_j \in \mathcal{U}. \tag{1}$$

The above maximization problem belongs to the classes of combinatorial and mixed integer problems and is hence very challenging and computationally intensive to solve. Apart from $L^\star$, the solution of (1) would provide the optimal operating modes $\boldsymbol{\alpha}^\star$ of all activated SNs and all optimal coverage chains $\mathcal{U}^\star$, which describes all activated SNs. We now set out to provide some insight by utilization of graph based methods.

A. Graph Based Investigations

The maximization problem (1) is a proper candidate to be implemented as a graph. Thus, we construct two different graphs $\mathcal{G}_1$ and $\mathcal{G}_2$, where the first one provides more insight to the problem (1) while the second one enables to solve problem (1) with less effort.

Each vertex of the graph $\mathcal{G}_1$ represents a single SN without considering its operating mode. Furthermore, two additional vertices s_ℓ and s_r are included in the graph, which represent a *virtual* SN located on the left guard area and a *virtual* SN located on the right guard area, respectively. An edge between any two vertices k_1 and k_2 exists, when the distance between the positions of the corresponding SNs is less than $d_{\max}$, i.e., (k_1, k_2) belongs to $\bigcup_{i=1}^{N} \mathcal{T}_i$. Moreover, an edge between the vertices s_ℓ and k_1 exists, when the SN k_1 is located on the left guard area. Similarly, an edge between the vertices k_2 and s_r exists, when the SN k_2 is located on the right guard area.

The vertices of the graph $\mathcal{G}_2$ represent all bistatic radars $R_{(k_1,k_2)}$ with $(k_1, k_2) \in \bigcup_{i=1}^{N} \mathcal{T}_i$ and both virtual radars R_ℓ and R_r. An edge between any two radars (vertices) $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$, R_ℓ and $R_{(k_3,k_4)}$, or $R_{(k_1,k_2)}$ and R_r exists, respectively, whenever $T_{(k_1,k_2),(k_3,k_4)} = 1$, $T_{\ell,(k_3,k_4)} = 1$, or $T_{(k_1,k_2),r} = 1$ holds.

Our goal is now to consider $\mathcal{G}_1$ and distinguish two special cases to provide more insight. The first case describes an upper bound on the maximum lifetime $L^\star$, while the second case provides a more realistic approximation of $L^\star$. Afterwards, we use $\mathcal{G}_2$ to suboptimally solve the optimization problem (1) by standard methods with lower computation effort.

B. Bound and Approximation for the Lifetime

By applying the Menger's theorem [7] and the max-flow min-cut theorem, we can deduce that the number M of disjoint paths between the vertices s_ℓ and s_r is equal to the cardinality $|\mathcal{U}^\star|$, if the capacity of each edge and each vertex is limited by one. Hence, we can compute M by searching the graph $\mathcal{G}_1$ for the minimum cut between the vertices s_ℓ and s_r, when simultaneously both vertex and edge capacities are limited by one. With this result, an achievable upper bound $L_{\max}$ on $L^\star$ is given by

$$L^\star \le M \left\lfloor \frac{P_{\text{budget}}}{\min\{P_{\text{radar}}, P_{\text{fusion}}\}} \right\rfloor . \tag{2}$$

Equality in (2) is attained, whenever $P_{\text{radar}} = P_{\text{fusion}}$ and $\left\lfloor \frac{P_{\text{budget}}}{\min\{P_{\text{radar}}, P_{\text{fusion}}\}} \right\rfloor = \frac{P_{\text{budget}}}{\min\{P_{\text{radar}}, P_{\text{fusion}}\}}$ hold. In (2), the expression within the floor function is responsible for the inequality sign, since both above theorems only provide the disjoint paths without any strategy for optimizing the operating modes of the activated SNs.

In practice, the operating modes of the SNs are more or less uniformly distributed. This results in the same number of transmitters and receivers. Thus, we can propose a more practical approximation L_{approx} of $L^\star$ by increasing the denominator of (2), which is described by

$$L^\star \approx M \left\lfloor \frac{2P_{\text{budget}}}{P_{\text{radar}} + P_{\text{fusion}}} \right\rfloor . \tag{3}$$

Note that in the case $P_{\text{radar}} = P_{\text{fusion}}$, both L_{approx} and $L_{\max}$ are equal. The approximation L_{approx} is usually a lower bound on $L^\star$, as we will see later.

An additional important insight is the fact, that M simultaneously describes an upper bound on the number n of disjoint paths for an n-barrier coverage situation. Since M is dependent on the specific distribution of the SNs, it is not possible to attain a required n-barrier coverage in a particular scenario, whenever $n > M$ for this scenario holds, cf. [2].

C. Maximization of the Lifetime

A direct optimization of (1) even by searching the entire graph $\mathcal{G}_2$ is often impossible, especially in sensor networks with huge number of nodes. A reduction of the computation effort is conceivable by searching subparts of $\mathcal{G}_2$ on the base of M. Since we efficiently can identify the edges belonging to the min-cut of $\mathcal{G}_1$ and in turn can select all corresponding SNs belonging to the identified edges, we extract the subset $\tilde{\mathcal{U}} \subseteq \mathcal{T}$, that include all bistatic radars consisting of the selected SNs. Then, we only need to search the subpart of $\mathcal{G}_2$ corresponding to $\tilde{\mathcal{U}}$. A further reduction of the computation effort is to obtain by applying standard and efficient methods like Dijkstra's algorithm [8]. However, for such algorithms specific cost functions are needed for the transition over the edges of $\mathcal{G}_2$. In our studies, it has turned out that exponential classes of cost functions achieve in most scenarios the best performance. Hence, we propose the cost function $w_{(k_1,k_2),(k_3,k_4)}$ for the directed edge pointing from the vertex (k_1, k_2) to (k_3, k_4). For the cost function we have to distinguish four different cases: *a*) if both bistatic radars $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$ share the same receiver, i.e., $k_2 = k_4$, and it holds that $P_{\text{budget}}^{(k_3,l)} \ge P_{\text{radar}}$, then $w_{(k_1,k_2),(k_3,k_4)} = \gamma v^{1-P_{\text{budget}}^{(k_3,l)}/P_{\text{budget}}}$; *b*) if both bistatic radars $R_{(k_1,k_2)}$ and $R_{(k_3,k_4)}$ share the same transmitter, i.e., $k_1 = k_3$, and it holds that $P_{\text{budget}}^{(k_4,l)} \ge P_{\text{fusion}}$, then $w_{(k_1,k_2),(k_3,k_4)} = (1-\gamma) v^{1-P_{\text{budget}}^{(k_4,l)}/P_{\text{budget}}}$; *c*) if k_1 and k_3 as well as k_2 and k_4 are different, and both $P_{\text{budget}}^{(k_3,l)} \ge P_{\text{radar}}$ and $P_{\text{budget}}^{(k_4,l)} \ge P_{\text{fusion}}$ simultaneously hold, then $w_{(k_1,k_2),(k_3,k_4)} = \gamma v^{1-P_{\text{budget}}^{(k_3,l)}/P_{\text{budget}}} + (1-\gamma) v^{1-P_{\text{budget}}^{(k_4,l)}/P_{\text{budget}}}$; *d*) otherwise $w_{(k_1,k_2),(k_3,k_4)} \mapsto \infty$. Recall, that after each network cycle the available power budget $P_{\text{budget}}^{(k,l)}$ of activated SNs is either reduced by P_{radar} or by P_{fusion} depending on their operating modes. Both quantities $v > 1$ and $0 < \gamma < 1$ are free parameters and must be optimized for the specific scenario.

By applying the above cost function to the graph $\mathcal{G}_2$ and searching paths with minimum costs between the vertices R_ℓ and R_r over the subset $\tilde{\mathcal{U}}$, we obtain suboptimal solutions for (1). The solution is suboptimal, since the true cost function is unknown and $w_{(k_1,k_2),(k_3,k_4)}$ is only a proper surrogate.

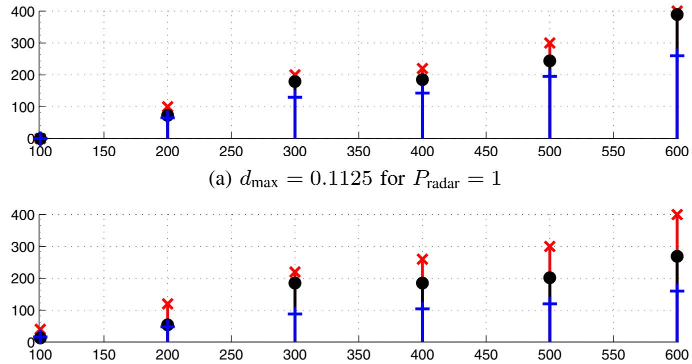
D. Additional Comments

In each network cycle, a connected path of coverage regions with minimum cost between the virtual radars R_ℓ on the left and R_r on the right side of the barrier is searched in $\mathcal{G}_2$ over $\tilde{\mathcal{U}}$ by the Dijkstra's algorithm. The sensors belonging to the determined path are subsequently activated with their respective operating mode and can now perform their tasks. Simultaneously, all vertices and edges corresponding to the determined path are excluded from the graph $\mathcal{G}_2$ for the next search procedure. This process is repeated until $n \leq M$ paths are found and an n-barrier coverage is established for the current network cycle. Note that applying this procedure mostly ensures for intersection-free chains of coverage regions, since in each iteration all activated sensors (vertices) are excluded from the whole set of sensors and they cannot further be considered as potential candidates for activation. Since in each network cycle activated sensors consume power for performing the radar task and for communication to the fusion center, the power budget of activated sensors is decreased either by P_{radar} or by P_{fusion}. Hence, after a specific network cycle, the search procedure is not anymore able to find n paths, since the power budget of essential sensors is exhausted and a further activation becomes impossible. In this state, the remaining power resources of the entire sensor network are too low to perform further observation cycles. At this point, the maximal lifetime of the sensor network is attained in the sense that an n-barrier coverage situation cannot be ensured anymore.

IV. Visualization and Numerical Results

In order to provide selected simulation results, we consider a squared region with unit side length in which $K = 100$ SNs are randomly distributed with uniform distribution. The width of each guard area is equal to 0.025. In order to investigate the lifetime over the number of SNs, we randomly add 100 SNs to the existing ones in successive simulations such that we in conclusion obtain $K \in \{100, 200, 300, 400, 500, 600\}$. For the other parameters we choose the following values: $P_{\text{radar}} \in \{1, 2\}$, $P_{\text{fusion}} = 0.5$, $P_{\text{budget}} = 10$, $\frac{C}{P_{\min}} = 0.00001$, $v = 1000$ and $\gamma = 0.8$. These values result in the maximum distances $d_{\max} = 0.1125$ and $d_{\max} = 0.1337$ for $P_{\text{radar}} = 1$ and $P_{\text{radar}} = 2$, respectively. We only consider a 1-barrier coverage situation. Note that since the computation effort over $\mathcal{G}_2$ due to the knowledge of $\tilde{\mathcal{U}}$ is low, we are able to simulate scenarios with up to $K = 600$ SNs in a short time. In contrast, other methods are only able to simulate scenarios with less than $K = 50$ SNs, cf. [4], [9].

For $P_{\text{radar}} = 1$ we obtain the values $M = (0, 5, 10, 11, 15, 20)$ by increasing the number K of SNs. It is noticeable that for $K = 100$ no connected coverage chains can be found in this specific scenario. In contrast, for $P_{\text{radar}} = 2$ we obtain the values $M = (2, 6, 11, 13, 15, 20)$. Since the maximum distance $d_{\max}$ is greater in case of $P_{\text{radar}} = 2$, the algorithm is able to even find $M = 2$ connected coverage chains for $K = 100$ SNs. Comparing both Figures 2a and 2b, we see that by increasing P_{radar} the lifetime for larger K becomes weak, since more power is used by each transmitter. In contrast, the lifetime becomes better by increasing P_{radar} for small K, since more bistatic radars can be established by the SNs.



(a) $d_{\max} = 0.1125$ for $P_{\text{radar}} = 1$

(b) $d_{\max} = 0.1337$ for $P_{\text{radar}} = 2$

Fig. 2. The upper bound $L_{\max}$ on the maximum lifetime, the lifetime L, and the approximated lifetime L_{approx} are visualized over the number of SNs by red crosses, black circles, and blue lines, respectively.

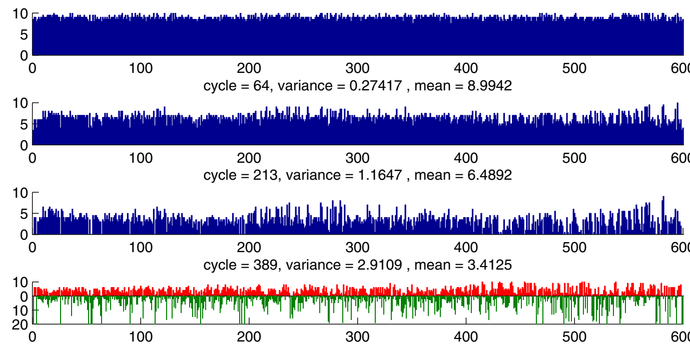


Fig. 3. Available power budget $P_{\text{budget}}^{(k,l)}$ of each SN for different network cycles is depicted in blue bars for 600 SNs. The maximum lifetime of the considered sensor network attains $L = 389$ cycles, although an expected power of 3.4125 remains unconsumed. The number of operating modes allocated to each SN is visualized in red and green bars for the transmit and receive mode, respectively.

As mentioned before, the approximated lifetime L_{approx} is usually a good lower bound on the maximum lifetime $L^\star$, as can be seen in Figure 2.

In Figure 3, we can observe the power distribution over the SNs and over different network cycles for $P_{\text{radar}} = 1$. A smart strategy for lifetime maximization seems to be the forcing of power consumption to be uniformly distributed over the SNs.

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