Lifetime and Power Consumption Analysis of Sensor Networks

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***Abstract*—Power consumption and lifetime are essential features of sensor networks. On the one hand, the power consumption should be as low as possible to enable an energy-aware system. On the other hand, the lifetime should be as long as possible to ensure for a comprehensive coverage. Especially, for application of sensor networks in extreme environments, it is also necessary to achieve high reliability over the whole lifetime. However, these features are contrary and they must be optimized simultaneously to achieve an optimal performance. In this paper, we thus study the minimization of the overall power consumption for any given lifetime and any required signal quality. First, a theoretical and challenging approach is proposed, which shows the feasible boundaries for both power reduction and achievability of a certain lifetime. Then, a practical approach is shown, which is nearly optimal and fits sufficiently together with the theoretical approach. Finally, selected results are visualized to show the performance of the new methods and to discuss the power consumption of the entire sensor network.**

I. INTRODUCTION

Due to the rise of Internet of things (IoT) and certain applications in 5th generation wireless systems (5G), sensor networks for various applications are drastically gaining importance. Since large-scale sensor networks usually utilize cheap and weak sensor nodes (SN), a complex signal processing within each SN is not applicable. Thus, mostly amplify-and-forward techniques are considered to relay an observed signal at each SN to a centralized unit, e.g., a fusion center. The task of the fusion center is then to perform a powerful processing with the aid of all transmitted signals in order to increase the reliability of the individual and independent observations. Several publications show that this reliability increases as the transmission power of each SN does. Naturally, the question arises how to guarantee for a minimum signal quality at the fusion center and simultaneously enable an energy-aware sensor network. Especially for sensor networks with huge number of SNs this question becomes essential, because the overall power consumption can drastically be reduced which will consequently prolong the network lifetime.

In this paper, we consider a common sensor network, which is used for sensing applications, i.e., target signal detection, localization, classification, and tracking. Figure 1 shows the target emitter, the sensing channel, independent and distributed SNs, the communication channel, and a fusion center. This scenario is well-investigated in many publications and will also serve as our framework in the present paper. The authors in [1] have considered a sensor network composed of microsensors and have described general architectural and algorithmic approaches to enhance the energy awareness of wireless sensor networks. In [2] a cluster-based approach and a centralized routing protocol is used to improve the network lifetime. A theoretical upper bound for the network lifetime is investigated in [3], which is in practice not achievable. A further notable publication is [4] in which different heuristics are used for lifetime maximization. The corresponding optimization problems are subsequently solved by numerical methods. In contrast, an analytical solution in closed-form to the power allocation problem is presented for several power constraints in [5], which improves the work [6]. This investigation is later extended in various ways in [7]–[14] and [15]. In the present paper, we also aim to extend our previous works and will answer to the question: 'Which amount of energy resources are needed to achieve a given lifetime in sensor networks?' This question is rarely investigated in the past, because of its mathematical challenges.

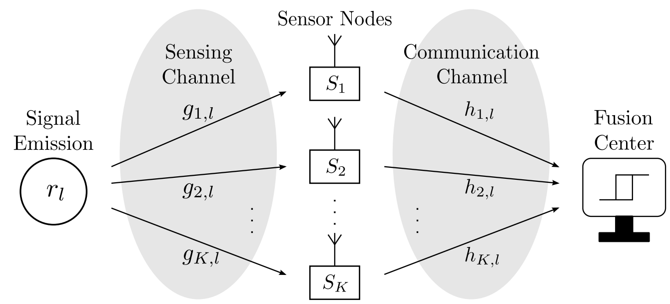

Fig. 1. A distributed sensor network.

In the present paper, we first introduce briefly the system model and formulate a power minimization problem for a given lifetime under several power constraints. Subsequently, the optimization problem is rewritten to show its convex nature and to enable its computation by standard numerical methods. Although the considered optimization problem is of theoretical nature and is in addition computationally intensive to solve, it provides sharp lower and upper bounds for the network power consumption and its lifetime, respectively. Second, we develop a practical method with less computation effort in order to achieve nearly optimal solutions to the power minimization problem for realistic scenarios. Finally, selected numerical results are visualized to show the performance of the new methods and to discuss the power consumption of the entire sensor network.

Mathematical Notations:

Throughout this paper, we denote the sets of natural, real and complex numbers by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$, respectively. Note that the set of natural numbers does not include the element zero. Moreover, $\mathbb{R}_+$ denotes the set of non-negative real numbers. Furthermore, we use the subset $\mathbb{F}_N \subseteq \mathbb{N}$, which is defined as $\mathbb{F}_N := \{1, \ldots, N\}$ for any given natural number N. We denote the absolute value of a real or complex-valued number z by $|z|$. The expected value of a random variable v is denoted by $\mathcal{E}[v]$. Moreover, the notation $V^\star$ stands for the value of an optimization variable V where the optimum is attained.

II. Overview and Technical System Description

In this paper, we use an extension of the system model that is described in [5]. The extended system model is depicted in Figure 2 and is briefly presented in the following.

We assume a discrete time system and denote the index of each observation process by $l \in \mathbb{F}_L$, where $L \in \mathbb{N}$ describes the lifetime of the sensor network. Hence, the network under consideration can only perform L consecutive observations with optimal performance and will be out of use afterwards. We consider a sensor network consisting of $K \in \mathbb{N}$ independent and spatially distributed SNs which receive random observations in each observation process. If a target signal $r_l \in \mathbb{C}$ with $R := \mathcal{E}[|r_l|^2]$ and $0 < R < \infty$ is present, then the received power at the SN S_k is a part of the emitted power from the actual target. Each received signal is weighted by the corresponding channel coefficient $g_{k,l} \in \mathbb{C}$ and is disturbed by additive white Gaussian noise (AWGN) $m_{k,l} \in \mathbb{C}$ with $M_k := \mathcal{E}[|m_{k,l}|^2] < \infty$. We assume that the coherence time of all sensing channels is much longer than the whole length of the observation process. Thus, the expected value and the quadratic mean of each coefficient during each observation step can be assumed to be equal to their instantaneous values, i.e., $\mathcal{E}[g_{k,l}] = g_{k,l}$ and $\mathcal{E}[|g_{k,l}|^2] = |g_{k,l}|^2$. Furthermore, the channel coefficients as well as the disturbances are assumed to be uncorrelated and jointly independent. The sensing channel is obviously wireless.

All SNs continuously take samples from the disturbed received signal and amplify them by $u_{k,l} \in \mathbb{R}_+$ without any additional data processing. Thus, the output signal and the expected value of its transmission power are described by

$$x_{k,l} := (r_l g_{k,l} + m_{k,l}) u_{k,l} \, , \; k \in \mathbb{F}_K \, , \; l \in \mathbb{F}_L \, , \qquad (1)$$

and

$$X_{k,l} := \mathcal{E}[|x_{k,l}|^2] = (R|g_{k,l}|^2 + M_k) u_{k,l}^2 \, , \; k \in \mathbb{F}_K \, , \; l \in \mathbb{F}_L \, , \qquad (2)$$

respectively. The local measurements are then transmitted to a fusion center which is placed at a remote location. The data communication between each SN and the fusion center can be either wired or wireless. In the latter case, a distinct waveform for each SN is used to distinguish the communication of different SNs and to suppress inter-user (inter-node) interferences at the fusion center. Hence, all K received signals at the fusion center are pairwise uncorrelated and are assumed to be conditionally independent. Each received signal at the fusion center is also weighted by the corresponding channel coefficient $h_{k,l} \in \mathbb{C}$ and is disturbed by additive white Gaussian noise $n_{k,l} \in \mathbb{C}$ with $N_k := \mathcal{E}[|n_{k,l}|^2] < \infty$, as well. We also assume that the coherence time of all communication channels is much longer than the whole length of the observation process. Thus, the expected value and the quadratic mean of each coefficient during each observation step can be assumed to be equal to their instantaneous values, i.e., $\mathcal{E}[h_{k,l}] = h_{k,l}$ and $\mathcal{E}[|h_{k,l}|^2] = |h_{k,l}|^2$. Furthermore, the channel coefficients as well as the disturbances are assumed to be uncorrelated and jointly independent.

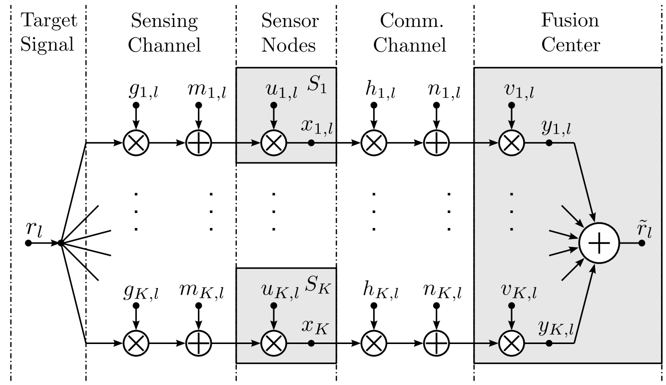

Fig. 2. System model of the distributed sensor network.

The noisy received signals at the fusion center are weighted by $v_{k,l} \in \mathbb{C}$ and combined together in order to obtain a single reliable observation $\tilde{r}_l$ of the actual target signal r_l. In this way, we obtain

$$y_{k,l} := \big((r_l g_{k,l} + m_{k,l}) u_{k,l} h_{k,l} + n_{k,l} \big) v_{k,l} \, , \; k \in \mathbb{F}_K \, , \; l \in \mathbb{F}_L \, , \qquad (3)$$

and hence,

$$\tilde{r}_l := \sum_{k=1}^{K} y_{k,l} = r_l \sum_{k=1}^{K} g_{k,l} u_{k,l} h_{k,l} v_{k,l} + \sum_{k=1}^{K} (m_{k,l} u_{k,l} h_{k,l} + n_{k,l}) v_{k,l} \, . \qquad (4)$$

Note that the fusion center can separate the input streams because the data communication is either wired or performed by distinct waveforms for each SN.

In order to obtain a single reliable observation at the fusion center, the value $\tilde{r}_l$ should be a good estimate of the present target signal r_l. Thus, the amplification factors $u_{k,l}$ and the weights $v_{k,l}$ should be chosen such as to minimize the average absolute deviation between $\tilde{r}_l$ and the true target signal r_l. The corresponding optimization program is elaborated in the next section.

III. Power and Lifetime Optimization

In this section, we first introduce the power minimization problem and subsequently develop its solution. Since the corresponding optimization problem is non-convex in its general form, we solve it by subsequent applications of the Lagrangian multipliers method with equality constraints, Karush-Kuhn-Tucker (KKT) conditions, and straightforward usage of mathematical analysis, see [16, pp. 323–335] and [17, pp. 243–244].

A. The Optimization Problem

As mentioned in the last section, the quantity $\tilde{r}_l$ should be a good estimate for the present target signal r_l. In particular, we aim at finding estimators $\tilde{r}_l$ of minimum mean squared error in the class of unbiased estimators for each r_l.

The estimate $\tilde{r}_l$ is unbiased simultaneously for each r_l if $\mathcal{E}[\tilde{r}_l - r_l] = 0$, i.e., from equation (4) we obtain the identity

$$\sum_{k=1}^{K} g_{k,l} u_{k,l} h_{k,l} v_{k,l} = 1 \,, \;\; l \in \mathbb{F}_L \,. \tag{5}$$

This identity is our first constraint in what follows. Note that the mean of the second sum in (4) vanishes since the noise is zero-mean. Furthermore, we do not consider the impact of both random variables $g_{k,l}$ and $h_{k,l}$ as well as their estimates in our calculations because the coherence time of both channels is assumed to be much longer than the target observation time. Note that equation (5) is complex-valued and may be separated as

$$\sum_{k=1}^{K} u_{k,l} |v_{k,l} g_{k,l} h_{k,l}| \cos(\vartheta_{k,l} + \phi_{k,l}) = 1 \,, \;\; l \in \mathbb{F}_L \,, \tag{6}$$

and

$$\sum_{k=1}^{K} u_{k,l} |v_{k,l} g_{k,l} h_{k,l}| \sin(\vartheta_{k,l} + \phi_{k,l}) = 0 \,, \;\; l \in \mathbb{F}_L \,, \tag{7}$$

where $\vartheta_{k,l}$ and $\phi_{k,l}$ are the phases of $v_{k,l}$ and the product $g_{k,l} h_{k,l}$, respectively.

The average power consumption of each node is approximately equal to its average output power $X_{k,l}$, if the input signal is negligible in comparison to the output signal and if the nodes have smart power components with low-power dissipation loss. We assume that equality between $X_{k,l}$ and the average power consumption of each node is ensured. In the present work, we assume that the average output power-range of each SN is limited by $P_{\text{min}} \in \mathbb{R}_+$ and $P_{\text{max}} \in \mathbb{R}_+$ with $0 \leq P_{\text{min}} < P_{\text{max}}$. The lower limit P_{min} denotes the minimum power which is needed to guarantee the awareness and presence of the SN while the upper limit P_{max} denotes the maximum allowed transmission power per SN due to power regulation standards or due to the functional range of the integrated circuit elements. In addition, each SN is usually powered by weak energy supplies, e.g., batteries, such that the operation time of the k^{th} SN is limited by an available power budget $P_{k,l,\text{bud}} \in \mathbb{R}_+$. Note that $l = 0$ describes the time point in which each SN has a full-power budget while after each observation process the new power budget $P_{k,l+1,\text{bud}}$ is equal to $P_{k,l,\text{bud}} - X_{k,l}$, with $X_{k,0} = 0$ for all $k \in \mathbb{F}_K$. In this way, the sensor network operates under the constraints

$$\begin{aligned} & P_{\text{min}} \leq X_{k,l} \leq P_{\text{max}} \\ \Leftrightarrow\;& P_{\text{min}} \leq (R|g_{k,l}|^2 + M_k) u_{k,l}^2 \leq P_{\text{max}} \,, \; k \in \mathbb{F}_K \,, \; l \in \mathbb{F}_L \,, \end{aligned} \tag{8}$$

and

$$\begin{aligned} & \sum_{l=1}^{L} X_{k,l} \leq P_{k,0,\text{bud}} \\ & \quad \Leftrightarrow\; \sum_{l=1}^{L} (R|g_{k,l}|^2 + M_k) u_{k,l}^2 \leq P_{k,0,\text{bud}} \,, \;\; k \in \mathbb{F}_K \end{aligned} \tag{9}$$

In order to guarantee for a certain signal quality at the fusion center, the mean squared error $\mathcal{E}[|\tilde{r}_l - r_l|^2]$ should not exceed a given maximum value $V_{\text{max}} \in \mathbb{R}_+$. By using equation (4) and the identity (5) we may write the next constraint as

$$\mathcal{E}\left[|\tilde{r}_l - r_l|^2\right] = \sum_{k=1}^{K} \big(M_k u_{k,l}^2 |h_{k,l}|^2 + N_k\big) |v_{k,l}|^2 \leq V_{\text{max}} \,, \tag{10}$$

which must hold for all $l \in \mathbb{F}_L$.

The objective is now to minimize the overall power consumption of the sensor network for a given lifetime L, i.e.,

$$\begin{aligned} P_{\text{over}}^{\star} :=& \underset{u_{k,l}, v_{k,l}}{\text{minimize}} \quad \sum_{k=1}^{K} \sum_{l=1}^{L} X_{k,l} \\ =& \underset{u_{k,l}, v_{k,l}}{\text{minimize}} \quad \sum_{k=1}^{K} \sum_{l=1}^{L} (R|g_{k,l}|^2 + M_k) u_{k,l}^2 \,. \end{aligned} \tag{11}$$

In summary, the optimization problem is to minimize the overall power consumption in (11) with respect to $u_{k,l}$ and $v_{k,l}$, subject to constraints (6), (7), (8), (9) and (10). Note that the optimization problem is a *signomial program*, which is a generalization of *geometric programming*, and is thus non-convex in general, see [18].

In order to avoid misunderstandings, we mention that the minimization of the overall power consumption for any given lifetime L is in general not equivalent to the maximization of the lifetime for a corresponding given overall power P_{over}, since the lifetime is of discrete nature while the power consumption is usually continuous. However, the solution difference between both optimization methods is at most only a single count in the lifetime and hence can be neglected in practice. Since the mathematical description of the lifetime maximization needs considerably more effort than the overall power minimization, we thus investigate the minimization of the power consumption in order to obtain insight for the maximization of the network lifetime.

B. Theoretical and Practical Solutions

For the sake of brevity, we define two new quantities $\alpha_{k,l}$ and $\beta_{k,l}$ by

$$\alpha_{k,l} := \sqrt{\frac{|g_{k,l}|^2}{M_k}} \quad \text{and} \quad \beta_{k,l} := \sqrt{\frac{N_k(R|g_{k,l}|^2 + M_k)}{M_k |h_{k,l}|^2}} \,. \tag{12}$$

TABLE I
VALUES OF FIXED PARAMETERS FOR ALL PLOTS.

Parameter:	R	$\mathcal{E}[\lvert g_{k,l}\rvert^2]$	$\mathcal{E}[\lvert h_{k,l}\rvert^2]$	M_k	N_k	P_{min}
Value:	1	1	1	1	1	0

TABLE II
DEFAULT PARAMETER VALUES FOR ALL PLOTS.

Parameter:	K	L	P_{max}	P_{bud}	V_{max}
Default value:	100	100	0.36	1.2	1.2

Since the above optimization problem is closely related to an optimization problem considered in [15], a first optimization over $v_{k,l}$ leads to the problem

$$\underset{X_{k,l}}{\text{minimize}} \quad \sum_{k=1}^{K}\sum_{l=1}^{L} X_{k,l} \tag{13a}$$

$$\text{subject to} \quad P_{\text{min}} \leq X_{k,l} \leq P_{\text{max}}\,,\ k \in \mathbb{F}_K\,,\ l \in \mathbb{F}_L\,, \tag{13b}$$

$$\sum_{l=1}^{L} X_{k,l} \leq P_{k,0,\text{bud}}\,,\ k \in \mathbb{F}_K\,, \tag{13c}$$

$$\sum_{k=1}^{K} \frac{X_{k,l}\alpha_{k,l}^2}{X_{k,l}+\beta_{k,l}^2} \geq V_{\text{max}}^{-1}\,,\ l \in \mathbb{F}_L\,, \tag{13d}$$

where equation (2) for the relation between $u_{k,l}$ and $X_{k,l}$ is used. It is easy to show that problem (13) is a convex optimization problem, see [5], and it can be solved by standard convex optimization tools.

The solution of (13) yields the overall power consumption $P_{\text{over}}^{\star}$ and all allocated powers $X_{k,l}^{\star}$ for a given lifetime L, signal quality V_{max}, minimum and maximum allowed transmission powers P_{min} and P_{max}, and power budgets $P_{k,0,\text{bud}}$. However, this solution in advance needs the knowledge about all channel realizations $g_{k,l}$ and $h_{k,l}$ and all noises $m_{k,l}$ and $n_{k,l}$, which are mostly unknown at the starting time of the sensor network. Nevertheless, the solution of (13) provides theoretical limits for the overall power consumption $P_{\text{over}}^{\star}$ and the network lifetime L, and furthermore, it enables comparisons of more practical methods. To provide a more realistic method, we highlight the following heuristic.

One possible approach is to optimize the power consumption per observation time. This means that at the beginning of the l^{th} observation process, a relaxed version of (13) is solved which neglects the impact of all upcoming observation steps. The relaxed version of (13) for the l^{th} observation process is described by

$$\underset{X_{k,l}}{\text{minimize}} \quad \sum_{k=1}^{K} X_{k,l} \tag{14a}$$

$$\text{subject to} \quad P_{\text{min}} \leq X_{k,l} \leq P_{\text{max}}\,,\ k \in \mathbb{F}_K\,, \tag{14b}$$

$$X_{k,l} \leq P_{k,l,\text{bud}}\,,\ k \in \mathbb{F}_K\,, \tag{14c}$$

$$\sum_{k=1}^{K} \frac{X_{k,l}\alpha_{k,l}^2}{X_{k,l}+\beta_{k,l}^2} \geq V_{\text{max}}^{-1}\,. \tag{14d}$$

The solution of the relaxed optimization problem (14) is well-investigated and even solved analytically in closed-form in [15], cf. also [5]. Since this solution is available in closed-form, the computation of the power allocation needs less effort and can simply be performed on-line for each observation process. However, due to the relaxation of (13), the solution of (14) is only a suboptimal solution. If we denote the suboptimal solution of (14) by $\tilde{P}_{\text{over}}^{\star}$ with the suboptimal powers $\tilde{X}_{k,l}^{\star}$, then the inequality

$$\tilde{P}_{\text{over}}^{\star} = \sum_{l=1}^{L}\left(\sum_{k=1}^{K}\tilde{X}_{k,l}^{\star}\right) = \sum_{l=1}^{L}\underset{X_{k,l}}{\text{minimize}}\sum_{k=1}^{K} X_{k,l} \tag{15}$$

$$\geq \underset{X_{k,l}}{\text{minimize}}\sum_{k=1}^{K}\sum_{l=1}^{L} X_{k,l} = \sum_{l=1}^{L}\sum_{k=1}^{K} X_{k,l}^{\star} = P_{\text{over}}^{\star} \tag{16}$$

obviously holds. This relation simply shows that due to the shortage of information, a sensor network which is optimized stepwise will consume more power and thus will have a shorter lifetime in comparison to an overall optimization by (13). Another difference is the development of the available power budget $P_{k,l,\text{bud}}$ over the time. Recall, that for each observation process, the new available power budgets must be updated as $P_{k,l+1,\text{bud}} = P_{k,l,\text{bud}} - \tilde{X}_{k,l}^{\star}$ while in contrast for an optimization by (13) the development would be $P_{k,l+1,\text{bud}} = P_{k,l,\text{bud}} - X_{k,l}^{\star}$. Fortunately, we will see later that both developments converge together in many scenarios. The convergence speed is certainly a function of all parameters, especially dominated by P_{min}, P_{max}, $P_{k,0,\text{bud}}$ and V_{max}.

It is self-evident that both optimizations (13) and (14) represent two extreme cases for a variety of optimization methods. Based on the optimization method in (14), other heuristics can be proposed to gain a better performance. For example, the optimization in (14) can be extended by a robust method, in which information about the channel states and noise values of upcoming observation steps is not needed. Another approach is to extend (14) by channel and noise estimation methods, e.g., based on Kalman filtering, in order to obtain sufficient information about the unknown parameters of upcoming observation steps. These and other smart optimization methods will lead to an improved performance at the cost of more complexity in comparison with the method proposed in (14). The investigation of these methods is devoted to future works.

IV. VISUALIZATION AND NUMERICAL RESULTS

In order to evaluate the performance of both optimization problems (13) and (14), we perform four simulations in the same scenario and for the same network. For all four simulations the values given in Table I are kept constant. Especially, all channel and noise realizations remain the same in all simulations to simplify subsequent comparisons. These realizations are drown randomly from independent Gaussian distributions. The available power budget $P_{k,0,\text{bud}}$ is assumed to be equal for all SNs and thus denoted by P_{bud}. Figure 3 shows simulation results with parameter values given in

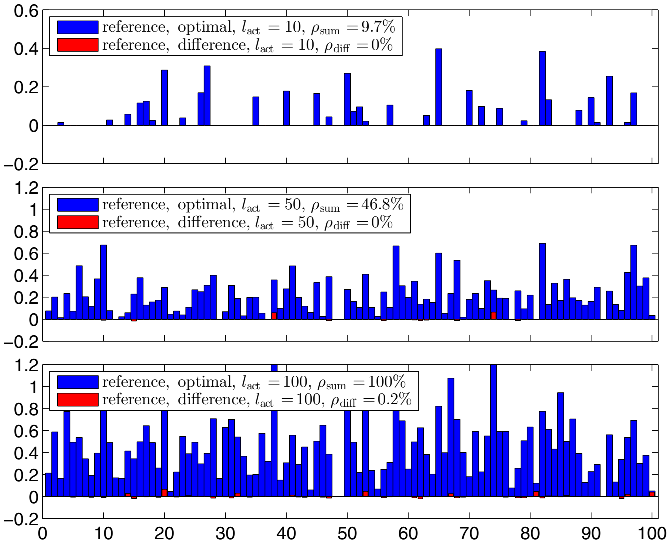

Fig. 3. Reference curves for comparisons with other simulations. Both minimum overall power consumptions, and the expected number of active SNs in each observation step are equal to $P^{\star}_{\text{over}} = 44.12$, $\tilde{P}^{\star}_{\text{over}} = 44.20$, and 4, respectively.

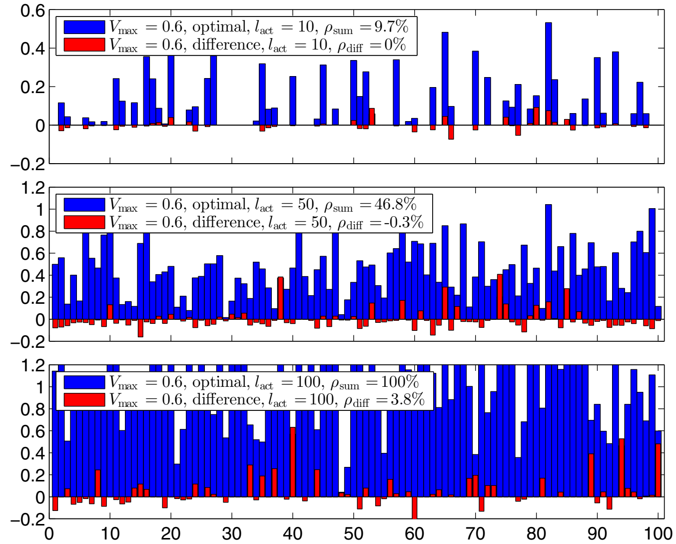

Fig. 4. Decreasing V_{max} worsens the lifetime, the power consumption, and the convergence speed between both optimization results. Both minimum overall power consumptions, and the expected number of active SNs in each observation step are equal to $P^{\star}_{\text{over}} = 95.15$, $\tilde{P}^{\star}_{\text{over}} = 98.72$, and 7, respectively.

Table II and these results serve as references for all other figures. In all other figures the change of only one of the parameters P_{max}, P_{bud} or V_{max} is shown. The specific new value of the changed parameter is noted in the legend of the corresponding figure. In each legend three other values are given. The first value is the actual observation process l_{act} that shows in which observation step the power distribution within the sensor network is illustrated. The second and third values are defined by $\rho_{\text{sum}} := \sum_{l=1}^{l_{\text{act}}} \sum_{k=1}^{K} X^{\star}_{k,l} / P^{\star}_{\text{over}}$ and $\rho_{\text{diff}} := \sum_{l=1}^{l_{\text{act}}} \sum_{k=1}^{K} (\tilde{X}^{\star}_{k,l} - X^{\star}_{k,l}) / P^{\star}_{\text{over}}$, respectively. The value of ρ_{sum} describes the amount of $P^{\star}_{\text{over}}$ in percent, which is already consumed by the network at the observation step l_{act} by utilization of the optimization (13). The value of ρ_{diff} reflects the percentage of $P^{\star}_{\text{over}}$, which is additionally consumed at the observation step l_{act} due to utilization of the suboptimal optimization (14). Hence, the absolute value of ρ_{diff} should be as small as possible for describing an accurate fit of the suboptimal solution to the global one. The index of all SNs is shown on the abscissa of each plot. In each figure, the sum $\sum_{l=1}^{l_{\text{act}}} X^{\star}_{k,l}$ of consumed powers for each SN is visualized in a blue bar. These bars together show how the power distribution over the SNs is, where a uniform distribution over all SNs is preferential. In addition, the differences $\sum_{l=1}^{l_{\text{act}}} \tilde{X}^{\star}_{k,l} - X^{\star}_{k,l}$ are depicted in red bars. In contrast to the blue bars, the red bars show how both theoretical and practical methods fit together, where smaller deviations of each bar from zero is more favorable. Moreover, the captions include the minimum achieved values $P^{\star}_{\text{over}}$ and $\tilde{P}^{\star}_{\text{over}}$ for each simulation. Furthermore, the expected number of active SNs in each observation step is stated in the caption, where for both optimization methods (13) and (14) the same number always results in our simulations.

In each figure, three states of the network lifetime are presented. The most upper, the ones in the middle, and the most lower illustrations represent the power distribution after $l_{\text{act}} = 10$, $l_{\text{act}} = 50$, and $l_{\text{act}} = 100$ observation cycles, respectively.

In Figure 3, we observe that by utilization of the suboptimal optimization (14) the overall power consumption is slightly increased form $P^{\star}_{\text{over}} = 44.12$ to $\tilde{P}^{\star}_{\text{over}} = 44.20$. This small increment of the power consumption depends on the specific values of the parameters P_{max}, P_{bud} and V_{max} and is in practice negligible for most scenarios. Additionally, it is visible from the few red bars that both optimization methods fit sufficiently together. These observations are generally valid and they can be verified by the aid of all other figures.

Figure 4 shows the variation of V_{max} and its impact on the performance of both optimization methods. This parameter has the most impact on the performance. Both the number of active SNs and the overall power consumption highly fluctuate with the variation of V_{max}.

In Figure 5 the performance change of both optimization methods over the parameter P_{bud} is depicted. The available power budget has the least influence on both the number of active SNs and the overall power consumption. The reason is that either the available power budget is large enough to achieve the given lifetime or it is too small such that the corresponding optimization problem becomes infeasible.

Figure 6 illustrates the effect of P_{max} on the performance. As expected, the lifetime is increasing in P_{max} while the power consumption is decreasing. Thus, any limitation of the transmission power by P_{max} and/or by P_{min} has always a negative effect on the performance of sensor networks.

In conclusion, the overall power consumptions $P^{\star}_{\text{over}}$ and $\tilde{P}^{\star}_{\text{over}}$ are monotonically decreasing in the parameters P_{max}, P_{bud} and V_{max} while the lifetime L is an increasing function of these parameters. Another important observation is that by decreasing P_{bud} or V_{max}, while the other parameters are kept constant, the value of ρ_{diff} increases. This shows that the stepwise optimization by (14) converges slower to the global solution by (13). The reason behind this statement is that the gap between the solutions (13) and (14) increases since

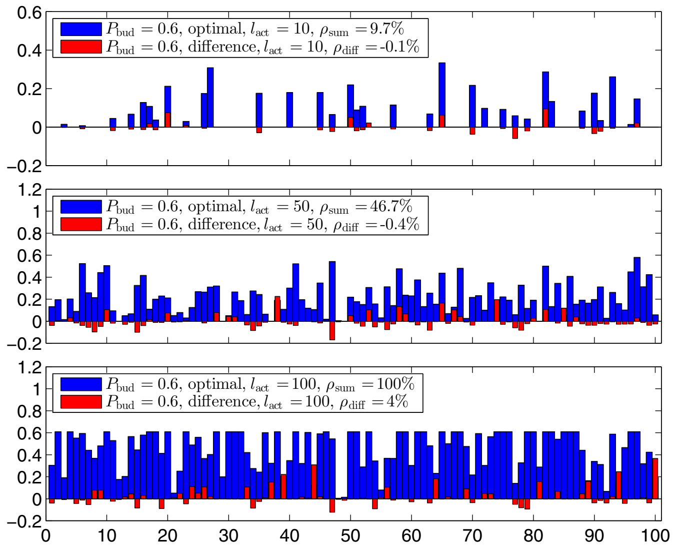

Fig. 5. Decreasing P_{bud} can worsen the lifetime, the power consumption, and the convergence speed. Both minimum overall power consumptions, and the expected number of active SNs in each observation step are equal to $P^{\star}_{\text{over}} = 44.64$, $\tilde{P}^{\star}_{\text{over}} = 46.41$, and 4, respectively.

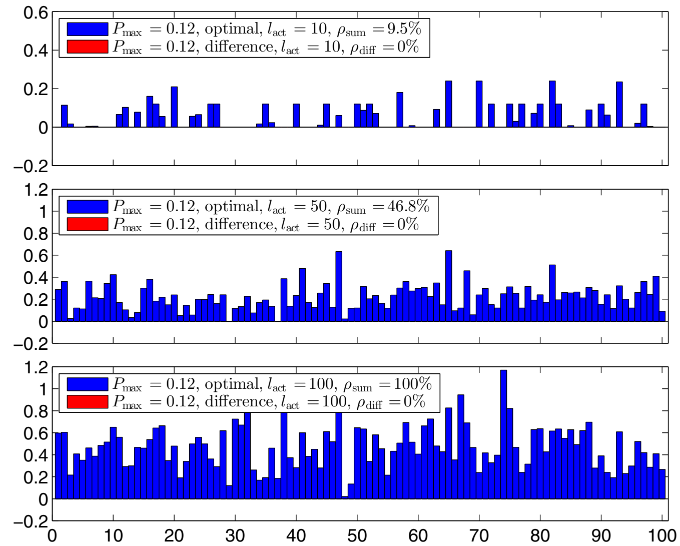

Fig. 6. Decreasing P_{max} always worsens both the lifetime and the power consumption, but in contrast it increases the convergence speed. Both minimum overall power consumptions, and the expected number of active SNs in each observation step are equal to $P^{\star}_{\text{over}} = 47.39$, $\tilde{P}^{\star}_{\text{over}} = 47.39$, and 6, respectively.

sharper constraints are to handle, especially by the relaxed optimization method. Conversely, decreasing P_{max}, while the other parameters are kept constant, will result in a decreased ρ_{diff} which in turn shows the increased convergence speed of the solution by (14) to the global one by (13). The fact behind this is that by decreasing P_{max} the power allocation engages more SN and the activation of SNs is more distributed.

Since the evaluation of (13) is highly computation intensive, we unfortunately were not able to simulate networks with larger values of L, that are more relevant in practice. Furthermore, a sensitivity analysis of both lifetime and power consumption under variation of other network parameters is also important and will be investigated in future works.

V. Conclusion

Power consumption and lifetime are pronounced features of sensor networks. This paper provides a theoretical and a practical method for minimizing the power consumption for any given lifetime. Both methods provide deep insight into the distribution of power among the sensor nodes over the time. Comparisons of both methods are performed via extensive simulations. Especially, it is shown that a power allocation by the practical method sufficiently converges to the power allocation performed by the aid of the theoretical method.

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