Robust Transceiver Design in Full-Duplex MIMO Cognitive Radios

Ali Cagatay Cirik*, Sudip Biswas*, Omid Taghizadeh†, An Liu‡, and Tharmalingam Ratnarajah*
* Institute for Digital Communications, School of Engineering, University of Edinburgh, Edinburgh EH9 3JL, U.K.
† Institute for Theoretical Information Technology, RWTH Aachen University, Aachen, 52074, Germany
‡ Department of Electrical and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong
Email: [a.cirik, sudip.biswas]@ed.ac.uk

***Abstract*—We study a full duplex (FD) multiple-input multiple-output (MIMO) cognitive cellular network, in which a secondary base-station (BS) operating in FD mode serves multiple uplink (UL) and downlink (DL) secondary users (SUs) operating in half-duplex (HD) mode simultaneously. The spectrum is shared between secondary and primary networks, and thus uplink SUs and secondary BS generate interference on primary users (PUs). We assume that the channel state information (CSI) available at the transmitters is imperfect, and the errors of the CSI are assumed to be norm bounded. Under the impact of channel uncertainty, we address the robust minimization of the sum of mean-squared-errors (MSE) of all estimated symbols subject to power constraints at the uplink SUs and secondary BS, and interference constraints projected to each PU. We show that this problem can be cast as a Semidefinite programming (SDP), and joint design of transceiver matrices can be obtained through an iterative algorithm. Numerical results are presented to show the effectiveness and robustness of the proposed robust algorithm.**

Keywords*—Cognitive radio, full-duplex, imperfect CSI, MIMO, MSE, multi-user, self-interference, transceiver designs.

I. Introduction

With the proliferation of wireless data traffic, how to effectively and efficiently utilize the scarce spectrum resources has become an extremely important issue. In currently deployed half-duplex (HD) wireless communication systems, transmission and reception are always orthogonal in time or in frequency. Among the emerging technologies for next-generation wireless networks, full-duplex communication (FD) is considered a way to potentially double the speed of wireless communication, since it enables available spectral resources to be fully utilized both in time and frequency [1], [2].

In addition to FD systems, cognitive radio system is also a promising technology to enhance spectrum efficiency. In underlay cognitive radio systems, a set of unlicensed secondary users (SUs) operate within the service range of licensed primary users (PUs) where the amount of interference from SUs to PUs must be constrained to meet the Quality-of-Service (QoS) requirements for the PUs. Since it is difficult to obtain the estimates of the channels between SUs and PUs (due to the lack of full SU-PU cooperation), it is important to consider the imperfect channel estimates, and develop robust beamforming schemes that ensure constrained interference on PUs [3]- [4].

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Resource allocation problems for FD non-cognitive cellular systems have been studied in [5]- [7]. A sum mean-squared-error (MSE) minimization problem for a multiple-input multiple-output (MIMO) FD underlay cognitive radio system has been studied in [8], in which the optimization problem has been cast as a second-order cone program (SOCP). But, the authors have not taken the channel estimation errors into account, and the SOCP-based algorithm proposed in [8] cannot be applied under norm-bounded deterministic imperfect channel-state-information (CSI). Therefore, in this paper, we consider a scenario where a secondary base-station (BS) operating in FD mode communicates with uplink (UL) and downlink (DL) SUs operating in HD mode simultaneously within the service range of multiple PUs. In addition to self-interference, co-channel interference (CCI) is also taken into account to design the optimum robust beamformers under a norm-bounded-error model, i.e., the instantaneous channel lies in a known set of possible values, which represents the amount of uncertainty on the channel [3]- [4]. We study the sum-MSE as the objective function to minimize subject to power constraints at the UL SUs and secondary BS, and interfering power constraints at the PUs. Since this problem is semi-infinite and non-convex, the semi-infinite constraints are first transformed into the tractable forms, and an iterative Semidefinite programming (SDP) algorithm which optimizes the transmit and receiving beamforming matrices in alternating manner is proposed. At each iteration, sum-MSE decreases monotonically, and is guaranteed to converge.

Notation: The following notations are used in this paper. Matrices and vectors are denoted as bold capital and lowercase letters, respectively. $(\cdot)^T$ is the transpose; $(\cdot)^*$ is the conjugate, and $(\cdot)^H$ is the conjugate transpose. $\mathbb{E}\{\cdot\}$ means the statistical expectation; $\mathbf{I}_N$ is the N by N identity matrix; $\mathbf{0}_{N\times M}$ is the N by M zero matrix; $\mathrm{tr}\{\cdot\}$ is the trace; $\mathrm{diag}\left(\mathbf{A}\right)$ is the diagonal matrix with the same diagonal elements as $\mathbf{A}$. $\mathcal{CN}\left(\mu,\sigma^2\right)$ denotes a complex Gaussian distribution with mean μ and variance σ^2. $vec(\cdot)$ stacks the elements of a matrix to one long column vector. The operator $\otimes$ denotes Kronecker product and $\perp$ denotes the statistical independence. $\|\mathbf{X}\|_F$ and $\|\mathbf{x}\|_2$ denote the Frobenius norm of a matrix $\mathbf{X}$ and the Euclidean norm of a vector $\mathbf{x}$, respectively. $\lfloor \mathbf{A}_i \rfloor_{i=1,\ldots,K}$ denotes a tall matrix (or vector) obtained by stacking the matrices $\mathbf{A}_i,\ i=1,\ldots,K$. $\Re\{\mathbf{X}\}$ denotes the real part of $\mathbf{X}$. $\mathbf{A}\succeq\mathbf{0}$ indicates that $\mathbf{A}$ is a positive semidenite matrix. $\mathcal{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote the ensemble of all $m\times n$ real and complex matrices, respectively.

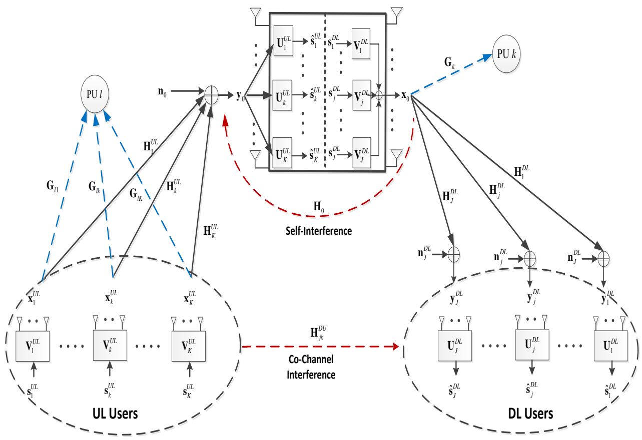

Fig. 1: An illustration of a Full-duplex multi-user MIMO system setup.

II. System Model

We study a cognitive cellular system, in which a secondary FD mode BS communicates with HD mode UL and DL SUs, simultaneously within the service range of PUs as seen in Fig. 1. The BS equipped with M_0 transmit and N_0 receive antennas serves K UL and J DL users simultaneously. The number of antennas of the k-th UL and the j-th DL user are denoted by M_k and N_j, respectively. The number of data streams transmitted from the k-th UL user (to the j-th DL user) is denoted by d_k^{UL} (d_j^{DL}).

The channels $\mathbf{H}_k^{UL} \in \mathbb{C}^{N_0 \times M_k}$ and $\mathbf{H}_j^{DL} \in \mathbb{C}^{N_j \times M_0}$ represent the k-th UL and the j-th DL channel, respectively. $\mathbf{H}_0 \in \mathbb{C}^{N_0 \times M_0}$ is the self-interference channel from the transmitter antennas of BS to the receiver antennas of BS. $\mathbf{H}_{jk}^{DU} \in \mathbb{C}^{N_j \times M_k}$ denotes the CCI channel from the k-th UL user to the j-th DL user.

The vector of source symbols transmitted by the k-th UL user is denoted as $\mathbf{s}_k^{UL}$. It is assumed that the symbols are independent and identically distributed (i.i.d.) with unit power, i.e., $\mathbb{E}\left[\mathbf{s}_k^{UL}\left(\mathbf{s}_k^{UL}\right)^H\right] = \mathbf{I}_{d_k^{UL}}$. Similarly, the transmit symbols for the j-th DL user is denoted by $\mathbf{s}_j^{DL}$, with $\mathbb{E}\left[\mathbf{s}_j^{DL}\left(\mathbf{s}_j^{DL}\right)^H\right] = \mathbf{I}_{d_j^{DL}}$. Denoting the precoders for the data streams of the k-th UL and j-th DL user as $\mathbf{V}_k^{UL} \in \mathbb{C}^{M_k \times d_k^{UL}}$, and $\mathbf{V}_j^{DL} \in \mathbb{C}^{M_0 \times d_j^{DL}}$, respectively, the transmit signal of the k-th UL user and the BS can be written, respectively, as

$$\mathbf{x}_k^{UL} = \mathbf{V}_k^{UL}\mathbf{s}_k^{UL}, \qquad \mathbf{x}_0 = \sum_{j=1}^{J} \mathbf{V}_j^{DL}\mathbf{s}_j^{DL}. \tag{1}$$

We consider a FD multi-user MIMO system that suffers from self-interference and CCI. The signal received by the BS and that received by the j-th DL user can be written, respectively, as

$$\mathbf{y}_0 = \sum_{k=1}^{K} \mathbf{H}_k^{UL}\left(\mathbf{x}_k^{UL} + \mathbf{c}_k^{UL}\right) + \mathbf{H}_0\left(\mathbf{x}_0 + \mathbf{c}_0\right) + \mathbf{e}_0 + \mathbf{n}_0, \tag{2}$$

$$\mathbf{y}_j^{DL} = \mathbf{H}_j^{DL}\left(\mathbf{x}_0 + \mathbf{c}_0\right) + \sum_{k=1}^{K} \mathbf{H}_{jk}^{DU}\left(\mathbf{x}_k^{UL} + \mathbf{c}_k^{UL}\right) + \mathbf{e}_j^{DL} + \mathbf{n}_j^{DL}, \tag{3}$$

where $\mathbf{n}_0 \in \mathbb{C}^{N_0}$ and $\mathbf{n}_j^{DL} \in \mathbb{C}^{N_j}$ denote the additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\mathbf{R}_0 = \mathbf{I}_{N_0}$ and $\mathbf{R}_j^{DL} = \mathbf{I}_{N_j}$ at the BS and the j-th DL user, respectively.[1]

Moreover, in (2)-(3), $\mathbf{c}_k^{UL}$ ($\mathbf{c}_0$) is the transmitter distortion at the k-th UL user (BS), which models the effect of limited transmitter DR, and closely approximates the effects of additive power-amplifier noise, non-linearities in the digital-to-analog converter (DAC)and phase noise. The covariance matrix of $\mathbf{c}_k^{UL}$ is given by κ ($\kappa \ll 1$) times the energy of the intended signal at each transmit antenna [11]. In particular $\mathbf{c}_k^{UL}$ can be modeled as

$$\mathbf{c}_k^{UL} \sim \mathcal{CN}\left(\mathbf{0}, \kappa \operatorname{diag}\left(\mathbf{V}_k^{UL}\left(\mathbf{V}_k^{UL}\right)^H\right)\right), \tag{4}$$

$$\mathbf{c}_k^{UL} \perp \mathbf{x}_k^{UL}. \tag{5}$$

Finally, in (3)((2)), $\mathbf{e}_j^{DL}$ ($\mathbf{e}_0$) is the receiver distortion at the j-th DL user (BS), which models the effect of limited receiver DR, and closely approximates the combined effects of additive gain-control noise, non-linearities in the analog-to-digital converter (ADC) and phase noise. The covariance matrix of $\mathbf{e}_j^{DL}$ is given by β ($\beta \ll 1$) times the energy of the undistorted received signal at each receive antenna [11]. In particular, $\mathbf{e}_j^{DL}$ can be modeled as

$$\mathbf{e}_j^{DL} \sim \mathcal{CN}\left(\mathbf{0}, \beta \operatorname{diag}\left(\mathbf{\Phi}_j^{DL}\right)\right), \tag{6}$$

$$\mathbf{e}_j^{DL} \perp \mathbf{u}_j^{DL}, \tag{7}$$

where $\mathbf{\Phi}_j^{DL} = \operatorname{Cov}\{\mathbf{u}_j^{DL}\}$ and $\mathbf{u}_j^{DL}$ is the undistorted received vector at the j-th DL user, i.e., $\mathbf{u}_j^{DL} = \mathbf{y}_j^{DL} - \mathbf{e}_j^{DL}$. The discussion on the transmitter/receiver distortion model holds for $\mathbf{c}_0$ and $\mathbf{e}_0$, as well. Note that this transmitter/receiver distortion model is valid, since it was shown by hardware measurements in [12] and [13] that the non-ideality of the transmitter and receiver chain can be approximated by an independent Gaussian noise model, respectively. This model has also been adopted in many papers in the literature [5], [8], [11], [14], [15].

The received signals are processed by linear decoders, denoted as $\mathbf{U}_k^{UL} \in \mathbb{C}^{N_0 \times d_k^{UL}}$, and $\mathbf{U}_j^{DL} \in \mathbb{C}^{N_j \times d_j^{DL}}$ by the BS and the j-th DL user, respectively. Therefore the estimate of data streams of the k-th UL user at the BS is given as $\hat{\mathbf{s}}_k^{UL} = \left(\mathbf{U}_k^{UL}\right)^H \mathbf{y}_0$, and similarly, the estimate of the data

[1] Since the SU receiver cannot differentiate the interference generated by the PUs from the background thermal noise, the noise vectors in (2) and (3) captures the background thermal noise as well as the interference generated by the PUs. This assumption is also adopted in [3] and [9]- [10], and the noise is modeled as zero mean with unit variance in [3], [9] as we have assumed in this paper.

streams of the j-th DL user is $\hat{\mathbf{s}}_j^{DL} = \left(\mathbf{U}_j^{DL}\right)^H \mathbf{y}_j^{DL}$. Using these estimates, the MSE of the k-th UL and j-th DL user can be written as in (8) and (9), respectively, shown at the bottom of the following page. In (8), $\mathbf{\Sigma}_k^{UL}$ is the covariance matrix of the aggregate interference-plus-noise terms for the k-th UL user, and can be approximated, similar to [11], as in (10), given at the bottom of the following page[2]. The covariance matrix of the aggregate interference-plus-noise terms of the j-th DL user, $\mathbf{\Sigma}_j^{DL}$ given in (9) can be defined similarly, i.e. by replacing $\mathbf{H}_j^{UL}$, $\mathbf{V}_j^{UL}$, and $\mathbf{H}_0$ in (10) with $\mathbf{H}_j^{DL}$, $\mathbf{V}_k^{DL}$, and $\mathbf{H}_{jk}^{DU}$, respectively.

Without loss of generality, we assume that there is only DL transmission over the considered frequency band in the primary network. Therefore, the power of the interference resulting from the secondary UL users and BS at the l-th PU, $l = 1, \ldots, L$ equipped with T_l receive antennas can be written as

$$\begin{aligned} I_l^{PU} = & \sum_{k=1}^{K} \operatorname{tr}\left\{\mathbf{G}_{lk}\left(\mathbf{V}_k^{UL}\left(\mathbf{V}_k^{UL}\right)^H\right.\right. \\ & \left.\left. + \kappa \operatorname{diag}\left(\mathbf{V}_k^{UL}\left(\mathbf{V}_k^{UL}\right)^H\right)\right)\mathbf{G}_{lk}^H\right\} \\ & + \sum_{j=1}^{J} \operatorname{tr}\left\{\mathbf{G}_l\left(\mathbf{V}_j^{DL}\left(\mathbf{V}_j^{DL}\right)^H\right.\right. \\ & \left.\left. + \kappa \operatorname{diag}\left(\mathbf{V}_j^{DL}\left(\mathbf{V}_j^{DL}\right)^H\right)\right)\mathbf{G}_l^H\right\}, \end{aligned} \quad (11)$$

where $\mathbf{G}_{lk} \in \mathbb{C}^{T_l \times M_k}$ $\left(\mathbf{G}_l \in \mathbb{C}^{T_l \times M_0}\right)$ is the channel between the l-th PU and k-th UL user (l-th PU and the BS).

A. Joint Beamforming Design

We take sum-MSE as the performance measure to design the transceivers. Upper limits on both transmit power of the secondary UL users and BS, and interfering power at the PUs are considered. The sum-MSE optimization problem is formulated as

$$\min_{\substack{\mathbf{V}_k^{UL}, \mathbf{U}_k^{UL} \\ \mathbf{V}_j^{DL}, \mathbf{U}_j^{DL}}} \sum_{k=1}^{K} \operatorname{tr}\left\{\mathbf{MSE}_k^{UL}\right\} + \sum_{j=1}^{J} \operatorname{tr}\left\{\mathbf{MSE}_j^{DL}\right\} \quad (12)$$

$$\text{s.t.} \quad \operatorname{tr}\left\{\mathbf{V}_k^{UL}\left(\mathbf{V}_k^{UL}\right)^H\right\} \le P_k,\ k = 1, \ldots, K, \quad (13)$$

$$\sum_{j=1}^{J} \operatorname{tr}\left\{\mathbf{V}_j^{DL}\left(\mathbf{V}_j^{DL}\right)^H\right\} \le P_0, \quad (14)$$

$$I_l^{PU} \le \lambda_l,\ l = 1, \ldots, L, \quad (15)$$

where P_k in (13) is the transmit power constraint at the k-th UL user, P_0 in (14) is the total power constraint at the BS, and λ_l is the upper bound of the interference allowed to be imposed on the l-th PU.

[2]Note that $\mathbf{\Sigma}_k^{UL}$ and $\mathbf{\Sigma}_j^{DL}$ are approximated under $\kappa \ll 1$ and $\beta \ll 1$, which is a practical assumption [2], [11]. Therefore, the terms including the multiplication of κ and β are negligible, and have been ignored in the approximation.

1) Simplification of Notations: To simplify the notations, we will combine UL and DL channels, similar to [7]. Let us use $\mathcal{S}^{UL}$ and $\mathcal{S}^{DL}$ to represent the set of K UL and J DL channels, respectively. Denoting $\mathbf{H}_{ij}$, $\mathbf{n}_i$, $\mathbf{G}_{lj}$ and receive (transmit) antenna numbers $\tilde{N}_i \left(\tilde{M}_i\right)$ as

$$\mathbf{H}_{ij} = \begin{cases} \mathbf{H}_j^{UL}, i \in \mathcal{S}^{UL},\ j \in \mathcal{S}^{UL}, \\ \mathbf{H}_0, \quad i \in \mathcal{S}^{UL},\ j \in \mathcal{S}^{DL}, \\ \mathbf{H}_{ij}^{DU}, i \in \mathcal{S}^{DL},\ j \in \mathcal{S}^{UL}, \\ \mathbf{H}_i^{DL}, i \in \mathcal{S}^{DL},\ j \in \mathcal{S}^{DL}, \end{cases} \mathbf{n}_i = \begin{cases} \mathbf{n}_0, & i \in \mathcal{S}^{UL}, \\ \mathbf{n}_i^{DL}, & i \in \mathcal{S}^{DL}, \end{cases}$$

$$\mathbf{G}_{lj} = \begin{cases} \mathbf{G}_{lj}, & j \in \mathcal{S}^{UL}, \\ \mathbf{G}_l, & j \in \mathcal{S}^{DL}, \end{cases} \tilde{N}_i\left(\tilde{M}_i\right) = \begin{cases} N_0\left(M_i\right), & i \in \mathcal{S}^{UL}, \\ N_i\left(M_0\right), & i \in \mathcal{S}^{DL}, \end{cases}$$

and referring to $\mathbf{V}_i^X$, $\mathbf{U}_i^X$, d_i^X and $\mathbf{\Sigma}_i^X$, $X \in \{UL, DL\}$ as $\mathbf{V}_i$, $\mathbf{U}_i$, d_i and $\mathbf{\Sigma}_i$, respectively, the MSE of the i-th link, $i \in \mathcal{S} \triangleq \mathcal{S}^{UL} \bigcup \mathcal{S}^{DL}$ can be written as

$$\begin{aligned} \mathbf{MSE}_i = & \left(\mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i - \mathbf{I}_{d_i}\right)\left(\mathbf{U}_i^H \mathbf{H}_{ii} \mathbf{V}_i - \mathbf{I}_{d_i}\right)^H \\ & + \mathbf{U}_i^H \mathbf{\Sigma}_i \mathbf{U}_i, \end{aligned} \quad (16)$$

where

$$\begin{aligned} \mathbf{\Sigma}_i = & \sum_{j \in \mathcal{S}, j \neq i} \mathbf{H}_{ij} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{ij}^H + \kappa \sum_{j \in \mathcal{S}} \mathbf{H}_{ij} \operatorname{diag}\left(\mathbf{V}_j \mathbf{V}_j^H\right) \mathbf{H}_{ij}^H \\ & + \beta \sum_{j \in \mathcal{S}} \operatorname{diag}\left(\mathbf{H}_{ij} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{ij}^H\right) + \mathbf{I}_{\tilde{N}_i}, \end{aligned} \quad (17)$$

and the interference power at the l-th PU I_l^{PU} in (11) can be rewritten as

$$I_l^{PU} = \sum_{j \in \mathcal{S}} \operatorname{tr}\left\{\mathbf{G}_{lj}\left(\mathbf{V}_j \mathbf{V}_j^H + \kappa \operatorname{diag}\left(\mathbf{V}_j \mathbf{V}_j^H\right)\right) \mathbf{G}_{lj}^H\right\}. \quad (18)$$

Using the simplified notations, the problem (12)-(15) can be rewritten as

$$\min_{\mathbf{V}, \mathbf{U}} \quad \sum_{i \in \mathcal{S}} \operatorname{tr}\left\{\mathbf{MSE}_i\right\} \quad (19)$$

$$\text{s.t.} \quad \operatorname{tr}\left\{\mathbf{V}_i \mathbf{V}_i^H\right\} \le P_i,\ i \in \mathcal{S}^{UL}, \quad (20)$$

$$\sum_{i \in \mathcal{S}^{DL}} \operatorname{tr}\left\{\mathbf{V}_i \mathbf{V}_i^H\right\} \le P_0, \quad (21)$$

$$I_l^{PU} \le \lambda_l,\ l = 1, \ldots, L, \quad (22)$$

where $\mathbf{V}$ and $\mathbf{U}$ are stacked matrices composed of $\mathbf{V}_i$ and $\mathbf{U}_i$, $i \in \mathcal{S}$, respectively.

B. Imperfect CSI Model

In this paper, the CSI for both the channels in secondary network, and the channels between secondary and primary network are assumed to be imperfectly known. The imperfect CSI is modeled using deterministic norm-bounded error model [3]-[4], which is expressed as

$$\mathbf{H}_{ij} \in \mathcal{H}_{ij} = \left\{\tilde{\mathbf{H}}_{ij} + \mathbf{\Delta}_i : \|\mathbf{\Delta}_i\|_F \le \delta_i,\ j \in \mathcal{S}\right\}, \quad (23)$$

$$\mathbf{G}_{lj} \in \mathcal{G}_{lj} = \left\{\tilde{\mathbf{G}}_{lj} + \mathbf{\Lambda}_l : \|\mathbf{\Lambda}_l\|_F \le \theta_l,\ j \in \mathcal{S}\right\}, \quad (24)$$

where $\tilde{\mathbf{H}}_{ij}$, $\tilde{\mathbf{G}}_{lj}$, and δ_i, λ_l denote the nominal value of the CSI and uncertainty bounds, respectively.

With the imperfect CSI, the optimization problem in (19)-(22) can be rewritten as

$$\min_{\mathbf{V},\mathbf{U}} \max_{\forall \mathbf{H}_{ij}\in\mathcal{H}_{ij}} \quad \sum_{i\in\mathcal{S}} \text{tr}\left\{\mathbf{MSE}_i\right\} \tag{25}$$

$$\text{s.t.} \quad \text{tr}\left\{\mathbf{V}_i\mathbf{V}_i^H\right\} \le P_i,\ i\in\mathcal{S}^{UL}, \tag{26}$$

$$\sum_{i\in\mathcal{S}^{DL}} \text{tr}\left\{\mathbf{V}_i\mathbf{V}_i^H\right\} \le P_0, \tag{27}$$

$$I_l^{PU} \le \lambda_l,\ \forall \mathbf{G}_{lj}\in\mathcal{G}_{lj},\ l=1,\ldots,L. \tag{28}$$

We assume that the secondary BS has the knowledge of nominal channels and the radius of uncertainty regions. We consider a centralized optimization where the secondary BS coordinates the calibration of channel matrices, collects all channel matrices, computes the beamforming matrices based on the imperfect CSI, and then distributes them to the SUs. In the next section, we will develop SDP-based robust algorithm to solve (25)-(28).

III. Robust Transceiver Design Based on SDP Method

Using epigraph form and introducing slack variables τ_i, the problem (25)-(28) is rewritten as

$$\min_{\mathbf{V},\mathbf{U},\boldsymbol{\tau}} \quad \sum_{i\in\mathcal{S}} \tau_i \tag{29}$$

$$\text{s.t.} \quad \text{tr}\left\{\mathbf{MSE}_i\right\} \le \tau_i,\ \forall \mathbf{H}_{ij}\in\mathcal{H}_{ij},\ i\in\mathcal{S}, \tag{30}$$

$$\text{tr}\left\{\mathbf{V}_i\mathbf{V}_i^H\right\} \le P_i,\ i\in\mathcal{S}^{UL}, \tag{31}$$

$$\sum_{i\in\mathcal{S}^{DL}} \text{tr}\left\{\mathbf{V}_i\mathbf{V}_i^H\right\} \le P_0, \tag{32}$$

$$I_l^{PU} \le \lambda_l,\ \forall \mathbf{G}_{lj}\in\mathcal{G}_{lj},\ l=1,\ldots,L, \tag{33}$$

where $\boldsymbol{\tau}$ is a stacked vector composed of $\tau_i,\ i\in\mathcal{S}$.

To solve the optimization problem (29)-(33), we need to write tr$\{\mathbf{MSE}_i\}$ and I_l^{PU} in vector forms. After some straightforward manipulations, the vector forms of tr$\{\mathbf{MSE}_i\}$ and I_l^{PU} can be written as tr$\{\mathbf{MSE}_i\} = \|\boldsymbol{\mu}_i\|_2^2$ and $I_l^{PU} = \|\boldsymbol{\iota}_l\|_2^2$, where $\boldsymbol{\mu}_i$ and $\boldsymbol{\iota}_l$ are given as

$$\boldsymbol{\mu}_i = \begin{bmatrix} \left(\mathbf{V}_i^T\otimes\mathbf{U}_i^H\right) vec\left(\mathbf{H}_{ii}\right) - vec\left(\mathbf{I}_{d_i}\right) \\ \left\lfloor \left(\mathbf{V}_j^T\otimes\mathbf{U}_i^H\right) vec\left(\mathbf{H}_{ij}\right)\right\rfloor_{j\in\mathcal{S},j\neq i} \\ \left\lfloor \left\lfloor \sqrt{\kappa}\left(\left(\boldsymbol{\Gamma}_\ell\mathbf{V}_j\right)^T\otimes\mathbf{U}_i^H\right) vec\left(\mathbf{H}_{ij}\right)\right\rfloor_{\ell\in\mathcal{D}_j^{(T)}}\right\rfloor_{j\in\mathcal{S}} \\ \left\lfloor \left\lfloor \sqrt{\beta}\left(\mathbf{V}_j^T\otimes\left(\mathbf{U}_i^H\boldsymbol{\Gamma}_\ell\right)\right) vec\left(\mathbf{H}_{ij}\right)\right\rfloor_{\ell\in\mathcal{D}_i^{(R)}}\right\rfloor_{j\in\mathcal{S}} \\ vec\left(\mathbf{U}_i\right) \end{bmatrix} \tag{34}$$

$$\boldsymbol{\iota}_l = \begin{bmatrix} \left\lfloor \left(\mathbf{V}_j^T\otimes\mathbf{I}_{T_l}\right) vec\left(\mathbf{G}_{lj}\right)\right\rfloor_{j\in\mathcal{S}} \\ \sqrt{\kappa}\left\lfloor \left\lfloor \left(\left(\boldsymbol{\Gamma}_\ell\mathbf{V}_j\right)^T\otimes\mathbf{I}_{T_l}\right) vec\left(\mathbf{G}_{lj}\right)\right\rfloor_{\ell\in\mathcal{D}_j^{(T)}}\right\rfloor_{j\in\mathcal{S}} \end{bmatrix}, \tag{35}$$

where $\mathcal{D}_j^{(R)}$ represents the set $\{1\cdots\tilde{N}_j\}$, $\mathcal{D}_j^{(T)}$ represents the set $\{1\cdots\tilde{M}_j\}$ and $\boldsymbol{\Gamma}_\ell$ is a square matrix with zero elements, except for the ℓ-th diagonal element, equal to 1. Using the vector forms (34) and (35), the problem (29)-(33) can be rewritten as

$$\min_{\mathbf{V},\mathbf{U},\boldsymbol{\tau}} \quad \sum_{i\in\mathcal{S}} \tau_i \tag{36}$$

$$\text{s.t.} \quad \|\boldsymbol{\mu}_i\|_2^2 \le \tau_i,\ \|\boldsymbol{\Delta_i}\|_F \le \delta_i,\ i\in\mathcal{S}, \tag{37}$$

$$\|vec\left(\mathbf{V}_i\right)\|_2^2 \le P_i,\ i\in\mathcal{S}^{UL}, \tag{38}$$

$$\|\lfloor vec\left(\mathbf{V}_i\right)\rfloor_{i\in\mathcal{S}^{DL}}\|_2^2 \le P_0, \tag{39}$$

$$\|\boldsymbol{\iota}_l\|_2^2 \le \lambda_l,\ \|\boldsymbol{\Lambda_l}\|_F \le \theta_l,\ l=1,\ldots,L. \tag{40}$$

To recast the semi-infinite problem (36)-(40) as a SDP problem, the Schur complement lemma [16] is used to rewrite the constraints (37) and (40) in Linear matrix inequalities (LMI) form. The resulting optimization problem is written as

$$\min_{\mathbf{V},\mathbf{U},\boldsymbol{\tau}} \quad \sum_{i\in\mathcal{S}} \tau_i \tag{41}$$

$$\text{s.t.} \quad \begin{bmatrix} \tau_i & \boldsymbol{\mu}_i^H \\ \boldsymbol{\mu}_i & \mathbf{I}_{A_i} \end{bmatrix} \succeq 0,\ \|\boldsymbol{\Delta_i}\|_F \le \delta_i,\ i\in\mathcal{S}, \tag{42}$$

$$\|vec\left(\mathbf{V}_i\right)\|_2^2 \le P_i,\ i\in\mathcal{S}^{UL}, \tag{43}$$

$$\|\lfloor vec\left(\mathbf{V}_i\right)\rfloor_{i\in\mathcal{S}^{DL}}\|_2^2 \le P_0, \tag{44}$$

$$\begin{bmatrix} \lambda_l & \boldsymbol{\iota}_l^H \\ \boldsymbol{\iota}_l & \mathbf{I}_{B_l} \end{bmatrix} \succeq 0,\ \|\boldsymbol{\Lambda_l}\|_F \le \theta_l,\ l=1,\ldots,L, \tag{45}$$

$$\mathbf{MSE}_k^{UL} = \left(\left(\mathbf{U}_k^{UL}\right)^H\mathbf{H}_k^{UL}\mathbf{V}_k^{UL} - \mathbf{I}_{d_k^{UL}}\right)\left(\left(\mathbf{U}_k^{UL}\right)^H\mathbf{H}_k^{UL}\mathbf{V}_k^{UL} - \mathbf{I}_{d_k^{UL}}\right)^H + \left(\mathbf{U}_k^{UL}\right)^H\boldsymbol{\Sigma}_k^{UL}\mathbf{U}_k^{UL}, \tag{8}$$

$$\mathbf{MSE}_j^{DL} = \left(\left(\mathbf{U}_j^{DL}\right)^H\mathbf{H}_j^{DL}\mathbf{V}_j^{DL} - \mathbf{I}_{d_j^{DL}}\right)\left(\left(\mathbf{U}_j^{DL}\right)^H\mathbf{H}_j^{DL}\mathbf{V}_j^{DL} - \mathbf{I}_{d_j^{DL}}\right)^H + \left(\mathbf{U}_j^{DL}\right)^H\boldsymbol{\Sigma}_j^{DL}\mathbf{U}_j^{DL}. \tag{9}$$

$$\begin{aligned}\boldsymbol{\Sigma}_k^{UL} \approx & \sum_{j\neq k}^{K}\mathbf{H}_j^{UL}\mathbf{V}_j^{UL}\left(\mathbf{V}_j^{UL}\right)^H\left(\mathbf{H}_j^{UL}\right)^H + \kappa\sum_{j=1}^{K}\mathbf{H}_j^{UL}\text{diag}\left(\mathbf{V}_j^{UL}\left(\mathbf{V}_j^{UL}\right)^H\right)\left(\mathbf{H}_j^{UL}\right)^H \\ & + \sum_{j=1}^{J}\mathbf{H}_0\left(\mathbf{V}_j^{DL}\left(\mathbf{V}_j^{DL}\right)^H + \kappa\text{diag}\left(\mathbf{V}_j^{DL}\left(\mathbf{V}_j^{DL}\right)^H\right)\right)\mathbf{H}_0^H + \beta\sum_{j=1}^{K}\text{diag}\left(\mathbf{H}_j^{UL}\mathbf{V}_j^{UL}\left(\mathbf{V}_j^{UL}\right)^H\left(\mathbf{H}_j^{UL}\right)^H\right) \\ & + \beta\sum_{j=1}^{J}\text{diag}\left(\mathbf{H}_0\mathbf{V}_j^{DL}\left(\mathbf{V}_j^{DL}\right)^H\mathbf{H}_0^H\right) + \mathbf{I}_{N_0}.\end{aligned} \tag{10}$$

where the dimensions of the identity matrices in (42) and (45) are given, respectively, as

$$A_i = d_i \left(\sum_{j\in\mathcal{S}} \left(d_j + \tilde{M}_j \right) + \tilde{N}_i \right) + \tilde{N}_i \sum_{j\in\mathcal{S}} d_j, \quad (46)$$

$$B_l = T_l \sum_{j\in\mathcal{S}} \left(d_j + \tilde{M}_j \right). \quad (47)$$

To further simplify the problem (41)-(45), the following lemma from [17] is used to relax the semi-infiniteness of the constraints (42) and (45).

Lemma 1. *Given matrices* $\mathbf{P},\ \mathbf{Q},\ \mathbf{A}$ *with* $\mathbf{A} = \mathbf{A}^H$*, the semi-infinite LMI of the form of*

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P}, \qquad \forall \mathbf{X} : \|\mathbf{X}\|_F \le \rho,$$

holds if and only if $\exists \epsilon \ge 0$ *such that*

$$\begin{bmatrix} \mathbf{A} - \epsilon\mathbf{Q}^H\mathbf{Q} & -\rho\mathbf{P}^H \\ -\rho\mathbf{P} & \epsilon\mathbf{I} \end{bmatrix} \succeq 0.$$

To apply Lemma (1), the LMI in (42) is first expressed as

$$\begin{bmatrix} \tau_i & \tilde{\boldsymbol{\mu}}_i^H \\ \tilde{\boldsymbol{\mu}}_i & \mathbf{I}_{A_i} \end{bmatrix} + \begin{bmatrix} 0 & \boldsymbol{\mu}_{\Delta_i}^H \\ \boldsymbol{\mu}_{\Delta_i} & \mathbf{0}_{A_i\times A_i} \end{bmatrix} \succeq 0, \quad (48)$$

where[3]

$$\tilde{\boldsymbol{\mu}}_i = \begin{bmatrix} \left(\mathbf{V}_i^T \otimes \mathbf{U}_i^H\right) vec\left(\tilde{\mathbf{H}}_{ii}\right) - vec\left(\mathbf{I}_{d_i}\right) \\ \left\lfloor \left(\mathbf{V}_j^T \otimes \mathbf{U}_i^H\right) vec\left(\tilde{\mathbf{H}}_{ij}\right) \right\rfloor_{j\in\mathcal{S}, j\neq i} \\ \left\lfloor \left\lfloor \sqrt{\kappa}\left((\boldsymbol{\Gamma}_\ell\mathbf{V}_j)^T \otimes \mathbf{U}_i^H\right) vec\left(\tilde{\mathbf{H}}_{ij}\right) \right\rfloor_{\ell\in\mathcal{D}_j^{(T)}} \right\rfloor_{j\in\mathcal{S}} \\ \left\lfloor \left\lfloor \sqrt{\beta}\left(\mathbf{V}_j^T \otimes (\mathbf{U}_i^H\boldsymbol{\Gamma}_\ell)\right) vec\left(\tilde{\mathbf{H}}_{ij}\right) \right\rfloor_{\ell\in\mathcal{D}_i^{(R)}} \right\rfloor_{j\in\mathcal{S}} \\ vec\left(\mathbf{U}_i\right) \end{bmatrix},$$

$$\boldsymbol{\mu}_{\Delta_i} = \underbrace{\begin{bmatrix} \left(\mathbf{V}_i^T \otimes \mathbf{U}_i^H\right) \\ \left\lfloor \left(\mathbf{V}_j^T \otimes \mathbf{U}_i^H\right) \right\rfloor_{j\in\mathcal{S}, j\neq i} \\ \left\lfloor \left\lfloor \sqrt{\kappa}\left((\boldsymbol{\Gamma}_\ell\mathbf{V}_j)^T \otimes \mathbf{U}_i^H\right) \right\rfloor_{\ell\in\mathcal{D}_j^{(T)}} \right\rfloor_{j\in\mathcal{S}} \\ \left\lfloor \left\lfloor \sqrt{\beta}\left(\mathbf{V}_j^T \otimes (\mathbf{U}_i^H\boldsymbol{\Gamma}_\ell)\right) \right\rfloor_{\ell\in\mathcal{D}_i^{(R)}} \right\rfloor_{j\in\mathcal{S}} \\ \mathbf{0}_{d_i\tilde{N}_i\times\tilde{N}_i\tilde{M}} \end{bmatrix}}_{\mathbf{D}_{\Delta_i}} vec\left(\boldsymbol{\Delta_i}\right).$$

By choosing

$$\mathbf{A} = \begin{bmatrix} \tau_i & \tilde{\boldsymbol{\mu}}_i^H \\ \tilde{\boldsymbol{\mu}}_i & \mathbf{I}_{A_i} \end{bmatrix}, \ \mathbf{P} = \left[\mathbf{0}_{\tilde{N}_i\tilde{M}\times 1},\ \mathbf{D}_{\Delta_i}^H\right], \quad (49)$$

$$\mathbf{X} = vec\left(\boldsymbol{\Delta_i}\right), \ \mathbf{Q} = \left[-1, \mathbf{0}_{1\times A_i}\right], \quad (50)$$

and using Lemma 1, the LMI in (42) is relaxed as

$$\underbrace{\begin{bmatrix} \tau_i - \epsilon_i & \tilde{\boldsymbol{\mu}}_i^H & \mathbf{0}_{1\times\tilde{N}_i\tilde{M}} \\ \tilde{\boldsymbol{\mu}}_i & \mathbf{I}_{A_i} & -\delta_i\mathbf{D}_{\Delta_i} \\ \mathbf{0}_{\tilde{N}_i\tilde{M}\times 1} & -\delta_i\mathbf{D}_{\Delta_i}^H & \epsilon_i\mathbf{I}_{\tilde{N}_i\tilde{M}} \end{bmatrix}}_{\mathbf{N}_i} \succeq 0, \ i\in\mathcal{S}, \quad (51)$$

$$\epsilon_i \ge 0, \ i\in\mathcal{S}. \quad (52)$$

[3]To simplify the presentation, from now on we will assume the number of transmit antennas at the BS is equal to number of transmit antennas at the UL users, i.e., $\tilde{M} = M_0 = M_i,\ i\in\mathcal{S}^{UL}$.

Using a similar procedure, the LMI in (45) is expressed as

$$\begin{bmatrix} \lambda_l & \tilde{\boldsymbol{\iota}}_l^H \\ \tilde{\boldsymbol{\iota}}_l & \mathbf{I}_{B_l} \end{bmatrix} + \begin{bmatrix} 0 & \boldsymbol{\iota}_{\Lambda_l}^H \\ \boldsymbol{\iota}_{\Lambda_l} & \mathbf{0}_{B_l\times B_l} \end{bmatrix} \succeq 0, \quad (53)$$

where

$$\tilde{\boldsymbol{\iota}}_l = \begin{bmatrix} \left\lfloor \left(\mathbf{V}_j^T \otimes \mathbf{I}_{T_l}\right) vec\left(\tilde{\mathbf{G}}_{lj}\right) \right\rfloor_{j\in\mathcal{S}} \\ \sqrt{\kappa}\left\lfloor \left\lfloor \left((\boldsymbol{\Gamma}_\ell\mathbf{V}_j)^T \otimes \mathbf{I}_{T_l}\right) vec\left(\tilde{\mathbf{G}}_{lj}\right) \right\rfloor_{\ell\in\mathcal{D}_j^{(T)}} \right\rfloor_{j\in\mathcal{S}} \end{bmatrix},$$

$$\boldsymbol{\iota}_{\Lambda_l} = \underbrace{\begin{bmatrix} \left\lfloor \left(\mathbf{V}_j^T \otimes \mathbf{I}_{T_l}\right) \right\rfloor_{j\in\mathcal{S}} \\ \sqrt{\kappa}\left\lfloor \left\lfloor \left((\boldsymbol{\Gamma}_\ell\mathbf{V}_j)^T \otimes \mathbf{I}_{T_l}\right) \right\rfloor_{\ell\in\mathcal{D}_j^{(T)}} \right\rfloor_{j\in\mathcal{S}} \end{bmatrix}}_{\mathbf{E}_{\Lambda_l}} vec\left(\boldsymbol{\Lambda_l}\right).$$

Then the LMI in (45) is relaxed as

$$\underbrace{\begin{bmatrix} \lambda_l - \eta_l & \tilde{\boldsymbol{\iota}}_l^H & \mathbf{0}_{1\times T_l\tilde{M}} \\ \tilde{\boldsymbol{\iota}}_l & \mathbf{I}_{B_l} & -\theta_l\mathbf{E}_{\Lambda_l} \\ \mathbf{0}_{T_l\tilde{M}\times 1} & -\theta_l\mathbf{E}_{\Lambda_l}^H & \eta_l\mathbf{I}_{T_l\tilde{M}} \end{bmatrix}}_{\mathbf{M}_l} \succeq 0, \ l = 1,\ldots,L, \quad (54)$$

$$\eta_l \ge 0, \ l = 1,\ldots,L. \quad (55)$$

Using the relaxed LMIs in (51) and (54), the SDP problem, which is equivalent to (25)-(28) is formulated as:

$$\min_{\mathbf{V},\mathbf{U},\boldsymbol{\tau},\epsilon_i\ge 0,\eta_l\ge 0} \quad \sum_{i\in\mathcal{S}} \tau_i \quad (56)$$

$$\text{s.t.} \quad \mathbf{N}_i \succeq 0, \ i\in\mathcal{S}, \quad (57)$$

$$\|vec\left(\mathbf{V}_i\right)\|_2^2 \le P_i, \ i\in\mathcal{S}^{UL}, \quad (58)$$

$$\|\left\lfloor vec\left(\mathbf{V}_i\right)\right\rfloor_{i\in\mathcal{S}^{DL}}\|_2^2 \le P_0, \quad (59)$$

$$\mathbf{M}_l \succeq 0, \ l = 1,\ldots,L. \quad (60)$$

Note that the problem (56)-(60) is not jointly convex over transmit beamforming matrices $\mathbf{V}$ and receiving beamforming matrices $\mathbf{U}$, but is component-wise convex over $\mathbf{V}$ and $\mathbf{U}$, i.e., for fixed $\mathbf{U}$, the problem is convex with respect to $\mathbf{V}$ and vice versa. Therefore, we will employ an iterative algorithm that finds the efficient solutions of $\mathbf{V}$ and $\mathbf{U}$ in an alternating fashion until convergence or a pre-defined number of iterations is reached. The algorithm for the sum-MSE optimization problem (25)-(28) that uses SDP method is given in Table I.

Since the proposed sum-MSE algorithm monotonically decreases the total MSE over each iteration by updating the transceivers in an alternating fashion, and the fact that MSE is bounded below (at least by zero), it is clear that the proposed sum-MSE minimization algorithm is convergent and is guaranteed to converge to a stationary minimum. Since the sum-MSE optimization problem is not jointly convex, the proposed algorithm is not guaranteed to converge to a global optimum point.

TABLE I: Sum-MSE Minimization using SDP Algorithm

1) Set the iteration number $n = 0$ and initialize $\mathbf{V}^{[n]}$.
2) $n \leftarrow n+1$. Update $\mathbf{U}_i^{[n]},\ i\in\mathcal{S}$ by solving convex SDP problem (56)–(60) under fixed $\mathbf{V}^{[n-1]}$.
3) Update $\mathbf{V}_i^{[n]},\ i\in\mathcal{S}$ by solving convex SDP (56)–(60) under fixed $\mathbf{U}^{[n]}$.
4) Repeat steps 2 and 3 until convergence or a predefined number of iterations is reached.

TABLE II: Complexity Parameters of SDP-based Method

	Number of variables (n)	Dimension of blocks (a_i)
$\mathbf{V}$	$\sum_{i\in\mathcal{S}} 2\tilde{M}d_i + 2\lvert\mathcal{S}\rvert + L$	$a_i = A_i + \tilde{N}_i\tilde{M} + 1,\ i \in \mathcal{S}$ $a_i = \tilde{M}d_i^{UL} + 1,\ i \in \mathcal{S}^{UL}$ $a_i = \tilde{M}\sum_{i\in\mathcal{S}^{DL}} d_i^{DL} + 1$ $a_l = B_l + T_l\tilde{M} + 1,\ l,\dots,L$
$\mathbf{U}_i$	$2\tilde{N}_i d_i + 2$	$a_i = A_i + \tilde{N}_i\tilde{M} + 1,\ i \in \mathcal{S}$

A. Computational Complexity

In this subsection, the computational complexity of the proposed SDP method in Table I is discussed. The number of arithmetic operations required to solve a standard real-valued SDP problem

$$\min_{\mathbf{x}\in\mathcal{R}^n} \mathbf{c}^T\mathbf{x} \qquad \text{s.t.}\ \mathbf{A}_0 + \sum_{i=1}^{n} x_i\mathbf{A}_i \succeq \mathbf{0}, \quad \|\mathbf{x}\|_2 \le R,$$

where $\mathbf{A}_i$ denotes the symmetric block-diagonal matrices with P diagonal blocks of size $a_l \times a_l,\ l = 1,\dots,P$, is upper-bounded by [18]

$$\mathcal{O}(1)\left(1+\sum_{l=1}^{P} a_l\right)^{1/2} n\left(n^2 + n\sum_{l=1}^{P} a_l^2 + \sum_{l=1}^{P} a_l^3\right). \quad (61)$$

Since the proposed algorithm in Table I solves a SDP problem in Step 2 and Step 3, the number of arithmetic operations required to compute optimal $\mathbf{V}_i$ and $\mathbf{U}_i$ is calculated from (61) as follows. In computing $\mathbf{V}_i$, the number of diagonal blocks P is equal to $\lvert\mathcal{S}\rvert+\lvert\mathcal{S}^{UL}\rvert+L+1$. For the MSE constraint of each user, the dimension of blocks are $a_i = A_i + \tilde{N}_i\tilde{M} + 1,\ i \in \mathcal{S}$. For the UL SU power constraint, the dimension of the blocks are $a_i = \tilde{M}d_i^{UL} + 1,\ i \in \mathcal{S}^{UL}$. For the BS power constraint, the dimension of the block is $a_i = \tilde{M}\sum_{i\in\mathcal{S}^{DL}} d_i^{DL} + 1$, and for the PU interference constraint, the dimension of the blocks are $a_l = B_l + T_l\tilde{M} + 1,\ l,\dots,L$. The unknown variables to be determined are of size $n = \sum_{i\in\mathcal{S}} 2\tilde{M}d_i + 2\lvert\mathcal{S}\rvert + L$, where the first term corresponds to the real and image parts of $\mathbf{V}_i$ and the other terms represent the additional slack variables. The calculation of the number of arithmetic operations required to $\mathbf{U}_i$ can be carried out similarly. The computational complexity parameters for solving the sum-MSE minimization problem using SDP method are given in Table II.

IV. Simulation Results

In this section, we numerically compare the proposed algorithm with the HD algorithm under the 3GPP LTE specifications for small cell deployments [19]. A single hexagonal cell having a BS in the center with $M_0 = 2$ transmit and $N_0 = 2$ receive antennas with randomly distributed $K = 3$ UL and $J = 3$ DL users equipped with 2 antennas is simulated. The cognitive radio system has $L = 2$ PUs, with the same maximum allowed interfering power (i.e., $\lambda_l = 0$dB). The channel between BS and users (both SUs and PUs) are assumed to experience the path loss model for line-of-sight (LOS), and the channel between UL and DL users are assumed to experience the path loss model for non-line-of-sight (NLOS) communications. Detailed simulation parameters are shown in Table III.

TABLE III: Simulation Parameters

Parameter	Settings
Cell Radius	40m
Carrier Frequency	2GHz
Bandwidth	10MHz
Thermal Noise Density	-174dBm/Hz
Noise Figure	BS: 13dB, User: 9dB
Path Loss (dB) between BS and users (d in km)	$103.8 + 20.9\log_{10} d$
Path Loss (dB) between users (d in km)	$145.4 + 37.5\log_{10} d$
Shadowing Standard Deviation	LOS: 3dB, NLOS: 4dB

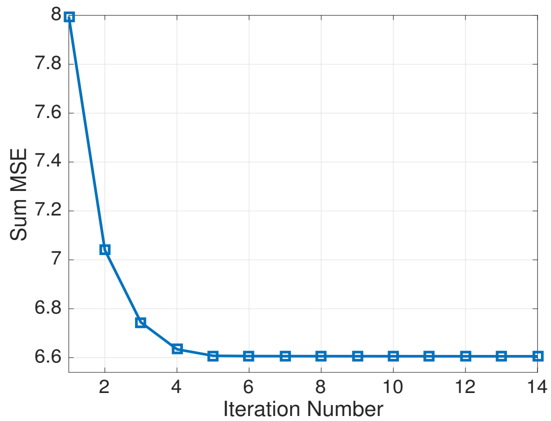

Fig. 2: Convergence of the proposed algorithm.

The estimated channel gain between the BS to kth UL user is given by $\tilde{\mathbf{H}}_k^{UL} = \sqrt{\kappa_k^{UL}}\hat{\mathbf{H}}_k^{UL}$, where $\hat{\mathbf{H}}_k^{UL}$ denotes the small scale fading following a complex Gaussian distribution with zero mean and unit variance, and $\kappa_k^{UL} = 10^{(-X/10)}$, $X \in \{\mathrm{LOS}, \mathrm{NLOS}\}$ represents the large scale fading consisting of path loss and shadowing, where LOS and NLOS are calculated from a specific path loss model given in Table III. The channels between BS and DL users, between UL users and DL users, between BS and PUs, and between UL users and PUs are defined similarly. We adopt the Rician model in [1], in which the self-interference channel is distributed as $\tilde{\mathbf{H}}_0 \sim \mathcal{CN}\left(\sqrt{\frac{K_R}{1+K_R}}\hat{\mathbf{H}}_0, \frac{1}{1+K_R}\mathbf{I}_{N_0} \otimes \mathbf{I}_{M_0}\right)$, where K_R is the Rician factor, and $\hat{\mathbf{H}}_0$ is a deterministic matrix[4] Unless stated otherwise, we consider $K = J = 2,\ \ \kappa = \beta = -70$dB and $\delta_i = \theta_i = 0.1,\ i \in \mathcal{S}$.

Fig. 2 shows the evolution of the proposed algorithm, i.e., the convergence of the algorithm in Table I. The monotonic decrease of the sum-MSE can be verified, and is seen to converge quite rapidly.

In our second example, we will compare FD with HD systems in terms of sum-rate performance for different $\kappa = \beta$ values. As seen in Fig. 3, the performance of HD system is not affected with κ and β values, and at high self-interference cancellation levels, FD systems achieves around 1.6 times more sum-rate than that of HD, and the performance of FD system drops below that of HD scheme around $\kappa = \beta = -55$dB.

[4]Similar to [6], without loss of generality, we set $K_R = 1$ and $\tilde{\mathbf{H}}_0$ to be the matrix of all ones for all experiments.

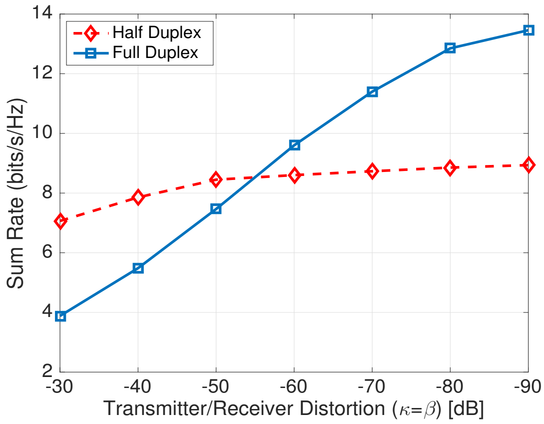

Fig. 3: Sum-rate comparison of FD and HD systems with respect to transmitter/receiver distortion, i.e., κ, β.

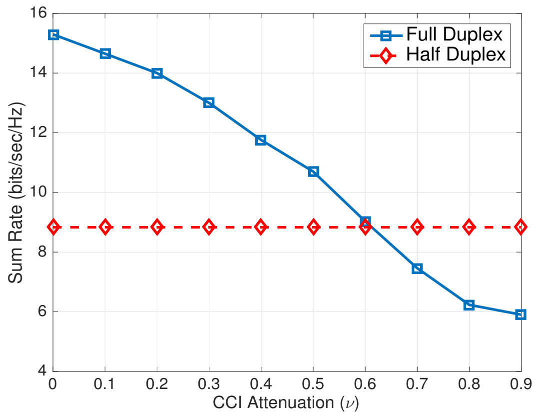

Fig. 4: Sum-rate comparison of FD and HD systems with respect to CCI attenuation factor, i.e., ν.

It is important to note that while the channel matrices are assumed to be given for each user, it is essential for a practical system to exploit a smart channel assignment algorithm prior to precoder/decoder design. This is particularly essential for a FD setup as the CCI can be reduced by assigning the users with weaker interference paths into the same channel. In order to incorporate the effect of channel assignment into our simulation, we assume an attenuation coefficient, namely ν, on the CCI channels, which represent the degree of isolation among UL and DL users due to channel assignment. In Fig. 4, the importance of the smart channel assignment, as a stage prior to the precoder/decoder design is depicted. The CCI attenuation represents the provided isolation among the UL and DL users. As the suppression level of CCI increases, the FD system starts outperforming the HD system, and thus isolation among the UL and DL users is essential for a successful coexistence of UL and DL users in a FD setup.

V. Conclusion

In this work, we have studied the robust MSE-based transceiver design problem for a FD MIMO cognitive cellular system that suffers from self-interference and co-channel interference under the limited DR at the transmitters and receivers, and norm-bounded channel uncertainties. Since the globally optimal solution is difficult to obtain due to the non-convex nature of the problems, an alternating SDP-based algorithm that iterates between transmit and receiving beamforming matrices while keeping the other fixed is first proposed. Simulation results confirmed the improved robustness of the proposed method. Moreover, it has been shown in simulations that the sum-rate achieved by FD system is higher than that of HD system under reasonable self-interference cancellation values.

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