Modern Heuristical Optimization Techniques for Power System State Estimation

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Abstract. The development of efficient and accurate algorithms for state estimation has come into the focus in power system research as the power grid becomes more decentralized. In this work, we apply the heuristical continuous optimization techniques *differential evolution*, *simulated annealing* and *particle swarm optimization* to power system state estimation problem, and provide a comparison between them in terms of convergence and optimality. Examining the results, we propose a hybrid algorithm combining particle swarm optimization and differential evolution.

Keywords: simulated annealing,particle swarm optimization,differential evolution,power system,meta-heuristic

1 Introduction

The accurate state estimation has been the most fundamental problem in power grids, since it delivers the system state as an input to all other applications in an energy management system. With the integration of renewable resources and the resulting decentralization of the power grids, an accurate and reliable information about the state of the system is required not only for the transmission level but also for the distribution level. This is a prerequisite to introduce new applications and to ensure a stable operation of the power system.

The traditional state estimation problem is formulated as a nonlinear weighted least square (WLS) problem, which can be solved iteratively by using gradient-based methods, e.g., the Gauss-Newton method. However, the success of these methods is based on a proper selection of a starting point, which is usually unknown. Thus, they often converge to a local minimum instead of a global one. On the other hand, the gradient is needed which is in practice replaced by an approximation, since the used objective functions and constraints are often discontinuous or very complicated to handle. In scenarios with a high number of distributed generators and consumers, this problem leads to Jacobian matrices [1], that are ill-conditioned. Considering the drawbacks of the basic state estimation approach, modern heuristical optimization techniques can provide an alternative to deliver an accurate snapshot of the system state for the decentralized structure of future distribution grids.

Since the seminal work of Schweppe et.al. [2], numerous formulations of the state estimation problem and different numerical solution techniques have been proposed. In [3] a comprehensive survey of different state estimation techniques is given. The authors in [4] have proposed a hybrid particle swarm optimization(PSO) algorithm with a natural selection mechanism for evolutionary computation of the system state. For the computation, measurements consisting of branch voltages and injection values are used. Similarly, Mallick et.al. [5] have proposed a PSO algorithm with an additional differential evolution(DE) update step and have shown that the PSO performance can be improved by this method and it outperforms the Gauss-Newton method while considering ill-conditioned networks. Basetti et.al. [6, 7] have applied a gravitational search algorithm considering both traditional measurements and phasor measurement units (PMU), as well as a Taguchi differential evolution algorithm. Their proposed method delivers satisfactory results at the cost of longer computation time. In [8] a genetic algorithm-based technique is used, and the authors point out that the heuristical algorithm converges prematurely for IEEE 14 bus network without giving a good estimate. The authors of [9] use a self-adaptive evolutionary approach and conclude that evolutionary programming provides an accurate estimation in tests with IEEE test networks for 14 and 30 bus.

This literature review reveals the interest in power system research to find effective heuristical computational methods with good convergence properties in order to overcome the drawbacks of traditional numerical methods. It is well-known that the performance of heuristical techniques is highly dependent on the structure of the considered optimization problem [10]. Hence, for a specific optimization problem, one has to try out different heuristical approaches in order to find the one with the best performance. This is our main goal in the present work to compare all three methods, namely DE, simulated annealing(SA) and PSO, and provide meaningful results for the state estimation of energy grids.

We start with a description of the power system model as well as the traditional state estimation problem. Next, we introduce the optimization techniques followed by the description of the test cases along with simulation details. Finally, we present our main results and conclude our achievements.

2 Power System State Estimation

State estimation in power systems tries to obtain a reliable estimate of the voltage phasors at all system buses in the network by using a set of measurements. Although recent PMUs can measure voltage and current values with a high accuracy, the wide deployment of PMUs is costly, especially when considering the scale of distribution grids. For this reason, the traditional formulation of the state estimation problem still plays a crucial role.

In the following, we introduce the basic formulation of the state estimation problem for a power system. We define the system state $\mathbf{x}$ of a power grid with

n buses by the vector $\mathbf{x} = [\theta_2, \dots, \theta_n, |V_1|, \dots, |V_n|]$, where θ_i and $|V_i|$ are the angle and the magnitude of the voltage phasor at node i, respectively. The angle θ_1 of the first bus is set to zero and used as the reference angle. The set of measurements is denoted by

$$\mathbf{z} = \begin{bmatrix} h_1(\mathbf{x}) \\ \vdots \\ h_n(\mathbf{x}) \end{bmatrix} + \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix} = \mathbf{h}(\mathbf{x}) + \mathbf{e}, \tag{1}$$

where $h_i(\mathbf{x})$ is a nonlinear function, which describes the relation between the value of measurement i and the state vector $\mathbf{x}$, while $\mathbf{e}$ is the vector of measurement errors. It is assumed that the measurement errors are independent and zero-mean Gaussian distributed, i.e., $\mathbf{e} \sim \mathcal{N}(0, \mathbf{R})$, where $\mathbf{R} = \mathrm{diag}(\sigma_1^2, \dots, \sigma_n^2)$ is the covariance matrix, with the variances σ_i^2 of the noise components e_i as its diagonal entries. The standard deviation σ_i of each measurement is modeled to take the accuracy of different measurements into account. The nonlinear measurement function $h_i(\mathbf{x})$ depends on the measurement type and location, and is formulated by the Kirchhoff rules for voltage and current. To illustrate the measurement function P_i for the real power injection at bus i, we may write

$$P_i = V_i \sum_{j=1}^{N} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}), \tag{2}$$

where $\mathbf{G}$ and $\mathbf{B}$ are the real and imaginary parts of the bus admittance matrix $\mathbf{Y}$ of the power system and $\theta_{ij} = \theta_i - \theta_j$. The WLS estimator tries to find the state vector, which minimizes the error in the measurements. The optimization problem reads as

$$\underset{\mathbf{x}}{\text{minimize}} \quad J(\mathbf{x}), \tag{3}$$

where $J(\mathbf{x})$ is defined by $(\mathbf{z} - \mathbf{h}(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{z} - \mathbf{h}(\mathbf{x}))$. Note that the optimization problem (3) is subject to implicit constraints of state variables due to the assumption of a stable operation. Of course, a solution to (3) can be found by the Gauss-Newton method, however as mentioned in the introduction, the Gauss-Newton method has its own challenges. For a solid treatment of this approach please refer to [11].

3 Optimization Techniques

The heuristical techniques, which are considered in this work, are well-known methods which have been applied to different problems in engineering and other natural sciences. In this section, we introduce our notation for the power system state estimation problem and present the parameters adopted in this work. A good overview of modern heuristical techniques along with their applications in power systems can be found in [12].

3.1 Differential Evolution

In this work, we exploit the key ideas of differential variation, crossover and mutation in DE, where mutation and crossover operations occur with a certain probability, which is given by the algorithm parameter CR. The optimization variable $\mathbf{x}_i^k$ is encoded as a vector of floating point values.

The generation of a new candidate $\mathbf{x}_i^{k+1}$ follows the update equation

$$\mathbf{x}_i^{k+1} = \mathbf{x}_p^k + (\mathbf{x}_i^k - \mathbf{x}_{r_1})\lambda + (\mathbf{x}_{r_2} - \mathbf{x}_{r_3})F \tag{4}$$

where $\mathbf{x}_i^k$ is i^{th} member of the current generation k, $\mathbf{x}_p^k$ is the population member to perturb, λ and F are the crossover and mutation coefficients, respectively, and $\mathbf{x}_{r_i}$ are randomly selected population members. We select the first term $\mathbf{x}_p^k$ as the best population member $\mathbf{x}_{\text{best}}^k$. This strategy has outperformed other options in the test problems considered in this work.

3.2 Simulated Annealing

The key concepts in SA are cooling schedule, state generation and state acceptance. In present work, initial temperature is chosen as 100 and updated by $T_{k+1} = 0.95^k T_k$, where k is the iteration number. A new candidate $\mathbf{x}_{k+1}$ is generated by perturbing the current point $\mathbf{x}_k$ as in

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{r}\sqrt{T_k}, \tag{5}$$

where $\mathbf{r}$ is a random unit vector with $|\mathbf{r}| = 1$. The acceptance of a candidate with an inferior objective function value occurs with a probability calculated by the acceptance function f_{accept} as

$$f_{\text{accept}} = \exp\Big(\frac{J(\mathbf{x}_i) - J(\mathbf{x}_j)}{T_k}\Big). \tag{6}$$

Table 1. Summary of algorithm parameters and functions in this work

Algorithm	Parameters	Functions
DE	$CR = 0.7$ $F = 0.7$, $\lambda = 0.5$	$\mathbf{x}_i^{k+1} = \mathbf{x}_{\text{best}}^k + (\mathbf{x}_i^k - \mathbf{x}_{r_1})\lambda + (\mathbf{x}_{r_1} - \mathbf{x}_{r_2})F$
SA	$T_0 = 100$	$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{r}\sqrt{T_k}, \lvert\mathbf{r}\rvert = 1$ $T_{k+1} = 0.95^k T_k$, $f_{\text{accept}} = \exp\Big(\frac{J(\mathbf{x}_i)-J(\mathbf{x}_j)}{T_k}\Big)$
PSO	$c_{1,2} = 1.49$ $w = [0.4, 0.8]$	$\mathbf{v}_i^{k+1} = w\mathbf{v}_i^k + (\mathbf{p}_{\text{best},i} - \mathbf{x}_i^k)c_1 r_1 + (\mathbf{g}_{\text{best},i} - \mathbf{x}_i^k)c_2 r_2$ $\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^k$

3.3 Particle Swarm Optimization

In PSO, a population with N members have their positions $\mathbf{x}_i^k$ and velocities $\mathbf{v}_i^k$ where k is the iteration number and i is the member index. Each member knows its personal best value $\mathbf{p}_{\text{best},i}$ and the best value of its neighborhood $\mathbf{g}_{\text{best},i}$. The velocity $\mathbf{v}_i$ and the position $\mathbf{x}_i$ are updated by

$$\mathbf{v}_i^{k+1} = w\mathbf{v}_i^k + (\mathbf{p}_{\text{best},i} - \mathbf{x}_i^k)c_1 r_1 + (\mathbf{g}_{\text{best},i} - \mathbf{x}_i^k)c_2 r_2, \tag{7}$$
$$\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \mathbf{v}_i^k, \tag{8}$$

where w is called the weight, c_1, c_2 are cognitive and social acceleration coefficients, and r_1 and r_2 are independent and uniformly distributed random numbers between 0 and 1. If the new member $\mathbf{x}_i^{k+1}$ achieves a smaller value of the objective function, the member and its personal best are updated.

In Table 1 the chosen parameters for all algorithms are summarized.

4 Test Cases

In this section, we describe the test networks considered in the present work. We use the IEEE 14 and 30-bus test networks [13] for the comparison of the heuristical optimization techniques whose details are provided in the previous section. In this work, we consider only power flow and generation injection measurements. The power flow measurements are located on the branches of a spanning tree of the networks. With the injection measurements, this placement ensures full observability [11]. The exact measurement locations are listed in Table 2.

Table 2. Measurements used for calculations in IEEE 14- and 30-bus test networks

	Power Flow on Branches	Generation at Buses
14-Bus	1-2,1-5,2-3,2-4,4-7,4-9,5-6,6-11,6-12,6-13,7-8,9-10,9-14	1,2,3,6,8
30-Bus	1-2,1-3,2-4,3-4,2-5,2-6,6-7,6-8,6-9,6-10,23-24,25-26, 6-28,9-11,10-17,10-20,10-21,10-22,12-13,25-27,28-27, 12-15,12-16,14-15,15-18,15-23,18-19,22-24,27-29,27-30	1,2,5,8,11,13

We use the MATLAB package MATPOWER 5.1 to obtain the real (correct) measurement values and the values of the state variables by solving the optimal power flow problem [14]. The measurement values are then overlaid with additive Gaussian noise with the standard deviation σ_i of 0.02 and 0.015 for power flow and injection measurements, respectively.

Two different approaches are used for the initialization of the first candidates. In the first approach, we set the starting candidate in SA and one member of the first populations of DE and PSO to a candidate obtained by overlaying the true system state along with a noise which has the same statistics as in the measurement creation step. In other words, the algorithms start from a point in the search space which is close to the true system state. This approach, which we call *near start* in the following, is reasonable since the power system state changes gradually and the result of the last estimation can be used as the starting point in the next estimation. For the sake of completeness, in the second approach, we apply the same procedure with a flat start, where all voltage phasor magnitudes $|V_i|$ are set to 1 p.u. and all voltage phasor angles θ_i to 0°. In both approaches, other candidates in DE and PSO are randomly generated over the search space. For each optimization algorithm, we perform Monte Carlo simulations with 30 runs with an iteration limit of 1000 iterations.

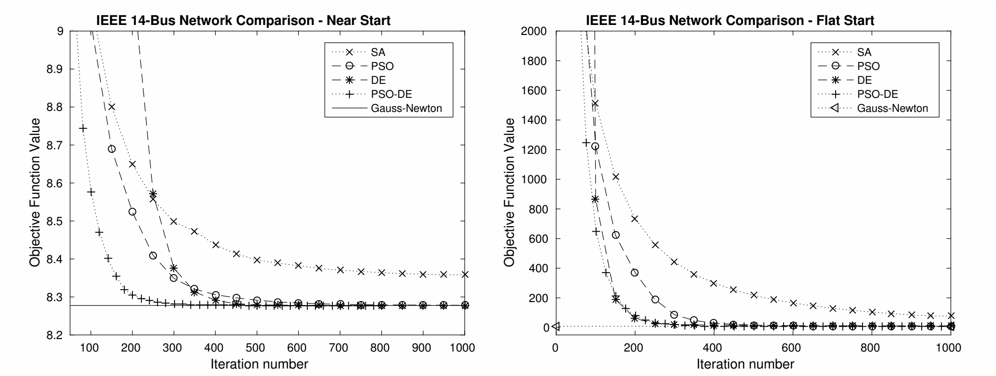

Fig. 1. Values of the objective function at certain iteration points for the IEEE 14-bus network for all three heuristical optimization techniques DE, SA, PSO and hybrid PSO-DE algorithm. Left: Near Start, Right: Flat Start.

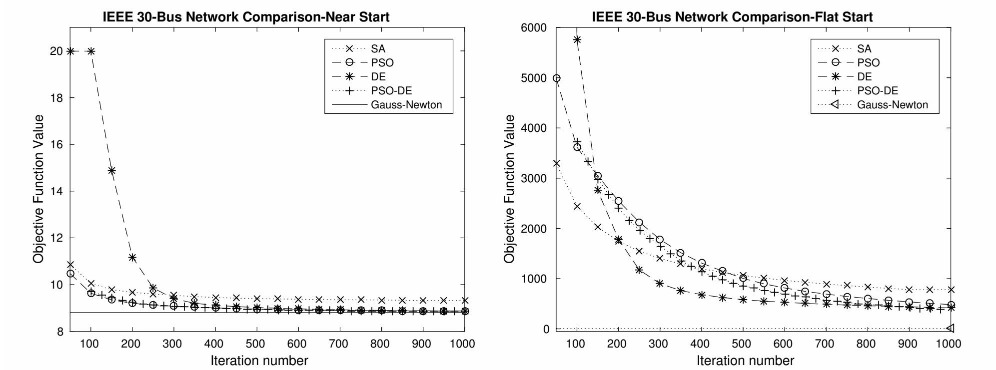

Fig. 2. Values of the objective function at certain iteration points for the IEEE 30-bus network for all three heuristical optimization techniques DE, SA, PSO and hybrid PSO-DE algorithm. Left: Near Start, Right: Flat Start.

5 Results

The results for IEEE 14-bus and 30-bus test networks are illustrated in Figure 1 and Figure 2, respectively. The graphs on the left provide a comparison of the expected values of the objective function for near start initializations, whereas the graphs on the right side show the results with flat start. It is noticeable in near start case in Figure 1 that PSO performs best in first iterations, but is overtaken by DE after 300 iterations. This observation has led us to propose an hybrid solution with PSO and DE to benefit from their superior performances in search and intensification, respectively. In this algorithm, we observe the rate of decrease in the best value of PSO until it reaches a threshold value. After the threshold is reached, the algorithm continues with the update steps of DE. We set the threshold to 5% of the improvement in the first 5 iterations, where the decrease in the best function value is compared with the threshold at every five iterations. As can be seen in Figure 1, the proposed hybrid solution outperforms all other algorithms considerably in near start case. On the other hand, we observe a very slight improvement in flat start initialization. In fact, DE outperforms PSO in flat start initialization, which can be attributed to the start from a worse population and the decrease in the search capability of PSO with increasing iteration number. Another observation is the inferior performance of SA compared with PSO and DE.

In Figure 2, the results of 30-bus test case enable the evaluation of the scalability of the algorithms in a larger problem size. We observe in near start initialization that the hybrid algorithm improves the performance of PSO marginally. This is reasonable as DE does not outperform PSO in later iterations as in 14-bus test case. In flat start case, we see that none of the algorithms can achieve acceptable objective function values comparable to the result of Gauss-Newton method, which can be reasoned by the larger problem size and the iteration limit of the algorithms. Interesting is that the hybrid algorithm improves the PSO performance only slightly, although the performance of DE is better than PSO in flat start initialization. Regarding the variation in the achieved results, PSO, DE and the hybrid algorithm have an average normalized standard deviation of 0.08, 0.1, and 0.09% in 14-bus near start case, and 0.26, 0.41, and 0.23% in 30-bus near start case, respectively. On the other hand, the variation in the flat start case is as high as 36 and 89% in 14- and 30-bus networks, respectively.

6 Conclusion

In this work, we have applied the modern heuristical optimization techniques DE, SA, and PSO to the problem of state estimation in power systems with traditional measurements. Based on the comparison results in 14-bus network test case, we have proposed a hybrid algorithm with PSO and DE, which has improved the convergence noticeably. On the other hand, we have deduced that a larger problem size poses a challenge for the hybrid algorithm, whereas none of the algorithms can deliver satisfactory results with flat start initialization.

Nevertheless, the decentralized structure of future distribution grids can enable the use of heuristical techniques in a distributed manner.

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