Finite Blocklength Performance of Multi-Hop Relaying Networks

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***Abstract*—In this paper, we study the multi-hop relaying performance with finite blocklengths under the quasi-static Rayleigh fading channel. The maximum throughput under the infinite blocklength regime (IBL-throughput) and the throughput under the finite blocklength regime (FBL-throughput) are derived, respectively. Moreover, we investigate the FBL-throughput under two different setups: We first consider transmissions with a target overall error probability through the multi-hop network while in the second scenario the coding rate is assumed to be fixed. By numerical analysis, we show the difference between the IBL-throughput and the FBL-throughput of multi-hop relaying. In particular, the optimal number of hops under the FBL regime and the IBL regime are different. In addition, we show that the FBL-throughput under the target error probability scenario is quasi-concave in the target error probability while it is also quasi-concave in the coding rate under the constant coding rate scenario. Moreover, the influences of blocklength on FBL-throughputs under the two scenarios are different. In particular, the performance of target error probability scenario is more sensitive to increasing the blocklength in comparison to the constant coding rate scenario.**

***Index Terms*—Finite blocklength, decode-and-forward, multi-hop relaying, throughput.**

I. Introduction

In wireless communication, relaying is an efficient way to achieve broader coverage and to mitigate wireless fading by reducing pathloss. Specifically, when multi-hop relaying system is available for data transmission, such as in device-to-device communications, the performance of the transmission is significantly improved [1] [2] [3]. However, all the above studies are under the Shannon's channel capacity with ideal assumption, i.e., coding is assumed to be performed using a block with an infinite length.

As the blocklength of coding is restricted to a small size, the error probability of the communication is no longer negligible. In [4], the authors identify a tight bound of the coding rate for a given error probability in the finite blocklength (FBL) regime, which is accurate even when the blocklength is as small as 100. The research in [4] shows that the performance loss due to a finite blocklength is considerable and becomes more significant when the blocklength is relatively short. This actually indicates a tradeoff in the multi-hop relaying system between reducing pathloss by introducing more hops and FBL performance loss due to more shorter blocklength at each hop. In the previous work of two authors of this paper, the performance model of decode-and-forward (DF) relaying with static channels [5] [6] and quasi-static channels [7] was investigated. However, all these works focused on exact two-hop relaying systems, leaving the analysis of multi-hop relaying (where the number of hops could be higher than two) in the FBL regime an open problem.

In this paper, we consider a under the quasi-static Rayleigh fading channel. We address the following fundamental questions of multi-hop relaying under the FBL regime: What is the performance difference between multi-hop relaying under the FBL regime and the one under the infinite blocklength (IBL) regime? What is the impact of the FBL on the system behavior? To address the questions, we first derive the maximum throughput in the IBL regime (IBL-throughput) and the maximum throughput in the FBL regime (FBL-throughput) of a multi-hop relaying. Subsequently, we investigate the FBL-throughput under the scenario with target overall error probability and the scenario with constant coding rate. In addition, we show the difference between the IBL performance and the FBL performance through numerical analysis. Moreover, we conclude a set of guidelines for design efficient multi-hop relaying networks.

The rest of this paper is organized as follows. Section II introduces the system model under consideration. In Section III, the IBL-throughput and the FBL-throughput of a multi-hop relaying are derived. Based on this, the FBL-throughput of a multi-hop relaying is studied under the target overall error probability scenario and the constant coding rate scenario, respectively. Finally, numerical results are provided and discussed in Section IV.

II. System Model

We consider a N-hop relaying system as shown in Fig. 1, consisting of a source node R_0, a destination node R_N, and $(N-1)$ DF relay nodes R_1, R_2, ..., R_{N-1}. We consider a simple linear topology where nodes are placed equidistantly in a line. The total transmission distance form R_0 to R_N is denoted by D. The entire system operates in time-slotted fashion and time is divided into frames of length, where M is total number of symbols for a transmission period, i.e.,

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transmitting the data blocklength from R_0 to R_N. In other words, the blocklength of each hop is given $m = M/N$. For simplification, we assume M is always the integral multiple of N. In other words, m is an integer.

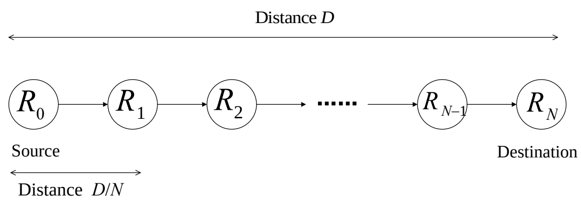

Fig. 1. A linear multihop relaying system with $N-1$ relays.

We consider a quasi-static Rayleigh fading channel model, i.e., channels are constant in a transmission period and vary from one period to the next. During a transmission period j, the received signal at ith hop is given by

$$\mathbf{y}_{i,j} = p_{tx} h_{i,j} \mathbf{x}_j + \mathbf{n}_{i,j}, \quad for\ i = 1, ..., N. \tag{1}$$

The transmitted signal $\mathbf{x}_j$ and received signal $\mathbf{y}_{i,j}$ are complex m-dimensional vectors. Besides, $\mathbf{n}_{i,j}$ $(i = 1, ..., N)$ denote the noise vectors for different hops in transmission period j, which are independent and identically distributed (i.i.d.) complex Gaussian vectors: $\mathbf{n} \sim \mathcal{N}\left(0, \sigma^2 \mathbf{I}_m\right)$, $\mathbf{n} \in \{\mathbf{n}_{1,j}, \ldots, \mathbf{n}_{N,j}\}$, where $\mathbf{I}_m$ denotes an $m \times m$ identity matrix. In addition, p_{tx} represents the transmit power for ith hop. We consider a total power constraint assumption, i.e., all nodes equally share the total power P. Furthermore, $h_{i,j}$ is the channel of ith hop during the transmission period j. Under the quasi-static channel model, the instantaneous channel gain for each hop has two components, including the average channel gain and the random fading. On the one hand, we denote the average channel gains (e.g., due to the path loss) of these hops over the transmission periods by $\left|\bar{h}_i\right|^2$ $(i = 1, ..., N)$. On the other hand, we assume that the gain due to Rayleigh fading is given by the exponential distribution: $f(z) = e^{-z}$. We denote $z_{i,j}$ as the gains due to Rayleigh fading in transmission period j for hop i. Hence, we have $|h_{i,j}|^2 = z_{i,j}\left|\bar{h}_i\right|^2$, $i = 1, ..., N$. Moreover, these channel fading gains at different hops during the same transmission period are assumed to be independent and identically distributed (i.i.d). We assume perfect channel state information (CSI) at the receivers and in particular at the source. Thus, the received signal to noise ratio (SNR) at ith hop in transmission period j is given by

$$\gamma_{i,j} = \frac{|h_{i,j}|^2 p_{tx}}{\sigma^2} = \frac{z_{i,j}\left|\bar{h}_i\right|^2 P}{\sigma^2 N}, \quad for\ i = 1, ..., N. \tag{2}$$

We consider the scenario with finite blocklengths, where decoding errors may occur due to noise.[1] We assume that all relay nodes and the destination node can detect the errors reliably. On this protocol, an relay does not forward the block when an error occurs in link to the relay. In addition, if a decoding error occurs at a relay node or the destination node, the throughput of the multi-hop relaying system equals zero.

III. Multi-hop Relaying Performance

In this section, the multi-hop relaying performance is investigated under both the IBL regime and the FBL regime.

A. The IBL-throughput of Multi-hop Relaying

Under the IBL regime, according to the Shannon capacity theory a coding rate being higher than the Shannon capacity definitely leads to an error/outage. Hence, the maximum coding rate at ith hop during the transmission period j is subject to the corresponding Shannon capacity

$$\mathcal{C}\left(|h_{i,j}|^2\right) = \log_2\left(1 + \frac{|h_{i,j}|^2 p_{tx}}{\sigma^2}\right) = \log_2\left(1 + \frac{z_{i,j}\left|\bar{h}_i\right|^2 P}{\sigma^2 N}\right). \tag{3}$$

Then, the IBL-throughput during the transmission period j is subject to the link with the worst channel quality and given by

$$\begin{aligned} C_{\mathrm{IBL},j} &= \tfrac{1}{N} \min_{i=1,\ldots,N} \left\{\mathcal{C}\left(|h_{i,j}|^2\right\}\right) \\ &= \tfrac{1}{N}\mathcal{C}\left(\min_{i=1,\ldots,N} \left\{z_{i,j}\left|\bar{h}_i\right|^2\right\}\right). \end{aligned} \tag{4}$$

Hence, the (average) IBL-throughput (in bits per channel use) of multi-hop relaying over channel fading is

$$C_{\mathrm{IBL}} = \frac{1}{N} \mathop{\mathbb{E}}_{z}\left[\mathcal{C}\left(\min_{i=1,\ldots,N}\left\{z_{i,j}\left|\bar{h}_i\right|^2\right\}\right)\right], \tag{5}$$

where $\mathop{\mathbb{E}}_{z}[*]$ is the expectation over the distribution of channel fading z.

B. The FBL-throughput of Multi-hop Relaying

For the real additive white Gaussian noise (AWGN) channel, Theorem 54 in [4] derives a tight bound on the coding rate for a single-hop transmission system. With SNR γ, error probability ε, and blocklength m, the coding rate r (in bits per channel use) is given by: $r \approx \frac{1}{2}\log_2(1+\gamma) - \sqrt{\frac{1}{2m}\left(1 - \frac{1}{(1+\gamma)^2}\right)} Q^{-1}(\varepsilon)\log_2 e$, where $Q(w) = \int_w^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$ is the Gaussian Q-function.

The above result for a real AWGN channel was extended to a complex quasi-static fading channel in [7]–[9]. With the error probability $\varepsilon_{i,j}$ and blocklength $m = M/N$, the coding rate (in bits per channel use) at the ith hop during the transmission period j is given by

$$\begin{aligned} &r_{i,j} = \mathcal{R}\left(|h_{i,j}|^2, \varepsilon_{i,j}, m\right) \\ &= \mathcal{C}\left(|h_{i,j}|^2\right) - \sqrt{\tfrac{1}{m}\left(1 - 2^{-2\mathcal{C}\left(|h_{i,j}|^2\right)}\right)} Q^{-1}(\varepsilon_{i,j})\log_2 e. \end{aligned} \tag{6}$$

[1]Recall that under the infinite blocklength regime, the transmission is error-free if the coding rate is lower than the shannon capacity.

In the other way round, if the coding rate $r_{i,j}$ is given, the decoding error probability at the ith hop is given by

$$\begin{aligned}\varepsilon_{i,j} &= \mathcal{P}\left(|h_{i,j}|^2, r_{i,j}, m\right) \\ &= Q\left(\frac{\mathcal{C}\left(|h_{i,j}|^2\right)-r_{i,j}}{\sqrt{\frac{1}{m}\left(1-2^{-2\mathcal{C}\left(|h_{i,j}|^2\right)}\right)}\log_2 e}\right).\end{aligned} \tag{7}$$

Hence, the overall error probability during the transmission period j is given by

$$\varepsilon_j = 1 - \prod_{i=1}^{N}(1-\varepsilon_{i,j}). \tag{8}$$

Thus, the general overall FBL-throughput of multi-hop relaying at transmission period j is given by

$$C_{\mathrm{FBL},j} = \frac{1}{N}\varepsilon_j \min_{i=1,\ldots,N}\{r_{i,j}\}. \tag{9}$$

Finally, the average FBL-throughput over channel fading is

$$C_{\mathrm{FBL}} = \mathop{\mathbb{E}}_{z}[C_{\mathrm{FBL},j}]. \tag{10}$$

So far, we have derive the FBL-throughput for a general case. Then, we further study the FBL-throughput of multi-hop relaying by considering the following two specific scenarios. In practice, it is possible that traffics have target error probability, e.g., control signals. The first scenario we considered is corresponding to the above service, where we assume that the overall error probability of multi-hop relaying is target. Under this scenario, the target error probability at each hop is influenced by the number of hops. In addition, applications such as VoIP likely generate date with constant rate. In our second setup, we consider a scenario where the coding rate at each hop is constant. Hence, due to the independent random channel behaviors at different links, the error probabilities at different hops are more likely different. It should be mentioned that these two scenarios are the same under the IBL regime where the target error portability is always zero.

1) Scenario with target overall error probability: Under a scenario with target overall error probability ε_*, i.e., the equivalent error probability at each hop is $\sqrt[N]{1-\varepsilon_*}$. Hence, the coding rate $r_{i,j}$ at each hop is given by $r_{i,j} = \mathcal{R}\left(|h_{i,j}|^2, 1-\sqrt[N]{1-\varepsilon_*}, \frac{M}{N}\right)$.

Thus, the (average) FBL-throughput (in bits per channel use) of multi-hop relaying over time in this scenario is

$$C_{\mathrm{FBL}} = (1-\varepsilon_*)\mathop{\mathbb{E}}_{z}\left[\frac{1}{N}\left(\min_{i=1,\ldots,N}\{r_{i,j}\}\right)\right]. \tag{11}$$

2) Scenario with constant coding rate: In this scenario we assume the coding rate at each hop is the same (i.e. r_j), the FBL-throughput (in bits per channel use) of multi-hop relaying during transmission period j is given by

$$C_{\mathrm{FBL},j} = \frac{1}{N}\left(1-\prod_{i=1}^{N}(1-\varepsilon_{i,j})\right)\min_{i=1,\ldots,N}\{r_{i,j}\}, \tag{12}$$

where $\varepsilon_{i,j}$ is given by $\mathcal{P}\left(|h_{i,j}|^2, r_j, m\right)$. Then, the average FBL-throughput can be obtained according to (10).

IV. Numerical Results And Discussion

In this section, we investigate the multi-hop relaying systems by numerical analysis where we consider the following parameterization of the system model: The total transmitted power and noise power are set to 22 dBm and -85 dBm, respectively. Distance is set to $D \in [0.5km, 7km]$, while blocklength is set to $M \in [200, 6000]$.

We study the FBL-throughput of multi-hop relaying while varying the blocklength. We show the results in Fig. 2 where both the target overall error probability scenario and the constant coding rate scenario are considered. We can observe that

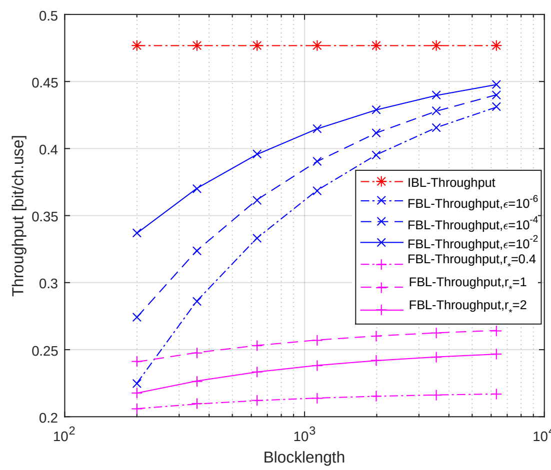

Fig. 2. The performance comparison between IBL-throughput and FBL-throughput under the scenario with target overall error probability and the scenario with constant coding rate while varying blocklength. In the analysis, $D = 6km$.

FBL-throughputs of both these two scenarios are increasing in the blocklength, while IBL-throughput is not influenced by the blocklength. Under the target error probability scenario, the FBL-throughput with a low overall error probability target increasing rapidly in blocklength in comparison to the one with a relatively high overall error probability target. For the constant coding rate scenario, the FBL-throughput with a high coding rate increasing in blocklength more significantly than the one with a low coding rate. More interesting, the influences of blocklength on FBL-throughputs under the two scenario are different. In particular, the performance of target error probability scenario is more sensitive to increasing the blocklength.

In the following, we study the target error probability scenario and the constant coding rate scenario, respectively. We first consider the target error probability. We show the relationship between FBL-throughput and target overall error probability in Fig. 3. From the figure, we observe a interesting results that the FBL-throughput is quasi-concave in the target overall error probability. In other wards, under the target error probability scenario the FBL performance can be optimized by choosing an optimal target overall error probability.

In addition, under the target error probability scenario the relationship between the FBL-throughput and the total transmission distance is shown in Fig. 4. From the figure,

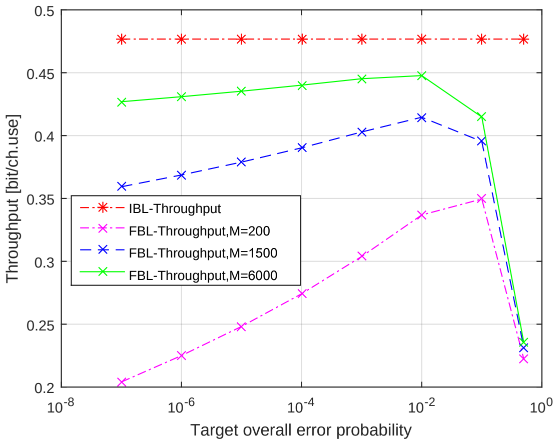

Fig. 3. Target error probability scenario: The FBL-throughput vs. the overall error probability. In the analysis, $D = 6km$.

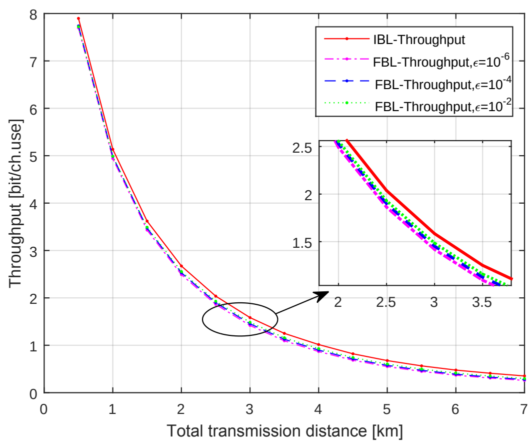

Fig. 4. Target error probability scenario: The throughput vs. distance. In the analysis, $M = 1500$.

the FBL-throughput with large overall error probability has a similar performance as IBL-throughput, while the FBL-throughput with small overall error probability is significantly different from the IBL-throughput. Moreover, the relationship between the optimal number of hops and the total transmission distance is shown in Fig. 5. The optimal number of hops is strictly increasing in the total transmission distance. Moreover, the optimal number of hops with small overall error probability is bigger than that with large overall error probability.

The last study under the target error probability scenario is regarding the relationship between the optimal number of hops and the blocklength. We show the results in Fig. 6 and observe that if the target overall error probability is not extremely high, the optimal number of hops under the FBL regime is higher than the ones under the IBL regime. Under this case, the optimal number of hops under the FBL regime is decreasing the blocklength. More interestingly, when the overall error probability is significantly high, which makes the target error probability at each hop be higher than 0.5, the optimal number of hops under the FBL regime becomes lower than the IBL regime, while the optimal number of hops under

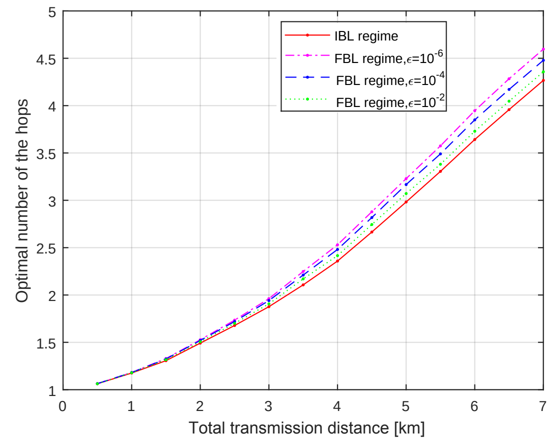

Fig. 5. Target error probability scenario: The optimal number of hops vs. total transmission distance. In the analysis, $M = 1500$.

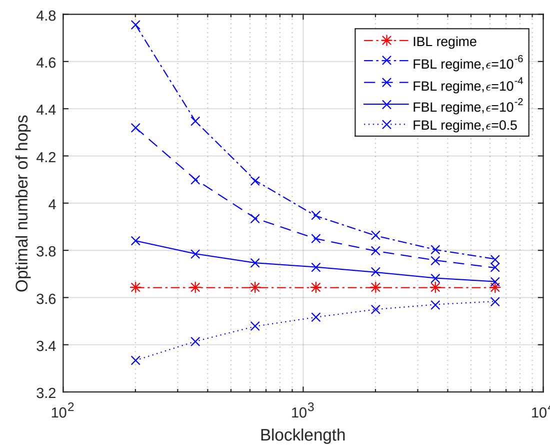

Fig. 6. Target error probability scenario: The optimal number of hops vs. blocklength. In the analysis, $D = 6km$.

the FBL regime under this case is increasing the blocklength.

Then, we focus on the scenario with constant coding rate. We show in Fig. 7 the relationship between the coding rate and the FBL-throughput. We plot the corresponding IBL-

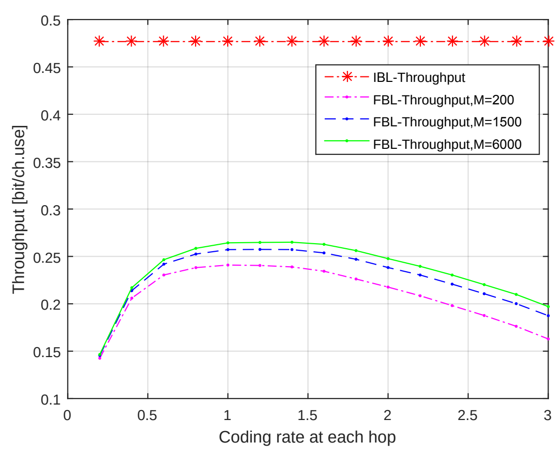

Fig. 7. Constant coding rate scenario: The FBL-throughput vs. the coding rate at each hop. In the analysis, $D = 6km$.

throughput of multi-hop relaying as a performance reference. We find an interesting result that the FBL-throughput is quasi-concave in the coding rate. Therefore, under the constant coding rate scenario, the FBL-throughput of multi-hop relaying has a global maximum which can be achieved by choosing an appropriate coding rate.

In addition, the relationship between FBL-throughput and the total transmission distance is shown in Fig. 8. From the

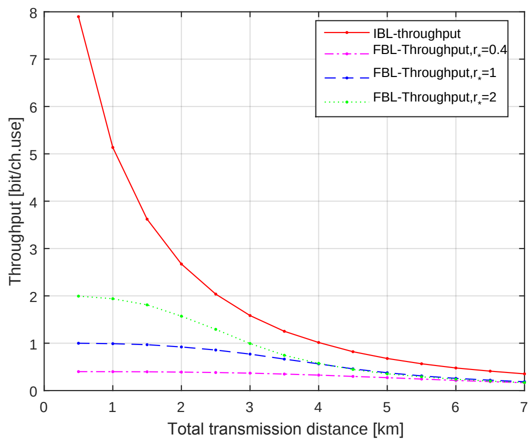

Fig. 8. Constant coding rate scenario: The throughput vs. total transmission distance. In the analysis, $M = 1500$.

figure, we observe that a low coding rate is more suitable for long transmission distances. More interesting, the performance of FBL-throughputs under the constant coding rate scenario are significantly different form the IBL-throughput. This is totally different from the FBL performance under the target error probability scenario. In particular, setting an overaggressive coding rate, e.g., to be higher than the Shannon capacity, leads to a significant overall error probability.

Furthermore, the relationship between the optimal number of hops and the total transmission distance is shown in Fig. 9. We observe from the figure that for some cases the optimal

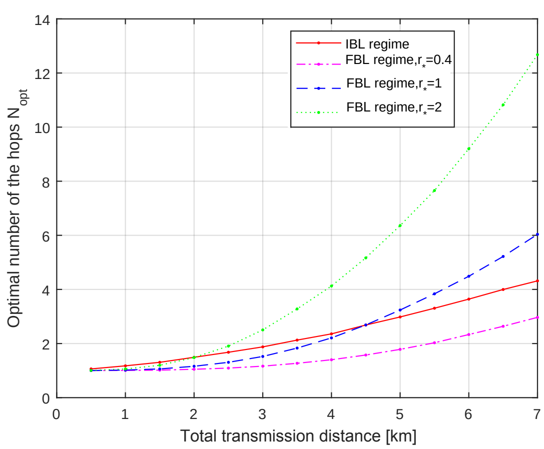

Fig. 9. Constant coding rate scenario: The optimal number of hops vs. total transmission distance. In the analysis, $M = 1500$.

number of hops under the FBL regime is less than the optimal number of hops under the IBL regime while for the rest cases are not. The boundary between these two different type of cases is actually the special case under which the coding rate equals the Shannon capacity. In particular, with a coding rate being lower than the Shannon capacity, the optimal number of hops under the FBL regime is smaller than that under the IBL regime. This is the reason why all IBL curves are lower than the IBL curve when the distance is short. On the other hande, with a high coding rate being higher than the Shannon capacity the optimal number of hops under the FBL regime becomes larger than the one under the IBL regime. In particular, the curve with the most aggressive coding rate $r = 2$ only lower than the IBL curve for a small region with short distances, while the one with a prudential coding rate $r = 0.4$ always lower than the IBL curve in the figure.

V. Conclusion

In this paper, we investigate the multi-hop relaying performance in the FBL regime. Both the target error probability scenario and the constant coding rate scenario are considered. We derived the FBL-throughputs under the two scenarios. We show the performance difference of multi-hop relaying with FBLs and IBLs through numerical analysis. In addition, we conclude a set of guidelines for design efficient multi-hop relaying networks with FBLs: i. The optimal number of hops under the FBL regime and the IBL regime are different. ii. The FBL-throughput under the target error probability scenario is quasi-concave in the target error probability while it is also quasi-concave in the coding rate under the constant coding rate scenario. iii. The influences of blocklength on FBL-throughputs under the two scenarios are different. In particular, the performance of target error probability scenario is more sensitive to increasing the blocklength in comparison to the constant coding rate scenario.

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