Linear Precoder and Decoder Design for Bidirectional Full-Duplex MIMO OFDM Systems

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***Abstract*—In this paper we address the linear precoding and decoding design problem for a bidirectional orthogonal-frequency-division-multiplexing (OFDM) communication system, between two multiple-input-multiple-output (MIMO) full-duplex (FD) nodes. The effects of hardware distortions, leading to residual self-interference and inter-carrier leakage, are taken into account. In the first step, the operation of a FD MIMO OFDM transceiver is modeled under the impact of known hardware impairments. An alternating quadratic convex program (AltQCP) is then provided to obtain a minimum-mean-squared-error (MMSE) design for the defined system. The proposed design is then extended to maximize the system sum rate, applying the weighted-MMSE (WMMSE) method. The proposed AltQCP methods result in a monotonic improvement, leading to a necessary convergence to a stationary point. Finally, the performance of the defined system is evaluated under various system conditions, and in comparison to the other approaches in the literature. A significant gain is observed via the application of the proposed method as the hardware inaccuracy, and consequently inter-carrier leakage, increases.**

I. Introduction

Full-duplex (FD) transceivers are known for their capability to transmit and receive at the same time and frequency, and hence have the potential to enhance the spectral efficiency [1]. Nevertheless, such systems suffer from the inherent self-interference from their own transmitter. Recently, specialized self-interference cancellation (SIC) techniques, e.g., [2], have demonstrated an adequate level of isolation between transmit (Tx) and receive (Rx) directions to facilitate a FD communication and motivated a wide range of related studies, see [1]. Nevertheless, such methods are not perfect in a realistic environment mainly due to *i)* aging and inherent inaccuracy of the hardware (analog) elements, as well as *ii)* inaccurate channel state information (CSI) in the self-interference path, due to limited channel coherence time. In this regard, inaccuracy of the analog hardware elements used in subtracting the dominant self-interference path in RF domain may result in severe degradation of SIC quality. This issue becomes more relevant in a realistic scenario, where unlike the demonstrated setups in the lab environment, analog components are prone to aging, temperature fluctuations, and occasional physical damage.

In order to combat the aforementioned issues, a FD transceiver may adapt its transmit/receive strategy to the accuracy of the chain elements, e.g., by dedicating less task to the chains with noisier elements. In this regard, a widely used model for the operation of a multiple-antenna FD transceiver is proposed in [3], assuming a single carrier communication system, where the impact of hardware impairments are taken into account. A gradient-projection-based method is then proposed in the same work for maximizing the sum rate in a FD bidirectional setup. Building upon the proposed benchmark, a convex optimization design framework is introduced in [4] by defining a price/threshold for the self-interference power, assuming an accurate transceiver operation. While this approach reduces the design computational complexity, it does not provide a reliable performance under the impact of hardware impairments. Consequently, the consideration of transceiver error in a FD bidirectional system is further studied in [5]–[7] by maximizing the system sum rate, and in [8]–[10] for minimizing the system power consumption under a required quality of service.

The aforementioned works focus on modeling and design methodologies for single-carrier FD bidirectional systems, under frequency-flat channel assumptions. In this regard, the importance of extending the previous designs for a multi-carrier (MC) system with a frequency selective channel is threefold. Firstly, due to the increasing rate demand of the wireless services, the usage of larger bandwidths becomes necessary. This, in turn, invalidates the usual frequency-flat assumption and calls for updated design methodologies. Secondly, unlike the half-duplex (HD) systems where the operation of different subcarriers can be safely separated in the digital domain, an FD system is highly prone to the inter-carrier leakage due to the impact of hardware distortions on the strong self-interference channel[1]. This, in particular, calls for a proper modeling of the inter-carrier leakage

[1]For instance, a high-power transmission in one of the subcarriers will result in a higher residual self-interference in all of the subcarriers due to, e.g., a higher quantization and power amplifier noise levels.

as a result of non-linear hardware distortions for FD transceivers. And finally, the frequency diversity on the frequency selective channels shall be opportunistically exploited to enhance the system performance.

In this paper we study a bidirectional FD MIMO OFDM system, where the impacts of hardware distortions are taken into account. In Section II the operation of a FD MC transceiver is modeled, where explicit impact of hardware inaccuracies on the inter-carrier leakage is formulated in relation to the intended transmit/receied signal. In Section III, an alternating quadratic convex program (QCP), denoted as AltQCP, is proposed in order to obtain a minimum weighted MSE transceiver design. The weighted-minimum-MSE (WMMSE) method [11] is then utilized to extend the AltQCP framework for maximizing the system sum rate. A monotonic performance improvement is observed at each step, leading to a necessary convergence. Numerical results show that the gain of utilizing a design which takes into account the impacts of inter-carrier leakage, becomes significant as transceiver inaccuracy increases.

II. System Model

We consider a bidirectional OFDM communication system between two MIMO FD transceivers. Each communication direction is associated with N_i transmit and M_i receive antennas, where $i \in \mathbb{I}$, and $\mathbb{I} := \{1, 2\}$ represents the set of the communication directions. The desired channel in the communication direction i and subcarrier $k \in \mathbb{F}_K$ is denoted as $\mathbf{H}_{ii}^k \in \mathbb{C}^{M_i \times N_i}$ where K is the number of subcarriers. The self-interference channel from i to j-th communication direction is denoted as $\mathbf{H}_{ji}^k \in \mathbb{C}^{M_j \times N_i}$. All channels are quasi-static[2], and frequency-flat in each subcarrier. The transmitted signal in the direction i, subcarrier k is formulated as

$$\mathbf{x}_i^k = \underbrace{\mathbf{V}_i^k \mathbf{s}_i^k}_{=:\mathbf{v}_i^k} + \mathbf{e}_{\mathrm{t},i}^k, \quad \sum_{k \in \mathbb{F}_K} \mathbb{E}\left\{ \|\mathbf{x}_i^k\|_2^2 \right\} \leq P_i, \qquad (1)$$

where $\mathbb{F}_K := \{1, \ldots, K\}$, and $\mathbf{s}_i^k \in \mathbb{C}^{d_i}$, $\mathbf{V}_i^k \in \mathbb{C}^{N_i \times d_i}$ and $P_i \in \mathbb{R}^+$ respectively represent the vector of the data symbols, the transmit precoding matrix, and the maximum affordable transmit power. The number of the data streams in each subcarrier, and in direction i is denoted as d_i, and $\mathbb{E}\{\mathbf{s}_i^k {\mathbf{s}_i^k}^H\} = \mathbf{I}_{d_i}$. Moreover, $\mathbf{v}_i^k \in \mathbb{C}^{N_i}$ represents the desired signal to be transmitted, where $\mathbf{e}_{\mathrm{t},i}^k$ models the inaccurate behavior of the transmit chain elements, i.e, transmit distortion, see Subsection II-A for more details. The received signal at the destination can be consequently written as

$$\mathbf{y}_i^k = \underbrace{\mathbf{H}_{ii}^k \mathbf{x}_i^k + \mathbf{H}_{ij}^k \mathbf{x}_j^k + \mathbf{n}_i^k}_{=:\mathbf{u}_i^k} + \mathbf{e}_{\mathrm{r},i}^k, \qquad (2)$$

where $\mathbf{n}_i^k \sim \mathcal{CN}\left(\mathbf{0}_{M_i}, \sigma_{i,k}^2 \mathbf{I}_{M_i}\right)$ is the additive thermal noise. Similar to the transmit signal model, $\mathbf{e}_{\mathrm{r},i}^k$ represents the receiver distortion, and models the inaccuracies of the receive chain elements. The *known*, i.e., distortion-free, part of the self-interference signal is then subtracted from the received signal. This is formulated as

$$\tilde{\mathbf{y}}_i^k := \mathbf{y}_i^k - \mathbf{H}_{ij}^k \mathbf{V}_j^k \mathbf{s}_j^k = \mathbf{H}_{ii}^k \mathbf{V}_i^k \mathbf{s}_i^k + \boldsymbol{\nu}_i^k, \qquad (3)$$

where $\tilde{\mathbf{y}}_i^k$ is the received signal in direction i and subcarrier k, after the self-interference cancellation. Moreover, the aggregate interference-plus-noise term is denoted as $\boldsymbol{\nu}_i^k \in \mathbb{C}^{M_i}$ where

$$\boldsymbol{\nu}_i^k = \mathbf{H}_{ij}^k \mathbf{e}_{\mathrm{t},j}^k + \mathbf{H}_{ii}^k \mathbf{e}_{\mathrm{t},i}^k + \mathbf{e}_{\mathrm{r},i}^k + \mathbf{n}_i^k, \;\; j \neq i. \qquad (4)$$

Finally, the estimated data vector is obtained at the receiver as

$$\tilde{\mathbf{s}}_i^k = \left(\mathbf{U}_i{}^k\right)^H \tilde{\mathbf{y}}_i^k, \qquad (5)$$

where $\mathbf{U}_i^k \in \mathbb{C}^{M_i \times d_i}$ is the linear receive filter.

A. Limited dynamic range in an FD OFDM system

The inaccurate function of the transmit chain elements, e.g., digital-to-analog converter (DAC) error, power amplifier noise and oscillator phase noise, are jointly modeled for each antenna as an additive distortion, and written as $x_l(t) = v_l(t) + e_{\mathrm{t},l}(t)$, see Fig. 1, such that

$$e_{\mathrm{t},l}(t) \sim \mathcal{CN}\Big(0, \theta_{\mathrm{tx},l} \mathbb{E}\{\, |v_l(t)|^2 \,\}\Big), \qquad (6)$$

$$e_{\mathrm{t},l}(t) \perp v_l(t), \;\; e_{\mathrm{t},l}(t) \perp e_{\mathrm{t},l'}(t), \;\; e_{\mathrm{t},l}(t) \perp e_{\mathrm{t},l}(t'), \quad l \neq l' \in \mathbb{L}_T, \;\; t \neq t', \qquad (7)$$

where v_l, x_l, and $e_{\mathrm{t},l} \in \mathbb{C}$ respectively represent the intended transmit signal, the actual transmit signal, and the additive transmit distortion at the l-th transmit chain, and t denotes the instance of time. The set $\mathbb{L}_T$ represents the set of all transmit chains. Moreover, $\theta_{\mathrm{tx},l} \in \mathbb{R}^+$ represents the distortion coefficient for the l-th transmit chain, relating the collective power of the distortion signal, over the active spectrum, to the intended transmit power. A detailed elaboration of the used distortion model is given in [12, Section II].

In the receiver side, the combined effect of the inaccurate hardware elements, i.e., ADC error, oscillator phase noise and automatic gain control noise, are presented as additive distortion terms and written as $y_l(t) = u_l(t) + e_{\mathrm{r},l}(t)$ such that

$$e_{\mathrm{r},l}(t) \sim \mathcal{CN}\Big(0, \theta_{\mathrm{rx},l} \mathbb{E}\{\, |u_l(t)|^2 \,\}\Big), \qquad (8)$$

$$e_{\mathrm{r},l}(t) \perp u_l(t), \;\; e_{\mathrm{r},l}(t) \perp e_{\mathrm{r},l'}(t), \;\; e_{\mathrm{r},l}(t) \perp e_{\mathrm{r},l}(t'), \quad l \neq l' \in \mathbb{L}_R, \;\; t \neq t', \qquad (9)$$

where u_l, y_l, and $e_{\mathrm{r},l} \in \mathbb{C}$ respectively represent the intended (distortion-free) receive signal, additive receive distortion, and the received signal from the l-th receive antenna. The set $\mathbb{L}_R$ represents the set of all receive chains. Similar to the transmit chain characterization, $\theta_{\mathrm{rx},l} \in \mathbb{R}^+$ is the distortion coefficient for the l-th receive chain, see Fig. 1.

In this work we consider a general framework where the transmit (receive) distortion coefficients are not necessarily identical for all transmit (receive) chains

[2]It indicates that the channel is constant in each communication frame, but may vary from one frame to another frame.

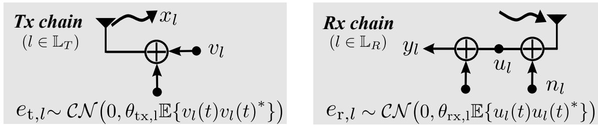


Figure 1. A FD transceiver model. Limited dynamic range is modeled by injecting additive distortion terms at each transmit or receive chain.

belonging to the same transceiver, i.e., different chains may hold different accuracy due to occasional damage and aging. This assumption is important in practice since it enables the design algorithms to reduce communication task on the chains with noisier elements. The statistics of the distortion terms, introduced in (1), (2) can be hence inferred as

$$\mathbf{e}_{\text{t},i}^k \sim \mathcal{CN}\left(\mathbf{0}_{N_i}, \mathbf{\Theta}_{\text{tx},i}^k \mathbf{P}_{\text{tx},i}\right), \; \mathbf{\Theta}_{\text{tx},i}^k := B^k \mathbf{\Theta}_{\text{tx},i} / B_{\text{tot}}, \tag{10}$$

$$\mathbf{e}_{\text{r},i}^k \sim \mathcal{CN}\left(\mathbf{0}_{M_i}, \mathbf{\Theta}_{\text{rx},i}^k \mathbf{P}_{\text{rx},i}\right), \; \mathbf{\Theta}_{\text{rx},i}^k := B^k \mathbf{\Theta}_{\text{rx},i} / B_{\text{tot}}, \tag{11}$$

and

$$\mathbf{P}_{\text{tx},i} := \sum_{k \in \mathbb{F}_K} \text{diag}\left(\mathbb{E}\left\{\mathbf{v}_i^k {\mathbf{v}_i^k}^H\right\}\right), \tag{12}$$

$$\mathbf{P}_{\text{rx},i} := \sum_{k \in \mathbb{F}_K} \text{diag}\left(\mathbb{E}\left\{\mathbf{u}_i^k {\mathbf{u}_i^k}^H\right\}\right), \tag{13}$$

where B^k and B_{tot} respectively represent the bandwidth associated with each subcarrier, and the total bandwidth of the system. In the above formulations, $\mathbf{\Theta}_{\text{tx},i} \in \mathbb{R}^{N_i \times N_i}$ ($\mathbf{\Theta}_{\text{rx},i} \in \mathbb{R}^{M_i \times M_i}$) is a diagonal matrix including distortion coefficients $\theta_{\text{tx},l}$ ($\theta_{\text{rx},l}$) for the corresponding chains[3]. Similarly, $\mathbf{P}_{\text{tx},i}$ ($\mathbf{P}_{\text{rx},i}$) is a diagonal matrix with each diagonal element representing the intended transmit (receive) signal power at the corresponding chain.

Via the application of (10)-(13) on (4) we conclude

$$\begin{aligned}
\mathbf{\Sigma}_i^k &:= \mathbb{E}\left\{\boldsymbol{\nu}_i^k {\boldsymbol{\nu}_i^k}^H\right\} \\
&= \sum_{j \in \mathbb{I}} \mathbf{H}_{ij}^k \mathbf{\Theta}_{\text{tx},j}^k \text{diag}\left(\sum_{l \in \mathbb{F}_K} \mathbf{V}_j^l {\mathbf{V}_j^l}^H\right) {\mathbf{H}_{ij}^k}^H \\
&+ \mathbf{\Theta}_{\text{rx},i}^k \text{diag}\left(\sum_{l \in \mathbb{F}_K}\left(\sigma_{i,l}^2 \mathbf{I}_{M_i} + \sum_{j \in \mathbb{I}} \mathbf{H}_{ij}^l \mathbf{V}_j^l {\mathbf{V}_j^l}^H {\mathbf{H}_{ij}^l}^H\right)\right) \\
&+ \sigma_{i,k}^2 \mathbf{I}_{M_i}, \;\; k \in \mathbb{F}_K,
\end{aligned} \tag{14}$$

where $\mathbf{\Sigma}_i^k \in \mathbb{C}^{M_i \times M_i}$ is the covariance of the received collective interference-plus-noise signal, and is obtained considering $0 \le \theta_{\text{rx},l} \ll 1$, $0 \le \theta_{\text{tx},l} \ll 1$, and hence ignoring the terms containing higher orders of the distortion coefficients in (14). As expected, the role of the distortion signals on the residual self-interference, including the resulting inter-subcarrier leakage, is evident from (14). This is the main goal of the remaining parts of this paper to incorporate and evaluate this impact on the design of the defined MC system.

[3] A simpler mathematical presentation can be obtained by assuming the same transceiver accuracy over all antennas. In such a case, the defined diagonal matrices can be replaced by a scalar.

III. Linear Transceiver Design for FD Multi-Carrier Communications

Via the application of $\mathbf{V}_i^k$ and $\mathbf{U}_i^k$, as the linear transmit precoder and receive filters, the mean-squared-error (MSE) matrix of the defined system is calculated as

$$\begin{aligned}
\mathbf{E}_i^k &:= \mathbb{E}\left\{\left(\tilde{\mathbf{s}}_i^k - \mathbf{s}_i^k\right)\left(\tilde{\mathbf{s}}_i^k - \mathbf{s}_i^k\right)^H\right\} + {\mathbf{U}_i^k}^H \mathbf{\Sigma}_i^k \mathbf{U}_i^k \\
&= \left({\mathbf{U}_i^k}^H \mathbf{H}_{ii}^k \mathbf{V}_i^k - \mathbf{I}_{d_i}\right)\left({\mathbf{U}_i^k}^H \mathbf{H}_{ii}^k \mathbf{V}_i^k - \mathbf{I}_{d_i}\right)^H,
\end{aligned} \tag{15}$$

where $\mathbf{\Sigma}_i^k$ is given in (14). In the following we propose two design strategies for the defined system, proposing an alternating QCP framework.

A. Weighted MSE minimization via Alternating QCP

An optimization problem for minimizing the weighted sum MSE is written as

$$\min_{\mathbb{V},\mathbb{U}} \sum_{i \in \mathbb{I}} \sum_{k \in \mathbb{F}_K} \text{tr}\left(\mathbf{S}_i^k \mathbf{E}_i^k\right) \tag{16a}$$

$$\text{s.t. } \text{tr}\left(\left(\mathbf{I}_{N_i} + \mathbf{\Theta}_{\text{tx},i}\right) \sum_{l \in \mathbb{F}_K} \mathbf{V}_i^l {\mathbf{V}_i^l}^H\right) \le P_i, \;\; \forall i \in \mathbb{I}, \tag{16b}$$

where $\mathbb{X} := \{\mathbf{X}_i^k, \; \forall i \in \mathbb{I}, \; \forall k \in \mathbb{F}_K\}$, with $\mathbb{X} \in \{\mathbb{U}, \mathbb{V}\}$, and (16b) represents the transmit power constraint. It is worth mentioning that the application of $\mathbf{S}_i^k \succ 0$, as a weight matrix associated with $\mathbf{E}_i^k$ is two-folded. Firstly, it may appear as a diagonal matrix, emphasizing the importance of different data streams and different users. Secondly, it can be applied as an auxiliary variable which later relates the defined weighted MSE minimization to a sum-rate maximization problem, see Subsection III-B.

It is observed that (16) is not a jointly convex problem. Nevertheless, it holds a QCP structure separately over the sets $\mathbb{V}$ and $\mathbb{U}$, in each case when other variables are fixed. In this regard, the objective (16a) can be decomposed over $\mathbb{U}$ for different communication directions, and for different subcarriers. The optimal minimum MSE (MMSE) receive filter can be hence calculated in closed form as

$$\mathbf{U}_{i,\text{mmse}}^k = \left(\mathbf{\Sigma}_i^k + \mathbf{H}_{ii}^k \mathbf{V}_i^k {\mathbf{V}_i^k}^H {\mathbf{H}_{ii}^k}^H\right)^{-1} \mathbf{H}_{ii}^k \mathbf{V}_i^k. \tag{17}$$

Nevertheless, the defined problem is coupled over $\mathbf{V}_i^k$, due to the impact of inter-carrier leakage, as well as the power constraint (16b). The Lagrangian function, corresponding to the optimization (16) over $\mathbb{V}$ is expressed as

$$\mathcal{L}\left(\mathbb{V}, \boldsymbol{\iota}\right) := \sum_{i \in \mathbb{I}}\left(\iota_i \mathcal{P}_i\left(\mathbb{V}\right) + \sum_{k \in \mathbb{F}_K} \text{tr}\left(\mathbf{S}_i^k \mathbf{E}_i^k\right)\right), \tag{18}$$

$$\mathcal{P}_i\left(\mathbb{V}\right) := -P_i + \text{tr}\left(\left(\mathbf{I}_{N_i} + \mathbf{\Theta}_{\text{tx},i}\right) \sum_{l \in \mathbb{F}_K} \mathbf{V}_i^l {\mathbf{V}_i^l}^H\right), \tag{19}$$

where $\boldsymbol{\iota} := \{\iota_i, \; i \in \mathbb{I}\}$ is the set of dual variables. The dual function, corresponding to the above Lagrangian is defined as $\mathcal{F}\left(\boldsymbol{\iota}\right) := \min_{\mathbb{V}} \; \mathcal{L}\left(\mathbb{V}, \boldsymbol{\iota}\right)$, where the optimal

$\mathbf{V}_i^k$ is obtained as

$$\mathbf{V}_i^{k\star}=\Big(\mathbf{J}_i^k+\iota_i\left(\mathbf{I}_{N_i}+\mathbf{\Theta}_{\text{tx},i}\right)+\mathbf{H}_{ii}^{k\,H}\mathbf{U}_i^k\mathbf{S}_i^k\mathbf{U}_i^{k\,H}\mathbf{H}_{ii}^k\Big)^{-1}\times\mathbf{H}_{ii}^{k\,H}\mathbf{U}_i^k\mathbf{S}_i^k, \qquad (20)$$

and

$$\mathbf{J}_i^k:=\sum_{l\in\mathbb{F}_K}\sum_{j\in\mathbb{I}}\left(\mathbf{H}_{ji}^{k\,H}\text{diag}\left(\mathbf{U}_j^l\mathbf{S}_j^l\mathbf{U}_j^{l\,H}\mathbf{\Theta}_{\text{rx},j}^l\right)\mathbf{H}_{ji}^k+\text{diag}\left(\mathbf{H}_{ji}^{l\,H}\mathbf{U}_j^l\mathbf{S}_j^l\mathbf{U}_j^{l\,H}\mathbf{H}_{ji}^l\mathbf{\Theta}_{\text{tx},i}^l\right)\right). \qquad (21)$$

Due to the convexity of the original problem (16) over $\mathbb{V}$, the defined dual problem is a concave function over $\boldsymbol{\iota}$, with $\mathcal{P}_i(\mathbb{V})$ as a subgradient, see [13, Eq. (6.1)]. As a result, the optimal $\boldsymbol{\iota}$ is obtained from the maximization

$$\boldsymbol{\iota}^\star=\underset{\boldsymbol{\iota}\geq 0}{\text{argmax}}\;\mathcal{F}\left(\boldsymbol{\iota}\right), \qquad (22)$$

following a standard subgradient update, [13, Subsection 6.3.1].

Utilizing the proposed optimization framework, the alternating optimization over $\mathbb{V}$ and $\mathbb{U}$ is continued until a stable point is obtained. Note that due to the monotonic decrease of the objective in each step, and the fact that (16a) is non-negative and hence bounded from below, the defined procedure leads to a necessary convergence.

B. WMMSE design for sum rate maximization

Via the utilization of $\mathbf{V}_i^k$ as the transmit precoders, the resulting communication rate for the k-th subcarrier and for the i-th communication direction is written as

$$I_i^k=B^k\log_2\left|\mathbf{I}_{d_i}+\mathbf{V}_i^{k\,H}\mathbf{H}_{ii}^{k\,H}\left(\mathbf{\Sigma}_i^k\right)^{-1}\mathbf{H}_{ii}^k\mathbf{V}_i^k\right|, \qquad (23)$$

where B^k and $\mathbf{\Sigma}_i^k$ are defined in (10) and (14). The sum rate maximization problem can be written as

$$\max_{\mathbb{V}}\quad\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{F}_K}I_i^k,\;\;\text{s.t.}\;\;(16\text{b}). \qquad (24)$$

The optimization problem (24) is intractable in the current form. In the following we propose an iterative optimization solution, following the WMMSE method [11]. Via the application of the MMSE receive filters from (17), the resulting MSE matrix is obtained as

$$\mathbf{E}_{i,\text{mmse}}^k=\left(\mathbf{I}_{d_i}+\mathbf{V}_i^{k\,H}\mathbf{H}_{ii}^{k\,H}\left(\mathbf{\Sigma}_i^k\right)^{-1}\mathbf{H}_{ii}^k\mathbf{V}_i^k\right)^{-1}. \qquad (25)$$

By recalling (23), and upon utilization of $\mathbf{U}_{i,\text{mmse}}^k$, we observe the following useful connection to the rate function $I_i^k=-B^k\log_2\left|\mathbf{E}_{i,\text{mmse}}^k\right|$ which facilitates the decomposition of rate function via the following lemma, see also [11, Eq. (9)].

Lemma III.1. *Let $\mathbf{E}\in\mathbb{C}^{d\times d}$ be a positive definite matrix. The maximization of the term $-log\left|\mathbf{E}\right|$ is equivalent to the maximization*

$$\max_{\mathbf{E},\mathbf{S}}-tr\left(\mathbf{SE}\right)+log\left|\mathbf{S}\right|+d, \qquad (26)$$

where $\mathbf{S}\in\mathbb{C}^{d\times d}$ is a positive definite matrix, and we have $\mathbf{S}=\mathbf{E}^{-1}$ at the optimality.

Proof: See [14, Lemma 2]. ∎

The original optimization problem over $\mathbb{V}$ can be hence equivalently formulated as

$$\max_{\mathbb{V},\mathbb{U},\mathbb{S}}\sum_{k\in\mathbb{F}_K}B^k\sum_{i\in\mathbb{I}}\Big(\log\left|\mathbf{S}_i^k\right|+d_i-\text{tr}\left(\mathbf{S}_i^k\mathbf{E}_i^k\right)\Big)\;\text{s.t. (16b)}, \qquad (27)$$

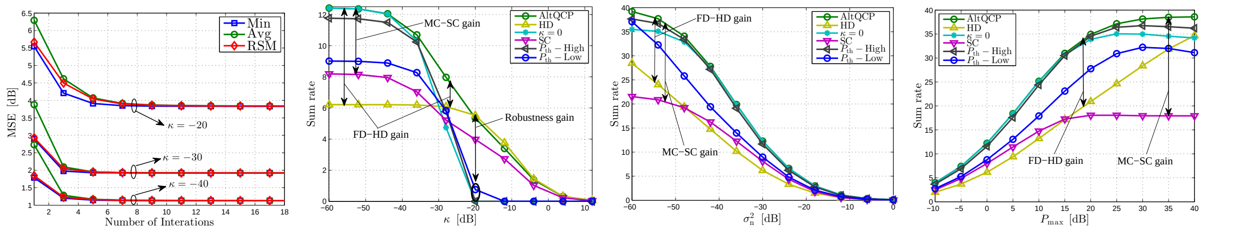
where $\mathbb{S}:=\{\mathbf{S}_i^k\succ 0,\;\forall i\in\mathbb{I},\;\forall k\in\mathbb{F}_K\}$. The obtained optimization problem (27) is not a jointly convex problem. Nevertheless, it is a QCP over $\mathbb{V}$ when other variables are fixed, and can be obtained with a similar structure as for (16). Moreover, the optimization over $\mathbb{U}$ and $\mathbb{S}$ is obtained from (17), and $\mathbf{S}_i^k=\mathbf{E}_i^{k^{-1}}$. This facilitates an alternating optimization where in each step the corresponding problem is solved to optimality. The defined alternating optimization steps results in a necessary convergence due to the monotonic increase of the objective in each step, and the fact that the eventual system sum rate is bounded from above.

IV. Simulation Results

In this section we evaluate the behavior of the studied FD MC system via numerical simulations. In particular, we evaluate the proposed AltQCP design in Section III for different levels of noise, transceiver accuracy, and transmit power. Communication channels $\mathbf{H}_{ii}^k$ follow an uncorrelated Rayleigh flat fading model with variance ρ. For the self-interference channel we follow the characterization reported in [15]. In this respect we have $\mathbf{H}_{ij}\sim\mathcal{CN}\left(\sqrt{\frac{\rho_{\text{si}}K_R}{1+K_R}}\mathbf{H}_0,\frac{\rho_{\text{si}}}{1+K_R}\mathbf{I}_{M_i}\otimes\mathbf{I}_{N_j}\right)$ where ρ_{si} represents the self-interference channel strength, $\mathbf{H}_0$ is a deterministic term,[4] and K_R is the Rician coefficient. The overall system performance is then averaged over 100 channel realizations. Unless otherwise is stated, the following values are used to define our default setup: $K=4$, $K_R=10$, $M:=M_i=N_j=2$, $\rho=-20$ dB, $\rho_{\text{si}}=1$, $\sigma_{\text{n}}^2:=\sigma_{i,k}^2=-30$ dB, $P_{\max}=P_i=1$, $d_i=2$, $\kappa=-50$ dB where $\mathbf{\Theta}_{\text{rx},i}^k=\kappa\mathbf{I}_{M_i}$ and $\mathbf{\Theta}_{\text{tx},i}^k=\kappa\mathbf{I}_{N_i}$, $\forall i,j\in\mathbb{I},\;k\in\mathbb{F}_K$. The implemented comparison benchmarks are defined in deatil in [12, Subsection VI.B].

In Fig. 2 (a) the average convergence behavior of the AltQCP algorithm is depicted. Note that due to the alternating solution structure, the convergence behavior is meaningful both for validating the algorithm function, as well as an indicator for the required computational complexity. In this respect, the objective of the AltQCP algorithm is depicted in Fig. 2 (a) by employing the right singular initialization (RSM), as well as the average (AVG) and the minimum value of objective (Min) among 20 number of random initializations. It is observed that the used right singular matrix initialization outperforms the average performance of random

[4] For simplicity, we choose $\mathbf{H}_0$ as a matrix of all-1 elements.



(a) Avg. convergence of AltQCP (b) Sum rate vs. κ (c) Sum rate vs. noise variance (d) Sum rate vs. max. trans. power

Figure 2. Average sum rate for different system conditions. The gain of a distortion-aware design is apparent for a high SNR, or a high distortion region.

initialization, and operates close to the best case, particularly as κ is small. Moreover, independent from the used initialization strategy, the algorithm converges in approximately 15 iterations.

In Fig. 2 (b) the resulting system sum rate is evaluated for different values of transceiver accuracy. It is observed that a higher κ results in a smaller sum rate. Moreover, the obtained gains via the application of the defined MC design in comparison to the designs with frequency-flat assumption, and via the application of FD setup in comparison to HD setup, are evident for a system with accurate hardware operation. Conversely, it is observed that a design with consideration of hardware impairments is essential as κ increases.

In Fig. 2 (c) and (d) the opposite impact of noise level, and the maximum transmit power are observed on the system sum rate. It is observed that the system sum rate increases as noise level decreases, or as the maximum transmit power increases. In both cases, the gain of AltQCP method, in comparison to the methods which ignore the impact of hardware distortions are apparant for a high SNR region, i.e., as transmit power increases or as noise level decreases.

V. Conclusion

The application of bi-directional FD communication presents a potential for improving the spectral efficiency. Nevertheless, such systems are limited due to the impact of residual self-interference. This issue becomes more crucial in a multi-carrier system, where the residual self-interference spreads over multiple carriers, due to the impact of hardware distortion. In this work we have presented a modeling and design framework for a FD MIMO OFDM system, taking into account the impact of hardware distortions leading to inter-carrier leakage. It is observed that the application of a distortion-aware design is essential, as transceiver accuracy degrades. Moreover, a significant gain is observed compared to the usual single-carrier design, for a channel with a high frequency selectivity.

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