ABSTRACT

Leveraging a relay for communication is a promising approach for improving throughput, coverage, and energy efficiency in wireless networks. If the destination device is nomadic, transmitting through a relay that is always at the same location is usually suboptimal in terms of maximizing the benefits of relaying. A mobile relay that is capable of positioning itself at different locations opens the possibility for dynamic optimization of the path quality between the source and the nomadic destination. How to optimally position the mobile relay in order to maximize the path quality, however, remains a challenging task. Under the assumption that the physical location information of the devices are either known or can be estimated, we propose a mechanism for positioning of the mobile relay with the aim of maximizing the Signal-to-Noise Ratio (SNR) between the source and the destination. The proposed mechanism takes into account the practically unavoidable inaccuracies of estimated locations, as well as the propagation characteristics of the served environment. Using WiFi as an example technology, we experimentally evaluate the proposed mechanism in a complex indoor environment with the support of a specifically designed testbed infrastructure. For relatively small localization errors, our results show less than 4 dB average difference between the measured SNR at optimal locations of the mobile relay vs. the SNR at locations yielded by our positioning mechanism. Our results also illustrate how the quality of the paths created by the proposed positioning mechanism degrades in the face of increasing localization errors.

1 INTRODUCTION

Leveraging a relaying device to transmit information from a source to a destination is shown to enhance throughput, coverage, and energy efficiency of wireless communication [27]. If the destination device is nomadic, however, a fixed relaying device usually significantly constrains the achievable benefits. This fact motivates the use of mobile relaying in which the relaying device is capable of positioning itself in a served environment in a way that optimizes the path quality between the end-devices [10]. However, it is still unclear how to find an optimal location of the mobile relay for such a scenario.

In this paper, we propose a mechanism for positioning of the mobile relay. Under the assumption that an environment is serviced by a localization service, we ground the decision about the optimal relay location on physical location information of the source and the destination. The proposed mechanism accounts for the errors in location information provided by the localization service, as well as for the propagation characteristics (i.e., path-loss and large-scale fading) of the served environment.

Due to its practical relevance, we assume a scenario in which the source device is unmovable with perfect information about its location. We further assume that the destination device is nomadic and its location information can be estimated by a localization service, but this information is burdened with a certain localization error. Moreover, we assume the availability of a mobile relaying device, where its location in the served environment can also be estimated with a certain level of accuracy. Since the contribution of this work is not the relaying scheme itself, as an example, in our scenario we assume a simple repeater-like relay, i.e., there is no physical-layer combining of the original and relayed transmissions.
However, in combination with our location-based mechanism for positioning of the mobile relay, more advanced relaying schemes are also applicable. Furthermore, the mobile relay is considered to be a part of the wireless infrastructure, hence it is a trusted entity. Therefore, the usual security and privacy related pitfalls of relaying in wireless networks [22] do not apply in this scenario, in contrast to opportunistic relaying through generally untrusted mobile devices, e.g., [25].

For the assumed scenario, we derive closed form equations for the Euclidean distance between two devices for the case when the location information of both devices are burdened with errors, as well as for the case location information of one device is perfectly accurate. By leveraging a complex model for the radio propagation and the derived equations for the Euclidean distances, we further derive closed form equations for the expected SNR between two devices whose location information are known with a certain level of accuracies and for the case the location information of one device is perfectly accurate. Finding the optimal location of the mobile relay in the proposed mechanism is then based on leveraging the derived equations for the expected SNR at potential locations of the mobile relay in order to obtain the relay location that maximizes the expected SNR between the source and the destination. Note that our mechanism is not limited to the assumed scenario, but can be utilized more broadly, e.g., in case of multi-hop relaying, in case the location information of the source is burdened with errors, in case the location information of the destination is perfectly accurate or in case the location information of both the source and the destination devices are burdened with errors. Furthermore, the proposed mechanism works for both downlink and uplink transmission paths.

We evaluate the proposed mechanism in a complex indoor environment by leveraging a flexible testbed infrastructure for supporting such experimentation. We do in the 2.4 GHz Industrial, Scientific and Medical (ISM) frequency band using WiFi as an example technology, although the proposed mechanism can be applied for various other technologies for wireless communication. To demonstrate the baseline of the achievable performance of the proposed mechanism, we first evaluate the mechanism under relatively small localization errors, i.e., the smallest ones obtainable in our testbed environment. In case of relatively small per-coordinate localization errors of 10 cm, our results demonstrate less than 4 dB in average difference between the measured SNRs of optimal transmission paths and the measured SNRs of transmission paths through locations yielded by the proposed mechanism. Our results also characterize the loss of communication path quality as a function of increasing localization errors. Finally, we characterize the SNR enhancements due to mobile relaying supported by the proposed mechanism in comparison to direct transmission between end-devices.

2 RELATED WORK

Location information has a potential for improving the performance of wireless networks, as discussed e.g., in [7, 9]. Location information can be beneficial as an input to a decision-making mechanism in relaying, i.e., in the decision if and consequently which opportunistic relay should be utilized for transmitting information from the source to the destination, as discussed in [26, 29]. However, the aforementioned contributions assume that wireless propagation can be characterized with path-loss only, as well as that a perfectly accurate and instantaneous estimation of location information of the devices can be performed. Both assumptions are unrealistic in practice, which has already been recognized in the community. Hence, the authors in [25] consider the influence of information delay on location-based relaying. Furthermore, the influence of path-loss inaccuracies on location-based relaying is considered in [23]. The most similar to our contribution is the one made in [24], where the authors consider the joint influence of erroneous and delayed location information for optimizing location-based relaying.

In contrast to these contributions that are focused on deciding if a relay should be used and consequently on selecting the optimal relay, we focus on finding an optimal location of the mobile relaying device. Furthermore, we focus on environments with complex propagation in which propagation models based on path-loss only are not sufficient for accurate characterization of the propagation. Hence, in this work we leverage a more complex and presumably more accurate multi-wall model for indoor radio propagation [4]. Moreover, we do not assume high mobility of the destination device because of the fact that for scenarios with high mobility very frequent changes in the relay selection or location are needed. Those frequent changes introduce a large signaling overhead and essentially reduce or in some cases even remove the benefits of relaying, as discussed in [14]. Therefore, we assume location-based relaying in scenarios where the destination is a nomadic device. We believe that relaying is a suitable option for improving the path between the source and the destination only in scenarios with limited mobility. In the considered scenarios with slowly changing mobility, the practically unavoidable latency resulting from generating and reporting location information of the devices participating in the communication (as discussed in e.g., [17]) does not play an important role. Hence, in contrast to some of the aforementioned contributions, we do not consider delayed location information to be of significant importance. Finally, we assume that location information of both the destination and of the relay are burdened with errors, while in [24] the authors assumed only one of them is burdened with localization errors. Due to this fundamental differences in assumptions, a comparative evaluation of the mechanism proposed in [24] and our mechanism is not possible, hence in the evaluation we focus only at the performance benchmarking of our mechanism for positioning of the mobile relay.

3 RELAYING SCENARIO

The envisioned relaying scenario is presented in Figure 1. As depicted in the figure, the aim of the source is to transmit information to the destination, which is achieved through the use of a mobile relay. Since the source is static, perfect location information of the source is assumed to be known to the network infrastructure. The environment is assumed to be serviced by a localization service deployed in the infrastructure. The localization service is able to estimate locations of the mobile relay and of the destination with a certain level of accuracy.

The goal of the mechanism for positioning of the mobile relay is to decide at which location among multiple potential locations to position the relay so that the path quality between the source and
the destination is maximized. We specify the path quality between
the source and the destination by leveraging the Policy 1 from [2].
However, instead of taking the instantaneous Channel-State In-
formation (CSI), we take the expected SNR between the source
and the relay at a given location (i.e., \( \text{SNR}_{S,R} \)) and the ex-
pected SNR between the relay at a given location and the destination
(i.e., \( \text{SNR}_{R,D} \)). This is done because in the assumed scenario the relay
remains at the same location during a communication session be-
tween the source and the destination. The path quality for a certain
location \( i, i = 1, ..., N \), is then given by:

\[
\text{SNR}_i = \min(\text{SNR}_{S,R}, \text{SNR}_{R,D}).
\]  

(1)

The optimal relay location among a set of \( N \) candidate locations,
denoted as \( l^* \), is selected according to the following criterion:

\[
l^* = \arg \max_{i=1, \ldots, N} \{ \text{SNR}_i \}.
\]  

(2)

![Figure 1: Overview of the envisioned scenario](image)

Location information of all nomadic devices in the environment
are specified with their x and y coordinates in a 2-Dimensional (2D)
coordinate system. A per-coordinate error of location information
of a device is modeled as a zero-mean normally distributed random
variable. Modeling per-coordinate localization errors with a Ga-
ussian distribution is a well established procedure, with some examples
being [12, 30, 33]. All per-coordinate localization errors have the
same standard deviation \( \sigma \). This assumption is made because loca-
tion information of the devices in a single environment are usually
provided by the same localization service, hence statistically the
same localization errors should be expected for all of them.

4 LOCATION-BASED MECHANISM FOR
POSITIONING OF A MOBILE RELAY

4.1 Euclidean Distance Between Devices

In the following, we derive expressions for the Euclidean distance
between two devices in case one device’s location is erroneous
and the other’s location is perfectly accurate, as well as in case both
devices’ locations are erroneous.

4.1.1 Euclidean Distance Between Source and Relay. Let us
assume that the correct location information of the source is \((x_S, y_S)\).
Furthermore, let us assume that the potential location of the mobile
relay provided by the localization service is \((X_R, Y_R)\), while its cor-
rect location is \((\mu_{XR}, \mu_{YY})\). We assume that each 2D coordinate
of the location information of the relay provided by the localization
service is a normally distributed random variable specified by its
mean value \( \mu \) and standard deviation \( \sigma \):

\[
X_R \sim N(\mu_{XR}, \sigma^2), \quad Y_R \sim N(\mu_{YY}, \sigma^2).
\]  

(3)

Euclidean distance between the source and the mobile relay is
therefore estimated by the following equation:

\[
d = \sqrt{(X_R - x_S)^2 + (Y_R - y_S)^2}.
\]  

(4)

**Proposition 4.1.** If the location of the mobile relay is estimated
according to Equation 3 and the correct location of the source is known,
The Euclidean distance between the source and the relay is a random
variable distributed according to Rice distribution \( \text{Rice}(\lambda, \sigma) \)
with parameters \( \sigma \) and \( \lambda \) with the Cumulative Distribution Function (CDF)
given as follows:

\[
F_{S,R}(d) = \mathcal{P}(d \leq \delta) = 1 - Q_1\left(\frac{\delta}{\sigma}, \frac{\lambda}{\sigma}\right),
\]  

(5)

where:

\[
\lambda = (X_R - x_S)^2 + (Y_R - y_S)^2.
\]  

(6)

Furthermore, \( Q_1 \) is the Marcum 1 function given by [3]:

\[
Q_1(a, b) = \int_{b}^{\infty} x \exp\left(-\frac{x^2 + a^2}{2}\right) I_0(ax) \, dx,
\]  

(7)

for \( a, b \geq 0 \), where \( I_0 \) is a well-known modified Bessel function
of the first kind.

**Proof.** The proof is given in Appendix 6. \( \square \)

4.1.2 Euclidean Distance Between Relay and Destination. Sup-
pose that the potential location information of the relay and of the
location information of the destination are provided by the localiza-
tion service as \((X_R, Y_R)\) and \((X_D, Y_D)\), while the correct locations are
\((\mu_{XR}, \mu_{YY})\) and \((\mu_{XD}, \mu_{YD})\). Same as before, we assume that each
coordinate of the location information provided by the localization
service is a normally distributed random variable specified by its
mean value \( \mu \) and standard deviation \( \sigma \). It follows:

\[
X_R \sim N(\mu_{XR}, \sigma^2), \quad Y_R \sim N(\mu_{YY}, \sigma^2)
\]  

(8)

\[
X_D \sim N(\mu_{XD}, \sigma^2), \quad Y_D \sim N(\mu_{YD}, \sigma^2).
\]  

(9)

Euclidean distance between the relay and the destination is given
as:

\[
d = \sqrt{(X_R - X_D)^2 + (Y_R - Y_D)^2}.
\]  

(10)

**Proposition 4.2.** If the locations of both the relay and the desti-
nation are estimated according to Equations 8 and 9, the Euclidean
distance between the devices is a random variable distributed accord-
ing to a Rice distribution \( d \sim \text{Rice}(\nu, \sqrt{2}\sigma) \) with the CDF given as follows:
4.2 Propagation Modeling

For modeling wireless propagation in the served environment we use the COST 231 multi-wall model [4]. In the model, path-loss and large-scale fading (shadowing) are considered. For the envisioned scenario, we believe that it is important to model only the long-term behavior of the wireless channel. Small-scale (multi-path) fading could, due to destructive interference, create deep fades that will affect the quality of a communication path, as discussed in e.g. [18]. However, due to small-scale mobility of the destination device (e.g. the device being held by a person) and due to the fact that we are predominantly considering complex indoor environments in which there is a constant change in small-scale fading (e.g. people mobility, doors opening/closing, etc.), these deep fades are expected to have a short-time span. Additionally, in praxis there is a certain time required by the mobile relay to position itself to a location yielded by the proposed mechanism. Since we want to position a relay at a long-term optimal location, we do not model small-scale fading, which is a well-established procedure in location-based mechanisms for the selection of opportunistic relays, e.g., [23, 24].

The applicability of the used model for complex indoor environments has been demonstrated e.g. in [5, 15]. In the model, the signal attenuation $L(d)$ in dB is given by:

$$L(d) = l_c + 10 \gamma \log(d) + M_w.$$  

$l_c$ is a constant value related to the model fitting procedure discussed below. The attenuation $L(d)$ is dependent on the distance $d$ from the transmitting device, path-loss coefficient $\gamma$ of the environment, and the total attenuation from all walls $M_w$ in the direct path between the devices. Each wall has its attenuation contribution $l_w$, hence for the number of walls $N_w$ in the direct path between the devices total attenuation from all walls $M_w$ is given as:

$$M_w = \sum_{i=1}^{N_w} l_w.$$  

Therefore, the total attenuation in Watt is given by:

$$\ell(d) = 10^{\frac{L(d)}{10}} = d^{\gamma} 10^{\frac{l_c + \gamma N_w l_w}{10}} = \kappa d^{\gamma}.  

$$

The SNR between two devices is then given by:

$$\text{SNR} = \frac{P_{rx}}{N \kappa d^{\gamma}}.$$  

where $N$ is the noise power. Note that the SNR between the transmitter and the receiver is affected by the random parameter $d$. To use this SNR value for positioning of the relay later, one option is to use the expected SNR. However, since $d$ is Rician, $d^2$ is noncentral Chi squared distribution and the expected value of $\frac{1}{d^2}$ does not exist for $\gamma \geq 2$ [28, p. 345]. Instead, we assume that SNR is measured in dB. Therefore, in the following we derive expressions for the expected value of logarithm of SNR between two devices in case both devices’ location information are burdened with errors, as well as for the case one device’s location information is perfectly accurate, while the other’s location information is burdened with errors.

4.2.1 Expected SNR Between Source and Relay.

**Proposition 4.3. Assuming the Euclidean distance between the source and the relay is a Rician, i.e., a non-central Chi distributed random variable, as specified with Equation 5, the expected logarithm of SNR between the source and the relay at a certain location is given as follows:**

$$\text{SNR}_{S,R} = \frac{\ln \left( \frac{P_{tx}}{N_k \kappa \sqrt{2} \sigma^2} \right)}{\gamma^2} - \ln \left( \frac{\lambda^2}{2 \sigma^2} \right).$$  

where the function $g(\cdot)$ is defined as:

$$g(\xi) = \exp \left( \int_{-\gamma}^{\gamma} \frac{\exp(-t)}{t} \, dt \right).$$  

**Proof.** See Appendix 6.  

4.2.2 Expected SNR Between Relay and Destination.

**Proposition 4.4. Assuming the Euclidean distance between the relay at a certain location and the destination is Rice distributed random variable, as given with Equation 11, the expected SNR between the relay and the destination is given as follows:**

$$\text{SNR}_{R,D} = \frac{\ln \left( \frac{P_{tx}}{N_k \kappa (\sqrt{2} \sigma)^2} \right)}{\gamma^2} - \ln \left( \frac{\lambda^2}{2 \sigma^2} g \left( \frac{\lambda^2}{\sigma^2} \right) \right).$$  

where the function $g(\cdot)$ is given by Equation 18.

**Proof.** See Appendix 6.  

4.3 Discussion

In the two hop transmission through a relay, the expected SNR is a minimum one among the expected SNR between the source and the relay (i.e., SNR$_{S,R}$) and the expected SNR between the relay and the destination (i.e., SNR$_{R,D}$), as specified by Equation 1. The proposed mechanism yields a location of the relay by comparing the expected SNRs of transmissions through relays at different potential locations in the served environment, as defined by Equation 2. The location yielded by the mechanism is the one that maximizes the expected SNR. The relay can then be instructed to position itself at that location. The potential locations of the relay can be defined in a grid-like fashion or in any other constellation. This decision is currently left to the network administrator. Note that the decision if a direct or a relayed transmission should take place can also be based on the expected SNR by leveraging modified Equation 17, where the location of the destination should be used instead of the location of the relay.
The proposed mechanism for positioning of a mobile relay leverages the multi-wall model for modeling large-scale fading and path-loss. The multi-wall model assumes that a floor plan of the served environment is used for determining the number of walls in the direct path between the devices. The requirement for the availability of a floor-plan does not pose a challenge for the deployment of the proposed mechanism. The floor-plan will usually be available because the environment is serviced by a localization service and such a service usually requires a floor plan of the served environment.

Furthermore, the multi-wall model requires an estimation of model parameters, i.e., the wall attenuation $I_w$, the path-loss coefficient $\gamma$, and the constant $I_c$. To position the relay device at different locations in the environment, a setup, with the evaluation results reported in [6] (Section 6.1). The evaluation procedure involved use of Equations 17 and 19 for calculating the multi-wall parameters. The procedure yielded wall attenuation $I_w$ of 4.72 dBm, path-loss coefficient $\gamma$ of 1.74, and constant $I_c$ of 46.73 dBm. The second collection of measurements was used for the evaluation of the proposed mechanism. The evaluation procedure involved use of Equations 17 and 19 for calculating the expected SNRs between the source and the mobile relay.

In the evaluation, we aimed at examining the difference between the measured SNRs at an optimal among the potential locations and the measured SNRs at locations yielded by the proposed mechanism for positioning of the mobile relay. We performed our experimental evaluation using WiFi as an example technology, although the proposed mechanism can conceptually be used with other technologies for wireless communications. Note that, although that would have been a natural first step in the evaluation, we did not evaluate the accuracy of modeling of SNR values using the COST 231 multi-wall model. This evaluation has been carried out previously for the same setup, with the evaluation results reported in [6] (Section 6.1).

For our evaluation we selected the TWIST testbed environment, with a footprint given in Figure 2. The TWIST testbed is deployed in an office environment with typical usage patterns. At the same time, the testbed features a highly controllable infrastructure for supporting various experimental scenarios with Radio Frequency (RF) technologies [16]. Using an autonomous mobility platform, which is a part of the experimentation infrastructure, we were able to position the relay device at different locations in the environment. The locations were defined in a grid-like fashion, as indicated with red dots in the figure (with some small deviations due to obstacles in the environment). The per-coordinate accuracy of positioning of the autonomous mobility platform in the testbed environment is in average roughly 10 cm [16].

We used six WiFi Access Points (APs) in our evaluation, with their locations as indicated in Figure 2. The locations of the APs were presurveyed using a sophisticated Tachymeter Typ TS 06 Plus (Leica) device and were, hence, known with a very high level of accuracy, i.e., with the average localization errors of less than 2 mm [16]. In the evaluation, each AP was in turn used as the source of information, while all the others were used as destinations. To support the previously discussed scenario, we added a certain level of localization inaccuracies to the location information of APs that were used as destinations. These inaccuracies were introduced by adding a number drawn from a zero-mean Gaussian distribution with a given $\sigma$ to the perfect location information obtained through presurveying. As the result, the location of the source was in each instance of the experiment perfectly accurate, while the location of the destination and the potential locations of the mobile relay were burdened with inaccuracies characterized by the parameter $\sigma$.

At each measurement location we performed 40 scans for WiFi beacon packets from the six APs, followed by extracting Received Signal Strength Indicator (RSSI) values from the obtained beacon packets. 40 scans were taken to reduce the temporal variability due to small-scale fading from the measurements. The APs were TL-WDR4300 wireless routers operating in the 2.4 GHz ISM frequency band (channel 11) with their transmission powers set to 20 dBm (100 mW). The used routers feature 3 transmitting antennas, thus the spatial variability due to small-scale fading is reduced in the measurements. The receiver of beacon packets transmitted by the APs was a MacBook Pro notebook with the AirPort Extreme network interface card. The experiments were performed during a weekend, when no people were present. Furthermore, in the TWIST testbed all neighboring uncontrollable WiFi nodes are operating in the 5 GHz ISM frequency band [11], thus the interference was minimized during the experimentation.

Two measurement collections were performed at separate days. The first one was used for the least square fitting procedure (Equation 20), i.e., for calculating the multi-wall parameters. The procedure yielded wall attenuation $I_w$ of 4.72 dBm, path-loss coefficient $\gamma$ of 1.74, and constant $I_c$ of 46.73 dBm. The locations of the APs and evaluation points in the testbed environment are shown in Figure 2.
We selected the range of localization errors \( \sigma \) were in turn used as the destination. The procedure yielded 15 (Figure 2) provides a high enough safety margin for the “dri/ft” in distance between two potential and neighboring locations is 2.4 m. The expected per-axis location error \( \sigma \) caused by the localization errors. For example, let us assume that between the potential locations of the mobile relay of roughly 2.4 m localization errors in the performance of the proposed mechanism for the per-axis roughly 4 to around 8 dB. In addition, there is a very little change of communication path quality is doubled, i.e., it increases from 10 cm to 1 m in localization errors \( \sigma \). However, if the primary design goals are throughput and coverage enhancements, then the mobile relaying provides benefits in comparison to direct communication between the source and the destination. It is also possible that the relay is not able to reach the desired location, for example due to an obstacle. Moreover, due to relaying an additional delay, as well as an increase in jittering can be expected [13]. Finally, the mobile relaying imposes an added complexity and it increases the utilization of radio resources [32]. Hence, the potential user could question the initiative for using such a communication paradigm, in comparison to directly transmitting the information from the source to the destination.

However, if the primary design goals are throughput and coverage enhancements, then the mobile relaying provides benefits in comparison to direct communication between the source and the destination. To characterize the benefits of the proposed mechanism in terms of throughput enhancements, using the previous setup we evaluate the difference between measured averaged SNR of the path through the mobile relay at a location yielded by the mechanism and the measured averaged SNR of the direct path between the source and the destination. We do that for different per-axis localization errors \( \sigma \). The results are depicted in Figure 4. As visible from the figure, for the relatively small per-axis localization errors \( \sigma \) of 10 cm, the SNR of a path through the mobile relay is

![Figure 3: Loss of path quality in the proposed mechanism due to the selection of relay locations that do not maximize the measured averaged SNR as a function of increased per-axis localization errors \( \sigma \)](image)
in average more than 9 dB higher than the SNR of a direct path between the source and the destination. As the expected localization errors increase, this difference becomes smaller, since the locations yielded by the proposed mechanism become less optimal in terms of SNR maximization. Example-wise, for the per-axis localization error increase from 10 cm to 1 m, the averaged difference between the SNR of a relayed and of a direct path reduces from roughly 10 to around 5 dB. Note that, as depicted in the figure, this difference can be a negative number. This can happen for multiple reasons. First, the proposed mechanism for positioning of the mobile relay can yield a non-optimal location, hence the achieved SNR through the relayed path can be smaller than the SNR of the direct path. Second, it can happen that the distance between the source and the destination is smaller than the distance between either the source or the destination and the potential location of the mobile relay. Third, if all paths through a relay are heavily obstructed and only the direct provides a Line of Sight (LoS) connectivity (e.g., in a hallway), it can happen that the SNR of the direct path is larger than the SNR of any path through the relay.

Our evaluation results demonstrate a small difference between the SNR of optimal paths and the SNR of paths through locations yielded by the proposed mechanism. We further demonstrate the benefits of relaying using the proposed mechanism in terms of the SNR enhancements, in comparison to direct communication between the source and the destination. For the current commercial off-the-shelf state-of-the-art localization approaches with expected localization errors of roughly 0.5 m per-axis [19], the mechanism can already provide very good performance in terms of selecting a close-to-optimal location where the relay should be positioned and in terms of enhancements in the SNR, in comparison to direct transmission between the end-devices.

Currently, the specification of potential locations of the mobile relay is left to the network administrators, which could result in either over-provisioning on the number of locations or in failing to specify locations that could maximize the benefits of relaying. Obviously, this specification should define potential relay locations so that the whole environment is considered for positioning of the relay. However, the “drifts” in potential locations of the mobile relay should not occur. To avoid these drifts, we believe the expected localization accuracy in the served environment should be taken into account in the specification of potential relay locations. We speculate that the per-axis distance between neighboring potential locations of the mobile relay should be three times larger than the per-axis localization error \( \sigma \), so that the loss of path quality is contributed only to the inaccuracies of the propagation model. Future work will be oriented toward evaluating this hypothesis by examining an interplay between the density of potential relay locations, expected localization accuracy, and the performance of the proposed mechanism. We hope these future efforts will yield a methodology for optimal specification of potential locations of the mobile relay. Moreover, while in this work we considered a scenario with one relay and one destination, future work will also consider scenarios in which one or multiple relays have to be positioned in a way that optimizes path qualities for multiple destinations.

### APPENDIX

#### Proof of Proposition 4.1

**Proof.** Note that \((X_R - x_S) \sim \mathcal{N}(\mu_{xR} - x_S, \sigma^2)\) and \(Y_R - y_S \sim \mathcal{N}(\mu_{yR} - y_S, \sigma^2)\). Therefore the Euclidean distance \(d\) between the source and the relay follows:

\[
d = \sqrt{(X_R - x_S)^2 + (Y_R - y_S)^2} \sim \text{Rice}(\lambda, \sigma),
\]

where \(\lambda\) is given by Equation 6.

Note that \(\frac{d}{\sigma}\) has unit variance and is indeed a noncentral chi distribution with degrees of freedom equal to 2 and the noncentrality parameter is given by \(\frac{\lambda}{\sigma}\), denoted by \(\chi(2, \frac{\lambda}{\sigma})\).

#### Proof of Proposition 4.2

**Proof.** \(X_R - X_D\) is difference of two independent Gaussian random variable which is itself a Gaussian random variable with variance \(2\sigma^2\) and mean value \(\mu_{xR} - \mu_{xD}\). Similarly, one can see that \(Y_R - Y_D\) is a Gaussian random variable with \(2\sigma^2\) and mean value \(\mu_{yR} - \mu_{yD}\). Therefore, the distance \(d\) follows Rice distribution \(d \sim \text{Rice}(\nu, \theta)\) with parameters \(\nu = \sqrt{(\mu_{xR} - \mu_{xD})^2 + (\mu_{yR} - \mu_{yD})^2}\).
and \( \theta = \sqrt{2 \sigma} \). Note that similar proofs of this and previous propositions are given in [1], however for the distance between two univariate Gaussian distributions.

\[ \square \]

**Proof of Proposition 4.3**

**Proof.** From Equation 16, \( d \) being Rice distributed random variable, the expected logarithm of SNR is given as follows:

\[ \mathbb{E} \left( \ln \left( \frac{P_{rx}}{N_k \sigma^2} \right) \right) = \ln \left( \frac{P_{rx}}{N_k \sigma^2} \right) - \frac{\gamma}{2} \mathbb{E} \left( \frac{d^2}{\sqrt{2 \sigma}} \right). \]  
(22)

Note that \( \frac{d^2}{\sqrt{2 \sigma}} \) has noncentral Chi distribution \( \chi^2(2, \frac{\gamma}{2}) \). Therefore \((d/\sigma)^2\) is a non-central Chi square distributed random variable. In [21, Theorem 3], the expected value of logarithm of general noncentral Chi square random variable \( X \sim \chi^2(2, \xi) \) was derived as:

\[ \mathbb{E} \left( \ln(X) \right) = \ln \xi g(\xi), \]  
(23)

where:

\[ g(\xi) = \exp \left( \int_{\xi/2}^{\infty} e^{-t} \frac{dt}{\sqrt{2 \pi}} \right). \]  
(24)

The proof follows by using \( X = \frac{d^2}{\sqrt{2 \sigma}} \) and \( \xi = \frac{\gamma^2}{2 \sigma^2} \).

\[ \square \]

**Proof of Proposition 4.4**

**Proof.** The proof follows a similar procedure as above with different normalization. The expected SNR is given as:

\[ \mathbb{E} \left( \ln \left( \frac{P_{rx}}{N_k \sigma^2} \right) \right) = \ln \left( \frac{P_{rx}}{N_k (\sqrt{2} \sigma)^2} \right) - \frac{\gamma}{2} \mathbb{E} \left( \frac{d^2}{\sqrt{2} \sigma} \right). \]  
(25)

The expected value is evaluated using Equation 23 with \( \xi = \frac{\gamma^2}{2 \sigma^2} \).

The proposition follows accordingly.

\[ \square \]

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