Energy Saving in Heterogeneous Cellular Networks with User Classification

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***Abstract*—Having massive databases about mobile user activities, cellular network operators can employ data analytics to extract information about user profiles (e.g. rate requirements, traffic type, etc) and provide awareness for better adaption of network parameters and resources. In this paper, it is investigated how simple data-driven information about users can improve performance of cellular networks with focus on energy efficiency. Users are assumed to belong to two classes differing in their rate requirements in heterogeneous network setting. Both Base Station (BS) and users are modeled according to independent homogeneous Poisson Point Process (PPP). Two sleeping strategies are considered for Small Cell (SC) namely random and strategic sleeping. Using stochastic geometry framework, it is shown that using rate-based user classification in devising sleeping strategies provides better energy consumption and fair resource allocation compared to oblivious resource allocation for all the users.**

I. Introduction

Unprecedented increase in mobile data traffic has sparked significant efforts toward improving performances of cellular networks to cope with new traffic demands. Heterogeneous network (Heterogeneous Network (HetNet)) architecture[1] is one of the solutions to this problem, particularly relying on densification of base stations and leveraging multiple radio access technologies. However with no information about user requirements, the network is designed for worst-case situation, leading to possibly inefficient resource allocation. This shortcoming can be circumvented by extracting information from enormous amount of data gathered about mobile users like Call Detail Records (CDR). Recent breakthrough in learning algorithms and particularly deep learning, increasing of computation power and advances in storage capacity have made possible to extract useful information in real time from massive datasets. In that respect, user databases in HetNet can be similarly used by operators for better understanding of users, their general requirements and their usage pattern as well as network features such as temporal and spatial traffic variation. In recent years, there is a growing interest to harness the potential of big data from industry as well as academia to operate the network efficiently [2]. One idea is to classify the users based on usage profile (e.g. rate requirement, type of traffic, location etc) and allocate the network resources accordingly [3]. This information seems to lead to better network performance in terms of quality of service, energy saving or other relevant performance metrics.

Among plenty of parameters to improve in an increasingly complex wireless network, one big issue is energy consumption. Studies show that around 5% of CO_2 is contributed by Information and Communication Technology (ICT) alone. Interestingly it is estimated that wireless technology makes up around 75% of ICT by 2020 [4]. Hence energy efficiency has come into priority list for network operators due to global movement of energy saving. Taking a closer look at wireless network, studies such as [5] show that most of the energy consumption takes place at BS. Hence recently sleep strategies of BSs [6], [7] has become popular approach to save energy by switching off BSs when traffic demand is low.

In this work, the goal is to provide insights about the ways in which simple information about users can be utilized for better network design. The multi-tier heterogeneous network architecture is considered and similar to [8], [9], [10], the network is modeled using stochastic geometry framework. Both BSs and users are assumed to be distributed according to independent homogeneous PPPs. Further the users are classified into two classes which differ in their rate requirements. These classes are modeled by marked PPP from previously defined parent PPP. It is assumed that operators can derive this information about the users based on big databases, however we do not discuss the algorithms behind this task. The goal is to show how this information can be used to intelligently and fairly allocate the resources to different users based on their profile. If an equal resource is allocated to all users regardless of their actual demand, the fairness problem arises where the network favors those with less demanding data profile. In this work, the bandwidth, as the main resource without controlling Signal-to-Interference-Noise Ratio (SINR) directly, is allocated to users according to their demand. The variation of rate coverage is analyzed with respect to bandwidth allocated to the classes. We further analyze small cell sleeping strategies in heterogeneous network for two classes of users. Finally two-class system is compared with single class user system where each user is obliviously served with maximum rate. Results show that more energy can be saved when difference between rate requirements of the classes increases.

The paper is organized as follows. The system model is discussed in Section II. Section III discusses the rate coverage probability and Section IV discusses sleeping strategies for energy efficiency. Numerical results are provided in Section V where the benefits of users information are demonstrated.

II. System Model

In this work, a K-tier heterogeneous network is considered where each tier differs in terms of transmit power and density, essentially following the work in [9]. Let $\mathcal{V} = \{1, 2, ..K\}$ denote the set of indices of K tiers where one tier belong to small cells. Tier i BSs are assumed to be distributed according to independent homogeneous PPP Φ_i with density λ_i. SC densities are specifically denoted by λ_K. Mobile users are also assumed to be distributed according to independent homogeneous PPP Φ_u with density λ_u. In this work, no single rate threshold is assumed for all users but instead, users belong to two different classes, say $\mathcal{C}_1$ and $\mathcal{C}_2$, based on their rate requirements. Users in each class $\mathcal{C}_1$ and $\mathcal{C}_2$ require different rate thresholds, ρ_1 and ρ_2 respectively. These two classes are modeled as marked PPP Φ_{u_1} and Φ_{u_2} from parent PPP Φ_u with corresponding densities λ_{u_1} and λ_{u_2}, such that $\lambda_{u_1} + \lambda_{u_2} = \lambda_u$. These classes can be detected by applying properly trained classification algorithms on CDR data, which is not considered in this work. The received power from tier i BS at a distance x is given by $P_i h x^{-\alpha}$, where h is the channel power gain and α is the path loss exponent. The channel coefficient is assumed to be Rayleigh distributed with unit average power, i.e., the channel power gain h is exponentially distributed, $h \sim \exp(1)$. The noise is assumed to be additive Gaussian with power σ^2. Table I gives quick access to the symbols used in this work.

TABLE I: List of Symbols

Symbol	**Description**
$\mathcal{V}$	set of K-tier BS
$\mathcal{V}_q$	set $\mathcal{V}$ with small cell density is reduced from λ_K to $q \times \lambda_K, \quad (0 \leq q \leq 1)$
$\mathcal{V}_s$	set $\mathcal{V}$ with small cell density is reduced from λ_K to $\mathbb{E}(S)\lambda_K$
$\mathcal{C}_n$	class n
ρ_c	rate threshold of the user belong to class c
$\mathcal{R}$;$\mathcal{R}^{(c)}$	rate coverage of entire system; rate coverage of class c
P_k	Transmit power of tier-k BS
τ_k^c	SINR threshold of user associated with tier-k and class c.
$\bar{N}_k^c$	average number of users in c^{th} class associated with an BS of tier-k
κ	fraction of total spectrum used to serve first class users
λ_{u_c}	PPP density of users belong to c^{th} class
W	available bandwidth (universal reuse)

It is assumed that users are connected to the BS with maximum average received power. Suppose Z_k denotes the distance of the user from nearest BS in K^{th} tier, then the user connects to tier j where $j = \arg\max_{k \in \mathcal{V}} P_k Z_k^{-\alpha}$. Without loss of generality, the user is assumed to be at the origin and referred to as typical user. The SINR of a typical user with serving BS that belongs to j^{th}tier at a distance y is given by

$$\text{SINR}_j(y) = \frac{P_j h y^{-\alpha}}{\sum_{k \in \mathcal{V}} \sum_{x \in \Phi_k \setminus y} P_k h_x x^{-\alpha} + \sigma^2}. \quad (1)$$

All BSs in all tiers operate in same spectrum of bandwidth W. Moreover, at each BS, the bandwidth is split to serve two classes such that κW is allocated to the first class $\mathcal{C}_1$ and $(1-\kappa)W$ to the second class $\mathcal{C}_2$ where $(0 \leq \kappa \leq 1)$. κ is called class resource share factor. If the typical user connected to tier-j BS belongs to $\mathcal{C}_1$, its achievable rate is given by

$$R_j^{(1)} = \frac{\kappa W}{\overline{N}_j^1} \log_2(1 + \text{SINR}_j) \quad (2)$$

Similarly if the user belongs to $\mathcal{C}_2$ connected to tier-j BS, corresponding rate is given by,

$$R_j^{(2)} = \frac{(1-\kappa)W}{\overline{N}_j^2} \log_2(1 + \text{SINR}_j) \quad (3)$$

where, $\overline{N}_j^1$ and $\overline{N}_j^2$ denotes the average number of users served by tier-j BS of class $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively. A trivial extension of [9, Corollary 1] yields

$$\overline{N}_j^1 = 1 + \frac{1.28 \lambda_{u_1} P_j^{\frac{2}{\alpha}}}{\sum_{k \in \mathcal{V}} \lambda_k P_k^{\frac{2}{\alpha}}}, \quad \overline{N}_j^2 = 1 + \frac{1.28 \lambda_{u_2} P_j^{\frac{2}{\alpha}}}{\sum_{k \in \mathcal{V}} \lambda_k P_k^{\frac{2}{\alpha}}}. \quad (4)$$

Note that similar to [9], mean load approximation $\overline{N}_j$ has been used in this work instead of actual load N_j to simplify the results. By dividing the bandwidth κW and $(1-\kappa)W$, two-class HetNet turns into two parallel single class HetNet.

III. Rate Coverage for Two Classes of Users

In this work, the rate coverage is considered as the performance metric for evaluating HetNet. This is defined as the probability that a random user in (Φ_u, λ_u) achieves rate greater than a certain threshold ρ, in HetNet with tiers $\mathcal{V}$ and bandwidth W. It is denoted by $\mathcal{R}(\lambda_u, \rho, W, \mathcal{V})$. The rate coverage probability of a typical user in a single class HetNet with assumption that users connect to the BS with maximum average power is derived in [9, Corollary 1] and given by

$$\begin{aligned}\mathcal{R}(\lambda_u, \rho, W, \mathcal{V}) = \sum_{j \in \mathcal{V}} 2\pi\lambda_j \int_0^\infty y \exp\left(\frac{-\tau_j \sigma^2 y^\alpha}{P_j}\right) \\ \exp\left(-\pi \sum_{k \in \mathcal{V}} D_j(k, \tau_j) y^2\right) \exp\left(-\pi \sum_{k \in \mathcal{V}} G_j(k) y^2\right) \mathrm{d}y\end{aligned} \quad (5)$$

where

$$D_j(k, \tau_j) = \left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}} \lambda_k Z(\tau_j, \alpha, 1), \quad G_j(k) = \lambda_k \left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}},$$

$$Z(a, b, c) = a^{\frac{2}{b}} \int_{(\frac{c}{a})^{\frac{2}{b}}}^\infty \frac{\mathrm{d}u}{1 + u^{\frac{b}{2}}}, \qquad \tau_j = 2^{\left(\frac{\rho \overline{N}_j}{W}\right)} - 1,$$

$$\overline{N}_j = 1 + \frac{1.28 \lambda_u P_j^{\frac{2}{\alpha}}}{\sum_{k \in \mathcal{V}} \lambda_k P_k^{\frac{2}{\alpha}}}.$$

For two-class case, it is assumed that κ is fixed for all BSs. Following theorem provides the rate coverage probability for two-class HetNet, denoted by $\mathcal{R}_\kappa = \mathcal{R}(\{\lambda_{u_i}\}_{i=1,2}, \{\rho_i\}_{i=1,2}, \kappa, W, \mathcal{V})$.

Theorem 1. *Rate coverage of a typical user in HetNets with two classes of users and class resource share factor of κ is given by*

$$\begin{aligned}\mathcal{R}_\kappa = &\left(\frac{\lambda_{u_1}}{\lambda_{u_1}+\lambda_{u_2}}\right)\mathcal{R}(\lambda_{u_1}, \rho_1, \kappa W, \mathcal{V}) + \\ &\left(\frac{\lambda_{u_2}}{\lambda_{u_1}+\lambda_{u_2}}\right)\mathcal{R}(\lambda_{u_2}, \rho_2, (1-\kappa) W, \mathcal{V})\end{aligned} \tag{6}$$

Proof. See Appendix A. □

Note that $\mathcal{R}(\lambda_{u_1}, \rho_1, \kappa W, \mathcal{V})$ and $\mathcal{R}(\lambda_{u_2}, \rho_2, (1-\kappa) W, \mathcal{V})$ are class-wide rate coverage probabilities, denoted by $\mathcal{R}^{(1)}$ and $\mathcal{R}^{(2)}$. It is obvious from equation (2) and (3) that the rates $R_j^{(1)}$ and $R_j^{(2)}$ are increasing and decreasing with κ respectively. The following proposition, provide the Pareto optimal κ, i.e., the resource sharing factor for which both classes achieve equal rate coverage.

Proposition 1. *Pareto optimal κ, denoted by κ_p for bandwidth allocation among the two classes of users is given by*

$$\kappa_p = \frac{\rho_1 \overline{N}_j^1}{\rho_1 \overline{N}_j^1 + \rho_2 \overline{N}_j^2}. \tag{7}$$

If $\lambda_{u_1}, \lambda_{u_2} \gg 1$, then

$$\kappa_p \approx \frac{\rho_1 \lambda_{u_1}}{\rho_1 \lambda_{u_1} + \rho_2 \lambda_{u_2}}.$$

Proof. See Appendix B. □

IV. Small Cell Sleeping Strategies

Generally HetNets are analyzed for maximum rate requirement. However studies such as [11] show that traffic varies both in space and in time. Operators can potentially reduce the energy consumption by switching off BSs when traffic is low in certain area or in specific times. Similar to [6], two sleeping strategies for small cells are studied for HetNet with two classes, namely *random sleeping* and *strategic sleeping*.

A. Random Sleeping

Random sleeping strategy for HetNet accounts for the temporal variation of traffic. If the traffic is not at its peak, then some BSs can be turned off. Each Small Cell Base Stations (Small Cell Base Station (SCBS)s) are operated with probability q and switched off (sleep) with probability $1-q$ independently of other SCBSs. Hence SC density in the system reduces from λ_K to $q\lambda_K$ after random sleeping strategy is applied. Objective of this strategy is to adapt the density of SCBS to traffic variation.

Theorem 2. *In a multi-tier system with two user classes using random sleeping strategy, rate coverage of a random user is given by*

$$\begin{aligned}\mathcal{R}_{RS}(q) = &\left(\frac{\lambda_{u_1}}{\lambda_{u_1}+\lambda_{u_2}}\right)\mathcal{R}(\lambda_{u_1}, \rho_1, \kappa W, \mathcal{V}_q) + \\ &\left(\frac{\lambda_{u_2}}{\lambda_{u_1}+\lambda_{u_2}}\right)\mathcal{R}(\lambda_{u_2}, \rho_2, (1-\kappa) W, \mathcal{V}_q)\end{aligned} \tag{8}$$

where $\mathcal{V}_q$ is the set $\mathcal{V}$ with small cell density reduced from λ_K to $q\lambda_K$, $\quad (0 \le q \le 1)$.

Proof. This is a simple extension of Theorem 1 with updated SC density. During random sleeping, density of SCBS reduces from λ_K to $q\lambda_K$ $(0 \le q \le 1)$ and other tier BS remains the same. □

B. Strategic Sleeping

Strategic sleeping accounts for both temporal and spatial traffic variation. In this model, similar to [6], the activity level of each SC is modeled by random variable $a \in [0,1]$ with density f_A. It determines the probability that each user is active inside its cell. Knowing the activity level a, a monotonically increasing strategic function $S(a)$ is defined (also in range $[0,1]$) and each SCBS remains active with probability $S(a)$ and is turned off otherwise. After applying this sleeping strategy, the remaining SCBS density reduces from λ_K to $\lambda_K \mathbb{E}[S]$ where $\mathbb{E}[S] = \int_0^1 S(a) f_A(a) \mathrm{d}a$. The following theorem characterizes the rate coverage for this case.

Theorem 3. *In a multi-tier system with two user classes using strategic sleeping strategy, rate coverage of a random user is given by*

$$\begin{aligned}\mathcal{R}_{SS} = &\left(\frac{\lambda_{u_1}}{\lambda_{u_1}+\lambda_{u_2}}\right)\mathcal{R}_{SS}(\lambda_{u_1}, \rho_1, \kappa W, \mathcal{V}, S) + \\ &\left(\frac{\lambda_{u_2}}{\lambda_{u_1}+\lambda_{u_2}}\right)\mathcal{R}_{SS}(\lambda_{u_2}, \rho_2, (1-\kappa) W, \mathcal{V}, S)\end{aligned} \tag{9}$$

where $\mathcal{R}_{SS}$ is given by (25).

Proof. See Appendix C. □

C. Optimization to minimize SCBS energy consumption

In this section, we try to minimize energy consumption of SC for both sleep strategies in HetNet while keeping each classes necessary QoS. Note that rate coverage probability is considered as QoS metric.

1) Optimization with Random Sleeping: For HetNet in which random sleeping is applied for SC, problem can be formulated as,

$$\begin{aligned}\underset{q}{min} \quad & q\lambda_K P_a + (1-q)\lambda_K P_s \\ s.t. \quad & \mathcal{R}(\lambda_{u_1}, \rho_1, \kappa W, \mathcal{V}_q) \ge \epsilon_1 \\ & \mathcal{R}(\lambda_{u_2}, \rho_2, (1-\kappa) W, \mathcal{V}_q) \ge \epsilon_2\end{aligned} \tag{11}$$

Here q denotes the fraction of active SCBS. Moreover P_a and P_s represents power consumed at SCBS during active and sleep state respectively. To solve this optimization problem, similar to conclusions in [12, Theorem 3] or in [6, Corollary 1], it can be seen that the rate coverage increases monotonically with density of SC, hence with the value of q as well.

$$\mathcal{R}^*_{SS}(\lambda_u,\rho,W,\mathcal{V},S)=\frac{A_K}{\mathbb{E}[a]}\left\{\frac{2\pi\lambda_K}{A_K}T_K(\tau_K,\mathcal{V}_s,\mathcal{V})\int_{a^*}^{1}af_A(a)\mathrm{d}a+\left\{\sum_{j\in\mathcal{V}_s}2\pi\lambda'_j\;\;T_j\left(\tau_j,\mathcal{V}_s,\mathcal{V}_s\right)\right\}\int_0^{a^*}af_A(a)\mathrm{d}a\right\}$$
$$+\sum_{j\in\mathcal{V}/\{K\}}2\pi\lambda_j\;\;T_j(\tau_j,\mathcal{V}_s,\mathcal{V}) \tag{10}$$

Since rate coverage increase with q, optimum value of q will be the one which satisfies the constraints tightly. As seen from Figure 2, value of κ influences the rate coverage as well. Hence every increase in q, follows search of value of κ in $[0,1]$ as well. The main steps are shown in Algorithm 1.

Algorithm 1 Algorithm to find optimum q in random sleeping

Initialize, q=0, κ=0, set step size stp_q,stp_κ
repeat
  repeat
    find $\mathcal{R}^{(1)}$ and $\mathcal{R}^{(2)}$
    $\kappa \leftarrow \kappa + stp_\kappa$
  until κ=1 or ($\mathcal{R}^{(1)} \geq \epsilon_1$ and $\mathcal{R}^{(2)} \geq \epsilon_2$)
  if (κ=1) then (reset κ=0 and $q \leftarrow q + stp_q$)
until q=1 or ($\mathcal{R}^{(1)} \geq \epsilon_1$ and $\mathcal{R}^{(2)} \geq \epsilon_2$)

2) Optimization with Strategic Sleeping: Similarly for strategic sleeping problem is formulated as,

$$\begin{aligned} \underset{S}{min}\quad & \mathbb{E}[S]\lambda_K P_a + (1-\mathbb{E}[S])\lambda_K P_s \\ s.t.\quad & \mathcal{R}_{SS}(\lambda_{u_1},\rho_1,\kappa W,\mathcal{V},S) \geq \epsilon_1 \\ & \mathcal{R}_{SS}(\lambda_{u_2},\rho_2,(1-\kappa)W,\mathcal{V},S) \geq \epsilon_2 \end{aligned} \tag{12}$$

Here $\mathbb{E}[S]$ is the fraction of SC which are active after sleep strategy is applied. Again, the rate coverage is an increasing function with the density of SCBS. In case of strategic sleeping, density of SCBS is determined by strategic function $S(a)$. However for a fixed $\mathbb{E}[S]$, it has been proven [6, Lemma 4] that optimum function of S takes the form of threshold function based on activity a. That means $S(a)=1$ if $(a \geq a^*)$ where a^* is a threshold value and $S(a)=0$ otherwise. Then constraint functions in 12 with $S(a)$ as threshold function is given by 10. As function S becomes threshold function, SCBS density increases with decrease in a^*. Hence optimization problem reduces to finding value of a^* that satisfies the constraints tightly. Similar to random switching case, every new value of a^* follows search of all value of κ.

V. Numerical Results

In the following, we consider a heterogeneous network with 3-tiers where one tier belong to SCBS and all the numerical results are done for the same setting. $\mathcal{V}=\{1,2,3\}$ represents three set of tiers in the network where 3^{rd} tier represents SCBS. Transmit power for each tier is given by $P_1=43$ dBm, $P_2=38$ dBm and $P_3=21$ dBm. Original density of each tiers before any sleeping strategies is applied are $\lambda_1=1$, $\lambda_2=5$ and $\lambda_3=10$. Path loss exponent α is assumed be to 4 for all the tiers. Bandwidth W is set 10 MHz unless another value is specified. To make a fair comparison between strategic sleeping and random sleeping, we choose strategic sleeping model so that both strategies have same fraction of SCBS active. For strategic sleeping model, SCs have activity (A) equal to 1 with probability q, and activity 0 with probability $1-q$. Sleeping strategy (S) is 1 if the activity is 1, else 0. Figure 1 shows rate coverage of whole system at κ_p. Figure 2 shows variation of rate coverage of two classes as a function of κ. Note that $\kappa_p \approx 0.6$ (in line with Proposition (1)) at which rate coverage for both user classes is equal. Figure 3 compares required SC density for single class and two classes system for random sleeping case. This can be found as value of q by Algorithm 1 by setting $\lambda_K=1$ and relaxing condition $q=1$ in the outer-loop. Note that we assumed higher value of bandwidth W to make comparison a bit more fair as two class system assumes two fixed users per cell where as in single class only one typical user is assumed per cell. It can be seen that difference in required SCBS density increases as the difference in required rate threshold becomes large. Hence classifying the users clearly has advantage of saving energy rather than assuming all the users need maximum rate. Finally figure 4 compares sleep strategies where strategic sleeping clearly has advantage over random sleeping in terms of energy efficiency. Energy efficiency is given by,

$$EE=\frac{\lambda_{u_1}\rho_1\mathcal{R}^{(1)}+\lambda_{u_2}\rho_2\mathcal{R}^{(2)}}{E_t} \tag{13}$$

where E_t is total energy consumption and is given by:

$$E_t=\left(\sum_{j\in\mathcal{V}/\{K\}}\lambda_j(P_j+\Delta_j P_{j,0})+q\lambda_K(P_K+\Delta_K P_{K,0})\right)\times W+(1-q)\lambda_K P_s. \tag{14}$$

Total Energy Consumption indicates overall energy consumed at all tiers in unit area. Energy consumption at BS includes transmit power as well as static power consumption. Assumed values are $P_{1,0}=130W, P_{2,0}=56W, P_{K,0}=6.8W$, $\Delta_1=4.7, \Delta_2=2.6, \Delta_K=4$, $P_s=4.3W$, according to [11].

Appendix

A. Proof of Theorem 1

If the rate of typical random user u is denoted by R_u, then we have

$$\begin{aligned}\mathbb{P}(R_u \geq \rho(u)) &= \mathbb{P}(R_u \geq \rho_1)\mathbb{P}(u \in \mathcal{C}_1) \\ &+ \mathbb{P}(R_u \geq \rho_2)\mathbb{P}(u \in \mathcal{C}_2)\end{aligned}$$

As a result of bandwidth division using κ, $\mathbb{P}(R_u \geq \rho_i)=\mathcal{R}(\lambda_{u_i},\rho_i,\kappa_i W,\mathcal{V})$ for $i=1,2$ and $\kappa_1=\kappa$ and $\kappa_2=1-\kappa$. Moreover since each classes are modeled using marked PPP, the probability that a random user belongs to $\mathcal{C}_1$ or $\mathcal{C}_2$ is given

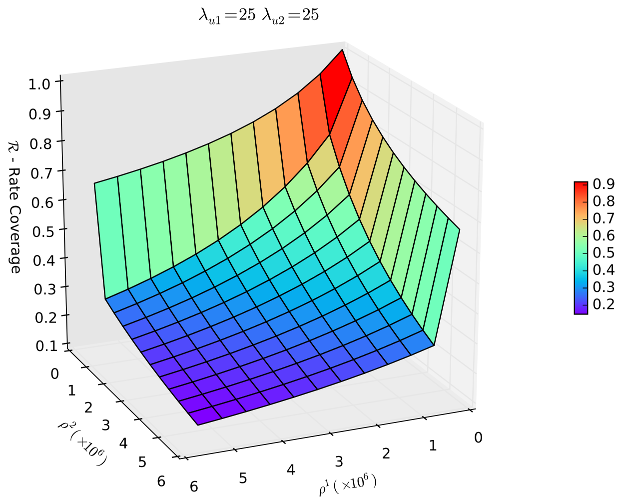

Fig. 1: rate coverage for 3-tier HetNet with two classes of users

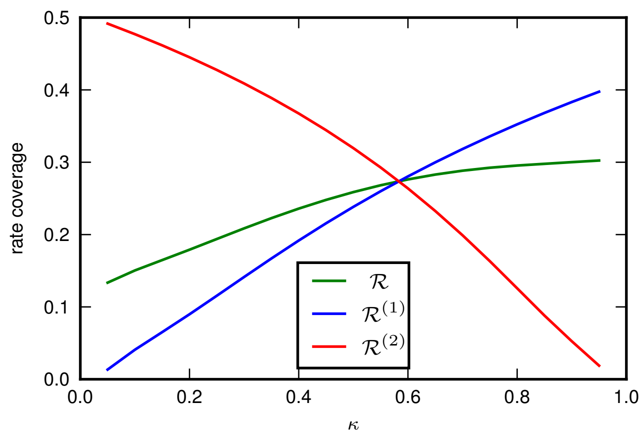

Fig. 2: Variation of rate coverage of classes as function of κ for $\lambda_{u_1} = 300, \lambda_{u_2} = 100, \rho_1 = 2 \times 10^6, \rho_2 = 4 \times 10^6, W = 50 \times 10^6$.

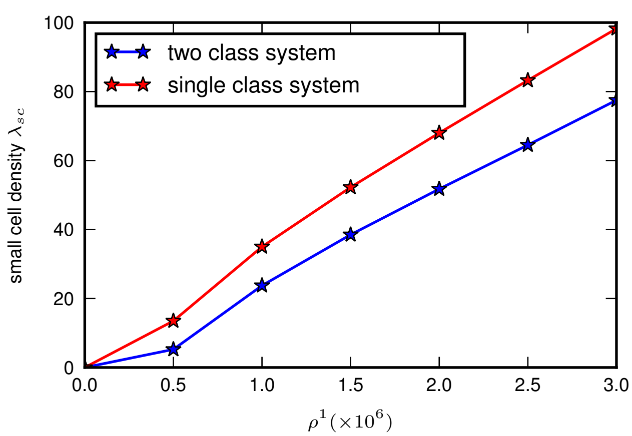

Fig. 3: Comparison of required SC density for single class vs two classes system $\lambda_{u_1} = \lambda_{u_2} = 250, \mathcal{R}_1 = \mathcal{R}_2 = 0.5, \rho_2 = 2 \times \rho_1$ and rate threshold for single class = $\rho = \max(\rho_1, \rho_2).W = 50 \times 10^6$

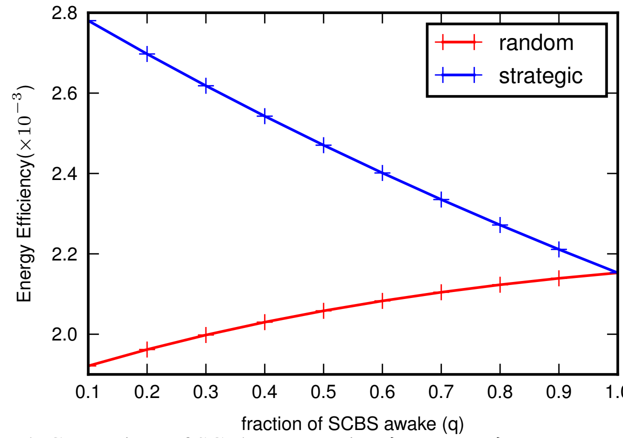

Fig. 4: Comparison of SC sleep strategies, $\lambda_{u_1} = 30, \lambda_{u_2} = 10, \rho_1 = 2 \times 10^6, \rho_2 = 4 \times 10^6$

by $\frac{\lambda_{u_1}}{\lambda_{u_1}+\lambda_{u_2}}$ or $\frac{\lambda_{u_2}}{\lambda_{u_1}+\lambda_{u_2}}$ respectively. Plugging in these results to the rate coverage, the theorem is obtained.

B. Proof of Proposition 1

Rate coverage for a typical user is actually derived from SINR coverage in [9, Theorem 1]. Hence rate coverage for two user class will be same when SINR thresholds for both classes are the same. This means

$$\tau_j^1 = \tau_j^2 \implies \left(\frac{\rho_1 \overline{N}_j^1}{\kappa W}\right) = \left(\frac{\rho_2 \overline{N}_j^2}{(1-\kappa)W}\right).$$

Solving this for κ, yields $\kappa_{\text{p}} = \frac{\rho_1 \overline{N}_j^1}{\rho_1 \overline{N}_j^1 + \rho_2 \overline{N}_j^2}$. Note that since $\overline{N}_j^1 = \frac{1.28\lambda_{u_1} P_j^{\frac{2}{\alpha}}}{\sum_{k\in\mathcal{V}} \lambda_k P_k^{\frac{2}{\alpha}}} + 1$, if λ_{u_1} is large enough then $\overline{N}_j^1$ can be approximated by $\frac{1.28\lambda_{u_1} P_j^{\frac{2}{\alpha}}}{\sum_{k\in\mathcal{V}} \lambda_k P_k^{\frac{2}{\alpha}}}$, and we have $\kappa_{\text{p}} = \frac{\rho_1 \lambda_{u_1}}{\rho_1 \lambda_{u_1} + \rho_2 \lambda_{u_2}}$.

C. Proof of Theorem 3

The rate coverage for single class and single tier system is obtained in [6]. First, this result is extended to multi-tier system and then to two user class system. Suppose that the user density is λ_u with required rate threshold ρ.

Lemma 1. *Association probability, i.e, probability that a typical user (before any sleep strategy is applied) connects to tier-J BS is given by,*

$$A_j = \frac{\lambda_j}{\sum_{k\in\mathcal{V}} \lambda_k \left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}}} \tag{15}$$

Proof. Proof can be obtained from Lemma 2 of [9] (assuming equal path loss exponent for all tiers). □

After strategic sleeping, the density of small cell is reduced to $\mathbb{E}(S)\lambda_K$. The new set of base stations is denoted by $\mathcal{V}_s$ and updated density of tier-j BS is denoted by λ_j'. Following theorem characterizes the association probability of a disconnected typical user of small cell after sleep strategy is applied, i.e., probability that a disconnected typical user connects to BS of tier-j.

Lemma 2. *The probability that a typical user who is disconnected from a switched off SCBS after strategic sleeping, now connects to an active tier-j BS is given by*

$$A_j^K = \frac{\lambda_j'}{\sum_{k\in\mathcal{V}_s}\lambda_k\left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}}} \tag{16}$$

where, λ_j' denotes the updated density of BS of tier-j after sleep strategy is applied.

Proof. Suppose that a user is connected to a SCBS at a distance of Z_K. The nearest k-tier BS is placed at Z_k. According to the association policy, it can be given as $P_K Z_K^{-\alpha} \geq P_k Z_k^{-\alpha}$. This means nearest active k-tier BS must be at distance

$$Z_k \geq \left(\frac{P_k}{P_K}\right)^{\frac{1}{\alpha}} Z_K \tag{17}$$

Next we find the distance distribution of nearest k-tier BS from the typical user who was previously connected to a switched-off SC. Suppose that there is no BS within a distance of d_1. If the distance of nearest k-tier BS is denoted by D, we have for $d > d_1$

$$\begin{aligned}
&\mathbb{P}(D > d|\text{no BS within } d_1) = \\
&\quad = \mathbb{P}(\text{No BS present within } d|\text{no BS within } d_1)\\
&\quad = \mathbb{P}\left(\Phi_k \cap (\mathcal{B}(0,d) - \mathcal{B}(0,d_1)) = \emptyset\right)\\
&\quad = \exp(-\pi\lambda_k(d^2 - d_1^2))
\end{aligned} \tag{18}$$

where $\mathcal{B}(0,x)$ is a ball centered at origin with radius x. Now density function of distance D is given by,

$$\begin{aligned}
f_D(d|\text{no BS within r1}) &= \frac{\mathrm{d}}{\mathrm{d}r}\left\{1 - \mathbb{P}[D > d|\text{no BS within } d_1]\right\}\\
&= 2\pi\lambda_k d\exp(-\pi\lambda_k(d^2 - d_1^2)).
\end{aligned} \tag{19}$$

The association probability A_j^K, the probability of a disconnected typical user connects to an active BS of tier-j after strategic sleeping, is given as:

$$\begin{aligned}
A_j^K &= \mathbb{P}\left(\bigcap_{k\in\mathcal{V}_s,k\neq j}\left\{P_j Z_j^{-\alpha} \geq P_k Z_k^{-\alpha}\right\}|\text{SC is off at } Z_K\right)\\
&\overset{(a)}{=} \prod_{k\in\mathcal{V}_s,k\neq j}\mathbb{P}\left\{P_j Z_j^{-\alpha} \geq P_k Z_k^{-\alpha}|\text{SC is off at } Z_K\right\}\\
&= \int_{s=0}^{\infty}\prod_{k\in\mathcal{V}_s,k\neq j}\mathbb{P}\left(Z_k \geq \left(\frac{P_k}{P_j}\right)^{\frac{1}{\alpha}} Z_j\right) f_{Z_K}(s)\mathrm{d}s\\
&\overset{(b)}{=} \int_{s=0}^{\infty}\int_{z=\left(\frac{P_j}{P_K}\right)^{\frac{1}{\alpha}}s}^{\infty}\\
&\prod_{k\in\mathcal{V}_s,k\neq j}\mathbb{P}\left(Z_k \geq \left(\frac{P_k}{P_j}\right)^{\frac{1}{\alpha}} z\right) f_{Z_j}(z)\mathrm{d}z\, f_{Z_K}(s)\mathrm{d}s\\
&\overset{(c)}{=} \int_{s=0}^{\infty} 2\pi\lambda_j' \int_{z=\left(\frac{P_j}{P_K}\right)^{\frac{1}{\alpha}}s}^{\infty} z\\
&\prod_{k\in\mathcal{V}_s,k\neq j}\exp\left(-\pi\lambda_k\left(\left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}} z^2 - \left(\frac{P_k}{P_K}\right)^{\frac{2}{\alpha}} s^2\right)\right)\\
&\exp\left(-\pi\lambda_j'\left(z^2 - \left(\frac{P_j}{P_K}\right)^{\frac{2}{\alpha}} s^2\right)\right)\mathrm{d}z \quad f_{Z_K}(s)\mathrm{d}s\\
&\overset{(d)}{=} \int_{s=0}^{\infty} 2\pi\lambda_j' \int_{z=\left(\frac{P_j}{P_K}\right)^{\frac{1}{\alpha}}s}^{\infty} z\\
&\exp\left(-\pi\sum_{k\in\mathcal{V}_s}\lambda_k\left(\left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}} z^2 - \left(\frac{P_k}{P_K}\right)^{\frac{2}{\alpha}} s^2\right)\right)\mathrm{d}z f_{Z_K}(s)\mathrm{d}s\\
&\overset{(e)}{=} \frac{\lambda_j'}{\sum_{k\in\mathcal{V}_s}\lambda_k\left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}}}
\end{aligned} \tag{20}$$

where, $f_{Z_K}(s)$ is given by [9, Lemma 4]:

$$f_{Z_K}(s) = \frac{2\pi\lambda_K}{A_K} s\exp\left(-\pi\sum_{k\in\mathcal{V}}\lambda_k\left(\frac{P_k}{P_K}\right)^{\frac{2}{\alpha}} s^2\right) \tag{21}$$

(a) follows from independence of $\Phi_k,\forall k \in \mathcal{V}$. Lower limit in inner integral of (b) follows from equation (17). First exponential and second exponential in (c) follow from equation (18) and (19). With simplification of expression we get (d). Finally, substituting distribution $f_{Z_K}(s)$ and with simplification of expression, we get (e) or equation (16).

An alternative proof follows easily from [9, Lemma 2] by reducing density from $\mathcal{V}$ to $\mathcal{V}_s$, however this derivation corroborates the independence property of PPPs. □

Lemma 3. *Approximated mean number of disconnected small cell users after strategic sleeping connects to active BS of tier-j is given by,*

$$\bar{N}_j^K = \bar{N}_K\left(\lambda_K\left[1 - \mathbb{E}[S]\right]\right)\frac{\mathcal{A}_j^K}{\lambda_j'}\int_0^1 a f_A(a)da \tag{22}$$

Proof. Association probability $\mathcal{A}_j^K$ can be interpreted as fraction of disconnected SC users connected to tier-j BS. Since we assume that activity of a SC is independent of other SC, switched off SCs can be interpreted as marked PPP with density $\lambda_K\left[1 - \mathbb{E}[S]\right]$. Because marked PPPs and PPPs are ergodic, average number of disconnected users associating to active tier-j BS can be approximated in an unit area (similar to Remark(2) in [9]). Along with the activity of small cells, average number of users can be approximated as,

$$\bar{N}_j^K = \bar{N}_K\big(\lambda_K\big[1-\mathbb{E}[S]\big]\big)\frac{\mathcal{A}_j^K}{\lambda_j'}\int_0^1 a f_A(a)da$$

where, $\bar{N}_K$ denotes the average number of SCBS users before sleep strategy is applied. $\lambda_K\big[1-\mathbb{E}[S]\big]$ denotes the number of switched off SC in an unit area. λ_j' denotes the updated density of BS of tier-j after sleep strategy. Finally integration in equation follows from taking into account the activity of the SC users. □

Lemma 4. *Rate coverage of a typical user connected (excluding disconnected users) to a BS of tier-j is given by,*

$$\mathbb{P}_j(R\geq\rho) = \frac{2\pi\lambda_j}{\mathcal{A}_j}\int_{y=0}^{\infty} y\exp\left(\frac{-\tau_j\sigma^2 y^\alpha}{P_j}\right) \exp\left(-\pi\sum_{k\in\mathcal{V}_s} D_j(k,\tau_j)y^2\right)\exp\left(-\pi\sum_{k\in\mathcal{V}} G_j(k)y^2\right)\mathrm{dy} \tag{23}$$

where, $\tau_j = 2^{\left(\frac{\rho\bar{N}_j^{\chi}}{W}\right)} - 1$ with $\bar{N}_j^{\chi} = \bar{N}_j + \bar{N}_j^K$
$\bar{N}_j = 1 + \frac{1.28\lambda_u\mathcal{A}_j}{\lambda_j}$

Proof. Result can be easily obtained from extending equation (26) in [9] by dividing the resource with extra associated SC users. Note that second exponential term represents Laplace transform of aggregated interference and hence has updated density of BS $\mathcal{V}_s$. Third exponential term corresponds to distance distribution. Distance distribution remains same in this case mainly because sleep strategy is applied for only small cell and small cells are assumed to be transmitting with least transmit power. Hence any typical user connected to tier-j BS (excluding switched off SC user) remains connected to same BS even after sleep strategy is applied. □

Lemma 5. *Rate coverage of a disconnected typical user given it is connected to active BS of tier-j (after sleep strategy is applied) is given by,*

$$\mathbb{P}_j^K(R\geq\rho) = \frac{2\pi\lambda_j'}{A_j^K}\int_{y=0}^{\infty} y\exp\left(\frac{-\tau_j\sigma^2 y^\alpha}{P_j}\right) \exp\left(-\pi\sum_{k\in\mathcal{V}_s} D_j(k,\tau_j)y^2\right)\exp\left(-\pi\sum_{k\in\mathcal{V}_s}\lambda_k\left(\frac{P_k}{P_j}\right)^{\frac{2}{\alpha}} y^2\right)\mathrm{dy} \tag{24}$$

where, $\tau_j = 2^{\left(\frac{\rho\bar{N}_j^{\chi}}{W}\right)} - 1$ with $\bar{N}_j^{\chi} = \bar{N}_j + \bar{N}_j^K$
$\bar{N}_j = 1 + \frac{1.28\lambda_u\mathcal{A}_j}{\lambda_j}$

Proof. Proof is obtained similar to Lemma 4. Note that distance distribution (third exponential term) changes and can be simply obtained from reducing densities (as points of PPP are independent). □

Now, using the previously stated results, conditioning on the activity of typical SC and with total law of probability, overall rate coverage of a typical user in the system is derived:

$$\begin{aligned}
&\mathcal{R}_{SS}(\lambda_u,\rho,W,\mathcal{V},S) \overset{(a)}{=} \frac{A_K}{\mathbb{E}[a]}\int_0^1 a\mathbb{P}_K(R\geq\rho|a)f_A(a)\mathrm{d}a + \sum_{j\in\mathcal{V}/sc} A_j\mathbb{P}_j(R\geq\rho)\\
&= \frac{A_K}{\mathbb{E}[a]}\int_0^1\Big\{a\mathbb{P}_K(R\geq\rho|\text{SC is actv})\mathbb{P}(\text{SC is actv})\\
&+ a\mathbb{P}_K(R\geq\rho|\text{SC is Inactv})\mathbb{P}(\text{SC is Inactv})\Big\}f_A(a)\mathrm{d}a\\
&+\sum_{j\in\mathcal{V}/sc} A_j\mathbb{P}_j(R\geq\rho)\\
&= \frac{A_K}{\mathbb{E}[a]}\int_0^1\Big\{a\mathbb{P}_K(R\geq\rho|\text{SC is actv})S(a)\\
&+ a\mathbb{P}_K(R\geq\rho|\text{SC is Inactv})(1-S(a))\Big\}f_A(a)\mathrm{d}a\\
&+\sum_{j\in\mathcal{V}/sc} A_j\mathbb{P}_j(R\geq\rho)\\
&\overset{(b)}{=} \frac{A_K}{\mathbb{E}[a]}\int_0^1\Big\{a\mathbb{P}_K(R\geq\rho|\text{SC is actv})S(a)\\
&+ a\Big\{\sum_{j\in\mathcal{V}_s} A_j^K\mathbb{P}_j^K(rate\geq\rho|\text{SC is inactive})\Big\}(1-S(a))\Big\}f_A(a)\mathrm{d}a\\
&+\sum_{j\in\mathcal{V}/sc} A_j\mathbb{P}_j(R\geq\rho)\\
&\overset{(c)}{=} \frac{A_K}{\mathbb{E}[a]}\Big\{\frac{2\pi\lambda_K}{A_K}T_K(\tau_K,\mathcal{V}_s,\mathcal{V})\int_0^1 aS(a)f_A(a)\mathrm{d}a\\
&+\Big\{\sum_{j\in\mathcal{V}_s} 2\pi\lambda_j'\ T_j(\tau_j,\mathcal{V}_s,\mathcal{V}_s)\Big\}\int_0^1 a(1-S(a))f_A(a)\mathrm{d}a\Big\}\\
&+\sum_{j\in\mathcal{V}/sc} 2\pi\lambda_j\ T_j(\tau_j,\mathcal{V}_s,\mathcal{V})
\end{aligned} \tag{25}$$

where,

$$T_j(\tau_j,v_1,v_2) = \int_0^\infty y\exp\left(\frac{-\tau_j\sigma^2y^\alpha}{P_j}\right)\exp\left(-\pi\sum_{k\in v_1}D_j(k,\tau_j)y^2\right) \exp\left(-\pi\sum_{k\in v_2}G_j(k)y^2\right)\mathrm{d}y$$

$$\tau_j = 2^{\left(\frac{\rho\overline{N}_j^{\chi}}{W}\right)} - 1 \qquad \overline{N}_j^{\chi} \approx \overline{N}_j + \overline{N}_j^K$$

and (a) shows conditioning on the load 'a' of SC. (b) follows by offloading disconnected users from switched-off SC (due to sleep strategy) to other active tier BSs. Since we offload switched off SC users, other active cells share its resources to serve offloaded users. Hence total mean number of users served by a BS of tier-j is given by $\overline{N}_j^{\chi}$. Finally (c) follows by substituting and simplifying derived result in this Appendix.

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