RSS-based Location and Transmit Power Estimation of Multiple Co-Channel Targets

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***Abstract*—In this paper we consider the problem of multi-target localization using sensor networks, where the received signal strength of targets are measured at sensor nodes and are processed at a fusion node to estimate the location as well as the transmit power of all targets. As we do not consider any kind of division multiplexing, e.g., TDMA or FDMA, targets cause each other co-channel interference. This is a very hard problem, especially because the transmit power of targets can be different. The classical methods in which the power of a single target cancels out by dividing the received power at different sensors, do not apply to our problem, as the received power of each sensor is the superposition of more than one term, each of which corresponding to one target. We tackle the problem by a grid based approach, i.e., discretizing the area, and proposing an adaptive scheme to refine the position of grid points. The proposed algorithms are based on mixed integer optimization approach.**

***Index Terms*—multi-source localization, compressed sensing, mixed-integer programming, internet of things**

I. Introduction

It is envisaged that the majority of applications in the context of internet of things and 5G mobile networks depend on the location awareness to deliver better services. That is why the research topic of localization, in spite of being a quit old topic, is not yet outdated. In the literature a variety of techniques have been exploited to solve the problem of localization. Among all those techniques we here pick the one based on received signal strength (RSS) of the receiving nodes for its simplicity and lower cost of implementation compared to techniques such as time difference of arrival (TDOA) or angle of arrival (AoA), [1]. Despite its vulnerability against uncertainties of path-loss model, RSS-based localization is still popular in the applications where precision can be compromised on for price.

The RSS-based localization for a single target with unknown transmit power is studied in many publications, such as [2], where the transmit power cancels out by dividing the RSS of two different receivers. The remaining of the problem is a standard multilateration problem. This technique is known as differential or ratio of RSS which is not applicable in our case, since we consider that there are more then one transmitter on the same channel which causes co-channel interference problem.

The work [3] also considers a multi-target scenario, but it does not assume co-channel interference since each receiver knows the RSS corresponding to each transmitter, separately. To the best of our knowledge, the only paper which considers the multi-user case with co-channel interference is [4]. The authors, nevertheless, do not exploit the explicit path-loss model but perform fingerprinting, instead. On the one hand, fingerprinting avoids the model uncertainty of the path-loss model. It, on the other hand, involves the difficulty of building the radio maps which can be costly and time consuming. The built radio map can also be different in reality, when moving humans or objects effect the propagation profile of the environment. The authors exploit the techniques of ℓ_1-minimization which needs a sufficiently enough number of observations. This translates into a radio map with high granularity. To surmount this problem, they use not only the RSS but also cross-correlation of the received signal at different sensors. This improves performance, but at the cost of more expensive sensors as well as higher power consumption. Please refer to Remark 1 in Sec. II. We, on the contrary, consider the explicit attenuation model to skip the radio map creation. We intend to deal with the problem of uncertainty associated with our knowledge path-loss exponent and large-scale shadowing in our future publications. We also resort to grid based solution to keep the number of sensor nodes (SNs) low and deploy techniques of mixed integer programming (MIP). Also, to maintain a low-complexity we keep the number of grid points (GPs) low and adapt the GPs in an iterative fashion.

The organization of this paper is as follows: the system model is described in Sec. II. We propose in Sec. III a mixed integer quadratic programming (MIQP) formulation, assuming the targets are located exactly at certain GPs. An adaptive scheme is proposed in Sec. IV to refine the GPs to overcome the problem of off-grid targets. The performance of the presented solutions will be justified by means of computer simulations in Sec. V. The Sec. VI concludes the paper.

Notations: All mathematical notations, symbols and variables of the system in this paper are summarized in Tab. I.

II. System Model

The system of consideration is consisted of $N \in \mathbb{N}$ active targets which must be localized using $K \in \mathbb{N}$ passive SNs. Each target $n \in \mathbb{F}_N$ transmits a signal with the unknown power p_n. We know that that transmit power of each target is bounded as follows

$$\underline{P} \leq p_n \leq \bar{P}\,,\ \forall n \in \mathbb{F}_N\,, \tag{1}$$

Table I: Summary of general mathematical notations

Notation	Description
$\mathbb{N}$	set of all integer positive and non-zero numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{R}_+$	set of all non-negative real numbers
$\mathbb{F}_{\mathrm{L}}$	the index-set $\mathbb{F}_{\mathrm{L}} = \{1, \cdots, L\}$ for any L$\in \mathbb{N}$
$\lvert\mathcal{G}\rvert$	the cardinality of set $\mathcal{G}$
δ_{lm}	Kronecker delta function, i.e., $\delta_{lm} = \begin{cases} 1, & l = m, \\ 0, & l \neq m. \end{cases}$

where $\underline{P}$, $\bar{P} \in \mathbb{R}_+$ are, respectively, the lowest and highest possible values for the transmit power of an active target.

We choose our channel model, based on the log-normal shadowing attenuation model presented in [5]. In a multi-source scenario, the RSS r_k at sensor k is the sum of different terms corresponding to the received power of each target signal, as follows

$$r_k = \sum_{n \in \mathbb{F}_N} c_0 \, p_n \, d_{kn}^{-\alpha} \, 10^{\frac{\zeta_{kn}}{10}} \, .$$

where d_{kn} is the distance between sensor k and n^{th} target, α is the path-loss exponent and $\zeta_{kn} \sim \mathcal{N}(0, \sigma_{kn}^2)$ is a zero-mean Gaussian random variable with power of σ_{kn}^2. This random variable corresponds to log-normal shadowing between each pair of sensor and target nodes and is assumed to be identically and independently distributed (iid). Also, c_0 is given by [5],

$$c_0 := \frac{G_t G_r \lambda^2}{(4\pi)^2} \, ,$$

where G_t, G_r and λ are the gains of transmit and receive antennae. The wavelength is denoted by λ. We assume that c_0 is known and without loss of generality and for the sake of simplicity, $c_0 = 1$. Here, we have neglected the thermal additive noise due to the fact that shadowing has much stronger effect on RSS compared to the thermal noise [6].

Remark 1. *The system of consideration is suitable for practical applications in the sense that SNs do not need to be costly and also do not need to be sophisticated, because:*

- *SNs can calculate the RSS via simple arithmetic operations without having huge storage capacity, e.g., using a shift register.*
- *The communication overhead between SNs and fusion center (FC) is minimal, since SNs relay only their RSS readings once within a certain time interval, instead of transmitting the whole signal samples. This keeps the communication overhead between the SNs and FC very low which in turn*
  1) *reduces the SNs power consumption. Note, transmitting an RF signal demands much more power consumption compared to simple arithmetic operation for calculating RSS.*
  2) *makes it possible to protect the few bits, conveying RSS reading, by long error correction codes. This results in reliable communication even in moderately low signal-to-noise ratio (SNR) between SNs and FC.*

  These two aforementioned facts, makes the current scenario more suitable than the one in [4], under the conditions that power consumption and reliable communication between SNs and FC are limiting factors. Though, we unlike [4] lose the valuable potential information lying in the cross-correlation between received signals at different sensors.
- *Unlike TDOA methods our sensor network does not need a very accurate synchronization.*
- *As a results such sensor network is of interest for battery-critical and budget-limited applications where the accuracy can be somewhat compromised.*

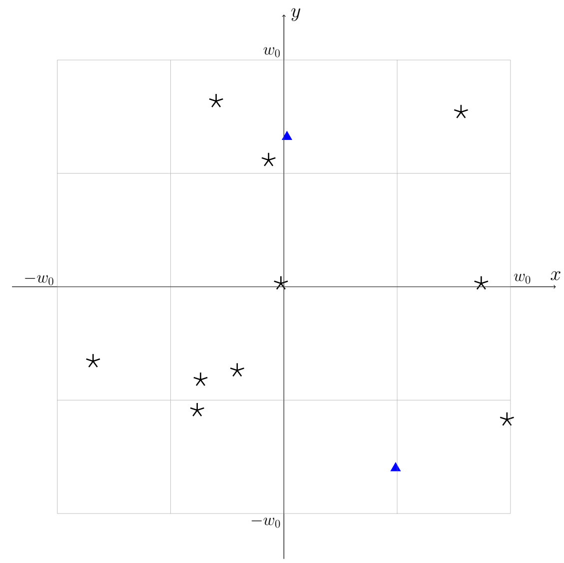

Figure 1. A wireless sensor network consisting of $K = 10$ passive sensors ($\star$) and $N = 2$ targets ($\blacktriangle$) . The grid granularity is $G = \sqrt{M} = 5$, which means the area of interest is divided to $(\sqrt{M} - 1)^2 = 16$ smaller squares. This leads to $M = 25$ GPs, around which we look for the targets. The size of each grid square is $\Delta_g = \frac{w_0}{2}$.

The area of observation is assumed to be an square in the range of $[-w_0, w_0]$, $w_0 \in \mathbb{R}_+$ in both x- and y- axes, in the Cartesian coordinate system. The targets and sensors are randomly distributed within the area. The ordered pair $(\check{x}_k, \check{y}_k)$ stands for the coordinate of k^{th} sensor node, while target n is located at the unknown position (x_n, y_n). Assuming that FC acquires the value of RSS r_k of the k^{th} sensor error-freely upon successful communication from SN, it has to solve the following system of nonlinear and non-convex equation to find the position (x_n, y_n) of each target:

$$r_k = \sum_{n \in \mathbb{F}_N} \frac{p_n \, 10^{\frac{\zeta_{kn}}{10}}}{\left(\sqrt{(x_n - \check{x}_k)^2 + (y_n - \check{y}_k)^2}\right)^{\alpha}} \, . \tag{2}$$

Unfortunately, such a system of equation is not tractable. Thus, we resort to solve it sub-optimally by means of MIQP. We, then, discretize the area by a grid of granularity of $\sqrt{M} \in \mathbb{N}$ which means M GPs in total. Let $\mathcal{G}_M^w(x_0, y_0)$ define the grid set centered at the point (x_0, y_0) of width $2w \in \mathbb{R}_+$ and M members. Indeed, it is the set of the equidistant GPs defined by

$$\mathcal{G}_M^w(x_0, y_0) := \\ \{(x_0 - w + (i-1)\Delta_g, y_0 - w + (j-1)\Delta_g) \,|\, i, j \in \mathbb{F}_G\} \,, \tag{3}$$

where $G = \sqrt{M}$ and $\Delta_g = \frac{2w}{\sqrt{M}-1}$ is the width of one grid square. Then, our defined grid consists of the GPs $(\tilde{x}_m, \tilde{y}_m) \in \mathcal{G}_M^{w_0}(0,0)$, $m \in \mathbb{F}_M$. Fig. 1 depicts an example grid given by $\mathcal{G}_{25}^{w_0}(0,0)$.

III. MIQP

The distance $\tilde{d}_{km}$ between sensor $k \in \mathbb{F}_K$ and the GP $m \in \mathbb{F}_M$ can be calculated using

$$\tilde{d}_{km} = \sqrt{(\tilde{x}_m - \check{x}_k)^2 + (\tilde{y}_m - \check{y}_k)^2} \,, \tag{4}$$

which in the absence of shadowing constitutes the following RSS at the sensor node k:

$$\tilde{r}_k = \sum_{m \in \mathbb{F}_M} \tilde{p}_m \tilde{d}_{km}^{-\alpha} \,. \tag{5}$$

The variable $\tilde{p}_m$ is equal to zero when no target is located at the vicinity of the GP m. It must be, otherwise, equal to the transmit power of the target whose closest GP is $(\tilde{x}_m, \tilde{y}_m)$. In case of no shadowing, i.e., $\sigma_{kn} = 0$ and when

- the targets are exactly at GP, i.e., for each $n \in \mathbb{F}_N$ there exists exactly one $m \in \mathbb{F}_M$ such that $(x_n, y_n) = (\tilde{x}_m, \tilde{y}_m)$,

$\tilde{p}_m$ can be chosen such that $r_k = \tilde{r}_k$. The first assumption is not realistic, when the targets are randomly distributed in the monitoring area. We, nevertheless, try to solve the following MIQP assuming the targets are exactly at the GPs. We deal with the problem of off-grid targets later in Sec. IV. We also assume that $\sigma_{kn} \to 0$. Later, we evaluate how the localization performance degrades in higher values of σ_{kn}, by means of computer simulations. Now, we aim at minimizing the mean square error (MSE)

$$\sum_{k \in \mathbb{F}_K} (r_k - \tilde{r}_k)^2 = \sum_{k \in \mathbb{F}_K} \left(r_k - \sum_{m \in \mathbb{F}_M} \tilde{p}_m \tilde{d}_{km}^{-\alpha} \right)^2 \,, \tag{6}$$

where $\tilde{d}_{km}$ can be pre-calculated by (4), r_k is the RSS reading at k^{th} sensor and $\tilde{p}_m$ is the optimization variable. We propose the following optimization problem for joint estimation of the location and transmit power of each target:

$$\min_{\substack{s_m, \tilde{p}_m \\ m \in \mathbb{F}_M}} \sum_{k \in \mathbb{F}_K} \left(r_k - \sum_{m \in \mathbb{F}_M} \tilde{p}_m \tilde{d}_{km}^{-\alpha} \right)^2 \tag{7a}$$

$$\text{s.t.} \;\; \tilde{p}_m \in \mathbb{R}_+ \,,\; \forall m \in \mathbb{F}_M \,, \tag{7b}$$

$$s_m \, \underline{P} \le \tilde{p}_m \le s_m \, \bar{P} \,,\; \forall m \in \mathbb{F}_M \,, \tag{7c}$$

$$s_m \in \{0, 1\} \,,\; \forall m \in \mathbb{F}_M \,, \tag{7d}$$

$$\sum_{m \in \mathbb{F}_M} s_m = N \,. \tag{7e}$$

While the constraint (7b) together with (7c) relate to the fact that we know the upper and lower bound on transmit power, i.e., eq. (1), of active targets, the constraints (7d) and (7e) guarantee that exactly N GPs are chosen as the candidate position of the N targets.

Despite the quadratic and convex form of the objective function (7a), the optimization problem is non-convex as it involves the binary variables s_m. On the other hand, it also has continuous variables $\tilde{p}_m$ and, therefore, belongs to the family of MIQP. Such problems have a combinatorial nature and are, hence, $\mathcal{NP}$-hard [7], [8]. So, the number of GPs is a serious challenge in terms of complexity. Consequently, many papers in the literature tackle such problems by means of compressed sensing, e.g., [3], [4], [9], which relaxes the norm zero to norm one. This though necessitates more number of observations, i.e., number of sensor nodes in our case, compared to the combinatorial solution. Subsequently, we rather stick to the MIQP as increasing the number of sensors increases the overhead of the communication to the FC, hardware costs as well as power consumption and in total implementation cost. Instead, we deal with the problem of complexity by choosing low number of GPs, M, and proposing a heuristic instead.

IV. Adaptive Grid Refinement

In case the targets are at off-grid positions the complexity of the optimization (7) does not allow us to increase the number of GPs M. Therefore, the induced inaccuracy makes our approach useless, unless we devise an smart solution to the problem. Our solution to this complication is to adapt the GPs in an iterative manner by defining additional variables which allow for displacement w.r.t to GPs. These new variables are then used to update the GP in the next iteration. We first derive the first order Taylor's series expansion of the function $f_k(\tilde{p}_1, \cdots, \tilde{p}_M, \tilde{x}_1, \cdots, \tilde{x}_M, \tilde{y}_1, \cdots, \tilde{y}_M)$ or f_k in short form

$$f_k := r_k - \sum_{m \in \mathbb{F}_M} \frac{\tilde{p}_m}{\left(\sqrt{(\tilde{x}_m - \check{x}_k)^2 + (\tilde{y}_m - \check{y}_k)^2} \right)^{\alpha}} \tag{8}$$

Let us first calculate the derivatives of function f_k, i.e.,

$$\frac{\partial f_k}{\partial \tilde{x}_m} = \frac{\alpha\, \tilde{p}_m(\tilde{x}_m - \check{x}_k)}{\left(\sqrt{(\tilde{x}_m - \check{x}_k)^2 + (\tilde{y}_m - \check{y}_k)^2}\right)^{\alpha+2}}\,, \tag{9}$$

$$\frac{\partial f_k}{\partial \tilde{y}_m} = \frac{\alpha\, \tilde{p}_m(\tilde{x}_m - \check{x}_k)}{\left(\sqrt{(\tilde{x}_m - \check{x}_k)^2 + (\tilde{y}_m - \check{y}_k)^2}\right)^{\alpha+2}}\,, \tag{10}$$

$$\frac{\partial f_k}{\partial \tilde{p}_m} = \frac{-1}{\left(\sqrt{(\tilde{x}_m - \check{x}_k)^2 + (\tilde{y}_m - \check{y}_k)^2}\right)^{\alpha}}\,. \tag{11}$$

Also, let a_{km}^{i-1}, b_{km}^{i-1} and c_{km}^{i-1} represent the values of derivative ∂f_k w.r.t, respectively, $\tilde{x}_m$, $\tilde{y}_m$ and $\tilde{p}_m$ at the point $(\tilde{p}_1^{i-1}, \cdots, \tilde{p}_M^{i-1}, \tilde{x}_1^{i-1}, \cdots, \tilde{x}_M^{i-1}, \tilde{y}_1^{i-1}, \cdots, \tilde{y}_M^{i-1})$, thus the Taylor's expansion of f_k at i^{th} iteration reads

$$f_k(\tilde{p}_1, \cdots, \tilde{p}_M, \tilde{x}_1, \cdots, \tilde{x}_M, \tilde{y}_1, \cdots, \tilde{y}_M) \approx$$
$$f_k^{i-1} + \sum_{m\in\mathbb{F}_M} a_{km}^{i-1}\mathrm{d}\tilde{x}_m + b_{km}^{i-1}\mathrm{d}\tilde{y}_m + c_{km}^{i-1}\mathrm{d}\tilde{p}_m\,, \tag{12}$$

$$f_k^{i-1} =: f_k(\tilde{p}_1^{i-1}, \cdots, \tilde{p}_M^{i-1}, \tilde{x}_1^{i-1}, \cdots, \tilde{x}_M^{i-1}, \tilde{y}_1^{i-1}, \cdots, \tilde{y}_M^{i-1})\,, \tag{13}$$

$$\mathrm{d}\tilde{x}_m =: \tilde{x}_m - \tilde{x}_m^{i-1}\,, \tag{14}$$

$$\mathrm{d}\tilde{y}_m =: \tilde{y}_m - \tilde{y}_m^{i-1}\,, \tag{15}$$

$$\mathrm{d}\tilde{p}_m =: \tilde{p}_m - \tilde{p}_m^{i-1}\,. \tag{16}$$

The GPs at iteration $i-1$ are updated by

$$\tilde{x}_m^i = \tilde{x}_m^{i-1} + \mathrm{d}\tilde{x}_m\,, \tag{17a}$$

$$\tilde{y}_m^i = \tilde{y}_m^{i-1} + \mathrm{d}\tilde{y}_m\,. \tag{17b}$$

Then, at i^{th} iteration, given the values of $\tilde{x}_m^{i-1}$, $\tilde{y}_m^{i-1}$ and $\tilde{p}_m^{i-1}$, we solve the following optimization problem, instead of (7)

$$\min_{\substack{s_m,\,\mathrm{d}\tilde{x}_m,\\ \mathrm{d}\tilde{y}_m,\,\mathrm{d}\tilde{p}_m\\ m\in\mathbb{F}_M}} \sum_{k\in\mathbb{F}_K}(f_k^{i-1} + \sum_{m\in\mathbb{F}_M} a_{km}^{i-1}\mathrm{d}\tilde{x}_m + b_{km}^{i-1}\mathrm{d}\tilde{y}_m + c_{km}^{i-1}\mathrm{d}\tilde{p}_m)^2 \tag{18a}$$

$$\text{s.t.}\quad \mathrm{d}\tilde{x}_m,\, \mathrm{d}\tilde{y}_m,\, \mathrm{d}\tilde{p}_m \in \mathbb{R}\,, \tag{18b}$$

$$s_m\,\underline{P} - p_m^{i-1} \le \mathrm{d}\tilde{p}_m \le s_m\,\bar{P} - p_m^{i-1}\,, \tag{18c}$$

$$-\,s_m\,(w + x_m^{i-1}) \le \mathrm{d}\tilde{x}_m \le s_m\,(w - x_m^{i-1})\,, \tag{18d}$$

$$-\,s_m\,(w + y_m^{i-1}) \le \mathrm{d}\tilde{y}_m \le s_m\,(w - y_m^{i-1})\,, \tag{18e}$$

$$-\,s_m\,\delta_i \le \mathrm{d}\tilde{x}_m \le s_m\,\delta_i\,, \tag{18f}$$

$$-\,s_m\,\delta_i \le \mathrm{d}\tilde{y}_m \le s_m\,\delta_i\,, \tag{18g}$$

$$s_m \in \{0,1\}\,, \tag{18h}$$

$$\sum_{m\in\mathbb{F}_M} s_m = N\,. \tag{18i}$$

The constraint (18c) guarantees that transmit power stays in the admissible range of $[\underline{P}, \bar{P}]$, while the constraints (18d) and (18e) assure that the update points $\tilde{x}_m^i$ and $\tilde{y}_m^i$ stay in the range of $[-w, w]$, i.e., in the observation area. On the other hand, the constraints (18f) and (18g) allow the update points x_m^i and y_m^i be anywhere within the grid square whose width at i^{th} iteration is the given value δ_i.

The main advantage of optimization problem (18) over (7) is that the variables $\mathrm{d}\tilde{x}_m$ and $\mathrm{d}\tilde{y}_m$ enable us to look for targets at off-grid coordinate values. Even though, (18) is yet of the class of MIQP, but choosing a low number of GPs, i.e., M, and repeating it in an iterative manner leads to a reduced complexity.

The proposed iterative solution of ours is summarized in the Alg. 1. The locations of the targets along with the values of their transmit power are represented by the sets $\mathcal{X}$ and $\mathcal{P}$, respectively. In the proposed algorithm, at each iteration a grid set of size of $M_0, \sqrt{M_0} \in \mathbb{N}$ is formed around each newly found point corresponding to one target. Indeed, in each iteration the grid is updated via (19) in an adaptive fashion. The number of GPs increases to NM_0 from iteration $i=2$, which increases the complexity. Therefore, we propose a second algorithms in which the number of GPs is always M_0. The performance of both algorithms will be compared via simulations in Sec. V.

Algorithm 1 Adaptive grid refinement for joint estimate of the transmit power and the location of multiple targets

initialization:
- set the number of GPs $M_0, \sqrt{M_0} \in \mathbb{N}$
- set the area size $w_0 \in \mathbb{R}_+$
- define the set $\mathcal{G} := \mathcal{G}_{M_0}^{w_0}(0,0)$ using (3)
- set the number of iterations $I \in \mathbb{N}$
- $i \leftarrow 1$

while $i \le I$ **do**
  $M \leftarrow |\mathcal{G}|$
  $w_i \leftarrow \frac{w_0}{2}$
  $\delta_i \leftarrow \frac{w_i}{\sqrt{M}-1}$
  let $(\tilde{x}_m^{i-1}, \tilde{y}_m^{i-1}) \in \mathcal{G}$, $\forall m \in \mathbb{F}_M$
  $p_m^{i-1} \leftarrow \frac{1}{2}(\underline{P} + \bar{P})\,, \forall m \in \mathbb{F}_M$
  find optimal values of s_m, $\mathrm{d}\tilde{x}_m$, $\mathrm{d}\tilde{y}_m$, $\mathrm{d}\tilde{p}_m$ using (18)
  calculate the values of $\tilde{x}_m^i$ and $\tilde{y}_m^i$ using (17)
  update the grid set

$$\mathcal{G} \leftarrow \bigcup_{m\in\mathbb{F}_M | s_m=1} \mathcal{G}_{M_0}^{w_i}(\tilde{x}_m^i, \tilde{y}_m^i) \tag{19}$$

  $i \leftarrow i+1$
end while
$\mathcal{X} := \left\{(\tilde{x}_m^I, \tilde{y}_m^I) \,\middle|\, s_m = 1, \forall m \in \mathbb{F}_M\right\}$
$\mathcal{P} := \left\{p_m^{I-1} + \mathrm{d}\tilde{p}_m \,\middle|\, s_m = 1, \forall m \in \mathbb{F}_M\right\}$
return $\mathcal{X}$ and $\mathcal{P}$

In Alg. 2 the total number of GPs is always M_0 and at each iteration GPs get updated according to (17). The reason that Alg. 1 outperforms the Alg. 2 is that it has more GPs. Consequently, its best GPs, compared to those of Alg. 2, are more likely to be closer to the real position of targets. This, in turns, means the Taylor series expansion is more accurate around the optimal points at each iteration, i.e., the optimal values of $\mathrm{d}\tilde{x}_m^i$ and $\mathrm{d}\tilde{y}_m^i$ are smaller, in general, compared to the second algorithm.

A. Complexity Analysis

In each possible combination of the combinatorial problem (18), only N out of M binary variables s_m are non-zero So,

Algorithm 2 Joint estimate of the transmit power and location of multiple targets

initialization:
- set the number of GPs $M_0, \sqrt{M_0} \in \mathbb{N}$
- $M \leftarrow M_0$
- set the area size $w_0 \in \mathbb{R}_+$
- define the set $\mathcal{G} := \mathcal{G}_{M_0}^{w_0}(0,0)$, using (3)
- let $(\tilde{x}_m^0, \tilde{y}_m^0) \in \mathcal{G}, \forall m \in \mathbb{F}_M$
- set the number of iterations $I \in \mathbb{N}$
- $i \leftarrow 1$

while $i \leq I$ **do**
  $\delta_i \leftarrow \frac{w_0}{\sqrt{M}-1}$
  $p_m^{i-1} \leftarrow \frac{1}{2}(\underline{P} + \bar{P}), \forall m \in \mathbb{F}_M$
  find optimal values of s_m, $\mathrm{d}\tilde{x}_m$, $\mathrm{d}\tilde{y}_m$, $\mathrm{d}\tilde{p}_m$ using (18)
  calculate the values of $\tilde{x}_m^i$ and $\tilde{y}_m^i$ using (17)
  $i \leftarrow i+1$
end while
$\mathcal{X} := \left\{(\tilde{x}_m^I, \tilde{y}_m^I) \,\middle|\, s_m = 1, \forall m \in \mathbb{F}_M\right\}$
$\mathcal{P} := \left\{p_m^{I-1} + \mathrm{d}\tilde{p}_m \,\middle|\, s_m = 1, \forall m \in \mathbb{F}_M\right\}$
return $\mathcal{X}$ and $\mathcal{P}$

in the worst case for each possible combination a quadratic programming (QP) must be solved. The total number of possibilities is $\frac{M!}{N!(M-N)!} \propto e^{N\ln(\frac{M}{N})}$. For each combination Alg. 1 and Alg. 2 solve a QP which has a complexity of $\mathcal{O}(n^3)$, where n is the number of variables, [10]. In our case $n = 3N$, because of variables $\mathrm{d}\tilde{x}_m$, $\mathrm{d}\tilde{y}_m$ and $\mathrm{d}\tilde{p}_m$. Since the total number of iterations is I, the overall complexity of Alg. 1 and Alg. 2 are, respectively, $\mathcal{O}(IM_0^N)$ and $\mathcal{O}(I\frac{M_0^N}{N^N})$. We see that the complexity of Alg. 2 is reduced by an order of N^N in comparison with Alg. 1.

Also, assume for a given N instead of using Alg. 2, we increase the granularity of the original MIQP (7) by an order of I, i.e., the same as number of iterations in Alg. 2. This means increasing number of GPs from M_0 to $M_0 I$, then complexity will be $\mathcal{O}(I^N\frac{M_0^N}{N^N})$. Now, we see that Alg. 2 has a complexity reduction of order I^{N-1}. Besides, increasing the number of GPs from e.g., $M = 25$ to 500, i.e., $I = 20$, does not provide us enough granularity for successful localization.

V. Simulations

To quantify the performance of the proposed algorithm, we resort to numerical evaluation, since there is no reference solution as the benchmark. In our simulation setup we have assumed the parameters given in Tab. II.

In the table J shows the number simulation instances, in each of which the position and power of targets are the same, while sensor positions as well as realizations of ζ_{kn}s are random. Let $\delta_n^j := \left(\hat{x}_n^j - x_n\right)^2 + \left(\hat{y}_n^j - y_n\right)^2$ be the squared Euclidean distance between the estimated position $(\hat{x}_n^j, \hat{y}_n^j)$ of the n^{th} target , at j^{th} instance of the simulation, and its true location (x_n, y_n). Then, positioning root mean square error (PRMSE) in meters is defined by

$$\overline{\delta}_{\text{avg}} = \sqrt{\frac{1}{J}\sum_{j=1}^{J}\delta_{\text{avg}}^j}\,,\ \delta_{\text{avg}}^j := \frac{1}{N}\sum_{n\in\mathbb{F}_N}\delta_n^j \tag{20}$$

as root mean square error (RMSE) of average estimation error of all targets at j^{th} simulation instance, i.e., δ_{avg}^j, [4].

We believe this definition of PRMSE is not accurate enough in case of multiple targets, since it projects the average positioning error of all targets. In fact, it does not guarantee that each and every single target is localized with a certain desired quality. We, additionally, define the worst position estimation in each simulation instance, i.e., $\delta_{\max}^j := \max_{n\in\mathbb{F}_N}\delta_n^j$. Let $\sqrt{\delta_{\max}^j}$ be represented by the random variable Δ. Then, we define the error function P_{d_0} of the maximum positioning error

$$P_{d_0} := 1 - F_\Delta(d_0) = \Pr\left(\Delta > d_0\right)\,, \tag{21}$$

where F_Δ is the cumulative distribution function (cdf) of the error. Indeed, P_{d_0} stands for the probability that at least one of the targets is localized with an error of more than d_0 meters. Similar to (20), RMSE of the transmit power reads

$$\overline{\rho}_{\text{avg}} = \sqrt{\frac{1}{NJ}\sum_{j=1}^{J}\sum_{n\in\mathbb{F}_N}\rho_n^j}\,, \tag{22}$$

where $\rho_n^j := (p_n - \hat{p}_n^j)^2$ is the square error of the estimated power value of n^{th} target at j^{th} realization.

The simulation results for both algorithms are given in Tab. III and Fig. 2 for different values of M and σ, where σ is an indicator of how strong shadowing is. We assume $\sigma =: \sigma_{kn}$ for all pairs of targets/sensors are equal. The problem (18) can be optimally solved by branch and bound method, e.g., using Gurobi [11].

As the figure shows, given $M = 25$ and in the absence of shadowing, the probability that the estimation error is more than 10cm is less than 5% and 23% using Alg. 1 and Alg. 2, respectively. The main advantage here is faster solution at the cost of performance loss.

If we agree that the position error of 10m in an area of 2Km $\times$ 2Km is an acceptable threshold, we see that the two algorithms do violate this threshold with probabilities of 17% and 26%, respectively, for $G = 5$ and $\sigma = 0.01$. Of course, when the shadowing becomes stronger the performance decreases. This is more or less the result of the fact that RSS-based localization is prone to path-loss exponent uncerainties [1]. It does not depend on what we do in Alg. 1 and Alg. 2.

In the table the average execution time $\overline{T}$ in seconds is shown. As we clearly see, the complexity of the second algorithm is much less than the first one. Note, that both algorithms have been executed on the same computer. Even though execution time is not a very accurate measure of complexity, but it roughly shows how the complexity in the second algorithm is reduced.

Table II: Parameters of simulation setup

parameter	K	N	$\bar{P}$	$\underline{P}$	I	J	w_0
value	10	2	1	0.5	20	5000	1 Km

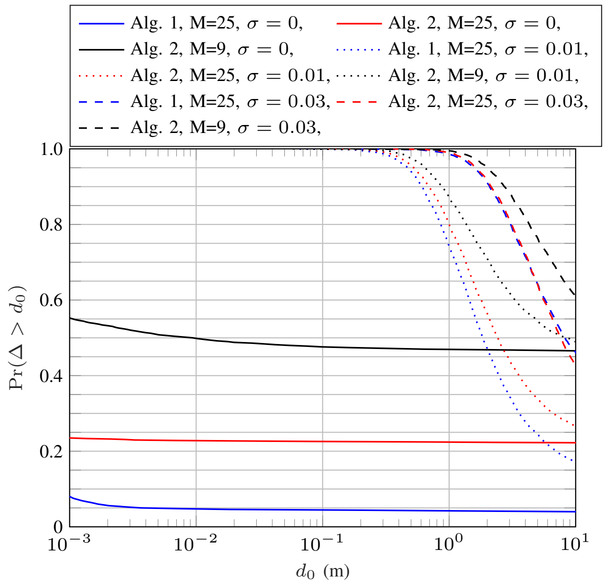

Figure 2. The error function P_{d_0} against values of d_0 in meters, for different values of σ.

Table III: Performance comparison between Alg. 1 & Alg. 2

	σ	M	$\overline{\delta}_{\text{avg}}$	$\overline{\rho}_{\text{avg}}$	$\overline{T}$
Alg. 1	0	25	144.72	0.0308	35.6
Alg. 2	0	25	246.48	0.0862	5.9
Alg. 2	0	9	458.54	0.1411	6.5
Alg. 1	0.01	25	137.04	0.0322	48.5
Alg. 2	0.01	25	245.10	0.0873	11.5
Alg. 2	0.01	9	457.93	0.1420	6.2
Alg. 1	0.03	25	149.39	0.0422	50.8
Alg. 2	0.03	25	245.78	0.0881	12.4
Alg. 2	0.03	9	463.65	0.1413	6.6

Also, we see from the table that the RMSE values $\overline{\delta}_{\text{avg}}$ (given the area size) and $\overline{\rho}_{\text{avg}}$ are relatively small. They could be much smaller, if the global optimality were guaranteed. For example Alg. 1 fails to hand in a reasonable estimation error, e.g., from 1m to 1Km and more, in 5% of the times. Anyhow, even such failure happens rarely and the algorithm delivers a low positioning error, e.g., in order of cm, in 95% of cases, but those rare cases increase the average error, tremendously. Therefore, we are more interested in thet cdf of error is for performance evaluation.

Also, the anomalies in the table, i.e., that RMSE values in some rows of the table, e.g., at $\sigma = 0.01$, are less than those of $\sigma = 0$ can be explained by the following fact. In order to get close to statistical expectation the averaging at higher values of d_0 needs to be done over a very big number of realization, i.e., $J \to \infty$ which is impossible to do. Therefore, $J = 5000$ introduces such anomalies.

VI. Conclusion

We have tackled the problem of multi-source localization based on the RSS readings at different sensor nodes, i.e., receivers, where the transmit power of the targets are assumed to be unknown. Due to the mathematical difficulty of the problem, caused by the superposition of several terms in the received power of each sensor, a closed-form solution is impossible. We, instead, have proposed two low complexity algorithms on the foundation of mixed integer optimization problem. To avoid the $\mathcal{NP}$-hardness associated with such problems, we keep the number of choices, i.e, grid granularity low and come up with adaptive refinement of the GPs. Though, the global optimality is not guaranteed by our solution, but the simulation results shows that in high SNR regime and weak shadowing scenarios the estimation error is in the order of millimeters, in an area of size 2Km $\times$ 2Km, with a high probability.

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