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Decoupled Power Allocation Through Pricing on a CDMA Reverse Link Shared by Energy-Constrained and Energy-Sufficient Data Terminals

Virgilio Rodriguez · Friedrich Jondral · Rudolf Mathar

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Abstract We perform market-oriented management of the reverse link of a CDMA cell populated by data terminals, each with its own data rate, channel gain, willingness to pay (wtp), and link-layer configuration, and with energy supplies that are limited for some, and inexhaustible for others. For both types of energy budgets, appropriate performance indices are specified. Notably, our solution is "decoupled" in that a terminal can choose optimally, irrespective from choices made by the others, because it pays in proportion to its *fraction* of the total power at the receiver, which directly determines its signal-to-interference ratio (SIR), and hence its performance. By contrast, in other similarly-sounding schemes terminals' optimal choices are interdependent, which leads to "games of strategy", and their practical and theoretical complications. We study two situations: pricing for maximal (i) network revenue, and (ii) social benefit. The socially-optimal price is common to all terminals of a given energy class, and an energy-constrained terminal pays in proportion to *the square* of its power fraction. By contrast, the revenue-maximising network sets for each terminal an individual price that drives the terminal to the "revenue per Watt" maximiser. The network price is higher, and drives each terminal to consume less. Distinguishing features of our model are: (i) the simultaneous consideration of both limited and unlimited energy supplies, (ii) the performance metrics utilised (one for each type of energy supply), (iii) the generality of our physical model, which can lead to an optimal link-layer configuration, and (iv) our pricing of the received power *fraction* which yields a "decoupled" solution.

Keywords power control · pricing · game theory · microeconomics · CDMA · revenue maximisation

1 Introduction

In telecommunication networks, pricing is a critical tool to both generate revenues, and induce efficient resource use. Herein, we propose and analyse a technical-economic scheme for the management of the reverse link of a CDMA cell populated by data terminals (each with its own data rate, channel gain, willingness to pay (wtp), and possibly individual link-layer configuration). Some terminals are battery-powered while others have boundless energy, and we specify pertinent utility functions for each type. The present paper builds upon [19], where the network sets prices that maximise its revenues, and [18], where a "social planner" uses pricing to drive the terminals to a socially-optimal allocation.

For reverse link CDMA power allocation, many useful price-based algorithms have been reported [1, 6, 10, 22, 23, 25]. Previous schemes rely on per-Watt pricing. However, a terminal's performance does not solely depend on the amount of power it buys. It also depends on

V. Rodriguez (✉)
Institute for Communication Networks (ComNets), RWTH, Aachen, Germany
e-mail: rodriguez@comnets.rwth-aachen.de

F. Jondral
Institut für Nachrichtentechnik, Karlsruher Institut für Technologie, Karlsruhe, Germany
e-mail: friedrich.jondral@kit.edu

R. Mathar
Institute for Theoretical Information Technology, RWTH, Aachen, Germany
e-mail: mathar@ti.rwth-aachen.de

the power choices made by the others. Thus, terminals choices are interdependent, which leads to a "game of strategy", a model in which several "selfish" agents make interdependent choices [7, 12].

Games engender both theoretical and practical problems. The typical solution concept is the Nash equilibrium (NE), which (i) may not exist, (ii) if existing, may not be unique, and (iii) is in general inefficient [5]. And even if a unique NE can be proved to exist, it may be unclear: (a) *how* will the players reach the NE, and (b) after how many "iterations". The facts that terminals frequently enter and exit the network at arbitrary times, and that they are "anonymous" to each other, further complicate matters. Additionally, if per-Watt pricing is implemented as a "true" billing scheme in a commercial network, it may face substantial consumer resistance, because in the common economic scenario with which the customer is familiar, his/her utility depends solely on how s/he spends her/his own income among a number of "goods", irrespectively of how others spend theirs.

Below we provide a "decoupled" solution. A terminal pays according to its fraction of the total power at the receiver. Because this fraction directly determines a terminal's signal-to-interference ratio (SIR), and hence its performance, for a given price, a terminal can make an optimal choice without worrying about the choices made by others. A decoupled solution is superior for both *technological* and *marketing* reasons: it shifts complexity from the terminal to the base station, it avoids Nash-equilibrium existence/convergence issues, and places the user in the familiar territory in which s/he "controls her/his own destiny". As a base line for performance evaluation, we do analyse a "game" in which each terminal chooses its power without economic incentives or a central controller. By also analysing the conditions yielding a "socially optimal" allocation, we conclude that our pricing scheme can, with relatively minor modifications, maximise "socially benefit", instead of network profit.

Additional distinguishing features of our model are: (i) the simultaneous consideration of both limited and unlimited energy budgets as in real networks, where battery-powered terminals coexist with those powered from a vehicle or the power-grid (including fixed wireless local loop), (ii) the performance metrics utilised (one for each type of energy budget), and (iii) the generality of our physical model (each terminal may have its own data rate, channel gain, willingness to pay (wtp), and even link-layer configuration).

Below, we first discuss the physical model, and the terminals' technical-economic rationale. Then, we address feasibility issues, and identify a terminal's power fraction as key. We then analyse how each terminal reacts under power-fraction pricing. Subsequently, we describe how the network can set prices to maximise its revenue. At that point, we shift to the zero-price game. Then, we describe the "socially optimal" allocation. Several numerical examples and figures are given throughout. In a final section, we summarise and discuss key contributions.

2 Generalities

2.1 Physical model

N terminals *upload* data *to* a CDMA base station (BS). The sub-index i identifies a terminal. z or x may be used as generic function arguments (e.g., $\sqrt{x}$ is a concave graph).

- p_0 is the average Gaussian noise power
- E_i is the energy budget
- $\hat{P}_i$ is the power constraint
- h_i is the channel gain
- $p_i = h_i P_i$ is the received power
- For convenience, we let

$$\bar{p} := \sum_{i=1}^{N} p_i \tag{1}$$

- $Y_i = \sum_{j \neq i} p_j + p_0$ is i's interference

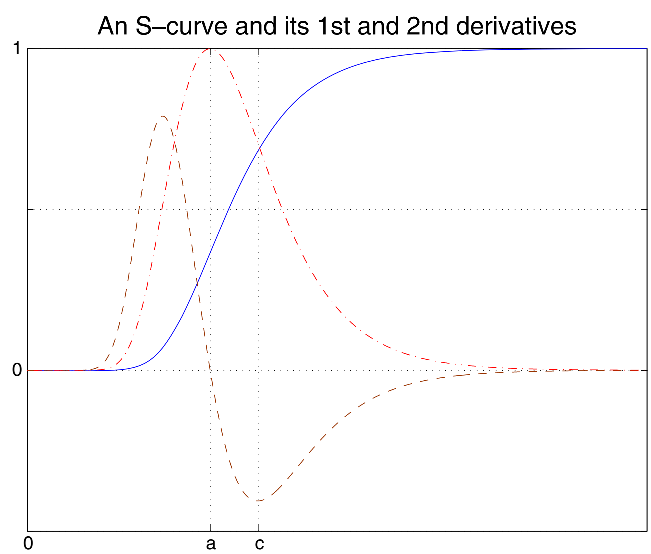


Fig. 1 An S-curve and scaled versions of the graphs of its 1st (*dash-dot*) and 2nd (*dash*) derivatives. The inflexion occurs at $x = a$

Table 1 Notation summary

Power	Ratios
p_0: noise	$\Gamma_i := W/R_i$: Spreading gain
$\hat{P}_i$: constraint	$\kappa_i := p_i/Y_i$ (carrier-to-interference)
$p_i := h_i P_i$: received	$\sigma_i := \Gamma_i \kappa_i$: (signal-to-interference)
$Y_i := p_0 + \sum_{j \neq i} p_j$	$\pi_i := \kappa_i/(1+\kappa_i)$ (power fraction)
$\bar{p} := \sum_{i=1}^N p_i$	$\bar{\pi} := \sum_{j=1}^N \pi_j$

- $\kappa_i = p_i/Y_i$ is the carrier-to-interference ratio (CIR)
- W is the available bandwidth, assumed equal to the common chip rate
- R_i is the data rate.
- $\Gamma_i = W/R_i$ is the spreading gain
- $\sigma_i = \Gamma_i \kappa_i$ is the signal-to-interference-plus-noise ratio (SIR)
- M_i-bit packets carrying $L_i < M_i$ information bits are used.
- F_i is the packet-success rate function (PSRF) giving the probability of correct reception of a data packet as a function of the SIR.
- For some technical reasons, $f_i(x) := F_i(x) - F_i(0)$ replaces F_i. Its graph is assumed to have the S-shape shown in Fig. 1. Our analysis does *not* rely on any specific PSRF.
- Furthermore, as described in the Appendix, f_i satisfies the technical Assumptions 1–2 that guarantee that each of certain graphs retains a desired shape ("S" or "bell") (Table 1).

2.2 Terminals' general objective

Two categories of data terminals are of interest: (i) energy-constrained (battery powered), and time-driven, referred as e-terminals and t-terminals, below. An **e**-terminal focuses on the total number of information bits transferred with its total **e**nergy budget. The **t**-terminal focuses on the number of information bits transferred over a reference **t**ime period (such as the time unit). The model below works for both categories.

The utility function has the "quasi-linear" form [24, Ch. 10]: $v_i B_i + y_i$ where:

- (i) v_i is the "willingness to pay" (wtp) (the monetary value to the terminal of one information bit successfully transferred),
- (ii) B_i is the (average) number of information bits the terminal has successfully transferred within a reference length of time, say τ, and
- (iii) y_i is the amount of money the terminal has left after any charges and rewards are computed.
- B_i generally depends on a vector of "resources" $\mathbf{z}$. When the terminal must pay $c_i(\mathbf{z})$, it chooses $\mathbf{z}$ to maximise $v_i B_i(\mathbf{z}) + [D_i - c_i(\mathbf{z})]$. D_i is the terminal's monetary budget, which limits its total expenditure and, if "large", need not be considered, in which case the terminal maximises benefit minus cost:

$$v_i B_i(\mathbf{z}) - c_i(\mathbf{z}) \tag{2}$$

3 Feasibility of power ratios

3.1 SIR/CIR feasibility

Definition 3.1 Let π_i be defined by

$$\pi_i = \frac{\kappa_i}{1+\kappa_i} \equiv \frac{\sigma_i}{\Gamma_i + \sigma_i} \tag{3}$$

Proposition 3.1 *π_i equals i's fraction of the total power at the receiver (including noise), that is,*

$$\frac{\kappa_i}{1+\kappa_i} = \frac{p_i}{\sum_{j=1}^N p_j + p_0} \tag{4}$$

Proof With $Y_i := \sum_{j=1}^N p_j - p_i + p_0$ (the interference experienced by terminal i), and $\kappa_i = p_i/Y_i$ (the CIR), Eq. 3 can be written as:

$$\frac{p_i/Y_i}{p_i/Y_i + 1} \equiv \frac{p_i}{p_i + Y_i} \equiv \frac{p_i}{\sum_{j=1}^N p_j + p_0} \tag{5}$$

□

Proposition 3.2 *There is a one-to-one correspondence between κ_i and π_i given by $\kappa_i = \kappa(\pi_i)$, where*

$$\kappa(z) := \frac{z}{1-z} \text{ for } z \in [0, 1) \tag{6}$$

κ is strictly increasing and convex.

Proof

$$\pi_i = \frac{\kappa_i}{1+\kappa_i} \implies \frac{1}{\pi_i} = \frac{1}{\kappa_i} + 1$$
$$\implies \kappa_i = \frac{1}{1/\pi_i - 1} \equiv \frac{\pi_i}{1-\pi_i}$$

$\kappa(z) = z/(1-z) \implies \forall\, z \in [0, 1)\ \kappa'(z) = (1-z)^{-2} > 0$ and $\kappa''(z) = 2(1-z)^{-3} > 0$ (see Fig. 2). □

Consider a system of N equations of the form

$$\frac{p_i}{\sum_{\substack{j=1 \\ j \neq i}}^N p_j + p_0} = \kappa_i \tag{7}$$

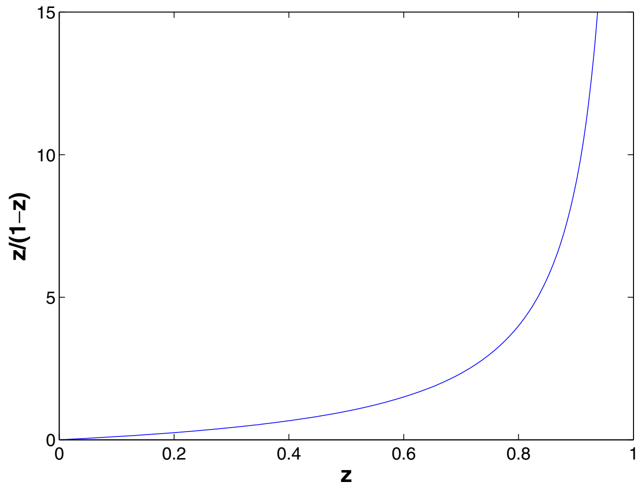


Fig. 2 Graph of $\kappa(z) = z/(1-z)$, the CIR as a function of the terminal's fraction of the total receiver power

Theorem 3.1 *If*

$$\bar{\pi} := \sum_{j=1}^{N} \pi_j < 1 \tag{8}$$

then the system defined by Eq. 7 has a positive solution given by

$$\frac{p_i}{p_0} = \frac{\pi_i}{1-\bar{\pi}} \tag{9}$$

Proof Condition (8) implies that Eq. 9 is positive. Replacing Eq. 9 into Eq. 7 yields:

$$\frac{p_i}{\sum_{\substack{j=1\\j\neq i}}^{N} p_j + p_0}$$
$$= \frac{p_0\pi_i/(1-\bar{\pi})}{p_0\sum_{j=1}^{N}\pi_j/(1-\bar{\pi}) - p_0\pi_i/(1-\bar{\pi}) + p_0}$$
$$= \frac{\pi_i}{\bar{\pi} - \pi_i + 1 - \bar{\pi}} = \frac{\pi_i}{1-\pi_i}$$

By Proposition 3.2, the last expression equals κ_i. □

Corollary 3.1 *If $p_0 \approx 0$ (cell is "interference limited"), and*

$$\sum_{j=1}^{N} \pi_j = 1 \tag{10}$$

then any power vector proportional to the vector $(\pi_1, \cdots, \pi_N)$ is a solution to the system defined by Eq. 7.

Proof Perform direct substitution into Eq. 7 with $p_0 = 0$ and $p_i = k\pi_i$ where $k > 0$:

$$\frac{p_i}{\sum_{\substack{j=1\\j\neq i}}^{N} p_j} = \frac{k\pi_i}{k\sum_{j=1}^{N}\pi_j - k\pi_i}$$
$$= \frac{\pi_i}{1-\pi_i}$$

By Proposition 3.2, the last expression equals κ_i. □

3.2 Power limits and the "ruling terminal"

Definition 3.2 Terminal k is a *ruling* terminal if

$$\frac{\pi_k}{h_k\hat{P}_k} \geq \frac{\pi_i}{h_i\hat{P}_i} \quad \forall i \in \{1, \ldots, N\} \tag{11}$$

Lemma 3.1 *Let k be a ruling terminal. If*

$$\bar{\pi} \leq 1 - \frac{\pi_k}{h_k(\hat{P}_k/p_0)} \tag{12}$$

then the power levels given by Eq. 9 obey the applicable power constraints (the system is feasible).

Proof By definition of k, Eq. 12 implies that

$$\bar{\pi} \leq 1 - \frac{\pi_i}{h_i(\hat{P}_i/p_0)} \quad \forall i \tag{13}$$

Since Eq. 12 $\Longrightarrow$ Eq. 8, Eq. 9 has a positive solution: $p_i^* = p_0\pi_i/(1-\bar{\pi})$.

From Eq. 13,

$$\frac{\pi_i}{h_i(\hat{P}_i/p_0)} \leq 1 - \bar{\pi} \implies \frac{\pi_i}{1-\bar{\pi}} p_0 \leq h_i\hat{P}_i \tag{14}$$

Thus, Eq. 12 implies that $p_i^* \leq h_i\hat{P}_i\ \forall i$. □

Remark 3.1 π_i represents terminal i's desired quality of service (QoS), and $h_i\hat{P}_i$ the highest power level that i can get to the receiver; thus the ratio $\pi_i/(h_i\hat{P}_i)$ yields i's desired QoS per unit of power available to i. The terminal with the highest such ratio wants, loosely speaking, "the most for the least", and hence is in the worst situation to achieve its QoS. If the ruling terminal can achieve its desired QoS, all others can as well.

4 Terminal's choice under pricing

4.1 Pie division through pricing

π_i is terminal i's fraction of the total power at the receiver. Thus, the network can view power allocation as a "pie division" problem: how big the "slice" to be allocated to a given terminal. The case of the interference-limited cell is the easiest to visualise: the "pie size" is necessarily 1 (Eq. 10), and the "slices" immediately indicate the received power levels. For a given value of π_i, the terminal can obtain directly the corresponding CIR (Eq. 6), and hence its performance. Thus, if the resource manager sets a price c_i at which terminal i can "buy" a "slice" π_i, for the given c_i, i can choose optimally π_i, *irrespective* of the choices made by others. In principle, the terminal's individual choices may exceed "capacity", but this may be avoided if the price is "right", or the network may select an "optimal" subset of terminals to be served.

In the general case, a terminal needs to know the total received power $\bar{p} + p_0$ in order to calculate its emission power, for its chosen power fraction. However, the analysis below shows that, for a given price, the total received power will not affect the terminal's optimal fraction.

4.2 General technical results

We first provide some technical results that are relevant to various parts of this work. Others more specific results will be inserted below where appropriate.

Lemma 4.1 *Let $h : [d, e] \to \Re_+$ be a continuous strictly quasi-concave function h with maximal value Y at X where $d < X < e$, that is, $d \leq x_1 < x_2 \leq X \implies h(x_2) > h(x_1)$ and $X \leq x_1 < x_2 \leq e \implies h(x_1) > h(x_2)$ (Fig. 3). Suppose $h(d) \geq h(e)$. Let $\mathcal{S} := \{x : h(x) = c\}$ (the set of solutions to $h(x) = c$).*

$$\mathcal{S} = \begin{cases} \emptyset & \text{if } c > Y \text{ or } c < h(e) \\ \{X\} & \text{if } c = Y \\ \{a, b\} \text{ with } a < X < b & \text{if } h(d) < c < Y \\ \{b\} \text{ with } X < b & h(e) < c < h(d) \end{cases}$$

Furthermore, $a < x < b \implies h(x) > c$.

Proof The first two cases are immediate, by definition. The next two cases follow from the fact that h is strictly increasing in (d, X) and strictly decreasing in (X, e). □

Lemma 4.2 *Suppose $S : \Re^+ \to [0, d]$ is an S-curve (Definition A.1) with inflexion at z_f. Let $\mathcal{B}(z) := S(z)/z$ with $\mathcal{B}(0) := \lim_{z \downarrow 0} \mathcal{B}(z) \equiv S'(0)$. Then, (i) there is a unique tangent line from the origin to $S(z)$, denoted as $c^* z$ and called the tangenu, with tangency point, genu, $(z^*, S(z^*))$, where $z^* > z_f$. (ii) $\mathcal{B}$ is strictly quasi-concave, and its unique maximiser in the interval $[0, Z]$ is $\min(z^*, Z)$.*

Proof See [16]. □

Remark 4.1 Figures 4 and 5 show the tangenu (tangent from (0,0)) and genu (tangency point) for some S-curves. Figure 6 shows the shape of the graph $S(z)/z$.

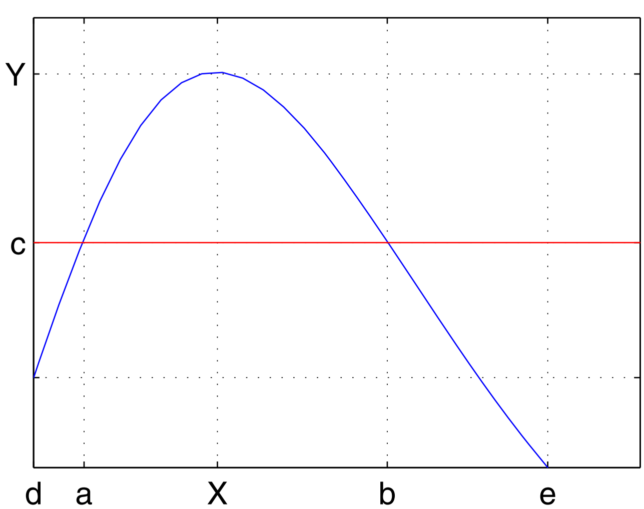


Fig. 3 Graph of a single-peaked function h with maximal value Y at X. a and b are the two solutions to $h(x) = c$, where $h(d) < c < Y$

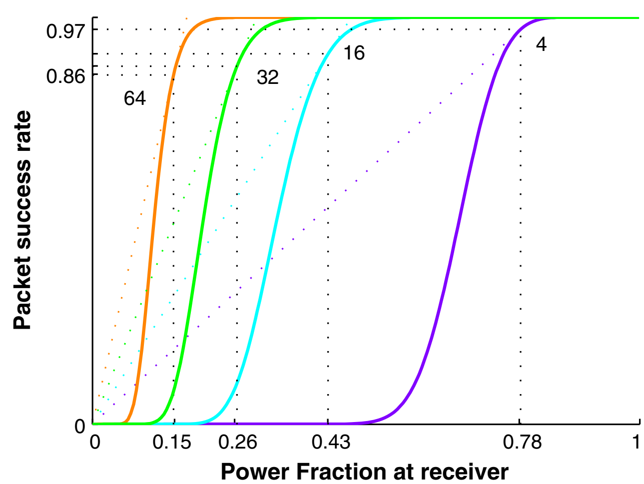


Fig. 4 The composite function $f(\Gamma\kappa(z)) = f(x)$, with f the PSRF, Γ the spreading gain, and $\kappa(z) = z/(1-z)$, plays a key role, especially its tangenu (tangent to (0,0)) and genu (tangency point). $f(x) = \left[1 - \exp\left(-x/2\right)/2\right]^{80} - 2^{-80}$ is displayed. The value of $\Gamma \in \{4, 16, 32, 64\}$ is indicated near the genu

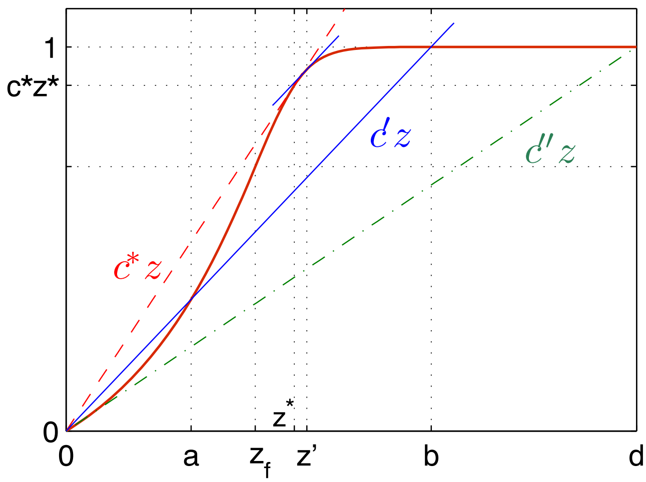


Fig. 5 Maximising $S(z) - cz$ subject to $z \le Z$, where $S(z)$ is an "S-curve". If $c > c^*$ or $c = c'$ and $Z < a$ then $z = 0$ is optimal. Otherwise $\min(Z, z')$ is the maximiser. At z', the curve's tangent (*short blue line*) is parallel to the cost line $c'z$

4.3 Choice by a time-driven terminal

4.3.1 Analysis in term of the power fraction

As discussed in Section 2.2, a t-terminal wishes to maximise its performance over some pre-specified length of time, say the time unit, considering both its benefit and its cost.

For a given π_i, the terminal's cost is $c_i\pi_i$, and (with $\kappa(z)$ given by Eq. 6) the average number of information bits transferred over a time unit is B_i:

$$\frac{L_i}{M_i} R_i f_i(\Gamma_i \kappa(\pi_i)) \tag{15}$$

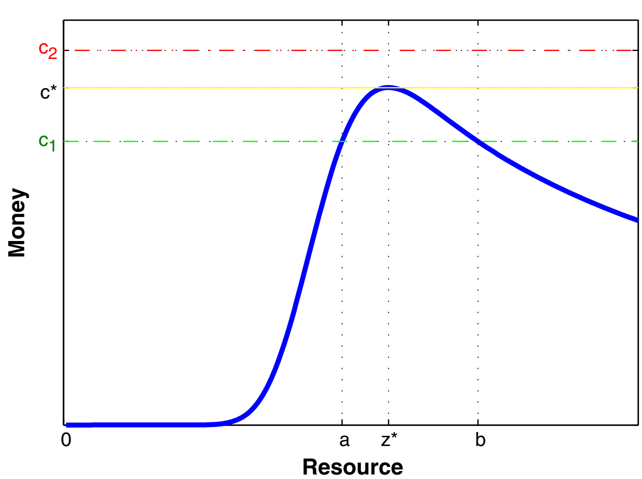


Fig. 6 For $c \le c^*$ the e-terminal chooses z^*; else $z = 0$ is optimal

Then, the terminal should choose the value of π_i that maximises benefit minus cost, that is:

$$v_i \frac{L_i}{M_i} R_i f_i(\Gamma_i \kappa(\pi_i)) - c_i \pi_i \tag{16}$$

f_i is the (slightly modified) packet-success-rate function, whose graph as a function of its argument, the SIR, is an S-curve. $f_i(\Gamma_i\kappa(z))$ is a composite function of f_i and κ, with independent variable z and parameter Γ_i. The graph of $f_i(\Gamma_i\kappa(z))$ inherits the S-shape of f_i (see Fig. 4 and the Appendix). Thus, the terminal maximises an expression of the form $S(z) - cz$, where S is some S-curve, a problem whose solution is given by Theorem 4.1:

Theorem 4.1 *Consider the problem of maximising $S(z) - cz$ subject to $0 \le z \le Z$ where c is a positive number and S is an S-curve (Definition A.1) with inflexion at z_f, and satisfying $c^*z^* = S(z^*)$. (i) If $c > c^*$ then $z = 0$ is the maximiser. (ii) For $c < c^*$, the equation $S'(z) = c$ has exactly one solution, z_c, that is greater than z_f. (iii) If $z_c \le Z$ then z_c is the maximiser. If $z_c > Z$ then the maximiser is either Z, if $S(Z) \ge cZ$, or zero, otherwise.*

Proof

(i) If $c > c^*$ then $\forall z > 0$, $cz > S(z) \implies S(z) - cz < 0$. Thus, $z = 0$ is the maximiser.
(ii) If $c < c^*$ then a maximiser of $S(z) - cz$ that is inside (0,Z) must satisfy the first-order necessary optimising condition: $S'(z) = c$. Because S is an S-curve, its derivative is strictly increasing between zero and z_f and strictly decreasing for $z > z_f$ (as shown in Fig. 1). If $S'(0) \le c < c^*$, then by Lemma 4.1, $S'(z) = c$ has 2 solutions z_o, z_c such that $z_0 < z_f < z_c$. If $c < S'(0)$ (as is c'' in Fig. 5) then, also by Lemma 4.1, $z_c > z_f$ is the only solution to $S'(z) = c$.
(iii) At z_c, S' is decreasing (S'' is negative) which implies that z_c is, if feasible, a maximiser. By contrast, at z_0, S' is increasing, therefore z_0 is a minimiser. If $z_c < Z$, z_c is the only feasible point that satisfies the necessary and sufficient conditions for a maximiser, and therefore it is the global maximiser. If $z_c > Z$, then one of the extreme points is the maximiser: 0 or Z. If $S(Z) - cZ > 0$ (e. g. $Z > a$ in Fig. 5) then Z is the maximiser. Otherwise zero maximises $S(z) - cz$. □

Remark 4.2 The case $c = c^*$, $Z \ge z^*$ is, in principle, indeterminate, because for $z = z^*$, $c^*z^* = S(z^*)$ which implies that the objective function evaluates to zero, just as it does for $z = 0$. Thus, in this case, either 0 or z^* are optimal. However, we shall assume that in any such situation the decision-maker prefers the positive value (i.e., to use the system) over zero (i.e., to remain inactive).

4.3.2 Analysis in terms of the SIR

One can perform an analysis similar to that of Section 4.3.1 in terms of the SIR, x. In this case, the benefit depends directly on x_i, and no composite function is needed. However, the cost is now a nonlinear function of x_i, namely $c_i x_i/(x_i + \Gamma_i)$ where Γ_i is the spreading gain.

Considering Eqs. 15 and 16, the terminal chooses x_i to maximise

$$v_i \frac{L_i}{M_i} R_i f_i(x_i) - c_i \frac{x_i}{x_i + \Gamma_i} \tag{17}$$

Thus, the terminal maximises an expression of the form $S(x) - c(x)$ subject to $0 \le x \le X$, where S is some S-curve and $c(x) := cx/(x + \Gamma)$ with c and Γ positive real numbers. Below we sketch a solution for this problem that follows closely the proof of Theorem 4.1. Some technical details are ignored.

An interior maximiser must satisfy $S'(x) = c'(x) = c\Gamma/(x + \Gamma)^2$, or

$$\Gamma \left(\frac{x}{\Gamma} + 1 \right)^2 S'(x) = c \tag{18}$$

By solving Eq. 18 together with the intersection condition $cx/(x + \Gamma) = S(x)$ one obtains c^* such that $c^* x/(x + \Gamma)$ is tangent to $S(x)$ at x^*.

It is clear that $c > c^*$ implies $\forall x > 0$, $S(x) - c(x) < 0$. Thus, $x = 0$ is the maximiser.

For $c < c^*$, it is also clear that if X is less than the abscissa of the point where the cost curve first intersects $S(x)$ (e.g., $X < a_2$ for $c = c_2$ in Fig. 6), $\forall x \in [0, X]$, $S(x) - c(x) < 0$. Then, $x = 0$ is again the maximiser.

For $c < c^*$ and $X > a_2$, let $h(x) := \Gamma(x/\Gamma + 1)^2 S'(x)$ (left side of Eq. 18). By Assumption 2 in the Appendix, h retains the single-peaked shape of S' (see Fig. 1).

Let $h(\hat{x})$ be h's maximal value. By Lemma 4.1, as long as $c < h(\hat{x})$, Eq. 18 has two solutions, x_1 and x_2 such that $x_1 < \hat{x} < x_2$, and $h(x) > c \; \forall x \in (x_1, x_2)$. Thus over (x_1, x_2) the derivative of the objective function $S'(x) - c\Gamma/(x + \Gamma)^2$ is positive, which implies that the largest feasible value in (x_1, x_2) is best. Hence, the maximiser is $\min(x_2, X)$.

The resulting payment by the terminal (network's revenue) at the chosen operating point is $cx/(x + \Gamma)$. If the x_2 is feasible, Eq. 18 is satisfied, and revenue reduces to:

$$x \left(\frac{x}{\Gamma} + 1 \right) S'(x) \tag{19}$$

Under suitable assumptions, the graph of Eq. 19 has the bell shape shown in Fig. 6.

4.4 Choice by an energy-limited terminal

We first establish some additional technical results about general maximisation problems involving an S-curve. Subsequently we explain how these results relate to the e-terminal problem.

4.4.1 Relevant technical results

Theorem 4.2 *Suppose S is an S-curve (Definition A.1) with inflexion at z_f, such that $c^* z^* = S(z^*)$. Let $\mathcal{B}(z) := S(z)/z$ with $\mathcal{B}(0) := \lim_{z \downarrow 0} \mathcal{B}(z) \equiv S'(0)$. Consider the problem of maximising $\mathcal{B}(z) - c$ subject to $0 \le z \le Z$ where $c > 0$ (see Fig. 7). If (ia) $c > c^*$ or (ib) $c \le c^*$, $z^* > Z$ and $\mathcal{B}(Z) < c$, then the maximiser is zero. (ii) If $c \le c^*$ and $z^* \le Z$ then z^* is optimal. (iii) If $c \le c^*$, $z^* > Z$ and $\mathcal{B}(Z) \ge c$, then $z = Z$ is optimal.*

Proof Since c is a constant independent of z, the $z > 0$ that maximises $\mathcal{B}(z) - c$ is the same that maximises $\mathcal{B}(z)$.

By Lemma 4.2 $\min(z^*, Z)$ maximises $\mathcal{B}(z)$.

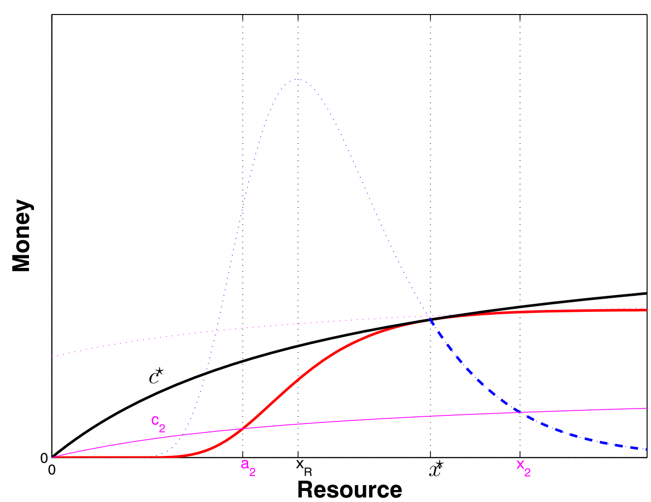


Fig. 7 Terminal chooses SIR x that maximises benefit minus cost: $S(x) - cx/(x + \Gamma)$; e.g., for $c = c_2$ it chooses x_2 where the derivative of the S-curve equals that of the cost-curve. The largest c for which the terminal will operate is c^*. The bell-shaped curve corresponds to the terminal's payment (network's revenue) as a function of its choice, over an appropriate range

By definition of z^*, $S(z^*)/z^* - c \equiv c^* - c$.

Therefore, $c > c^*$ implies that $z = 0$ maximises $\mathcal{B}(z) - c$, while $c \leq c^*$ and $z^* \leq Z$ implies that z^* is optimal.

If $c \leq c^*$ but $z^* > Z$ then the highest feasible value for the objective function is $S(Z)/Z - c$.

Therefore, if $c \leq c^*$, $z^* > Z$ and $S(Z)/Z \geq c$, then $z = Z$ is optimal; while $c \leq c^*$, $z^* > Z$ and $S(Z)/Z < c$, implies that $z = 0$ is optimal. □

Theorem 4.3 *Suppose S is an S-curve (Definition A.1) with inflexion at z_f, such that $c^*z^* = S(z^*)$. Let $\mathcal{B}(z) := S(z)/z$ with $\mathcal{B}(0) := \lim_{z\downarrow 0} \mathcal{B}(z) \equiv S'(0)$. By Lemma 4.2 $\mathcal{B}$ is strictly quasi-concave with a global maximum at z^*(i.e., it is single-peaked). Further assume that, as shown in Fig. 8, $\mathcal{B}(z)$ starts out convex and has a single inflexion point between 0 and z^*, at z_l. (i) Then, there is a unique line from the origin, denoted as $c_x z$, that is tangent to the graph of $\mathcal{B}$ (Fig. 8) at $z_x < z^*$. Consider the problem of maximising $\mathcal{B}(z) - cz$ subject to $0 \leq z \leq Z$ with $c > 0$. (ii) If $c > c_x$, then $z = 0$ is the maximiser. (ii) For $c < c^*$, the equation $\mathcal{B}'(z) = c$ has exactly one solution, z_M, that is greater than z_l. (iii) If $z_M \leq Z$ then z_M is the maximiser. If $z_M > Z$ then the maximiser is either Z, if $\mathcal{B}(Z) \geq cZ$, or zero, otherwise.*

Proof

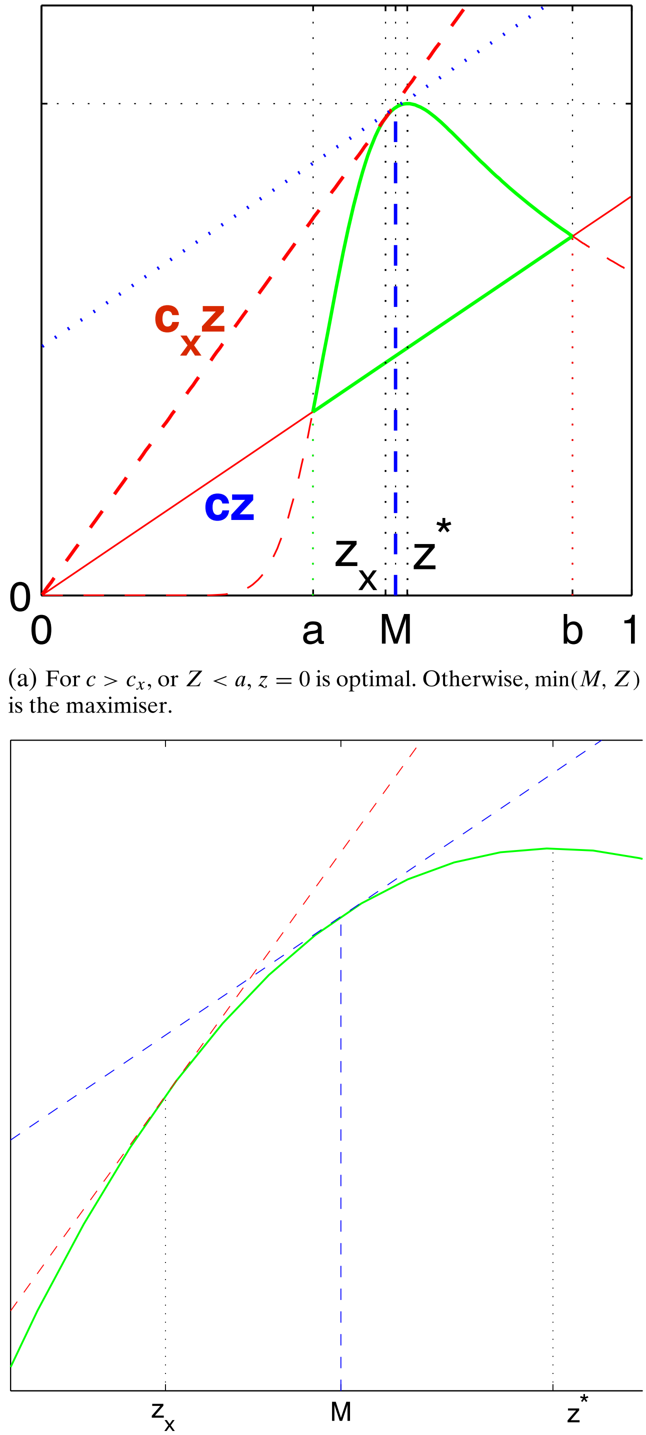
(i) By hypothesis, between 0 and z^*, $\mathcal{B}(z)$ has the S-shape, and the development in [16] implies the existence and uniqueness of the tangent $c_x z$. The reminder of the proof is very similar to that of Theorem 4.1.

(ii) If $c > c_x$, the line cz lies entirely over the curve except at the origin, thus, $\mathcal{B}(z) - cz < 0$ for any $z > 0$, and $z = 0$ is the maximiser.

(iii) For $0 < c < c_x$, a maximiser in $(0, Z)$ must satisfy $\mathcal{B}'(z) = c$. Any solution of $\mathcal{B}'(z) = c$ must be less than z^* because by strict quasi-concavity, $\mathcal{B}'(z) \leq 0$ for $z \geq z^*$. Also, since $\mathcal{B}(z)$ has, by hypothesis, an S-shape over $(0, z^*)$, $\mathcal{B}'$ is itself single-peaked over $(0, z^*)$, therefore by Lemma 4.1, $\mathcal{B}'(z) = c$ has at least one solution, z_M, to the right of z_l (where its peak is), and $\mathcal{B}''(z_M) < 0$.

If $z_M \leq Z$, then z_M is optimal, since it is feasible, and it is the only interior point that satisfies the necessary and sufficient condition for a maximiser.

If $z_M > Z$, then one of the extreme points is the maximiser. If $\mathcal{B}(Z) > cZ > 0$ (e.g., $Z > a$ in Fig. 8a), then Z is the maximiser. Otherwise zero maximises $\mathcal{B}(z) - cz$. □



(a) For $c > c_x$, or $Z < a$, $z = 0$ is optimal. Otherwise, $\min(M, Z)$ is the maximiser.

(b) Detailed view around the optimum.

Fig. 8 Maximising $\mathcal{B}(z) - cz$ subject to $0 \leq z \leq Z$

Remark 4.3 The case $c = c_x$ is indeterminate, but we assume that z_x is chosen (see Remark 4.2). Figure 8 illustrates the solution (z_M is denoted as M). If $c > c_x$, the cost line cz lies entirely over the curve except at the origin, and $z = 0$ is optimal. At the other extreme, if

$c \approx 0$, the maximiser is $\approx z^*$. If $c < c_x$, at the maximiser z_M a tangent to the curve is parallel to the cost line.

4.4.2 E-Terminal's choice with linear pricing

For an energy-limited terminal, the natural period of interest is battery life. For any given $\bar{p}$ the transmission power corresponding to π_i is:

$$P_i = p_i/h_i \equiv \pi_i(\bar{p} + p_0)/h_i \tag{20}$$

With energy E_i, battery life is

$$T_i = E_i/P_i \equiv \frac{E_i h_i}{\pi_i(\bar{p} + p_0)} \tag{21}$$

By Eq. 21, the terminal's total cost over the period of interest is given by

$$c_i \pi_i T_i \equiv c_i \frac{E_i h_i}{\bar{p} + p_0} \tag{22}$$

Notice that π_i drops out of the total cost expression.

The terminal's benefit is $v_i B_i$, with B_i the total (average) number of (energy-earned) information bits over the period T_i:

$$B_i(\pi_i) = \frac{L_i}{M_i} R_i f_i(\Gamma_i \kappa(\pi_i)) T_i \tag{23}$$

The terminal chooses π_i to maximise utility (total benefit minus total cost):

$$\frac{E_i h_i}{\bar{p} + p_0}\left(\frac{L_i}{M_i} v_i R_i \frac{f_i(\Gamma_i \kappa(\pi_i))}{\pi_i} - c_i\right) \tag{24}$$

c_i is a known constant. The composite function $f_i(\Gamma_i \kappa(z))$ retains the S-shape of f_i (see Fig. 4 and the Appendix). Thus, the terminal must maximise an expression of the form $S(z)/z - c$, a problem whose solution is given by Theorem 4.2.

Remark 4.4 Equation 24 indicates that the the total received power, $\bar{p} + p_0$ does affect the terminal's utility (benefit minus cost). But it appears as a factor of both the benefit and the cost; in the end it does not affect the terminal's optimal choice. Whatever the total received power turns out to be, the terminal still wants the quantity in brackets to be as large as possible. Thus, it chooses the π_i that maximises the bracketed expression in Eq. 24.

4.4.3 E-terminal's choice with quadratic pricing

As discussed in Section 4.4.2, with a linear price the independent variable drops out of the e-terminal's total cost expression. Thus, the price determines whether or not this terminal operates, but it does not affect its operating point. This is intuitively unappealing, and in fact undesirable, for reasons given below. In this section we consider a cost function of the form cz^2 (quadratic pricing).

With energy E_i, battery life T_i is $E_i h_i/(\pi_i(\bar{p} + p_0))$ (Eq. 21). Total cost is now $c_i \pi_i^2 T_i$, or

$$\frac{E_i h_i}{\bar{p} + p_0} c_i \pi_i \tag{25}$$

Now, π_i does *not* drop out (compare to Eq. 22).

The terminal chooses π_i to maximise total benefit minus total cost:

$$\frac{E_i h_i}{\bar{p} + p_0}\left(\frac{L_i}{M_i} v_i R_i \frac{f_i(\Gamma_i \kappa(\pi_i))}{\pi_i} - c_i \pi_i\right) \tag{26}$$

As before, $f_i(\Gamma_i \kappa(z))/z$ is the familiar ratio of an S-curve to its argument. The solution to the e-terminal's maximisation problem with quadratic pricing follows Theorem 4.3 (Section 4.4.1). See also Remark 4.4.

5 Revenue-maximising prices

5.1 Optimal single-terminal pricing

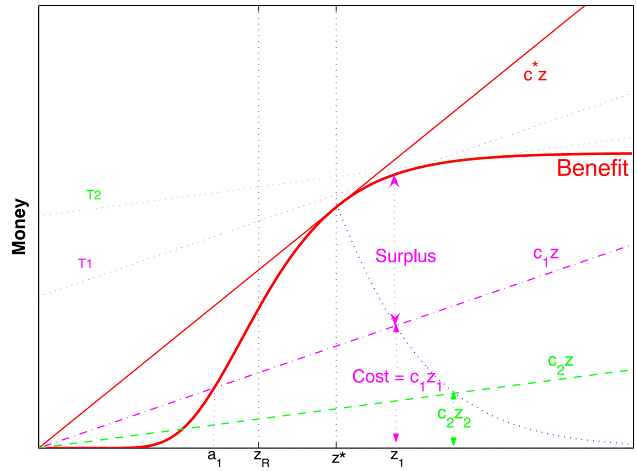
5.1.1 Pricing for a time-driven terminal

The analysis of Section 4.3 shows that as the price grows, the terminal consumes less; i.e., chooses smaller values of the resource. Since the network is interested in revenue (the product of the quantity purchased by the price), it is unclear what is the network's "best" price. To determine this, the network must observe how its revenue varies as a function of the price.

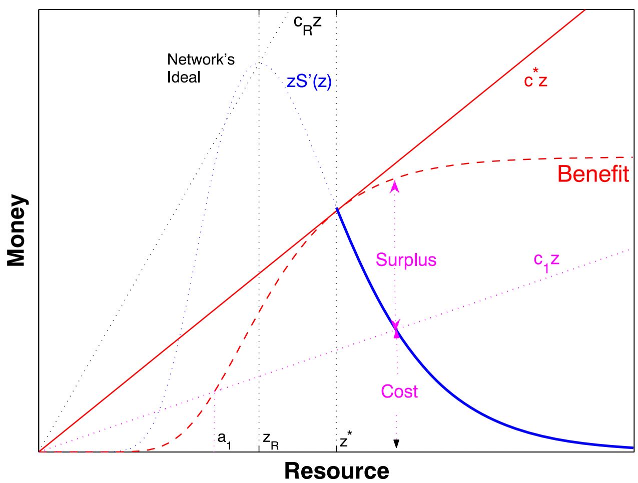
First, suppose that the resource constraint is "large", $Z \gg z^*$. By Theorem 4.1 and as illustrated by Fig. 9a, for a given $c_k < c^*$, the terminal chooses a value z_k that satisfies $S'(z_k) = c_k$. Then, the resulting network's revenue is $c_k z_k \equiv z_k S'(z_k)$. Thus, the network's revenue follows the curve $zS'(z)$.

The curve $zS'(z)$ has a single peak, say at z_R (Assumption 2 in the Appendix). In principle, the network would like to set a price such that the terminal chooses z_R. But it is shown below that $c_R > c^*$ which implies that $c_R z_R > S(z_R)$. Thus, c_R would be "too high" for the terminal, which would choose zero instead, yielding no revenue for the network.

Lemma 5.1 *The maximiser z_R of $zS'(z)$ satisfies $z_f < z_R \leq z^*$ where z_f and z^* denote respectively the inflexion point and the genu of the Benefit S-curve.*



(a) The terminal's side: With a power fraction of z, the terminal maximises benefits minus costs: $S(z) - cz$. The largest c for which the terminal choose to operate is c^*, the slope of the tangenu of S.



(b) The network's side: Revenue follows the graph $zS'(z)$. The network would prefer the price c_R but it is "too high" for the terminal. Thus, it settles for c^*.

Fig. 9 Pricing involves both a terminal's side and a network's side. Once the network understands how the terminal reacts to a price level (**a**), it can set the price that maximises its revenue (**b**)

Proof At z_R the derivative of $zS'(z)$ vanishes; that is, $S'(z_R) + zS''(z_R) = 0$. $S'(z_R)$ is necessarily positive, thus $S''(z_R) < 0$ which implies $z_R > z_f$.

If the increasing (rising) side of $zS'(z)$ intersected $S(z)$, its falling side would intersect $S(z)$ again. But by Lemma 4.2, the equation $zS'(z) = S(z)$ has only one positive solution, z^*. Thus, it is the falling side of the graph $zS'(z)$ that intersects $S(z)$. Therefore, $z_R \leq z^*$. □

Theorem 5.1 *Suppose that a t-terminal's benefit from utilising a power fraction z is given by the S-curve $S(z)$, which satisfies $c^*z^* = S(z^*)$. Then, the price that maximises the network's revenue when the resource constraint is Z is (i) c^* if $Z \geq z^*$ or (ii) $S(Z)/Z$ if $z^* > Z$.*

Proof

(i) By Lemma 5.1, with $Z > z^*$, it is the decreasing side of the revenue curve $zS'(z)$ that intersects $S(z)$ at z^*. Thus, $z > z^* \implies zS'(z) < z^*S'(z^*)$. Therefore, the network sets the price equal to c^* and the terminal chooses z^*. Then, the network's revenue equals $c^*z^* \equiv z^*S'(z^*) = S(z^*) =$

$$\frac{L}{M} v R f(\Gamma\kappa(z^*)) \tag{27}$$

(ii) By Theorem 4.1, if $c > S(Z)/Z$ then $cz > S(z)$ for $z \in (0, Z)$, and the terminal chooses $z = 0$ which produces no revenue for the network. Thus $S(Z)/Z$ is the largest price for which the terminal will operate. If the network choose a lower price, the terminal cannot choose $z > Z$, yielding a lower revenue for the network. Therefore the network sets the price equal to $S(Z)/Z$, the terminal chooses $z = Z$, and the network receives a revenue equal to $S(Z)$. □

5.1.2 Pricing for an energy-constrained terminal

Linear pricing By Theorem 4.2 and the discussion in Section 4.4, under linear pricing, the e-terminal will either decline to operate, or operate at the point that maximises "benefit per Watt" (the maximiser of $f_i(\Gamma_i\kappa(z))/z$).

The specific value of c_i plays a role indirectly, because it can make the cost exceed the benefit. It makes sense for the network to set the highest value of c_i that is acceptable to the terminal (see Eq. 24). That is:

$$c_i^* = \frac{L_i}{M_i} v_i R_i \frac{f_i(\Gamma_i\kappa(z^*))}{z^*} \tag{28}$$

At such level, the terminal's benefit equals its cost.

The total revenue provided by this terminal during the life of its battery equals (see Eq. 22):

$$\frac{E_i h_i}{\bar{p} + p_0} c_i^* \tag{29}$$

What the network receives from this terminal per time unit equals:

$$c_i^* z^* = \frac{L_i}{M_i} v_i R_i f_i(\Gamma_i\kappa(z^*)) \tag{30}$$

Quadratic pricing The development here follows closely that of Section 5.1.1. From Section 4.4.3, and Theorem 4.3, if the available resource $Z > z^*$, and with $\mathcal{B}(z) = S(z)/z$ the terminal's choice satisfies $\mathcal{B}'(z) = c$, and the network revenue is $cz = z\mathcal{B}'(z)$. By part (iv) of Assumption 2 from the Appendix, $z\mathcal{B}'(z)$ retains the single-peaked shape (see Fig. 10). Proceeding along those lines, one can prove the following theorem:

Theorem 5.2 *Suppose that an e-terminal's benefit from utilising a power fraction z is given by $\mathcal{B}(z)$, as defined in Theorem* 4.3. *Then, the price that maximises the network's revenue when the resource constraint is Z is (i) c_x if $Z \geq z_x$ or (ii) $\mathcal{B}(Z)/Z$ if $z_x > Z$.*

Remark 5.1 The argument to prove Theorem 5.2 is virtually identical to the proof of Theorem 5.1. The key observation is that, as pointed out in the proof of Theorem 4.3, only those values of z satisfying $0 < z \leq z^*$ play any role, and over this range $\mathcal{B}(z)$ (its "rising side") has—by assumption—the S-shape (see Fig. 8).

5.2 Serving many terminals

We assume that the network can set an individual price per terminal, and *in principle* treat each terminal independently, following Section 5.1. However, the sum of the individually chosen π_i^* may violate Eq. 12. From all the sets of terminals that satisfy Eq. 12, the network must choose the "best" set. This problem follows the pattern of the well-known "knapsack problem".

5.2.1 Which terminals to serve?

The (fractional) knapsack problem There is a finite set of items, each characterised by a "weight" and a "value". One seeks the combination of items that maximises the sum of the values, without exceeding a total weight constraint. The problem is in general NP-hard [13]. However, if one can include in the knapsack any desired fraction of any item, the problem admits a very simple and intuitive solution. Items are sorted by their "value to weight" ratio, and whole items are inserted in order. When no space is left for another whole item, the pertinent fraction of the next item is added to completely fill the knapsack [4]. In our problem, serving "a fraction" of a terminal is to admit it with a lowered π_i than it wants. However, the analogy is imperfect, because the "value" of the terminal is not linear with its "slice", π_i. Thus, the fractional knapsack solution yields a suboptimal choice in our case (which we neglect below).

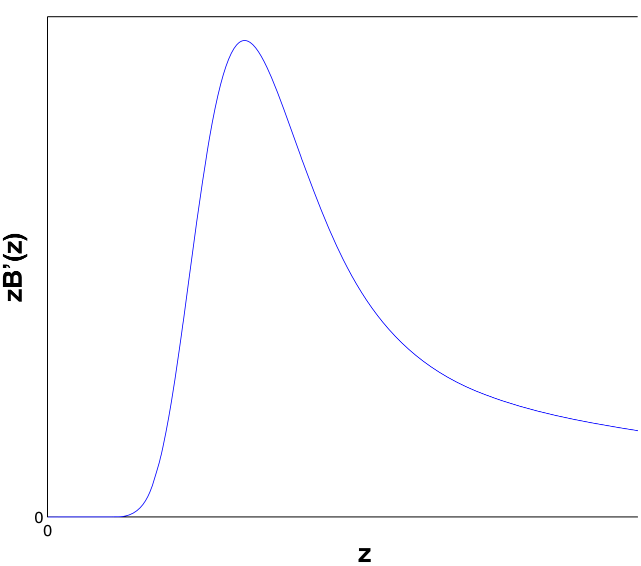


Fig. 10 With $\mathcal{B}(z) := S(z)/z$, $z\mathcal{B}'(z)$ retains the single-peakedness property

"Benefit per Watt" priority A terminal's "weight" should be (a function of) its service "slice", π_i, which is itself proportional to the terminal's received "Wattage" (Eq. 9). The obvious "value" measure (from the network's viewpoint) is revenue contribution, but over which period (time unit or battery life)?. The time unit is a natural choice for t-terminals. It turns out that it makes sense for the network to consider, for value-to-weight purposes, an e-terminal's revenue per second contribution. By doing so, the network measures both categories of terminal with the same yard stick. Furthermore, an e-terminal whose battery charge runs out is likely replaced by a new terminal which (statistically) has similar properties to the departing one. Thus, the network may as well focus on revenue per second.

Equations 27 and 30 are equivalent. Thus, the value/weight ratio for terminal i, while operating with a power fraction of $\pi_i > 0$ and paying the network an amount that equals the terminal's benefit, can be expressed as $v_i \hat{R}_i/\pi_i$, with

$$\hat{R}_i := \frac{L_i}{M_i} f_i(\Gamma_i \kappa(\pi_i)) R_i \tag{31}$$

Given the preceding pricing analysis, at the operating point, π_i should either be (i) z^*, the value at the genu of $f_i(\Gamma_i \kappa(z))$, or, if such value is "too high" for some reason, (ii) the highest reachable z, provided that at such z the terminal's cost does not exceed its benefit.

Optimal physical layer configuration Notice that $(L_i/M_i)(W/\Gamma_i) f_i(\Gamma_i \kappa(z^*))/z^*$ is determined by the physical-layer configuration (modulation, coding, data rate). If several such configurations are available, the network should impose the one that offers the largest:

$$\frac{L_i}{M_i} \frac{f_i(\Gamma_i \kappa(z^*))}{\Gamma_i z^*} \tag{32}$$

because it leads to greater "benefit per Watt" when the terminal operates optimally. Thus, two terminals that have a common spreading gain (or data rate), should have a common PSRF, f_i. This line of reasoning is explored further in a more general context in [20].

5.2.2 Power limited cell

The key is Section 3.2. If $\mathcal{A}$ are the indices of a certain set of terminals, they can occupy the cell each with a power fraction π_i, if the total fractional "slice" allocated to them satisfies a condition similar to Eq. 12: with $k \in \mathcal{A}$, and $\pi_k/\bar{p}_k \geq \pi_i/\hat{p}_i \quad \forall i \in \mathcal{A}$,

$$\sum_{i \in \mathcal{A}} \pi_i \leq 1 - \frac{\pi_k}{\hat{p}_k/p_0} \tag{33}$$

Terminal k is the "ruling terminal" of the set $\mathcal{A}$. Notice that π_k appears on the left side of constraint 33. Thus, with terminal k active, the total fractional "slice" left for possible companions is

$$1 - \pi_k - \frac{\pi_k}{\hat{p}_k/p_0} \tag{34}$$

The largest achievable value for π_k occurs when terminal k is alone in the cell, and equals $\hat{p}_k/(\hat{p}_k + p_0)$.

A terminal's "best subjects" Let $J(1)$, $J(2)$, …, $J(N)$ be indices such that

$$\frac{\pi_{J(1)}}{\hat{p}_{J(1)}} \geq \frac{\pi_{J(2)}}{\hat{p}_{J(2)}} \geq \cdots \geq \frac{\pi_{J(N)}}{\hat{p}_{J(N)}}$$

Thus, when all terminals are active, terminal $J(1)$ is the ruling terminal. When $J(1)$ is *not* active, terminal $J(2)$ becomes ruling, and so on. Evidently, terminal $J(N)$ "rules", *only* when no one else is active.

The network is interested in identifying the "best subjects" of terminal $J(m)$ ($m < N$), defined as the set of terminals that produces the most revenue (while satisfying the pertinent feasibility condition) when $J(m)$ is the ruling terminal, that is, with terminals $J(1)$, …, $J(m-1)$ *not* active. One can identify this set, by applying the knapsack solution described in Section 5.2.1, with "knapsack capacity" given by Eq. 34, that is, $1 - \pi_{J(m)}(1 + p_0/\hat{p}_{J(m)})$.

Let $\mathcal{A}_{J(m)}$ be the set that contains (the indices of) terminal $J(m)$ and its "best subjects" (with $\mathcal{A}_{J(N)} := \{J(N)\}$). The idea is to find $\mathcal{A}_{J(1)}$ and compute and store the combined revenue that $J(1)$ together with its "best subjects" produce. Subsequently, find $\mathcal{A}_{J(2)}$, and compute and store the combined revenue produced by the terminals in $\mathcal{A}_{J(2)}$. Then, proceed analogously with respect to $J(3)$, $J(4)$, and so on. Finally, from the previously obtained sets, choose the one that produces the most revenue, overall.

Table 2 Key parameters for each terminal

i	v_i	Γ_i	π_i^*	$g(\pi_i^*)$	$\hat{R}_i$	r_i	$\hat{p}_i$
1	3.5	32	0.26	0.88	3.52	46.6	1
2	4.0	64	0.15	0.86	1.72	46.3	5
3	3.1	32	0.26	0.88	3.52	41.3	4
4	3.5	64	0.15	0.86	1.72	40.5	3
5	3.7	128	0.08	0.85	0.85	39.7	1
6	3.5	128	0.08	0.85	0.85	37.6	4
7	3.0	64	0.15	0.86	1.72	34.7	4

Numerical illustrations Table 2 provides the key parameters for 7 terminals. Power limits are given as multiples of p_0. The common PSRF is that of Fig. 4, and units are such that $(L_i/M_i)R_i = 128/\Gamma_i$. r_i stands for "value to weight" ratio. The service SIR's $(\Gamma_i\kappa(\pi_i^*))$ are 11.5, 11.1 and 10.9 for spreading gains 32, 64, and 128 respectively, and $g(\pi_i) := f(\Gamma_i\kappa(\pi_i^*)$, the packet success rate.

Table 3 applies the solution procedure of Section 5.2.2 to these terminals. The first column has the indices of the terminals sorted in order of descending value/weight ratio (r_i in Table 2). The top row has the "slice" that can be allocated to all terminals when the terminal whose index is directly below "rules" $(d_j := \pi_j^*/\hat{p}_j)$. The second row has the terminals' indices sorted in descending order, from left to right, by ε_j (terminals 7, 2, and 6 are not shown for reasons

Table 3 A set of the terminals is chosen for service.

	0.74	0.92	0.93	0.95	$\overleftarrow{1-d_j}$	
i	**1**	**5**	**3**	**4**	$\downarrow \pi_i$	$\downarrow \hat{v}_i$
1	**[1]**	**[0]**	**[0]**	**[0]**	0.26	12.3
2	1	1	1	1	0.15	6.9
3	1	1	**[1]**	**[0]**	0.26	10.6
4	0,40	1	1	**[1]**	0.15	6.0
5	0	**[1]**	**[0]**	**[0]**	0.08	3.1
6	0	1	1	1	0.08	2.9
7	0	1	1	1	0.15	5.1
	0.74	0.87	0.79	0.52	$\overleftarrow{\sum \pi_i}$	
	30.1	**35.0**	31.9	21.0	$\overleftarrow{\sum \hat{v}_i}$	

explained below). Thus, terminal 1 has the greatest "value to weight" ratio, but coincidentally, it also has the highest $\pi_j^*/\hat{p}_j$ ratio (it is in the "worst situation"), which implies that terminal 1 "rules" when all are active. With terminal 1 absent, terminal 5 "rules"; and with 1 & 5 off, terminal 3 "rules", and so on. The last two columns have respectively π_i and $\hat{v}_i := v_i \hat{R}_i$ (the individual revenue contribution).

A "1" (resp. "0") in the position (i,j), (row, column), means that when terminal j "rules", terminal i is (resp. is *not*) among the "best subjects" of j. A number between 0 and 1 at such position indicates that when j "rules", terminal i is "fractionally served". The brackets denote mandatory (from the algorithm viewpoint) inclusions or exclusions.

Thus, when terminal 1 "rules", terminal 2 and 3 are "fully" served, and terminal 4 is also served but with less than half of its desired "slice". In this case, the sum of the slices is 0.74 and the total revenue brought by these terminals is 30.1. When terminal 5 "rules" (next column), terminal 1 is turned off by construction, and each of the other terminals can be "fully" served. The sum of the slices is 0.87 and the total revenue is 35.0. At this point the algorithm can be stopped. The group "ruled" by terminal 5 is chosen, because it produces more revenue than terminal 1 and its best subjects. Thus, even though terminal 1 itself produces more revenue (in absolute and relative terms) than any other terminal acting alone, it is the only terminal left out because of its "bad situation". The final two columns (3 and 4 rules, respectively) are provided as illustration, but not needed.

5.2.3 Analysis of the algorithm

The core algorithm that finds $\mathcal{A}_{J(m)}$ and computes revenue utilises the simple knapsack solution of Section 5.2.1, and is applied *at most* $N-1$ times ($\mathcal{A}_{J(N)} = \{J(N)\}$ by definition). Additionally, it becomes progressively simpler, because as m grows to N, fewer terminals need to be considered. E.g., $J(N)$ and $J(N-1)$ are the only potential members of $\mathcal{A}_{J(N-2)}$.

Furthermore, it may be unnecessary to run the core procedure $N-1$ times. The process can be stopped with terminal $J(m)$, if each one of terminals $J(m+1)$ to $J(N)$ can join $J(m)$, each operating at its optimal power fraction and satisfying constraint 33 with $k = J(m)$. Then, from the sets $\mathcal{A}_{J(1)}, \ldots, \mathcal{A}_{J(m)}$, the one that produces the most revenue is chosen. For instance, if $J(1)$ can be joined by all other terminals (each operating at π_i^* and satisfying Eq. 12) then, evidently, that is the best the network can do, and the process can be stopped right there.

Finally, this procedure is performed at the energy and processing-power rich base station. Thus, computational complexity does not seem to be a problem.

6 Game without direct cost

For performance comparison purposes, we discuss below a situation in which each terminal can choose its power without a direct cost. The network may still charge the terminal some conventional subscription fee. But if there is no connection between what the terminal pays and its resource usage, it will use resources as if they were "free".

This scenario is best modelled as a "game of strategy". Because there is neither pricing nor a central controller, this situation can be thought of a "worst case scenario", which provides a "base line" for performance evaluation.

6.1 Game of strategy

In a game of strategy, each of several "selfish" agents chooses a "strategy" in order to maximise its own "payoff", which depends on the strategies chosen by all players. Herein, the strategy is a transmission power level, which increases the payoff of the transmitting terminal, but degrades the performance of all others (for introductory treatments, see [7, 12]). The key solution concept is the Nash Equilibrium (NE); i.e., a strategy per player (power level) such that no player would be better off by *unilaterally* changing strategy. The key to identifying the NE(s) is to first characterise each player's "best response".

6.2 A terminal's best response

In our game, the "best response" of terminal i is the power level terminal i chooses given a vector representing one power level for each of the other terminals. In fact, terminal i only needs the sum of the components of such vector, which determines its level of interference, Y_i.

6.2.1 T-terminal best response

Since the benefit of a time-driven terminal is strictly increasing in its SIR, it is self-evident that, without cost considerations, its (selfish) best response to any level of interference or noise is to transmit at maximal power. The resulting received power can be added into the noise term.

6.2.2 "Statutory battery" discipline

A situation in which each t-terminal operates continuously at its maximal power is intuitively undesirable. Since power-pricing is now unavailable (by construction), a reasonable policy is to endow each terminal with a (virtual) "statutory battery", i.e., a limit on the total amount of energy that it can emit over a specified time period (comparable to a typical battery life). The base station can tell the power level received from a given terminal, its channel gain, the times it is active, and, hence, its total energy emitted. Under this policy, the t-terminal maximises its total benefit by the time its energy emission allowance is exhausted (see Section 2.2); that is, the t-terminal behaves as an e-terminal.

6.2.3 E-terminal best response

In any case, we can think that all players are e-terminals, experiencing a fixed level of "noise" p_0 (which may include power received from t-terminals that are not "virtual battery" powered, if any).

From Section 4.4, for a given Y_i and with $\sigma_i := \Gamma_i h_i P_i / Y_i$ (the SIR), a terminal with energy budget E_i chooses its transmission power, P_i, to maximise $E_i v_i R_i f_i(\Gamma_i h_i P_i / Y_i)/P_i$, or

$$\frac{h_i}{Y_i} E_i v_i \frac{L_i}{M_i} W \frac{f_i(\Gamma_i h_i P_i / Y_i)}{\Gamma_i h_i P_i / Y_i} \propto \frac{f_i(x)}{x} \tag{35}$$

Thus, the terminal's best response is to set the power level so that its SIR maximises $f_i(x)/x$. By Lemma 4.2, this ratio is single-peaked, and its unique maximiser, x^*, can be easily identified at the genu of the graph of the PSRF. If x^* cannot be reached because of power limitations, the terminal operates at its power limit.

All terminals with a common PSRF will aim at the same SIR (even if each has its own willingness to pay, data rate, channel state, packet size, and energy budget). If several link-layer configurations are available, the configuration whose PSRF has the highest ratio $f(x^*)/x^*$ should be chosen (see [20]). We assume below a common PSRF, thus, all terminals aim at the same SIR, x^*.

6.3 Equilibrium of the game

6.3.1 Symmetric equilibrium

From Section 3.2, it is feasible for each of N terminals to experience SIR x^* if and only if condition 12 is satisfied; that is, $\sum \pi_i \leq 1 - \pi_k/(\hat{p}_k/p_0)$, where $\pi_i = x^*/(x^* + \Gamma_i) \equiv (1 + \Gamma_i/x^*)^{-1}$, and k such that $\pi_k/\hat{p}_k \geq \pi_i/\hat{p}_i \quad \forall i \in \{1, \ldots, N\}$ (k is the "ruling terminal"). Thus, if condition 12 holds, each terminal can reach x^* with the feasible power level given Eq. 9: $h_j P_j =$

$$\frac{\pi_j p_0}{1 - \sum_{n=1}^{N} \pi_n} \equiv \frac{p_0 (1 + \bar{\Gamma}_j)^{-1}}{1 - \sum_{n=1}^{N} (1 + \bar{\Gamma}_n)^{-1}} \tag{36}$$

Since each terminal operates at its best-response SIR (for any level of interference), Eq. 36 describes a Nash equilibrium in closed form (with $\bar{\Gamma}_j := \Gamma_j / x^*$, each quantity in Eq. 36 is presumed known). Of course, condition 12 may not be satisfied.

6.3.2 Nash equilibrium in the general case

The general procedure to find the Nash equilibrium of this game is reminiscent of Section 5.2.2. Each terminal must obey inequality 14, which can be written (with $\bar{\pi} = \sum \pi_i$, and $\hat{p}_i := h_i \hat{P}_i$) as $\pi_i/\hat{p}_i \leq (1 - \bar{\pi})/p_0$. The right side is the same for all terminals, thus the terminal with the greatest ratio $\pi_i/\hat{p}_i$ has the most difficulty to meet this common constraint, therefore it is in the "worst situation" (or it is the "weakest"). If this terminal "gives up" on obtaining the optimal SIR, and simply operates at its maximal power, then constraint 14 no longer counts for this terminal, since it no longer aims for the optimal SIR (or any specific one). Then, the terminal with the second highest $\pi_i/\hat{p}_i$ can be thought of being in the worst situation (among those still aiming for the optimal SIR). Notice that when the data rate is common, $\Gamma_i = \Gamma$ and hence $\pi_i = \pi$, then, the terminal with the smallest $\hat{p}_i$ is in the worst situation. But a terminal with a large spreading gain, Γ_i, could have the smallest $\hat{p}_i$ and not be in the worst situation because it may achieve the desired SIR with a very small percentage of the receiver power ($\pi_i = 1/(1 + \Gamma_i/x^*)$).

Then, to find the Nash equilibrium in the general case, first sort the terminals by their $\pi_i/\hat{p}_i$ ratios, and re-label the terminals such that $\pi_1/\hat{p}_1 \leq \cdots \leq \pi_N/\hat{p}_N$. First check whether the symmetric NE (Section 6.3.1) exists, and if yes, stop. Otherwise, (i) find the terminal in the worst situation and assume it operates at its maximal power level, (ii) add its received power to the noise term, and then (iii) check whether the remaining $N-1$ terminals can achieve the desired SIR under this new scenario. If the answer is yes, stop. Otherwise, proceed recursively with two terminals maxed out.

In the second iteration (if necessary), to check whether terminals $1, \ldots, N-1$ can reach the desired SIR with terminal N maxed out, the condition is $\sum_{i=1}^{N-1} \pi_i \leq 1 - \pi_{N-1}/(\hat{p}_{N-1}/p_{0(1)})$ where $p_{0(1)} :=$

$p_0 + \hat{p}_N$. At the $(m+1)th$ iteration, to check whether terminals $1, \ldots, N-m$ can reach the desired SIR, while each of terminals $N-m+1, \ldots, N$ operates at its maximal transmission power, the condition becomes

$$\sum_{i=1}^{N-m} \pi_i \leq 1 - \frac{\pi_{N-m}}{\hat{p}_{N-m}/p_{0(m)}} \tag{37}$$

with $p_{0(m)} := p_0 + \sum_{j=N-m+1}^{N} \hat{p}_j$. If constraint 37 is satisfied, the procedure stops after $m+1$ iterations. Otherwise, it continues recursively.

In general, the game ends with N_* terminals achieving x^* ($N_* = 0$ is possible, of course), and each of the remaining $N - N_*$ operating at its maximal power level, and experiencing the resulting SIR. It is clear that an x^*-achieving terminal has no incentive to unilaterally change its power level. It is also clear that each of the remaining terminals would gain by unilaterally raising its power level, but cannot do so, because it is maxed out already. Thus, the procedure indeed stops at a Nash equilibrium.

6.3.3 Benefit at equilibrium

Once the equilibrium is found, one can apply Eq. 35 to calculate a terminal's total benefit while operating at equilibrium. With x_i° and p_i° denoting respectively the equilibrium SIR and received power, $\bar{p} = p_0 + \sum p_i^\circ$, and assuming a common energy budget, E, Eq. 35 yields:

$$\frac{L}{M} EW \frac{f(x_i^\circ)}{x_i^\circ} \frac{v_i h_i}{Y_i} := K \frac{f(x_i^\circ)}{x_i^\circ} \frac{v_i h_i}{\bar{p} + p_0 - p_i^\circ} \tag{38}$$

The constant K (measured in Joules) just gathers a product of parameters, and could be set to 1 by choosing convenient units of measurement. For an x^*-achieving terminal, $x_i^\circ = x^*$ and p_i° is given by Eq. 9. If these terminal's data rates are of the same order of magnitude, so will be the equilibrium power levels, and thus the denominator of Eq. 38 will be approximately constant for all these terminals. Then, the benefit of an x^*-achieving terminals will be, with convenient units, $\approx v_i h_i$. For terminals not achieving x^*, $p_i^\circ = \hat{p}_i$ and $x_i^\circ = \Gamma_i \hat{p}_i / (\bar{p} + p_0 - \hat{p}_i)$.

Equation 38 yields a terminal's benefit over its "battery life". This is of interest to the network, because what the terminal is willing to pay under any "conventional" fee structure is, ultimately, limited by what the terminal's "gets out of" the system. However, for reasons discussed in Section 5.2.1, the network may be more interested in benefit per time unit. While operating with SIR x_i°, the terminal successfully uploads $(L_i/M_i) R_i f_i(x_i^\circ)$ information bits per second, yielding a benefit of

$$v_i (L_i/M_i) R_i f(x_i^\circ) \tag{39}$$

6.3.4 Numerical illustration

In Table 4, we apply the procedure of Section 6.3.2, for the terminals of Table 2, assuming they are all energy-constrained. The common PSRF is that of Fig. 4. The maximiser of $f(x)/x$ is 10.75, hence, $\pi_i = 10.75/(10.75 + \Gamma_i)$. With the original indices, the $\pi_i/\hat{p}_i$ ratios are, respectively, 0.25, 0.03, 0.06, 0.05, 0.08, 0.02 and 0.04. This yields the sorting [6 2 7 4 3 5 1].

The first column of Table 4 has the terminals sorted by their "situations", from "worst" to "best". Each has been re-labelled (original indices shown in parenthesis). We use the notation of condition 37 to explain the next columns. The 2nd column corresponds to $p_{0(m)} = p_0 + \sum_{j=N-m+1}^{N} \hat{p}_j$, the third column has the ratio $\hat{p}_{N-m}/p_{0(m)}$, and the fourth column has $\pi_{N-m} p_{0(m)}/\hat{p}_{N-m}$. The fifth and sixth columns have, respectively, the left and right side of condition 37. The first time the left side is less than or equal to the right side, a NE has been found. Thus, this example ends at the 2nd iteration, with only the terminal 7(1) operating at maximal power, and all others enjoying their chosen SIR σ^*. The row corresponding to 5(3) is given as illustration, but not needed.

The equilibrium power levels can be obtained explicitly through Eq. 9 (with p_0 replaced by $p_{0(1)} = p_0 + \hat{p}_7 = 2$). $1 - \bar{\pi} = 0.16$, thus $h_i P_i = 2(\pi_i/0.16)$. E.g., $h_i P_i = 7/4$, for terminals 4, 3(7), and 2. Then, the equilibrium benefits can be calculated through Eq. 38.

Table 4 Finding a Nash equilibrium

i	p_0^+	$\frac{\hat{p}_w}{p_0^+}$	$\frac{\pi_w}{\hat{p}_w}$	$\sum z$	$1-d$	π_i	$\hat{p}_i$
7(1)	1	0.25	0.25	1.09	0.75	0.25	1
6(5)	2	0.50	0.15	0.84	0.85	0.08	1
5(3)	3	1.33	0.19	0.76	0,.81	0.25	4
4(4)	7	-	-		-	0.14	3
3(7)	-	-	-		-	0,.14	4
2(2)	-	-	-		-	0.14	5
1(6)	-	-	-		-	0.08	4

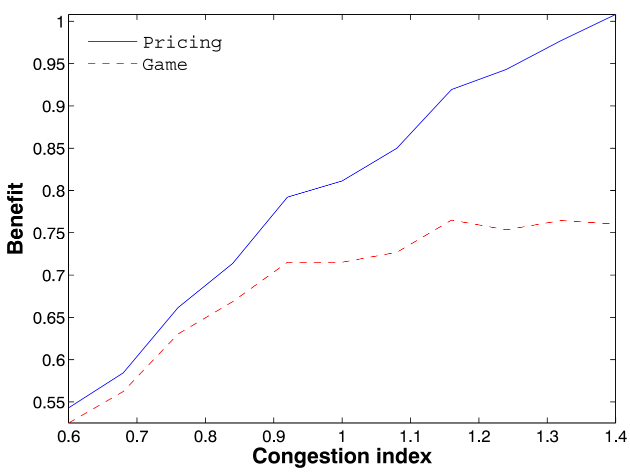
7 Performance experiments

Below we use the no-direct-cost game as a base line for performance. The key performance index is the total benefit derived by the terminals (which equals the network's revenue under the direct pricing scheme). Benefit is normalised by dividing it by "representative benefit", which is obtained assuming that (i) the number of terminals is the largest that can be served at the optimal SIR, (ii) when all have the average spreading gain, (iii) are uniformly distributed along the cell radius (1 Km), and (iv) have the average willingness to pay.

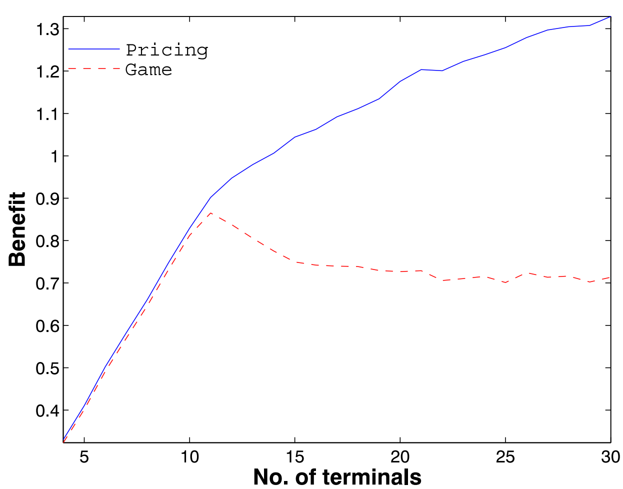
Figure 11 shows how performance changes with some measures of "congestion". The pricing scheme performs only marginally better than the game when the number of terminals is "small" (meaning that all can reach the optimal SIR in the game). However, as the number of terminals grow, the performance gap increases. This is reasonable because under pricing, the network always chooses to serve the "best" (revenue/Watt) users. Thus, having many users to choose from is actually good for the network.

The game, however, always "serves" everybody (at equilibrium all terminals transmit). With many terminals operating at full power, most get very poor performance. However the game performance does not "collapse" quickly, because each of many terminals does transmit and "gets something", and there are always a few of them that are very close to the base station, and get reasonable performance, which helps the game total. Thus, the specific "gain" of the pricing scheme depends on how "busy" the cell is. For instance, Fig. 11a indicates that with the "correct" arrival rate (that equal to the number of terminals that can be served at the optimal operating point), pricing outperforms the game significantly, but modestly (8 to 7). If the arrival rate drops to 60% of this level, the performance gap is only a few percentage points. Pricing outperforms the game about 4 to 3 (33%) when the arrival rate is 40% above the "right level".

Figure 12 describes how the "spread" of the willingness-to-pay values ("social classes") affects performance. The equally-likely wtp are the components of $4v/s$, where $v = [1\ a\ a^2\ a^3]$ and s is the sum of v's components (e.g., with a=1/2, $v = [1\ 1/2\ 1/4\ 1/8]$ and $s = 15/8$, but with $a = 1$ $v_i = 1\ \forall i$). With this choice, the expected value of the wtp vector ($4v/s$) is 1, for



(a) When the "congestion" index is 1, the arrival rate equals the cell "capacity".



(b) The number of terminals, N, is varied deterministically (with random spreading gains and locations). As N grows over 10, the game performance drops and stagnate, whereas performance continues to grow under pricing.

Fig. 11 Performance versus "congestion" under power-share pricing (*solid*) and at the equilibrium of a no-direct-cost game

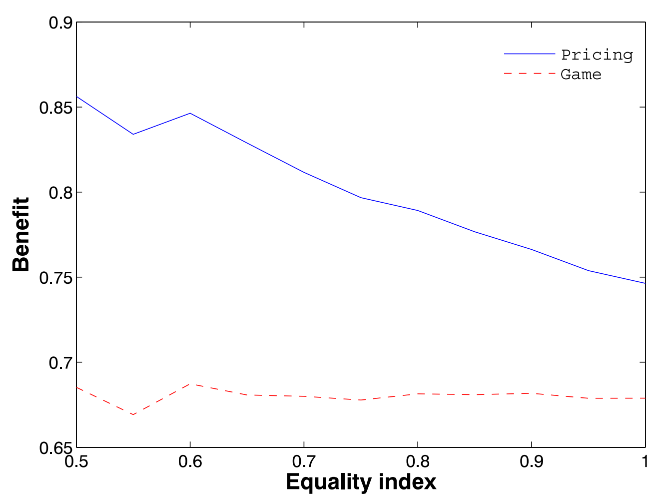


Fig. 12 The greater the inequality (lower a), the better the performance of the pricing scheme. But the game performance is unaffected

all a. With the arrival rate fixed at the “right level”, the pricing scheme advantage grows with “inequality” (doubles with $a = 1/2$), which is reasonable because the network considers a users’ wtp to decide whom to serve, but the game is blind to monetary considerations.

8 Socially optimal allocation

8.1 Power limits of data terminals

To simplify the exposition, we assume that there is a power-constrained media terminal with a service level agreement that specifies its spreading gain Γ_M and SIR, σ_M, and denote the total “slice” available for data terminals as $1-d$. In principle, any combination of π_i satisfying $\bar{\pi} = \sum \pi_i \leq 1-d$ can be assigned to the data terminals. The largest possible value of p_i occurs when $\pi_i = \bar{\pi} = 1-d$ (i is the *only* active data terminal), which yields $p_i = p_0(1-d)/d$ (Eq. 9). We assume below that for all i, $\hat{p}_i/p_0 \geq (1-d)/d$. Therefore, each data terminal can reach its resulting power level.

8.2 Objective function

With $V_i(\pi_i)$ denoting the appropriate benefit function (depends on the terminal’s energy class), a reasonable criterion for a social planner is to solve

$$\text{maximise:} \quad \sum_{i=1}^{N} \mathcal{V}_i(\pi_i) \tag{40}$$

subject to,

$$\sum_{i=1}^{N} \pi_i \leq 1-d \tag{41}$$

$$\pi_i \geq 0 \tag{42}$$

The necessary optimising conditions are [9, 11]:

$$\mathcal{V}_i'(\pi_i) - \mu_0 \leq 0 \text{ with equality for } \pi_i > 0 \tag{43}$$

8.3 Social optimum through pricing

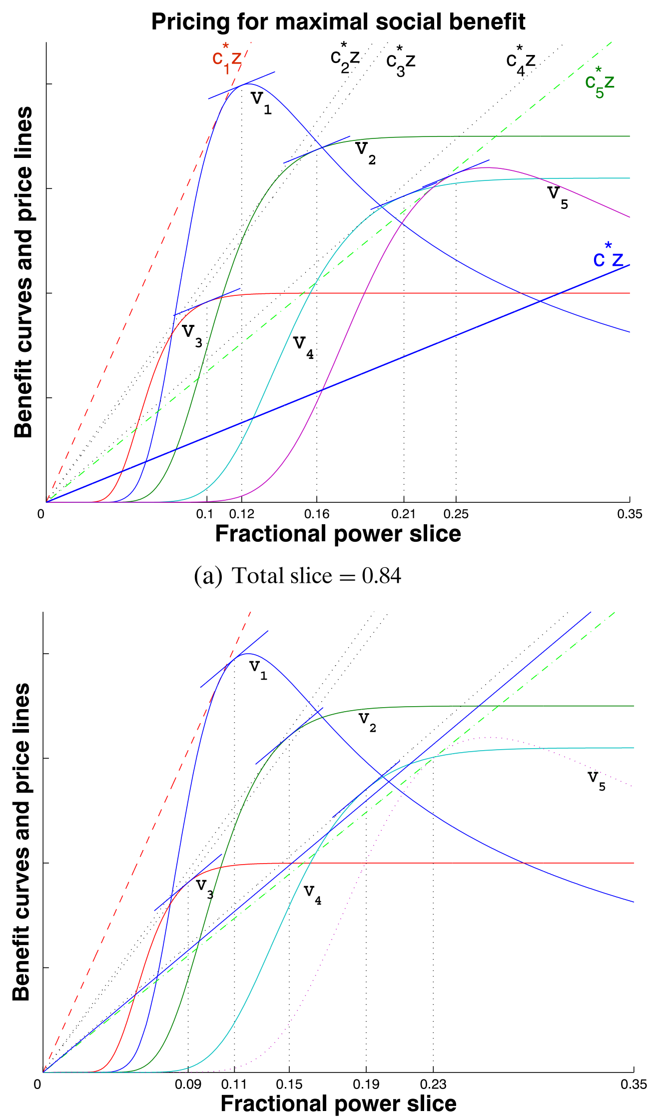
From Eq. 43, any terminal receiving a positive share of the power must satisfy $\mathcal{V}_i'(\pi_i) = \mu_0$. From Section 4.3, and Fig. 9b, a t-terminal can satisfy Eq. 43, and hence reach a “socially optimal” allocation, provided that a common price is set, which coincides with μ_0.

Per Section 4.4.2 and Fig. 7, in order for the e-terminal’s behaviour to lead to Eq. 43, quadratic pricing must be used. The corresponding analysis is in Section 4.4.3.

8.4 Numerical illustration

8.4.1 Finding the socially optimal price

One can solve the system of (nonlinear) Eq. 43 by algebraic or numerical means. But Fig. 13 provides greater insight (recall Figs 9b and 8). The planner sweeps a price line, from vertical to horizontal, until it finds the optimal slope. Any price greater than c_1^* (a line to the left of $c_1^* z$ (red, dash)) is “too high”. When the price



(a) Total slice = 0.84

(b) Total slice = 0,54

Fig. 13 Bell and S curves are benefit graphs. A terminal’s optimal allocation is identified by a short tangent, parallel to the *solid blue line* representing the socially optimal price, for the given resource constraint. All can be served when total resource is 0.84 (**a**), but terminal 5 is left out when the resource drops to 0.54

falls to c_1^*, terminal 1 chooses to operate, and as the price continues to drop, more terminals become active and/or those already active increase their purchase. The planner stops when the sum of "slices" equals $1 - d$.

8.4.2 Operating point: planner's versus network's

As shown by Fig. 13, the first terminals to become active are precisely those with the "steepest" tangenu, that is, those with the highest "value to weight" ratio, which is precisely the criterion used by the (monopolistic) network. However, the network chooses an individual price per terminal such that each operates at the genu ("knee"), where each maximises "benefit per Watt". The planner chooses a common price, and each active terminal ends up paying less that it would under the network's price. But at the lower planner's price, each active terminal consumes more, which may reduce the total number of terminals that can receive service.

9 Discussion

In the reverse link of a CDMA network with N terminals, each can receive service quality (SIR) σ_i only if $\sum \pi_i \leq 1 - d$ where $\pi_i = \sigma_i/(\sigma_i + \Gamma_i)$ (Γ_i is the spreading gain and $0 < d < 1$). π_i also equals i's share of the total power at the receiver. Thus, the uplink management can be approached as a "pie division" problem, where the total receiver power is the pie and π_i is the fractional "slice" assigned to i. We have proposed, analysed, and evaluated a technical-economic scheme based on the key variable chosen by nature: π_i. Each data terminal has its own data rate, channel gain, willingness to pay (wtp), and link-layer configuration. Some have limited energy (e-terminals), but others not (t-terminals) and we have specified appropriate performance metrics for both types. The receiver power fraction, π_i, immediately determines the carrier-to-interference ratio, $\kappa_i = \pi_i/(1 - \pi_i)$, which directly leads to the SIR, $\sigma_i = \Gamma_i \kappa_i$. Thus, given a price on π_i, the terminal can individually make an optimal choice irrespective of choices made by others. This is a major advantage of our proposal.

The network ultimately chooses for each terminal an individual price that forces it to operate where "revenue per Watt" is highest. Of course, the sum of the "slices" ordered by the terminals may exceed resource availability. Then, the network follows a "knapsack" approach [13] to find, among all sets of terminals that satisfy the constraint, the revenue maximiser. Thus, our proposal simplifies the terminal's choice at the expense of (reasonably) complicating the network's (a favourable trade-off, given the energy and/or computational limitations of a terminal).

As a base-line for performance evaluation, a "game" in which each terminal chooses its power to maximise its own performance without a direct cost is solved "from first principles". A "player" with unlimited energy will evidently always set its power to the maximal level, raising the "noise floor" of all. However, energy-conservation can be induced through a limit on total energy spent over an appropriate period of time ("statutory battery"). The existence of a Nash equilibrium is proved "constructively", by showing it in closed form. The pricing scheme always outperforms the game, and the performance gap grows with the number of terminals in the system, and also tends to increase with "social inequality". Moreover, a limited study of "social benefit" maximisation indicates that our proposal can achieve the social optimum, with a price common to all terminals, and with each energy-constrained terminal paying in proportion to the *square* of its power fraction. Thus, our analysis can equally lead to a "true" economic pricing scheme for network profit, or may be employed as a benefit-maximising algorithmic metaphor along the lines of [3].

We have captured the critical packet-success-rate function (PSRF) through an S-curve of unspecified algebraic form, and approach from [16, 17], which have been found useful in numerous scientific contributions, such as [14, 15]. This makes our analysis relevant to a wide variety of physical layer configurations, and in fact allows us to optimally (re)configure the link layer, as discussed in greater detail in [20].

For each of the three environments considered (network pricing, game, and "social" pricing) complete numerical examples have been given in tabular and/or graphical form. However, this work has an analytical core of three "general" maximisation problems: (1) $\mathcal{B}(x)$, (2) $S(x) - cx$ and (3) $\mathcal{B}(x) - cx$ ($S(x)$ is an S-curve and $\mathcal{B}(x) = S(x)/x$, a single-peaked curve). The 1st is key when an e-terminal faces linear or zero price (Fig. 7), and the 2nd is fundamental for the t-terminal's analysis (Fig. 9). The third arises when an e-terminal faces quadratic pricing, a requirement of the "social optimum" (Fig. 8). We have characterised the solutions through the geometrical properties of the graphs, under the presumption that "all we know" is that S is an S-curve, and that it satisfies some additional technical conditions—discussed in the Appendix—which guarantee that the shapes of certain graphs are as assumed ($S(x)/x$ has been proved to be "single-peaked" in [16, 17]).

The present work has no doubt limitations. As in [22] and many related works, we have only considered one cell (inter-cell interference can be added to noise). Furthermore, as commonly done in the power control literature, channel gains have been assumed fixed. This does not mean that our analysis only applies when users are immobile and channel are stable, but rather that there is a separation of timescale between power updates and changes in propagation conditions [8, Section 2.6.2]. In a practical system, channel state information is updated periodically. After each update, our analysis may lead to a revised resource allocation. We believe the single-terminal pricing analysis to be rigorous. However, the extension to a multi-terminal scenario through the knapsack approach should be viewed as an analytically-supported heuristic. Likewise, our solution to the interesting and challenging social planner problem is partial. And we have neglected inter-network price competition. These limitations suggest avenues for future work. Along these lines, in [21] we have applied a similar approach to fourth-generation cellular networks, by proposing an auction procedure to allocate frequency sub-channels combined with pricing for power allocation. We remain optimistic that, despite these limitations, (i) the advantages offered by our proposal, (ii) the generality of our model, and (iii) the innovative methodology we have utilised combine to produce a useful addition to existing scientific literature.

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Appendix: Mathematical issues

In the analysis we assume that certain functions involving an S-curve or the derivative of an S-curve retain the S-shape, or the "single-peakedness" of the derivative (see Fig. 1).

Definition 9.1 $S : \Re_+ \to [0, Y]$, is an S-curve with unique inflexion at x_f if (i) $S(0) = 0$, S is (ii) continuously differentiable, (iii) strictly increasing, (iv) convex over $[0, x_f)$ and concave over (x_f, ∞), and (v) surjective.

Remark A.1 In Definition A.1, S is strictly increasing and also surjective (for each $y \in [0, Y]$ there is an $x \in \Re_+$ such that $f(x) = y$). Therefore, S must approach Y asymptotically as x goes to infinity (i. e., this follows from the definition).

Definition 9.2 A function $h : \Re_+ \to [0, Y]$ is single-peaked over $\Re_+$ if h is continuous, surjective and has a global maximum at $X \in (0, \infty)$ (that is, $h(X) = Y$, $0 \leq x_1 < x_2 \leq X \implies h(x_2) > h(x_1)$ and $X \leq x_1 < x_2 \implies h(x_2) < h(x_1)$).

Remark A.2 In Definition A.2, h is strictly increasing up to $x = X$ and strictly decreasing thereafter. Since h is also surjective, it must approach 0 asymptotically as x goes to infinity.

Remark A.3 Definition A.2 is closely related to strict quasi-concavity. However, a strictly monotonic function satisfies strict quasi-concavity, but does not satisfy Definition A.2. For example, a function whose graph exhibits over the interval of interest the familiar "bell shape" of the Gaussian curve (as shown in Figs. 1 and 10, for example) is both strictly quasi-concave and single-peaked. On the other hand, the S-curve itself is strictly quasi-concave (since it is strictly increasing) but does not satisfy Definition A.2 (its "peak" occurs at infinity).

The specific assumptions are:

Assumption 1 *If S satisfies Definition* A.1*, then the composite function $s(z) := S(g(z))$ with $g(z) = \Gamma z/(1 - z)$, $\Gamma \geq 1$ and $z \in [0, 1)$ also satisfies Definition* A.1.

Assumption 2 *If S satisfies Definition* A.1*, then each of the following functions satisfies Definition* A.2*: (i)$xS'(x)$, and for $\Gamma \geq 1$(ii) $(x/\Gamma + 1)^2 S'(x)$, (iii) $x(x/\Gamma + 1)S'(x)$ and (iv) $x\mathcal{B}'(x)$ where $\mathcal{B}(x) := S(x)/x$ with $\mathcal{B}(0) := \lim_{x \downarrow 0} \mathcal{B}(x) \equiv S'(0)$.*

Remark A.4 By Lemma 4.2, if S satisfies Definition A.1 then $\mathcal{B}(x)$ satisfies Definition A.2 (i. e., this is a proved statement, not an assumption).

Below we formally describe some more primitive technical properties for the concerned S-curve that lead to the assumptions above. These properties have the "single crossing" form; i.e., the value of certain function crosses the origin exactly once, a notion that has proved quite useful in certain contexts, such as economics and game theory [2]. In the present work, a strong version of this notion is formalised as follows:

Definition 9.3 A function $f : D \to \Re$ with $D \subset \Re$ satisfies the unique-crossing from above condition (UCC) over $D' \subset D$ if $\exists t_0 \in D'$ such that $f(t_0) = 0$ and $\forall t \in D' \quad t < t_0 \implies f(t) > 0$ and $t > t_0 \implies f(t) < 0$.

Lemma A.1 *Consider the composite function $S(g(z))$, where $S : \Re_+ \to [0, d]$, is an S-curve with inflexion at x_f, and $g : [a, b] \to \Re_+$, with $0 \le a < b$, is a strictly increasing convex function such that $\lim_{z \to b} g(z) = \infty$.*
(i) *If the function $[g'(z)]^2 S''(g(z)) + g''(z)S'(g(z))$ satisfies the UCC over $[a, b]$, then:* (ia) *the composite function $s(z) := S(g(z))$ satisfies Definition* A.1, *and* (ib) *its inflexion abscissa z_f is such that $g(z_f) > x_f$.*
(ii) *With $g(z) = \Gamma z/(1 - z)$, $\Gamma \ge 1$ and $\mathcal{I} = [0, 1)$, conclusions (ia) and (ib) follow if the function $(x + \Gamma)S''(x) + 2S'(x)$ satisfies the UCC over the domain $\Re_+$.*

Proof

(ia) We only show below that the composite function has, under the hypothesis, the curvature properties required by Definition A.1; that it also has the other properties can also be shown.
The second derivative of $S(g(z))$ is $[g'(z)]^2 S''(g(z)) + g''(z)S'(g(z))$.
$g''(z)S'(g(z))$ is always positive because by hypothesis g'' is positive (convexity).
$[g'(z)]^2 S''(g(z))$ has the sign of S''; i.e. it is positive in $[0, x_f)$ and negative in (x_f, ∞).
Thus, the composite function starts out convex (its second derivative starts out positive).
If $[g'(z)]^2 S''(g(z)) + g''(z)S'(g(z))$ satisfies the UCC, then the composite function has exactly one inflexion point z^c_f.
The fact that s asymptotically goes to d as z goes to b is immediate.

(ib) z^c_f must satisfy $g(z^c_f) > x_f$ so that $S''(g(z_f))$ be negative.

(ii) If $g(z) = \Gamma z/(1 - z)$ then $g'(z) = \Gamma/(1 - z)^2$ and $g''(z) = 2\Gamma/(1 - z)^3$. Considering these expressions, the second derivative of the composite function becomes:

$$\frac{\Gamma^2}{(1 - z)^4} S''(g(z)) + \frac{2\Gamma}{(1 - z)^3} S'(g(z))$$

which has the same sign as $\Gamma S''(g(z)) + 2(1 - z)S'(g(z))$.
With $x := \Gamma z/(1 - z)$, $z = x/(x + \Gamma)$ and $1 - z = \Gamma/(x + \Gamma)$.
Therefore, the second derivative has the sign of $(x + \Gamma)S''(x) + 2S'(x)$. The thesis follows from part (i) of this proof. □

The next result involves the bell shape exhibited by the graph of S'.

Lemma A.2 *Consider the function $h(t) = g(t)S'(t)$ where $S : \Re_+ \to [0, d]$ is an S-curve with inflexion at t_f, and $g : \Re_+ \to \Re_+$ is a strictly increasing continuously differentiable function. Furthermore, $\lim_{t \to \infty} h(t) = 0$. If $g'(t)S'(t) + g(t)S''(t)$ satisfies the UCC at $t_0 \in (0, \infty)$ then h satisfies Definition* A.2, *and has its maximal value at $t_0 > t_f$.*

Proof The derivative $h'(t) = g'(t)S'(t) + g(t)S''(t)$. The first term is always positive, and the second term has the sign of S'' which is positive for $t < t_f$. Therefore, h starts out increasing.

If h' satisfies the UCC at t_0 then h reaches a global maximum at t_0.

$t_0 > t_f$ because $h'(t_0) = 0$ implies that $g(t_0)S''(t_0) < 0$. □

Remark A.5 If in Lemma A.2 $g(t) = t$, then the function that must satisfy the UCC reduces to $S'(t) + tS''(t)$, or, equivalently, $1 + tS''(t)/S'(t)$; that is, $tS''(t)/S'(t)$ must uniquely cross from above the horizontal line at ordinate negative 1. Division by $S'(t)$ is possible because $S'(t) > 0 \quad \forall t \in (0, \infty)$.

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