

# Joint Linear Receiver Design and Power Allocation Using Alternating Optimization Algorithms for Wireless Sensor Networks

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**Abstract**—In this paper, we consider a two-hop wireless sensor network (WSN) with multiple relay nodes where the amplify-and-forward (AF) scheme is employed. We present strategies to design jointly linear receivers and the power-allocation parameters via an alternating optimization approach subject to global, individual, and neighbor-based power constraints. Two design criteria are considered: The first criterion minimizes the mean-square error (MSE), and the second criterion maximizes the sum-rate (SR) of the WSN. We derive constrained minimum mean-square error (MMSE) and constrained maximum sum-rate (MSR) expressions for the linear receivers and the power-allocation parameters that contain the optimal complex amplification coefficients for each relay node. Computer simulations show good performance of our proposed methods in terms of bit error rate (BER) or SR compared with the method with equal power allocation and to a two-stage power-allocation technique. Furthermore, the methods with neighbor-based constraints bring flexibility to balance the performance against the computational complexity and the need for feedback information, which is desirable for WSNs to extend their lifetime.

**Index Terms**—Maximum sum-rate (MSR) criterion, minimum mean square error (MMSE) criterion, power allocation, wireless sensor networks (WSNs).

## I. INTRODUCTION

RECENTLY, there has been a growing research interest in wireless sensor networks (WSNs) because of their unique features that allow a wide range of applications in the areas of defense, environment, health, and home [1]. WSNs are usually composed of a large number of densely deployed sensing devices, which can transmit their data to the desired user through multihop relays [2]. Low complexity and high energy efficiency are the most important design characteristics of communication protocols [3] and physical layer techniques employed for WSNs. The performance and capacity of these networks can be significantly enhanced by exploiting the spatial

diversity with cooperation between the nodes [2], [4], [5]. In a cooperative WSN, nodes relay signals to each other to propagate redundant copies of the same signals to the destination nodes. Among the existing relaying schemes, the amplify-and-forward (AF) and decode-and-forward (DF) schemes are the most popular approaches [6]. In the AF scheme, the relay nodes amplify the received signal and rebroadcast the amplified signals toward the destination nodes. In the DF scheme, the relay nodes first decode the received signals and then regenerate new signals to the destination nodes subsequently.

Due to the limitations in sensor node power, computational capacity, and memory [1], some power-allocation methods have been proposed for WSNs to obtain the best possible SNR or the best possible quality of service (QoS) [7], [8] at the destinations. The majority of the previous literature considers a source–destination pair, with one or more randomly placed relay nodes. These relay nodes are usually placed with uniform distribution [9], equal distance [10], or in line [11] with the source and destination. The reason for these simple considerations is that they can simplify complex problems and obtain closed-form solutions. A single-relay AF system using mean-channel-gain channel state information (CSI) is analyzed in [12], where the outage probability is the criterion used for optimization. For DF systems, a near-optimal power-allocation strategy called the fixed-sum-power-with-equal-ratio scheme based on partial CSI has been developed in [9]. This near-optimal scheme allocates half of the total power to the source node and splits the remaining half equally among selected relay nodes. A node is selected for relaying if its mean channel gain to the destination is above a threshold. Simulation results show that this scheme significantly outperforms two traditional power-allocation schemes. The first scheme is the “constant-power scheme,” where all nodes serve as relay nodes, and all nodes including the source node and relay nodes transmit with the same power. The other scheme is the “best-select scheme,” where only one node with the largest mean channel gain to the destination is chosen as the relay node.

The bit-error-rate (BER) performance [13], [14], capacity [15], and outage probability [16], [17] are often used as the optimization criterion for the power-allocation performance. In [18], a power-allocation method is proposed to maximize the effective configuration duration in WSNs. It aims to minimize the signaling overhead for performing selection of relay nodes and power allocation, which can save the power significantly and thus extend the lifetime. Compared with traditional

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power-allocation schemes, this method jointly considers the residual energy of sensors and the mean channel gains. Therefore, the feedback burden is limited, and the stability of the topology is increased.

The alternating minimization procedure under the information geometry framework was proposed by Csiszar and Tusnady in 1984 [19], which have developed a proof for its global convergence in problems involving two variables. It is a very successful technique that has been used to solve optimization problems in applications that include signal processing, information theory, control, and finance because of its iterative nature and simplicity. A general set of sufficient conditions for its convergence and correctness were developed in [20] for adaptive problems.

In this paper, we consider a general two-hop WSN where the AF relaying scheme is employed. Our strategy is to design jointly the linear receivers and the power-allocation parameter vector that contains the optimal complex amplification coefficients for each relay node via an alternating optimization approach. Two kinds of receivers are designed: the minimum mean-square error (MMSE) receiver and the maximum sum-rate (MSR) receiver. They can be considered as solutions to constrained optimization problems where the objective function is the MSE cost function or the SR, and the constraint is a bound on the power levels among the relay nodes. Then, the constrained MMSE or MSR expressions for the linear receiver and the power-allocation parameter can be derived. For the MMSE receiver, a closed-form solution for the Lagrangian multiplier ( $\lambda$ ) that arises in the expressions of the power-allocation parameter can be achieved. For the MSR receiver, the novelty is that we make use of the generalized Rayleigh quotient [21] to solve the optimization problem in an alternating fashion. Finally, the optimal amplification coefficients are transmitted to the relay nodes through the feedback channel. In this paper, we first present the strategies where the power allocation is considered for all of the relay nodes. They are subject to the global or individual power constraints. Next, to reduce the computational complexity for the power allocation, we choose the relay nodes that have good channel coefficients (when a channel power gain is above a threshold) between them and the destination nodes called neighbor relay nodes. Only the power allocation for these nodes is required, and the remaining nodes use the equal power-allocation method [9]. Therefore, the computational complexity and feedback burden can be reduced. The main contributions of this paper can be summarized as follows.

- 1) Constrained MMSE expressions for the design of linear receivers and power-allocation parameters are discussed. The constraints include the global, individual, and neighbor-based power constraints. Some preliminary results of this part have been reported in [22].
- 2) Constrained MSR expressions for the design of linear receivers and power-allocation parameters are discussed. The constraints include the global and neighbor-based power constraints.
- 3) Alternating optimization algorithms that compute the linear receivers and power-allocation parameters in the

first and second contributions, to minimize the MSE or maximize the SR of the WSN are discussed.

- 4) Computational complexity and convergence analysis of the proposed optimization algorithms are discussed.

The rest of this paper is organized as follows. Section II describes the general two-hop WSN system model. Section III develops three joint MMSE receiver design and power-allocation strategies subject to three different power constraints. Section IV develops two joint MSR receiver design and power-allocation strategies subject to two different power constraints. Section V contains the analysis of the computational complexity and the convergence. Section VI presents and discusses the simulation results, whereas Section VII provides some concluding remarks.

## II. SYSTEM MODEL

Consider a general two-hop WSN with multiple parallel relay nodes, as shown in Fig. 1. The WSN consists of  $N_s$  source nodes,  $N_d$  destination nodes, and  $N_r$  relay nodes. We concentrate on a time-division scheme with perfect synchronization, for which all signals are transmitted and received in separate time slots. The sources first broadcast the  $N_s \times 1$  signal vector  $\mathbf{s}$  to all relay nodes. We consider an AF cooperation protocol in this paper. Each relay node receives the signal, and amplifies and rebroadcasts them to the destination nodes. In practice, we need to consider the constraints on the transmission policy. For example, each transmitting node would transmit during only one phase. Let  $\mathbf{H}_s$  denote the  $N_r \times N_s$  channel matrix between the source nodes and the relay nodes and  $\mathbf{H}_d$  denote the  $N_d \times N_r$  channel matrix between the relay nodes and the destination nodes, as given by

$$\mathbf{H}_s = \begin{bmatrix} \mathbf{h}_{s,1} \\ \mathbf{h}_{s,2} \\ \vdots \\ \mathbf{h}_{s,N_r} \end{bmatrix}, \quad \mathbf{H}_d = \begin{bmatrix} \mathbf{h}_{d,1} \\ \mathbf{h}_{d,2} \\ \vdots \\ \mathbf{h}_{d,N_d} \end{bmatrix} \quad (1)$$

where  $\mathbf{h}_{s,i} = [h_{s,i,1}, h_{s,i,2}, \dots, h_{s,i,N_s}]$  for  $i = 1, 2, \dots, N_r$  denotes the channel coefficients between the source nodes and the  $i$ th relay node, and  $\mathbf{h}_{d,i} = [h_{d,i,1}, h_{d,i,2}, \dots, h_{d,i,N_r}]$  for  $i = 1, 2, \dots, N_d$  denotes the channel coefficients between the relay nodes and the  $i$ th destination node. The received signal at the relay nodes can be expressed as

$$\mathbf{x} = \mathbf{H}_s \mathbf{s} + \mathbf{v}_r \quad (2)$$

$$\mathbf{y} = \mathbf{F} \mathbf{x} \quad (3)$$

where  $\mathbf{v}$  is a zero-mean circularly symmetric complex additive white Gaussian noise (AWGN) vector with covariance matrix  $\sigma_n^2 \mathbf{I}$ , and  $\mathbf{F} = \text{diag}\{(\sigma_s^2 |\mathbf{h}_{s,1}|^2 + \sigma_n^2), (\sigma_s^2 |\mathbf{h}_{s,2}|^2 + \sigma_n^2), \dots, (\sigma_s^2 |\mathbf{h}_{s,N_r}|^2 + \sigma_n^2)\}^{-1/2}$  denotes the normalization matrix, which can normalize the power of the received signal for each relay node. At the destination nodes, the received signal can be expressed as

$$\mathbf{d} = \mathbf{H}_d \mathbf{A} \mathbf{y} + \mathbf{v}_d \quad (4)$$

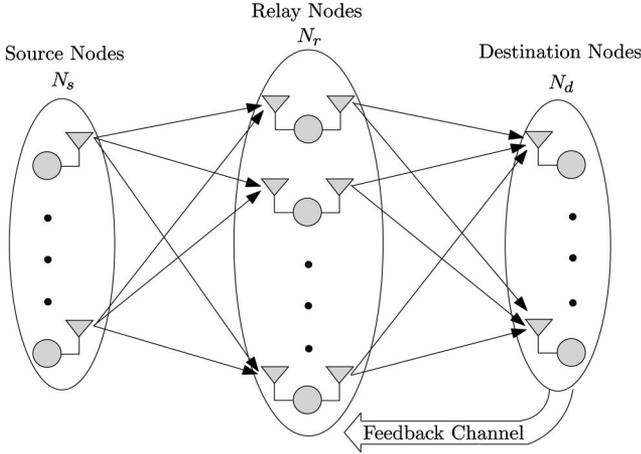


Fig. 1. Two-hop cooperative WSN with  $N_s$  source nodes,  $N_d$  destination nodes and  $N_r$  relay nodes.

where  $\mathbf{A} = \text{diag}\{a_1, a_2, \dots, a_{N_r}\}$  is a diagonal matrix whose elements represent the amplification coefficient of each relay node. Note that the property of the matrix vector multiplication  $\mathbf{A}\mathbf{y} = \mathbf{Y}\mathbf{a}$  will be used in the following, where  $\mathbf{Y}$  is the diagonal matrix form of the vector  $\mathbf{y}$ , and  $\mathbf{a}$  is the vector form of the diagonal matrix  $\mathbf{A}$ . In our proposed designs, the full CSI of the system is assumed to be known at all the destination nodes. In practice, a fusion center [23] that contains the destination nodes is responsible for gathering the CSI, computing the optimal linear filters and the optimal amplification coefficients. The fusion center also recovers the transmitted signal of the source nodes and transmits the optimal amplification coefficients to the relay nodes via a feedback channel.

### III. PROPOSED JOINT MINIMUM MEAN-SQUARE ERROR DESIGN OF THE RECEIVER AND POWER ALLOCATION

Here, three constrained optimization problems are proposed to describe the joint design of the MMSE linear receiver  $\mathbf{W}$  and the power-allocation parameter  $\mathbf{a}$  subject to a global, individual, and neighbor-based power constraints.

#### A. MMSE Design With a Global Power Constraint

We first consider the case where the total power of all the relay nodes is limited to  $P_T$ . The proposed method can be considered as the following optimization problem:

$$\begin{aligned} [\mathbf{W}_{\text{opt}}, \mathbf{a}_{\text{opt}}] &= \arg \min_{\mathbf{W}, \mathbf{a}} E [\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] \\ \text{subject to } N_d \mathbf{a}^H \mathbf{a} &= P_T \end{aligned} \quad (5)$$

where  $(\cdot)^H$  denotes the complex-conjugate (Hermitian) transpose. To solve this constrained optimization problem, we modify the MSE cost function using the method of Lagrange multipliers [24], which yields the following Lagrangian function:

$$\begin{aligned} \mathcal{L} &= E [\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] + \lambda (N_d \mathbf{a}^H \mathbf{a} - P_T) \\ &= E(\mathbf{s}^H \mathbf{s}) - E(\mathbf{d}^H \mathbf{W} \mathbf{s}) - E(\mathbf{s}^H \mathbf{W}^H \mathbf{d}) + E(\mathbf{d}^H \mathbf{W} \mathbf{W}^H \mathbf{d}) \\ &\quad + \lambda (N_d \mathbf{a}^H \mathbf{a} - P_T). \end{aligned} \quad (6)$$

By fixing  $\mathbf{a}$  and setting the gradient of  $\mathcal{L}$  in (6) with respect to the conjugate of the filter  $\mathbf{W}^*$  equal to zero, where  $(\cdot)^*$  denotes the complex conjugate, we get

$$\begin{aligned} \mathbf{W}_{\text{opt}} &= [E(\mathbf{d} \mathbf{d}^H)]^{-1} E(\mathbf{d} \mathbf{s}^H) \\ &= [\mathbf{H}_d \mathbf{A} E(\mathbf{y} \mathbf{y}^H) \mathbf{A}^H \mathbf{H}_d^H + \sigma_n^2 \mathbf{I}]^{-1} \mathbf{H}_d \mathbf{A} E(\mathbf{y} \mathbf{s}^H). \end{aligned} \quad (7)$$

The optimal expression for the power-allocation vector  $\mathbf{a}$  is obtained by equating the partial derivative of  $\mathcal{L}$  with respect to  $\mathbf{a}^*$  to zero, i.e.,

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{a}^*} &= -E \left( \frac{\partial \mathbf{d}^H}{\partial \mathbf{a}^*} \mathbf{W} \mathbf{s} \right) + E \left( \frac{\partial \mathbf{d}^H}{\partial \mathbf{a}^*} \mathbf{W} \mathbf{W}^H \mathbf{d} \right) + N_d \lambda \mathbf{a} \\ &= -E(\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{s}) \\ &\quad + E[\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H (\mathbf{H}_d \mathbf{Y} \mathbf{a} + \mathbf{v}_d)] + N_d \lambda \mathbf{a} \\ &= \mathbf{0}. \end{aligned} \quad (8)$$

Therefore, we get

$$\begin{aligned} \mathbf{a}_{\text{opt}} &= [E(\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{Y}) + N_d \lambda \mathbf{I}]^{-1} \\ &\quad \times E(\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{s}) \\ &= [\mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \circ E(\mathbf{y} \mathbf{y}^H)^* + N_d \lambda \mathbf{I}]^{-1} \\ &\quad \times [\mathbf{H}_d^H \mathbf{W} \circ E(\mathbf{y} \mathbf{s}^H)^* \mathbf{u}] \end{aligned} \quad (9)$$

where  $\circ$  denotes the Hadamard (elementwise) product, and  $\mathbf{u} = [1, 1, \dots, 1]^T$ . The expressions in (7) and (9) depend on each other. Thus, it is necessary to iterate them with an initial value of  $\mathbf{a}$  to obtain the solutions.

The Lagrange multiplier  $\lambda$  can be determined by solving

$$N_d \mathbf{a}_{\text{opt}}^H \mathbf{a}_{\text{opt}} = P_T. \quad (10)$$

Let

$$\phi = E(\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{W}^H \mathbf{H}_d \mathbf{Y}) \quad (11)$$

$$\mathbf{z} = E(\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{s}). \quad (12)$$

Equation (10) becomes

$$N_d \mathbf{z}^H (\phi + N_d \lambda \mathbf{I})^{-1} (\phi + N_d \lambda \mathbf{I})^{-1} \mathbf{z} = P_T. \quad (13)$$

Using an eigenvalue decomposition (EVD), we have

$$\phi = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^{-1} \quad (14)$$

where  $\mathbf{\Lambda} = \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_M, 0, \dots, 0\}$  consists of eigenvalues of  $\phi$  and  $M = \min\{N_s, N_r, N_d\}$ . Then, we get

$$\phi + N_d \lambda \mathbf{I} = \mathbf{Q} (\mathbf{\Lambda} + N_d \lambda \mathbf{I}) \mathbf{Q}^{-1}. \quad (15)$$

Therefore, (13) can be expressed as

$$N_d \mathbf{z}^H \mathbf{Q} (\mathbf{\Lambda} + N_d \lambda \mathbf{I})^{-2} \mathbf{Q}^{-1} \mathbf{z} = P_T. \quad (16)$$

Using the properties of the trace operation, (16) can be written as

$$N_d \text{tr} \{ (\mathbf{\Lambda} + N_d \lambda \mathbf{I})^{-2} \mathbf{Q}^{-1} \mathbf{z} \mathbf{z}^H \mathbf{Q} \} = P_T. \quad (17)$$

Defining  $\mathbf{C} = \mathbf{Q}^{-1}\mathbf{z}\mathbf{z}^H\mathbf{Q}$ , (17) becomes

$$N_d \sum_{i=1}^{N_r} (\Lambda(i, i) + N_d \lambda)^{-2} \mathbf{C}(i, i) = P_T. \quad (18)$$

Since  $\phi$  is a matrix with at most rank  $M$ , only the first  $M$  columns of  $\mathbf{Q}$  span the column space of  $E(\mathbf{Y}^H \mathbf{H}_d^H \mathbf{W} \mathbf{s})^H$ , which causes the last  $(N_r - M)$  columns of  $\mathbf{z}^H \mathbf{Q}$  to become zero vectors; thus, the last  $(N_r - M)$  diagonal elements of  $\mathbf{C}$  are zero. Therefore, we obtain the  $\{2M\}$ th-order polynomial in  $\lambda$  as follows:

$$N_d \sum_{i=1}^M (\alpha_i + N_d \lambda)^{-2} \mathbf{C}(i, i) = P_T. \quad (19)$$

### B. MMSE Design With Individual Power Constraints

Second, we consider the case where the power of each relay node is limited to some value  $P_{T,i}$ . The proposed method can be considered as the following optimization problem:

$$\begin{aligned} [\mathbf{W}_{\text{opt}}, a_{1,\text{opt}}, \dots, a_{N_r,\text{opt}}] &= \arg \min_{\mathbf{W}, a_1, \dots, a_{N_r}} E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] \\ \text{subject to } P_i &= P_{T,i}, \quad i = 1, 2, \dots, N_r \end{aligned} \quad (20)$$

where  $P_i$  is the transmitted power of the  $i$ th relay node, and  $P_i = N_d a_i^* a_i$ . Using the method of Lagrange multipliers, we have the following Lagrangian function:

$$\mathcal{L} = E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] + \sum_{i=1}^{N_r} \lambda_i (N_d a_i^* a_i - P_{T,i}). \quad (21)$$

Following the same steps described in Section III-A, we get the same optimal expression for the  $\mathbf{W}$  as in (7), and the optimal expression for the  $a_i$  is

$$a_{i,\text{opt}} = [\phi(i, i) + N_d \lambda_i]^{-1} \left[ \mathbf{z}(i) - \sum_{l \in I, l \neq i} \phi(i, l) a_l \right] \quad (22)$$

where  $I = \{1, 2, \dots, N_r\}$ ,  $\phi$ , and  $\mathbf{z}$  have the same expression as in (11) and (12). The Lagrange multiplier  $\lambda_i$  can be determined by solving

$$N_d a_{i,\text{opt}}^* a_{i,\text{opt}} = P_{T,i}, \quad i = 1, 2, \dots, N_r. \quad (23)$$

### C. MMSE Design With a Neighbor-Based Power Constraint

To reduce the computational complexity for power allocation and the need for feedback, we choose the relay nodes that have good channel coefficients between them and the destination nodes called neighbor relay nodes. Only the power allocation for these nodes is required, and the remaining nodes employ the equal power-allocation method. Therefore, the computational

complexity and feedback burden can be reduced. The received signal at the destination nodes can be rewritten as

$$\begin{aligned} \mathbf{d} &= \mathbf{H}_d \mathbf{A} \mathbf{y} + \mathbf{v}_d \\ &= \mathbf{H}_N \mathbf{A}_N \mathbf{y}_N + \mathbf{H}_o \mathbf{A}_o \mathbf{y}_o + \mathbf{v}_d \end{aligned} \quad (24)$$

where  $\mathbf{A}_N$  and  $\mathbf{y}_N$  denote the amplification matrix and the normalized signal for the neighbor relay nodes, and  $\mathbf{A}_o$  and  $\mathbf{y}_o$  denote the amplification matrix and the normalized signal for the nonneighbor relay nodes, respectively.

We consider the case where the total power of all the neighbor relay nodes is limited to  $P_N$  and  $P_N + N_d \mathbf{a}_o^H \mathbf{a}_o = P_T$ . The proposed method can be considered as the following optimization problem:

$$\begin{aligned} [\mathbf{W}_{\text{opt}}, \mathbf{a}_{N,\text{opt}}] &= \arg \min_{\mathbf{W}, \mathbf{a}_N} E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] \\ \text{subject to } N_d \mathbf{a}_N^H \mathbf{a}_N &= P_N. \end{aligned} \quad (25)$$

Using the method of Lagrange multipliers, we have the following Lagrangian function:

$$\mathcal{L} = E[\|\mathbf{s} - \mathbf{W}^H \mathbf{d}\|^2] + \lambda_N (N_d \mathbf{a}_N^H \mathbf{a}_N - P_N). \quad (26)$$

Following the same steps described in Section III-A, we get the same optimal expression for  $\mathbf{W}$  as in (7). Substituting (24) into (26), equating the partial derivative of  $\mathcal{L}$  with respect to  $\mathbf{a}_N^*$  to zero gives

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \mathbf{a}_N^*} &= -E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{s}) \\ &+ E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_N \mathbf{Y}_N) \mathbf{a}_N \\ &+ E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_o \mathbf{Y}_o \mathbf{a}_o) + N_d \lambda_N \mathbf{a}_N \\ &= \mathbf{0}. \end{aligned} \quad (27)$$

Therefore, we obtain the optimal expression for  $\mathbf{a}_N$  as follows:

$$\begin{aligned} \mathbf{a}_{N,\text{opt}} &= [E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_N \mathbf{Y}_N) + N_d \lambda_N \mathbf{I}]^{-1} \\ &\times [E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{s}) - E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_o \mathbf{Y}_o \mathbf{a}_o)]. \end{aligned} \quad (28)$$

The Lagrange multiplier  $\lambda_N$  can be determined by solving

$$N_d \mathbf{a}_{N,\text{opt}}^H \mathbf{a}_{N,\text{opt}} = P_N. \quad (29)$$

Let

$$\phi_N = E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_N \mathbf{Y}_N) \quad (30)$$

$$\mathbf{z}_N = E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{s}) - E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_o \mathbf{Y}_o \mathbf{a}_o). \quad (31)$$

Equation (29) becomes

$$N_d \mathbf{z}_N^H (\phi_N + N_d \lambda_N \mathbf{I})^{-1} (\phi_N + N_d \lambda_N \mathbf{I})^{-1} \mathbf{z}_N = P_N. \quad (32)$$

Using EVD

$$\phi_N = \mathbf{Q}_N \mathbf{\Lambda}_N \mathbf{Q}_N^{-1} \quad (33)$$

TABLE I  
SUMMARY OF THE PROPOSED MMSE DESIGN WITH GLOBAL, INDIVIDUAL, AND NEIGHBOUR-BASED POWER CONSTRAINTS

Global Power Constraint	Individual Power Constraints	Neighbour-based Power Constraint
Initialize the algorithm by setting: $\mathbf{A} = \sqrt{\frac{P_T}{N_r N_d}} \mathbf{I}$	Initialize the algorithm by setting: $a_i = \sqrt{\frac{P_{T,i}}{N_d}}$ for $i = 1, 2, \dots, N_r$	Initialize the algorithm by setting: $\mathbf{A} = \sqrt{\frac{P_T}{N_r N_d}} \mathbf{I}$
For each iteration: 1. Compute $\mathbf{W}_{\text{opt}}$ in (7). 2. Compute $\phi$ and $\mathbf{z}$ in (11) and (12). 3. Calculate the EVD of $\phi$ in (14). 4. Solve $\lambda$ in (19). 5. Compute $\mathbf{a}_{\text{opt}}$ in (9).	For each iteration: 1. Compute $\mathbf{W}_{\text{opt}}$ in (7). 2. Compute $\phi$ and $\mathbf{z}$ in (11) and (12). 3. For $i = 1, 2, \dots, N_r$ a) Solve $\lambda_i$ in (23). b) Compute $a_{i,\text{opt}}$ in (22).	For each iteration: 1. Compute $\mathbf{W}_{\text{opt}}$ in (7). 2. Compute $\phi_N$ and $\mathbf{z}_N$ in (30) and (31). 3. Calculate the EVD of $\phi_N$ in (33). 4. Solve $\lambda_N$ in (38). 5. Compute $\mathbf{a}_{N,\text{opt}}$ in (28).

where  $\mathbf{\Lambda}_N = \text{diag}\{\alpha_1, \alpha_2, \dots, \alpha_M, 0, \dots, 0\}$  consists of the eigenvalues of  $\phi_N$ , and  $M_N = \min\{N_s, N_N, N_d\}$  ( $N_N$  is the number of neighbor relay nodes), we get

$$\phi_N + N_d \lambda_N \mathbf{I} = \mathbf{Q}_N (\mathbf{\Lambda}_N + N_d \lambda_N \mathbf{I}) \mathbf{Q}_N^{-1}. \quad (34)$$

Therefore, (32) can be expressed as

$$N_d \mathbf{z}_N^H \mathbf{Q}_N (\mathbf{\Lambda}_N + N_d \lambda_N \mathbf{I})^{-2} \mathbf{Q}_N^{-1} \mathbf{z}_N = P_N. \quad (35)$$

Using the properties of the trace operation, (35) can be written as

$$N_d \text{tr} \{ (\mathbf{\Lambda}_N + N_d \lambda_N \mathbf{I})^{-2} \mathbf{Q}_N^{-1} \mathbf{z}_N \mathbf{z}_N^H \mathbf{Q}_N \} = P_N. \quad (36)$$

Defining  $\mathbf{C}_N = \mathbf{Q}_N^{-1} \mathbf{z}_N \mathbf{z}_N^H \mathbf{Q}_N$ , (36) becomes

$$N_d \sum_{i=1}^{M_N} (\Lambda_N(i, i) + N_d \lambda_N)^{-2} \mathbf{C}_N(i, i) = P_N. \quad (37)$$

Since  $\phi_N$  is a matrix with at most rank  $M_N$ , only the first  $M_N$  columns of  $\mathbf{Q}_N$  span the column space of  $E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W}_s)^H$  and  $E(\mathbf{Y}_N^H \mathbf{H}_N^H \mathbf{W} \mathbf{W}^H \mathbf{H}_o \mathbf{Y}_o \mathbf{a}_o)^H$ , which cause the last  $(N_N - M_N)$  columns of  $\mathbf{z}_N^H \mathbf{Q}_N$  to become zero vectors; thus, the last  $(N_N - M_N)$  diagonal elements of  $\mathbf{C}_N$  are zero. Therefore, we can obtain the  $\{2M\}$ -th-order polynomial in  $\lambda_N$  as follows:

$$N_d \sum_{i=1}^{M_N} (\alpha_i + N_d \lambda_N)^{-2} \mathbf{C}_N(i, i) = P_N. \quad (38)$$

We notice from the equations here that when all the relay nodes are chosen as the neighbor relay nodes, the MMSE design with a neighbor-based power constraint is equivalent to the MMSE design with a global power constraint. Therefore, the global approach can be considered as a specific case of the neighbor-based approach. Table I shows a summary of our proposed MMSE design with global, individual, and neighbor-based power constraints that will be used for the simulations. If the quasi-static fading channel (block fading) is considered in the simulations, we only need two iterations.

#### IV. PROPOSED JOINT MAXIMUM SUM-RATE DESIGN OF THE RECEIVER AND POWER ALLOCATION

Here, two constrained optimization problems are proposed to describe the joint MSR design of the linear receiver  $\mathbf{w}$

and the power-allocation parameter  $\mathbf{a}$  subject to global and neighbor-based power constraints. By the MSR designs, the best possible SNR and QoS can be obtained at the destinations. They will improve the spectrum efficiency that is desirable for the WSNs with the limitation in the sensor node computational capacity. The individual power constraints are not considered here, because for the MSR receiver, we make use of the generalized Rayleigh quotient which is only suitable to solve the optimization problems for the vectors.

##### A. MSR Design With a Global Power Constraint

We first consider the case where the total power of all the relay nodes is limited to  $P_T$ . By substituting (2) and (3) into (4), we get

$$\mathbf{d} = \mathbf{H}_d \mathbf{A} \mathbf{F} \mathbf{H}_s \mathbf{s} + \mathbf{H}_d \mathbf{A} \mathbf{F} \mathbf{v}_r + \mathbf{v}_d. \quad (39)$$

We focus on a system with one source node for simplicity. Therefore, the expression of the SR in terms of bits per second per Hertz (bps/Hz) for our two-hop WSN is

$$\text{SR} = \frac{1}{2} \log_2 \left[ 1 + \frac{\sigma_s^2}{\sigma_n^2} \frac{\mathbf{w}^H \mathbf{H}_d \mathbf{A} \mathbf{F} \mathbf{H}_s \mathbf{H}_s^H \mathbf{F}^H \mathbf{A}^H \mathbf{H}_d^H \mathbf{w}}{\mathbf{w}^H (\mathbf{H}_d \mathbf{A} \mathbf{F} \mathbf{F}^H \mathbf{A}^H \mathbf{H}_d^H + \mathbf{I}) \mathbf{w}} \right] \quad (40)$$

where  $\mathbf{w}$  is the linear receiver, and  $(\cdot)^H$  denotes the complex-conjugate (Hermitian) transpose. Let

$$\mathbf{\Phi} = \mathbf{H}_d \mathbf{A} \mathbf{F} \mathbf{H}_s \mathbf{H}_s^H \mathbf{F}^H \mathbf{A}^H \mathbf{H}_d^H \quad (41)$$

$$\mathbf{Z} = \mathbf{H}_d \mathbf{A} \mathbf{F} \mathbf{F}^H \mathbf{A}^H \mathbf{H}_d^H + \mathbf{I}. \quad (42)$$

Equation (40) becomes

$$\text{SR} = \frac{1}{2} \log_2 \left( 1 + \frac{\sigma_s^2}{\sigma_n^2} \frac{\mathbf{w}^H \mathbf{\Phi} \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} \right) = \frac{1}{2} \log_2(1 + ax) \quad (43)$$

where

$$a = \frac{\sigma_s^2}{\sigma_n^2} \quad (44)$$

$$x = \frac{\mathbf{w}^H \mathbf{\Phi} \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}}. \quad (45)$$

Since  $1/2 \log_2(1 + ax)$  is a monotonically increasing function of  $x$  ( $a > 0$ ), the problem of maximizing the SR is equivalent

to maximizing  $x$ . Therefore, the proposed method can be considered as the following optimization problem:

$$\begin{aligned} [\mathbf{w}_{\text{opt}}, \mathbf{a}_{\text{opt}}] &= \arg \max_{\mathbf{w}, \mathbf{a}} \frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} \\ \text{subject to } N_d \mathbf{a}^H \mathbf{a} &= P_T. \end{aligned} \quad (46)$$

We note that the expression  $\mathbf{w}^H \Phi \mathbf{w} / \mathbf{w}^H \mathbf{Z} \mathbf{w}$  in (46) is the generalized Rayleigh quotient. Thus, the optimal solution of our maximization problem can be solved [21]:  $\mathbf{w}_{\text{opt}}$  is any eigenvector corresponding to the dominant eigenvalue of  $\mathbf{Z}^{-1} \Phi$ .

To obtain the optimal power-allocation vector  $\mathbf{a}_{\text{opt}}$ , we rewrite  $\mathbf{w}^H \Phi \mathbf{w} / \mathbf{w}^H \mathbf{Z} \mathbf{w}$ , and the expression is given by

$$\frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} = \frac{\mathbf{a}^H \text{diag}\{\mathbf{w}^H \mathbf{H}_d \mathbf{F}\} \mathbf{H}_s \mathbf{H}_s^H \text{diag}\{\mathbf{F}^H \mathbf{H}_d^H \mathbf{w}\} \mathbf{a}}{\mathbf{a}^H \text{diag}\{\mathbf{w}^H \mathbf{H}_d \mathbf{F}\} \text{diag}\{\mathbf{F}^H \mathbf{H}_d^H \mathbf{w}\} \mathbf{a} + \mathbf{w}^H \mathbf{w}}. \quad (47)$$

Since the multiplication of any constant value and an eigenvector is still an eigenvector of the matrix, we express the receive filter as

$$\mathbf{w} = \frac{\mathbf{w}_{\text{opt}}}{\sqrt{\mathbf{w}_{\text{opt}}^H \mathbf{w}_{\text{opt}}}}. \quad (48)$$

Hence, we obtain

$$\mathbf{w}^H \mathbf{w} = 1 = \frac{N_d \mathbf{a}^H \mathbf{a}}{P_T}. \quad (49)$$

By substituting (49) into (47), we get

$$\frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} = \frac{\mathbf{a}^H \text{diag}\{\mathbf{w}^H \mathbf{H}_d \mathbf{F}\} \mathbf{H}_s \mathbf{H}_s^H \text{diag}\{\mathbf{F}^H \mathbf{H}_d^H \mathbf{w}\} \mathbf{a}}{\mathbf{a}^H \left( \text{diag}\{\mathbf{w}^H \mathbf{H}_d \mathbf{F}\} \text{diag}\{\mathbf{F}^H \mathbf{H}_d^H \mathbf{w}\} + \frac{N_d}{P_T} \mathbf{I} \right) \mathbf{a}}. \quad (50)$$

Let

$$\mathbf{M} = \text{diag}\{\mathbf{w}^H \mathbf{H}_d \mathbf{F}\} \mathbf{H}_s \mathbf{H}_s^H \text{diag}\{\mathbf{F}^H \mathbf{H}_d^H \mathbf{w}\} \quad (51)$$

$$\mathbf{N} = \text{diag}\{\mathbf{w}^H \mathbf{H}_d \mathbf{F}\} \text{diag}\{\mathbf{F}^H \mathbf{H}_d^H \mathbf{w}\} + \frac{N_d}{P_T} \mathbf{I}. \quad (52)$$

Equation (50) becomes

$$\frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} = \frac{\mathbf{a}^H \mathbf{M} \mathbf{a}}{\mathbf{a}^H \mathbf{N} \mathbf{a}}. \quad (53)$$

Likewise, we note that the expression  $\mathbf{a}^H \mathbf{M} \mathbf{a} / \mathbf{a}^H \mathbf{N} \mathbf{a}$  in (53) is the generalized Rayleigh quotient. Thus, the optimal solution of our maximization problem can be solved:  $\mathbf{a}_{\text{opt}}$  is any eigenvector corresponding to the dominant eigenvalue of  $\mathbf{N}^{-1} \mathbf{M}$  and satisfying  $\mathbf{a}_{\text{opt}}^H \mathbf{a}_{\text{opt}} = P_T / N_d$ . The solutions of  $\mathbf{w}_{\text{opt}}$  and  $\mathbf{a}_{\text{opt}}$  depend on each other. Thus, it is necessary to iterate them with an initial value of  $\mathbf{a}$  to obtain the optimum solutions.

### B. MSR Design With a Neighbor-Based Power Constraint

Similar to the steps described in Section III-C, we separate the relay nodes into neighbor relay nodes and nonneighbor

nodes in the expressions of the system model. Therefore, (2) and (3) can be rewritten as

$$\mathbf{x}_N = \mathbf{H}_{s,N} \mathbf{N} \mathbf{s} + \mathbf{v}_N \quad (54)$$

$$\mathbf{x}_o = \mathbf{H}_{s,o} \mathbf{s} + \mathbf{v}_o \quad (55)$$

$$\mathbf{y}_N = \mathbf{F}_N \mathbf{x}_N \quad (56)$$

$$\mathbf{y}_o = \mathbf{F}_o \mathbf{x}_o \quad (57)$$

where the subscript  $N$  is denoted for the neighbor relay nodes, and the subscript  $o$  is used for the nonneighbor relay nodes. By substituting (54)–(57) into (24), we get

$$\begin{aligned} \mathbf{d} &= (\mathbf{H}_N \mathbf{A}_N \mathbf{F}_N \mathbf{H}_{s,N} + \mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{H}_{s,o}) \mathbf{s} \\ &\quad + \mathbf{H}_N \mathbf{A}_N \mathbf{F}_N \mathbf{v}_N + \mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{v}_o + \mathbf{v}_d. \end{aligned} \quad (58)$$

We focus on the system which consists of one source node. Therefore, the expression of the SR in terms of bps/Hz for our two-hop WSN is

$$\text{SR} = \frac{1}{2} \log_2 \left( 1 + \frac{\sigma_s^2 \mathbf{w}^H \Phi \mathbf{w}}{\sigma_n^2 \mathbf{w}^H \mathbf{Z} \mathbf{w}} \right) \quad (59)$$

where

$$\begin{aligned} \Phi &= (\mathbf{H}_N \mathbf{A}_N \mathbf{F}_N \mathbf{H}_{s,N} + \mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{H}_{s,o}) \\ &\quad \times (\mathbf{H}_N \mathbf{A}_N \mathbf{F}_N \mathbf{H}_{s,N} + \mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{H}_{s,o})^H \end{aligned} \quad (60)$$

$$\begin{aligned} \mathbf{Z} &= \mathbf{H}_N \mathbf{A}_N \mathbf{F}_N \mathbf{F}_N^H \mathbf{A}_N^H \mathbf{H}_N^H \\ &\quad + \mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{F}_o^H \mathbf{A}_o^H \mathbf{H}_o^H + \mathbf{I}. \end{aligned} \quad (61)$$

We consider the case where the total power of all the neighbor relay nodes is limited to  $P_N$  and  $P_N + N_d \mathbf{a}_o^H \mathbf{a}_o = P_T$ . Following the same steps described in Section IV-A, the proposed method can be considered as the following optimization problem:

$$\begin{aligned} [\mathbf{w}_{\text{opt}}, \mathbf{a}_{N,\text{opt}}] &= \arg \max_{\mathbf{w}, \mathbf{a}_N} \frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} \\ \text{subject to } N_d \mathbf{a}_N^H \mathbf{a}_N &= P_N. \end{aligned} \quad (62)$$

We note that the expression  $\mathbf{w}^H \Phi \mathbf{w} / \mathbf{w}^H \mathbf{Z} \mathbf{w}$  in (62) is the generalized Rayleigh quotient. Thus, the optimal solution of our maximization problem can be solved:  $\mathbf{w}_{\text{opt}}$  is any eigenvector corresponding to the dominant eigenvalue of  $\mathbf{Z}^{-1} \Phi$ .

To obtain the optimal power-allocation vector for the neighbor relay nodes  $\mathbf{a}_{N,\text{opt}}$ , we rewrite  $\mathbf{w}^H \Phi \mathbf{w} / \mathbf{w}^H \mathbf{Z} \mathbf{w}$  as follows:

$$\frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} = \frac{\mathbf{a}_N^H \mathbf{M}_1 \mathbf{a}_N + \mathbf{a}_o^H \mathbf{M}_2 \mathbf{a}_o + \mathbf{a}_o^H \mathbf{M}_3 \mathbf{a}_N + \mathbf{a}_o^H \mathbf{M}_4 \mathbf{a}_o}{\mathbf{a}_N^H \mathbf{N}_1 \mathbf{a}_N + \mathbf{w}^H \mathbf{N}_2 \mathbf{w}} \quad (63)$$

where

$$\mathbf{M}_1 = \text{diag}\{\mathbf{w}^H \mathbf{H}_N \mathbf{F}_N\} \mathbf{H}_{s,N} \mathbf{H}_{s,N}^H \text{diag}\{\mathbf{F}_N^H \mathbf{H}_N^H \mathbf{w}\} \quad (64)$$

$$\mathbf{M}_2 = \text{diag}\{\mathbf{w}^H \mathbf{H}_N \mathbf{F}_N\} \mathbf{H}_{s,N} \mathbf{H}_{s,o}^H \text{diag}\{\mathbf{F}_o^H \mathbf{H}_o^H \mathbf{w}\} \quad (65)$$

$$\mathbf{M}_3 = \text{diag}\{\mathbf{w}^H \mathbf{H}_o \mathbf{F}_o\} \mathbf{H}_{s,o} \mathbf{H}_{s,N}^H \text{diag}\{\mathbf{F}_N^H \mathbf{H}_N^H \mathbf{w}\} \quad (66)$$

$$\mathbf{M}_4 = \text{diag}\{\mathbf{w}^H \mathbf{H}_o \mathbf{F}_o\} \mathbf{H}_{s,o} \mathbf{H}_{s,o}^H \text{diag}\{\mathbf{F}_o^H \mathbf{H}_o^H \mathbf{w}\} \quad (67)$$

$$\mathbf{N}_1 = \text{diag}\{\mathbf{w}^H \mathbf{H}_N \mathbf{F}_N\} \text{diag}\{\mathbf{F}_N^H \mathbf{H}_N^H \mathbf{w}\} \quad (68)$$

$$\mathbf{N}_2 = \mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{F}_o^H \mathbf{A}_o^H \mathbf{H}_o^H + \mathbf{I}. \quad (69)$$

TABLE II  
SUMMARY OF THE PROPOSED MSR DESIGN WITH GLOBAL AND NEIGHBOUR-BASED POWER CONSTRAINTS

Global Power Constraint	Neighbour-based Power Constraint
Initialize the algorithm by setting: $\mathbf{A} = \sqrt{\frac{P_r}{N_r N_d}} \mathbf{I}$ For each iteration: 1. Compute $\Phi$ and $\mathbf{Z}$ in (41) and (42). 2. Use the QR algorithm or the power method to compute the dominant eigenvector of $\mathbf{Z}^{-1}\Phi$ , denoted as $\mathbf{w}_{\text{opt}}$ . 3. Compute $\mathbf{M}$ and $\mathbf{N}$ in (51) and (52). 4. Use the QR algorithm or the power method to compute the dominant eigenvector of $\mathbf{N}^{-1}\mathbf{M}$ , denoted as $\mathbf{a}$ . 5. To ensure the power constraint $\mathbf{a}_{\text{opt}}^H \mathbf{a}_{\text{opt}} = \frac{P_r}{N_d}$ , compute $\mathbf{a}_{\text{opt}} = \sqrt{\frac{P_r}{N_d \mathbf{a}^H \mathbf{a}}}$ .	Initialize the algorithm by setting: $\mathbf{A} = \sqrt{\frac{P_r}{N_r N_d}} \mathbf{I}$ (include $\mathbf{a}_N$ and $\mathbf{a}_o$ ) For each iteration: 1. Compute $\Phi$ and $\mathbf{Z}$ in (60) and (61). 2. Use the QR algorithm or the power method to compute the dominant eigenvector of $\mathbf{Z}^{-1}\Phi$ , denoted as $\mathbf{w}_{\text{opt}}$ . 3. Compute $\mathbf{T}$ in (76). 4. Compute $\mathbf{M}$ and $\mathbf{N}$ in (78) and (73). 5. Use the QR algorithm or the power method to compute the dominant eigenvector of $\mathbf{N}^{-1}\mathbf{M}$ , denoted as $\mathbf{a}_N$ . 6. To ensure the power constraint $\mathbf{a}_{N,\text{opt}}^H \mathbf{a}_{N,\text{opt}} = \frac{P_N}{N_d}$ , compute $\mathbf{a}_{N,\text{opt}} = \sqrt{\frac{P_N}{N_d \mathbf{a}_N^H \mathbf{a}_N}} \mathbf{a}_N$ .

Since the multiplication of any constant value and an eigenvector is still an eigenvector of the matrix, we express the receive filter as

$$\mathbf{w} = \frac{\mathbf{w}_{\text{opt}}}{\sqrt{\mathbf{w}_{\text{opt}}^H (\mathbf{H}_o \mathbf{A}_o \mathbf{F}_o \mathbf{F}_o^H \mathbf{A}_o^H \mathbf{H}_o^H + \mathbf{I}) \mathbf{w}_{\text{opt}}}}. \quad (70)$$

Therefore, we obtain

$$\mathbf{w}^H \mathbf{N}_2 \mathbf{w} = 1 = \frac{N_d}{P_N} \mathbf{a}_N^H \mathbf{a}_N. \quad (71)$$

By substituting (71) into (63), we obtain

$$\frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} = \frac{\mathbf{a}_N^H \mathbf{M}_1 \mathbf{a}_N + \mathbf{a}_N^H \mathbf{M}_2 \mathbf{a}_o + \mathbf{a}_o^H \mathbf{M}_3 \mathbf{a}_N + \mathbf{a}_o^H \mathbf{M}_4 \mathbf{a}_o}{\mathbf{a}_N^H \mathbf{N} \mathbf{a}_N} \quad (72)$$

where

$$\mathbf{N} = \mathbf{N}_1 + \frac{N_d}{P_N} \mathbf{I}. \quad (73)$$

The expression in (72) can be divided into four terms, and only the first term is the generalized Rayleigh quotient. To make use of the generalized Rayleigh quotient to solve the optimization problem, our aim is to transform the remaining three terms into the generalized Rayleigh quotient. For the fourth term, we have

$$\begin{aligned} \mathbf{a}_o^H \mathbf{M}_4 \mathbf{a}_o &= \mathbf{a}_o^H \mathbf{M}_4 \mathbf{a}_o \frac{N_d \mathbf{a}_N^H \mathbf{a}_N}{P_N} \\ &= \mathbf{a}_N^H \left( \frac{N_d \mathbf{a}_o^H \mathbf{M}_4 \mathbf{a}_o}{P_N} \mathbf{I} \right) \mathbf{a}_N. \end{aligned} \quad (74)$$

For the second and third terms, we can achieve the generalized Rayleigh quotient by solving the following optimization problem:

$$\begin{aligned} [\mathbf{T}_{\text{opt}}, \mathbf{a}_{N,\text{opt}}] &= \arg \min_{\mathbf{T}, \mathbf{a}_N} (\mathbf{a}_N^H \mathbf{M}_2 \mathbf{a}_o + \mathbf{a}_o^H \mathbf{M}_3 \mathbf{a}_N - \mathbf{a}_N^H \mathbf{T} \mathbf{a}_N)^2 \\ &\text{subject to } N_d \mathbf{a}_N^H \mathbf{a}_N = P_N. \end{aligned} \quad (75)$$

By fixing  $\mathbf{a}_N$ , we obtain

$$\mathbf{T} = \frac{N_d}{P_N} (\mathbf{M}_2 \mathbf{a}_o \mathbf{a}_N^H + \mathbf{a}_N \mathbf{a}_o^H \mathbf{M}_3) \quad (76)$$

which satisfies the following equation:

$$\mathbf{a}_N^H \mathbf{M}_2 \mathbf{a}_o + \mathbf{a}_o^H \mathbf{M}_3 \mathbf{a}_N = \mathbf{a}_N^H \mathbf{T} \mathbf{a}_N \quad (77)$$

for any value of  $\mathbf{a}_N$ . Let us define

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{T} + \frac{N_d \mathbf{a}_o^H \mathbf{M}_4 \mathbf{a}_o}{P_N} \mathbf{I}. \quad (78)$$

Then, (72) becomes

$$\frac{\mathbf{w}^H \Phi \mathbf{w}}{\mathbf{w}^H \mathbf{Z} \mathbf{w}} = \frac{\mathbf{a}_N^H \mathbf{M} \mathbf{a}_N}{\mathbf{a}_N^H \mathbf{N} \mathbf{a}_N} \quad (79)$$

which is a generalized Rayleigh quotient. Therefore, the optimal solution of our maximization problem can be solved:  $\mathbf{a}_{N,\text{opt}}$  is any eigenvector corresponding to the dominant eigenvalue of  $\mathbf{N}^{-1}\mathbf{M}$  and satisfies  $\mathbf{a}_{N,\text{opt}}^H \mathbf{a}_{N,\text{opt}} = P_N/N_d$ .

Here, two methods are employed to calculate the dominant eigenvectors. The first method is the QR algorithm [25], which calculates all the eigenvalues and eigenvectors of a matrix. We can choose the dominant eigenvector among them. The second method is the power method [25], which only calculates the dominant eigenvector of a matrix. Hence, the computational complexity can be reduced. Table II shows a summary of our proposed MSR design with global and neighbor-based power constraints, which will be used for the simulations. If the quasi-static fading channel (block fading) is considered in the simulations, we only need two iterations.

## V. ANALYSIS OF THE PROPOSED ALGORITHMS

Here, an analysis of the computational complexity and the convergence of the algorithms are developed.

### A. Computational Complexity Analysis

Tables III and IV list the computational complexity per iteration in terms of the number of multiplications, additions, and divisions for our proposed joint linear receiver design (MMSE and MSR) and power-allocation strategies. For the joint MMSE designs, we use the QR algorithm to perform the eigendecomposition of the matrix. We set  $M = \min\{N_s, N_r, N_d\} = 1$  and  $M_N = \min\{N_s, N_N, N_d\} = 1$  to simplify the processing of solving the equations in (19) and (38). Note that, in this paper,

TABLE III  
COMPUTATIONAL COMPLEXITY PER ITERATION OF THE JOINT MMSE DESIGNS

Parameter	Power Constraint Type	Multiplications	Additions	Divisions
<b>W</b>	All	$N_d(N_d - 1)(4N_d + 1)/6$ $+(N_s + N_r)N_d^2 + N_r^2N_d + N_sN_rN_d$ $+N_rN_d$	$N_d(N_d - 1)(4N_d + 1)/6$ $+(N_s + N_r)N_d^2 + N_r^2N_d + N_sN_rN_d$ $-(N_d^2 + 2N_sN_d + N_rN_d) + N_d$	$N_d(3N_d - 1)/2$
	Global	$n_Q(\frac{13}{6}N_r^3 + \frac{3}{2}N_r^2 + \frac{1}{3}N_r - 2)$ $-N_r^3 + 3N_sN_r^2 + N_sN_rN_d$ $+N_r^2 + N_sN_r + 1$	$n_Q(\frac{13}{6}N_r^3 - N_r^2 - \frac{1}{6}N_r + 1)$ $-N_r^3 + 3N_sN_r^2 + N_sN_rN_d$ $-N_r^2 - 2N_sN_r - N_r + 1$	$n_Q(N_r - 1) + 1$
$\lambda$	Individual	$N_sN_r^2 + N_sN_rN_d + 2N_r^2 + N_sN_r + N_r$	$N_sN_r^2 + N_sN_rN_d - N_sN_r$	$N_r$
	Neighbour-based	$n_Q(\frac{13}{6}N_N^3 + \frac{3}{2}N_N^2 + \frac{1}{3}N_N - 2)$ $-N_N^3 + 2N_sN_N^2 + N_sN_rN_d + N_sN_rN_N$ $-N_N^2 + 2N_rN_N + N_sN_N + 1$	$n_Q(\frac{13}{6}N_N^3 - N_N^2 - \frac{1}{6}N_N + 1)$ $-N_N^3 + 2N_sN_N^2 + N_sN_rN_d + N_sN_rN_N$ $-N_N^2 - N_sN_N - N_sN_r - 2N_N + 2$	$n_Q(N_N - 1) + 1$
<b>a</b>	Global	$N_r(N_r - 1)(4N_r + 1)/6 + N_r^2 + 1$	$N_r(N_r - 1)(4N_r + 1)/6 + N_r^2$	$N_r(3N_r - 1)/2$
	Individual	$2N_r$	$N_r$	$N_r$
	Neighbour-based	$N_N(N_N - 1)(4N_N + 1)/6 + N_N^2 + 1$	$N_N(N_N - 1)(4N_N + 1)/6 + N_N^2$	$N_N(3N_N - 1)/2$

TABLE IV  
COMPUTATIONAL COMPLEXITY PER ITERATION OF THE JOINT MSR DESIGNS

Parameter	Power Constraint Type	Multiplications	Additions	Divisions
<b>w</b>	Global/Neighbour-based QR Algorithm	$n_Q(\frac{13}{6}N_d^3 + \frac{3}{2}N_d^2 + \frac{1}{3}N_d - 2)$ $+N_d(N_d - 1)(4N_d + 1)/6$ $+N_rN_d^2 + N_d^2 + 3N_rN_d$	$n_Q(\frac{13}{6}N_d^3 - N_d^2 - \frac{1}{6}N_d + 1)$ $+N_d(N_d - 1)(4N_d + 1)/6$ $+N_rN_d^2 - N_d^2 + N_rN_d$	$n_Q(N_d - 1)$ $+N_d(3N_d - 1)/2$
	Global/Neighbour-based Power Method	$n_PN_d^2$ $+N_d(N_d - 1)(4N_d + 1)/6$ $+N_d^3 + N_rN_d^2 + N_d^2 + 3N_rN_d$	$n_PN_d(N_d - 1)$ $+N_d(N_d - 1)(4N_d + 1)/6$ $+N_d^3 + N_rN_d^2 - 2N_d^2 + N_rN_d$	$N_d(3N_d - 1)/2$
<b>a</b>	Global QR Algorithm	$n_Q(\frac{13}{6}N_r^3 + \frac{3}{2}N_r^2 + \frac{1}{3}N_r - 2)$ $+N_r(N_r - 1)(4N_r + 1)/6$ $+N_r^2 + N_rN_d + 4N_r + N_d$	$n_Q(\frac{13}{6}N_r^3 - N_r^2 - \frac{1}{6}N_r + 1)$ $+N_r(N_r - 1)(4N_r + 1)/6$ $+N_rN_d + N_r + N_d - 2$	$n_Q(N_r - 1)$ $+N_r(3N_r - 1)/2$ $+N_d + 1$
	Global Power Method	$n_PN_r^2$ $+N_r(N_r - 1)(4N_r + 1)/6$ $+N_r^3 + N_r^2 + N_rN_d + 4N_r + N_d$	$n_PN_r(N_r - 1)$ $+N_r(N_r - 1)(4N_r + 1)/6$ $+N_r^3 - N_r^2 + N_rN_d + N_r + N_d - 2$	$N_r(3N_r - 1)/2$ $+N_d + 1$
	Neighbour-based QR Algorithm	$n_Q(\frac{13}{6}N_N^3 + \frac{3}{2}N_N^2 + \frac{1}{3}N_N - 2)$ $+N_N(N_N - 1)(4N_N + 1)/6$ $-N_N^3 + N_rN_N^2 + 2N_r^2 + 2N_N^2 + N_d^2$ $+N_rN_d - 2N_rN_N + 2N_r + 2N_N + N_d + 1$	$n_Q(\frac{13}{6}N_N^3 - N_N^2 - \frac{1}{6}N_N + 1)$ $+N_N(N_N - 1)(4N_N + 1)/6$ $-N_N^3 + N_rN_N^2 + N_r^2 + 2N_N^2 + N_d^2$ $+N_rN_d - 2N_rN_N - N_r + 3N_N - 3$	$n_Q(N_N - 1)$ $+N_N(3N_N - 1)/2$ $+N_d + 1$
	Neighbour-based Power Method	$n_PN_N^2$ $+N_N(N_N - 1)(4N_N + 1)/6$ $+N_rN_N^2 + 2N_r^2 + 2N_N^2 + N_d^2$ $+N_rN_d - 2N_rN_N + 2N_r + 2N_N + N_d + 1$	$n_PN_r(N_r - 1)$ $+N_N(N_N - 1)(4N_N + 1)/6$ $+N_rN_N^2 + N_r^2 + N_N^2 + N_d^2$ $+N_rN_d - 2N_rN_N - N_r + 3N_N - 3$	$N_N(3N_N - 1)/2$ $+N_d + 1$

the QR decomposition by Householder transformation [25], [26] is employed by the QR algorithms.  $n_Q$  and  $n_P$  denote the number of iterations of the QR algorithm and the power method, respectively. Because the multiplication dominates the computational complexity, to compare the computational complexity of our proposed joint MMSE and MSR designs, the number of multiplications versus the number of relay nodes for each iteration are displayed in Figs. 2 and 3. For the purpose of illustration, we set  $N_s = 1$ ,  $N_d = 2$ , and  $n_Q = n_P = 10$ .  $R$  denotes the averaged ratio of the number of neighbor relay nodes to the number of relay nodes. It can be seen that our pro-

posed MMSE and MSR receivers with a neighbor-based power constraint have a significant complexity reduction compared with the proposed receivers with a global power constraint. Obviously, lower  $R$  will lead to lower computational complexity. For the MMSE design, when the individual power constraints are considered, the computational complexity is lower than the other constraints because there is no need to compute the eigendecomposition for it. For the MSR design, employing the power method to calculate the dominant eigenvectors has lower computational complexity than employing the QR algorithm.

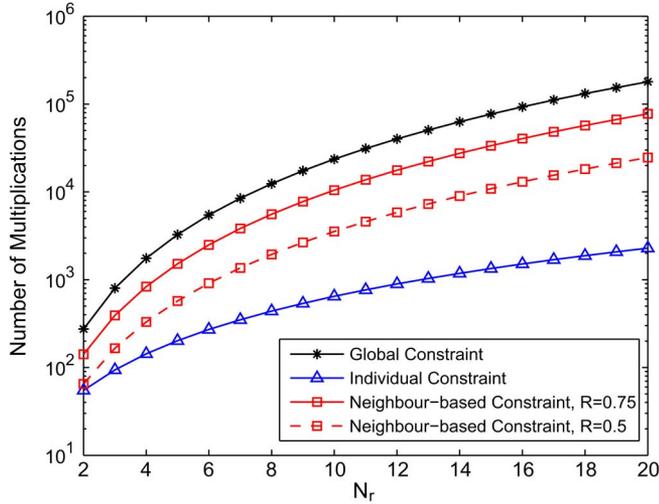


Fig. 2. Number of multiplications versus the number of relay nodes of our proposed joint MMSE design of the receiver and power allocation strategies.

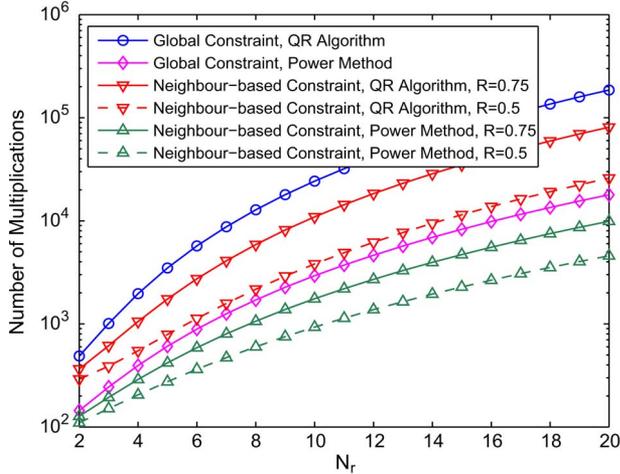


Fig. 3. Number of multiplications versus the number of relay nodes of our proposed joint MSR design of the receiver and power allocation strategies.

### B. Sufficient Conditions for Convergence

To develop the analysis and proofs, we need to define a metric space and the Hausdorff distance that will extensively be used. A metric space is an ordered pair  $(\mathcal{M}, d)$ , where  $\mathcal{M}$  is a nonempty set, and  $d$  is a metric on  $\mathcal{M}$ , i.e., a function  $d: \mathcal{M} \times \mathcal{M} \rightarrow \mathbb{R}$  such that, for any  $x, y$ , and  $z \in \mathcal{M}$ , the following conditions hold.

- 1)  $d(x, y) \geq 0$ .
- 2)  $d(x, y) = 0$  if and only if  $x = y$ .
- 3)  $d(x, y) = d(y, x)$ .
- 4)  $d(x, y) \leq d(x, z) + d(z, y)$ .

The Hausdorff distance measures how far two subsets of a metric space are from each other and is defined by

$$d_H(X, Y) = \max \left\{ \sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y) \right\}. \quad (80)$$

The proposed joint MMSE designs can be stated as an alternating minimization strategy based on the MSE defined

in (5) and expressed as

$$\mathbf{W}_n \in \arg \min_{\mathbf{W} \in \underline{\mathbf{W}}_n} \text{MSE}(\mathbf{W}, \mathbf{a}_{n-1}) \quad (81)$$

$$\mathbf{a}_n \in \arg \min_{\mathbf{a} \in \underline{\mathbf{a}}_n} \text{MSE}(\mathbf{W}_n, \mathbf{a}) \quad (82)$$

where the sets  $\underline{\mathbf{W}}, \underline{\mathbf{a}} \subset \mathcal{M}$ , and the sequences of compact sets  $\{\underline{\mathbf{W}}_n\}_{n \geq 0}$  and  $\{\underline{\mathbf{a}}_n\}_{n \geq 0}$  converge to the sets  $\underline{\mathbf{W}}$  and  $\underline{\mathbf{a}}$ , respectively. Although we are not given the sets  $\underline{\mathbf{W}}$  and  $\underline{\mathbf{a}}$  directly, we have the sequence of compact sets  $\{\underline{\mathbf{W}}_n\}_{n \geq 0}$  and  $\{\underline{\mathbf{a}}_n\}_{n \geq 0}$ . The aim of our proposed joint MMSE designs is to find a sequence of  $\mathbf{W}_n$  and  $\mathbf{a}_n$  such that

$$\lim_{n \rightarrow \infty} \text{MSE}(\mathbf{W}_n, \mathbf{a}_n) = \text{MSE}(\mathbf{W}_{\text{opt}}, \mathbf{a}_{\text{opt}}) \quad (83)$$

where  $\mathbf{W}_{\text{opt}}$  and  $\mathbf{a}_{\text{opt}}$  correspond to the optimal values of  $\mathbf{W}_n$  and  $\mathbf{a}_n$ , respectively. To present a set of sufficient conditions under which the proposed algorithms converge, we need the so-called three-point and four-point properties [19], [20]. Let us assume that there is a function  $f: \mathcal{M} \times \mathcal{M} \rightarrow \mathbb{R}$  such that the following conditions are satisfied.

- 1) *Three-point property*  $(\mathbf{W}, \widetilde{\mathbf{W}}, \mathbf{a})$ : For all  $n \geq 1$ ,  $\mathbf{W} \in \underline{\mathbf{W}}_n$ ,  $\mathbf{a} \in \underline{\mathbf{a}}_{n-1}$ , and  $\widetilde{\mathbf{W}} \in \arg \min_{\mathbf{W} \in \underline{\mathbf{W}}_n} \text{MSE}(\mathbf{W}, \mathbf{a})$ , we have

$$f(\mathbf{W}, \widetilde{\mathbf{W}}) + \text{MSE}(\widetilde{\mathbf{W}}, \mathbf{a}) \leq \text{MSE}(\mathbf{W}, \mathbf{a}). \quad (84)$$

- 2) *Four-point property*  $(\mathbf{W}, \mathbf{a}, \widetilde{\mathbf{W}}, \tilde{\mathbf{a}})$ : For all  $n \geq 1$ ,  $\mathbf{W} \in \underline{\mathbf{W}}_n$ ,  $\mathbf{a} \in \underline{\mathbf{a}}_n$ , and  $\tilde{\mathbf{a}} \in \arg \min_{\mathbf{a} \in \underline{\mathbf{a}}_n} \text{MSE}(\widetilde{\mathbf{W}}, \mathbf{a})$ , we have

$$\text{MSE}(\mathbf{W}, \tilde{\mathbf{a}}) \leq \text{MSE}(\mathbf{W}, \mathbf{a}) + f(\mathbf{W}, \widetilde{\mathbf{W}}). \quad (85)$$

These two properties are the mathematical expressions of the sufficient conditions for the convergence of the alternating minimization algorithms that are stated in [19] and [20]. This means that if there exists a function  $f(\mathbf{W}, \widetilde{\mathbf{W}})$  with the parameter  $\mathbf{W}$  during two iterations that satisfies the two inequalities about the MSE in (84) and (85), the convergence of our proposed MMSE designs that make use of the alternating minimization algorithm can be proved by the following theorem.

*Theorem:* Let  $\{(\underline{\mathbf{W}}_n, \underline{\mathbf{a}}_n)\}_{n \geq 0}$  and  $\underline{\mathbf{W}}, \underline{\mathbf{a}}$  be compact subsets of the compact metric space  $(\mathcal{M}, d)$  such that

$$\underline{\mathbf{W}}_n \xrightarrow{d_H} \underline{\mathbf{W}}, \quad \underline{\mathbf{a}}_n \xrightarrow{d_H} \underline{\mathbf{a}} \quad (86)$$

and let  $\text{MSE}: \mathcal{M} \times \mathcal{M} \rightarrow \mathbb{R}$  be a continuous function. Let conditions 1) and 2) hold. Then, for the proposed algorithms, we have

$$\lim_{n \rightarrow \infty} \text{MSE}(\mathbf{W}_n, \mathbf{a}_n) = \text{MSE}(\mathbf{W}_{\text{opt}}, \mathbf{a}_{\text{opt}}). \quad (87)$$

A general proof of this theorem is detailed in [19] and [20]. The proposed joint MSR designs can be stated as an alternating maximization strategy based on the SR defined in (40) that follows a similar procedure to that given earlier.

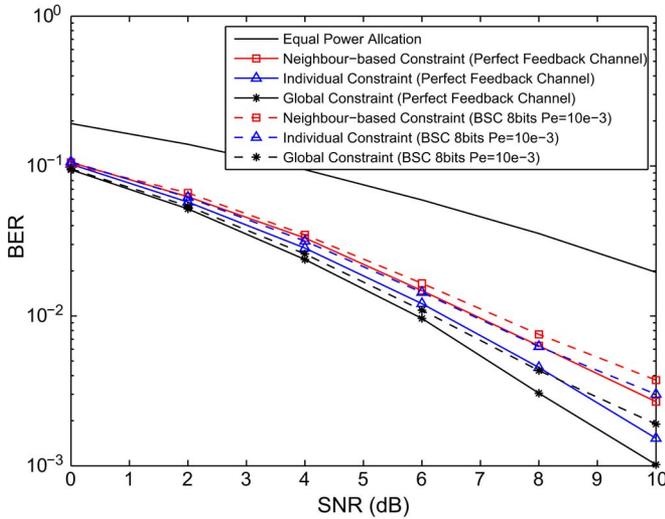


Fig. 4. BER performance versus SNR of our proposed joint MMSE design of the receiver and power allocation strategies, compared with the equal power allocation method.

VI. SIMULATIONS

Here, we numerically study the performance of our proposed joint designs of the linear receiver and the power-allocation methods and compare them with the equal power-allocation method [9] that allocates the same power level for all links between relay nodes and destination nodes. For the purpose of fairness, we assume that the total power for all relay nodes in the network is the same, which can be indicated as  $\sum_{i=1}^{N_r} P_{T,i} = P_T$ . We consider a two-hop WSN. The number of source nodes  $N_s$ , relay nodes  $N_r$ , and destination nodes  $N_d$  are 1, 4 and 2, respectively. We consider an AF cooperation protocol. The quasi-static fading channel (block fading channel) is considered in our simulations whose elements are Rayleigh random variables (with zero mean and unit variance) and assumed to be invariant during the transmission of each packet. In our simulations, the channel is assumed to be known at the destination nodes. For channel estimation algorithms for WSNs and other low-complexity parameter estimation algorithms, refer to [28] and [29]. During each phase, the source transmits the quadrature phase-shift keying-modulated packets with 1500 symbols. The noise at the relay and destination nodes is modeled as circularly symmetric complex Gaussian random variables with zero mean. A perfect (error-free) feedback channel between the destination nodes and the relay nodes is assumed to transmit the amplification coefficients.

For the MMSE design, it is shown Fig. 4 that our three proposed methods achieve a better BER performance than the equal power-allocation method. Among them, the method with a global constraint has the best performance. This result is what we expected because a global constraint provides the largest degrees of freedom for allocating the power among the relay nodes. For the method with a neighbor-based constraint, we introduce a bound  $B$ , which is set to 0.6, for the channel power gain between the relay nodes and the destination nodes to choose the neighbor relay nodes. Although it has a higher BER compared with the method with a global constraint, the averaged ratio of the number of neighbor relay nodes

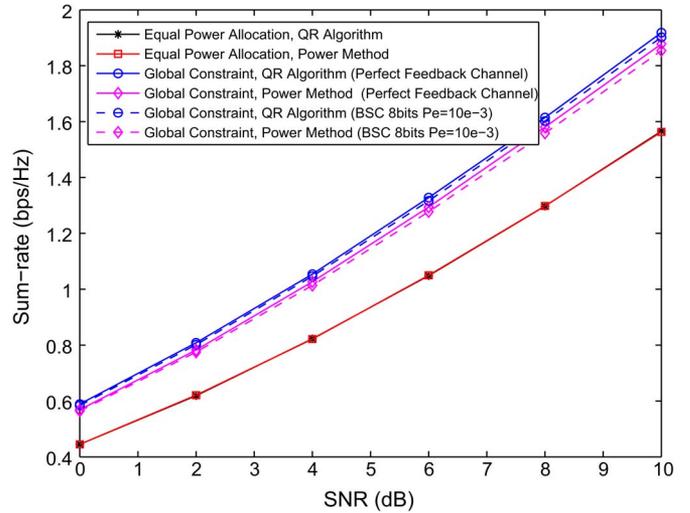


Fig. 5. Sum-rate performance versus SNR of our proposed joint MSR design of the receiver and power allocation strategies with a global constraint, compared with the equal power allocation method.

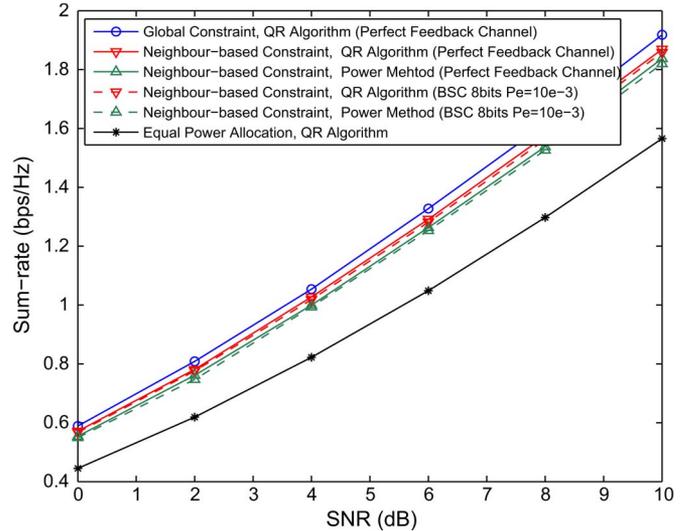


Fig. 6. Sum-rate performance versus SNR of our proposed joint MSR design of the receiver and power allocation strategies with a neighbor-based constraint, compared with the equal power allocation method.

to the number of relay nodes  $R$  is 0.7843, which indicates reduced computational complexity. For the MSR design, it is shown in Figs. 5 and 6 that our proposed methods achieve a better sum-rate performance than the equal power-allocation method. Using the power method to calculate the dominant eigenvector yields a very similar result to the QR algorithm but requires lower complexity. For the method with a neighbor-based constraint, when we introduce bound  $B = 0.6$ , a similar performance to the method with a global constraint can be achieved with reduced  $R$  (0.7830).

To show the performance tendency for other values of  $B$ , we fix the SNR at 10 dB and choose  $B$  ranging from 0 to 1.5. The performance curves are shown in Figs. 7 and 8, which include the BER and sum-rate performance versus  $B$ , and  $R$  versus  $B$  of the MMSE design and MSR design, respectively, with a neighbor-based power constraint. It can be seen that, along with the increase in  $B$ , their performance becomes worse, and  $R$  becomes lower. This demonstrates that for our joint designs of

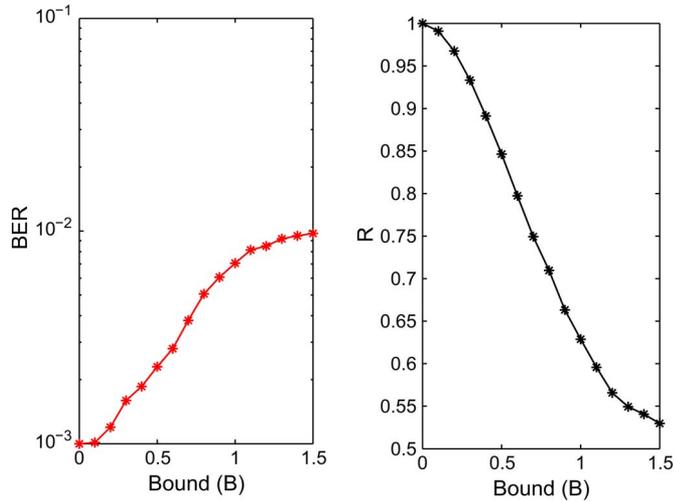


Fig. 7. (a) BER performance versus the bound and (b)  $R$  versus the bound of the MMSE design with a neighbor-based power constraint.

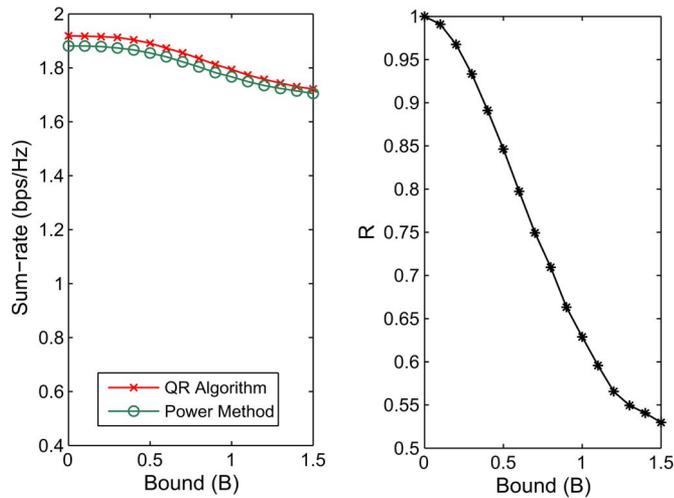


Fig. 8. (a) Sum-rate performance versus the bound and (b)  $R$  versus the bound of the MSR design with a neighbor-based power constraint.

the receivers with a neighbor-based power constraint, the value of  $B$  can be varied to trade off achievable performance against computation complexity.

In addition to the equal power-allocation scheme, the two-stage power-allocation scheme reported in [27] has also been used for comparison. It is shown in Fig. 9 that our proposed MMSE and MSR designs outperform the two-stage power-allocation scheme. Note that to have a fair comparison for which the sum power of all the relay nodes is constrained (global constraint), we only employ the second stage of the two-stage power-allocation scheme in the simulations.

In practice, the feedback channel cannot be error free. To study the impact of feedback channel errors on the performance, we employ the binary symmetric channel (BSC) as the model for the feedback channel and quantize each complex amplification coefficient to an 8-bit binary value (4 bits for the real part and 4 bits for the imaginary part). The error probability  $Pe$  of the BSC is fixed at  $10^{-3}$ . The dashed curves in Figs. 4–6 show the performance degradation compared with the performance when using a perfect feedback channel. To show the performance tendency of the BSC for other values of  $Pe$ , we

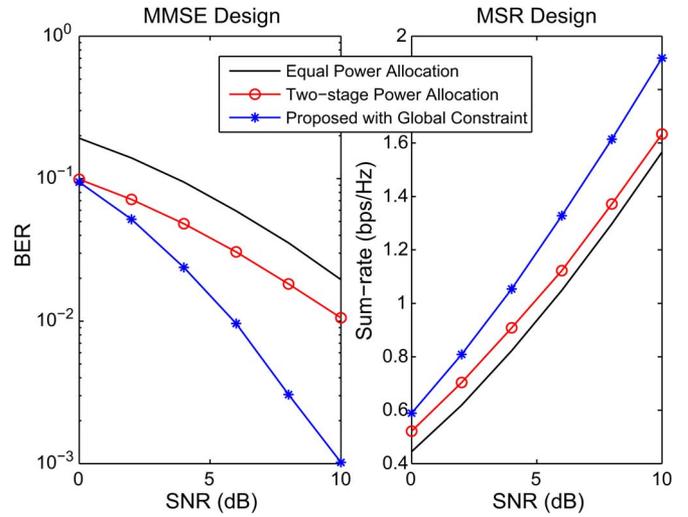


Fig. 9. (a) BER performance versus SNR of our proposed MMSE design. (b) Sum-rate performance versus SNR of our proposed MSR design with a global power constraint and compare with the two-stage power allocation and equal power allocation schemes.

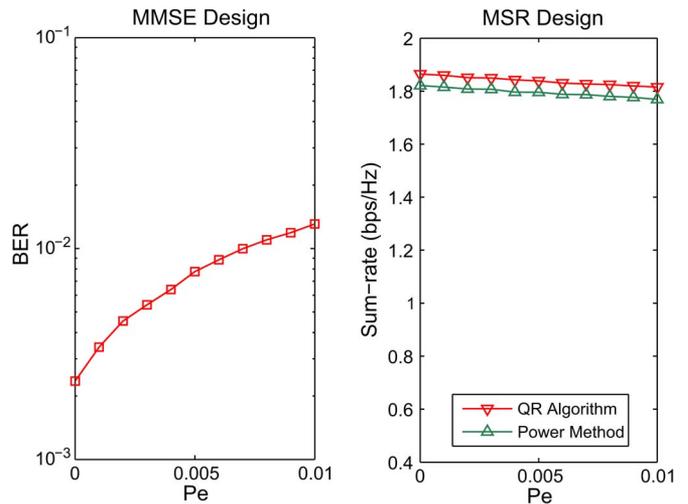


Fig. 10. (a) BER performance versus  $Pe$  of our proposed MMSE design. (b) Sum-rate performance versus  $Pe$  of our proposed MSR design with a neighbor-based power constraint when employing the BSC as the model for the feedback channel.  $B = 0.6$ .

fix the SNR at 10 dB and choose  $Pe$  ranging from 0 to  $10^{-2}$ . The performance curves are shown in Fig. 10, which illustrate the BER and the sum-rate performance versus  $Pe$  of our two proposed joint designs of the receivers with neighbor-based power constraints. It can be seen that along with the increase in  $Pe$ , their performance becomes worse.

Next, we replace the perfect CSI with the estimated channel coefficients to compute the receive filters and power-allocation parameters at the destinations. We employ the BEACON channel estimation, which is proposed in [28]. Fig. 11 shows the impact of the channel estimation on the performance of our proposed MMSE and SMR design with a global power constraint by comparing it with the performance of perfect CSI. The quantity  $n_t$  denotes the number of training sequence symbol per data packet. Note that, in these simulations, perfect feedback channel is considered, and the QR algorithm is used in the MSR design. For both the MMSE and MSR designs, it can be

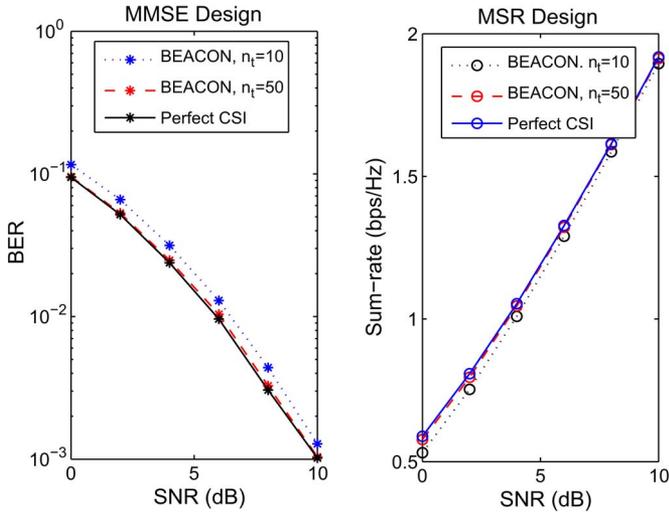


Fig. 11. (a) BER performance versus SNR of our proposed MMSE design (b) Sum-rate performance versus SNR our proposed MSR design with a global power constraint when employing the BEACON channel estimation, compared with the performance of perfect CSI.

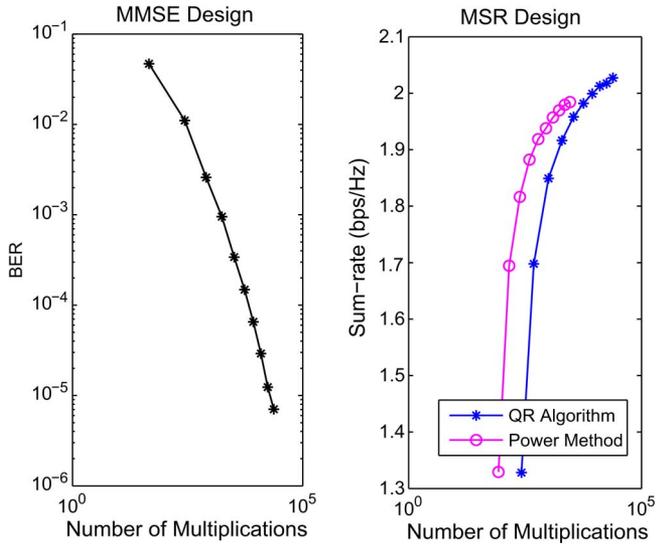


Fig. 12. (a) BER performance versus number of multiplications of our proposed MMSE design (b) Sum-rate performance versus number of multiplications of our proposed MSR design with a global power constraint.

seen that when  $n_t$  is set to 10, the BEACON channel estimation leads to an obvious performance degradation compared with the perfect CSI. However, when  $n_t$  is increased to 50, the BEACON channel estimation can achieve a similar performance to the perfect CSI. Other scenarios and network topologies have been investigated, and the results show that the proposed algorithms work very well with channel estimation algorithms and a small number of training symbols.

Finally, as the extension of this paper about the complexity analysis displayed in Figs. 2 and 3, we show that Fig. 12 indicates the performance/complexity tradeoff of our proposed MMSE and MSR designs when the global constraint is considered. We set  $N_s = 1$  and  $N_d = 2$ . The range of  $N_r$  is from 1 to 10. The SNR is fixed at 10 dB. It can be seen that, along with increasing the number of relay nodes, our proposed algorithms can achieve a better performance, which requires a higher number of multiplications and, consequently, higher complexity.

## VII. CONCLUSION

Two joint receiver design and power-allocation strategies have been proposed for two-hop WSNs. It has been shown that our proposed strategies achieve a significantly better performance than the equal power allocation and the two-stage power allocation. Moreover, when the neighbor-based constraint is considered, it allows a designer to trade off the performance against the computational complexity and the need for feedback information, which is desirable for WSNs to extend their lifetime. Possible extensions to this paper may include the development of these joint strategies in the general multihop WSNs, which can provide larger coverage than the two-hop WSNs. Finally, low-complexity adaptive algorithms can be used to compute the parameters of the receiver [30]–[35] and the power allocation, and nonlinear detection techniques [36]–[39] could be employed to improve the performance.

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