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Adaptive power allocation strategies for distributed space-time coding in cooperative MIMO networks

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Abstract: Adaptive power allocation (PA) algorithms with different criteria for a cooperative multiple-input multiple-output network equipped with distributed space-time coding are proposed and evaluated. Joint constrained optimisation algorithms to determine the PA parameters and the receive filter are proposed for each transmitted symbol in each link, as well as the channel coefficients matrix. Linear receive filter and maximum-likelihood detection are considered with amplify-and-forward and decode-and-forward cooperation strategies. In these proposed algorithms, the elements in the PA matrices are optimised at the destination node and then transmitted back to the relay nodes via a feedback channel. The effects of the feedback errors are considered. Linear minimum mean square error expressions and the PA matrices depend on each other and are updated iteratively. Stochastic gradient algorithms are developed with reduced computational complexity. Simulation results show that the proposed algorithms obtain significant performance gains as compared with existing PA schemes.

1 Introduction

Owing to the benefits of cooperative multiple-input and multiple-output (MIMO) systems [1], extensive studies of cooperative MIMO networks have been undertaken [2–6]. In [2], an adaptive joint relay selection and power allocation (PA) algorithm based on the minimum mean square error (MMSE) criterion is designed. A joint transmit diversity optimisation and relay selection algorithm for the decode-and-forward (DF) cooperating strategy [7] is designed in [3]. A transmit diversity selection matrix is introduced at each relay node in order to achieve a better MSE performance by deactivating some relay nodes. A central node which controls the transmission power for each link is employed in [4]. Although the centralised PA can improve the performance significantly, the complexity of the calculation increases with the size of the system. The works on the PA problem for the DF strategy measuring the outage probability in each relay node with a single antenna and determining the power for each link between the relay nodes and the destination nodes have been reported in [8–11]. The diversity gain can be improved by using the relay nodes with multiple antennas. When the number of relay nodes is the same, the cooperative gain can be improved by using the DF strategy compared with a system employing the amplify-and-forward (AF) strategy. However, the interference at the destination will be increased if the relay nodes forward the incorrectly detected symbols in the DF strategy. The PA optimisation algorithms in [12, 13] provide solid bit error rate (BER) against signal-to-noise ratio (SNR) performance with the cost of requirement of the eigenvalue decomposition to obtain the key parameters.

In this paper, we propose joint adaptive PA (JAPA) algorithms according to different optimisation criteria with a linear receiver or a maximum-likelihood (ML) detector for cooperative MIMO systems employing multiple relay nodes with multiple antennas to achieve cooperating strategies. This work was first introduced and discussed in [14, 15]. The PA matrices utilised in [14] are full rank, and after the optimisation all the parameters are transmitted back to the relay nodes and the source node with an error-free and delay-free feedback channels. In this paper, we employ the diagonal PA matrices in which the parameters stand for the power allocated to each transmit antenna. The requirement of the limited feedback is significantly reduced as compared with the algorithms in the previously reported works. It is worth to mention that the JAPA strategies derived in our algorithms are two-phase optimisation techniques, which optimised the power assigned at the source node and at the relay nodes in the first phase and the second phase iteratively, and the proposed JAPA algorithms can be used as a PA strategy for the second phase only.

Three optimisation criteria, namely, MMSE, minimum BER (MBER) and maximum sum rate (MSR), are employed in the proposed JAPA optimisation algorithms in this paper. We firstly develop joint optimisation algorithms of the PA matrices and the linear receive filter according to these three criteria, which require matrix inversions and bring a high computational burden to the receiver. In the proposed JAPA algorithms with the MMSE, MBER and MSR criteria, a stochastic gradient (SG) method from [16] is employed in order to lower the computational complexity of the proposed algorithms. The comparison of the computational complexity of the algorithms is considered in this paper. A normalisation process is employed by the optimisation algorithm in order to enforce the power constraint in both transmission phases. After the normalisation, the PA parameters will be transmitted back to each transmission node through a feedback channel. The effect of the feedback errors is considered in the analysis and in the simulation sections, where we indicate the increased MSE performance because of the feedback inaccuracy.

This paper is organised as follows. Section 2 introduces a two-hop cooperative MIMO system with multiple relays applying the AF strategy and the adaptive distributed space-time coding (DSTC) scheme. The constrained PA problems for relay nodes and linear detection method are derived in Section 3, and the proposed JAPA SG algorithms are derived in Section 4. Section 5 focuses on the computational complexity comparison between the proposed and the existing algorithms, and the effects of the feedback errors on the MSE of the system. Section 6 gives the simulation results and Section 7 provides the conclusion.

Notation: the italic, bold lower-case and bold upper-case letters denote scalars, vectors and matrices, respectively. The operators $E[\cdot]$ and $(\cdot)^{\mathrm{H}}$ stand for expected value and the Hermitian operator. The $N\times N$ identity matrix is written as $\boldsymbol{I}_N$. $\|\boldsymbol{X}\|_F=\sqrt{\mathrm{Tr}(\boldsymbol{X}^{\mathrm{H}}\cdot\boldsymbol{X})}=\sqrt{\mathrm{Tr}(\boldsymbol{X}\cdot\boldsymbol{X}^{\mathrm{H}})}$ is the Frobenius norm. $\Re[\cdot]$ stands for the real part, and $\mathrm{Tr}(\cdot)$ stands for the trace of a matrix. $\mathrm{sgn}(\cdot)$ denotes the sign function.

2 Cooperative system model

Consider a two-hop cooperative MIMO system in Fig. 1 with n_r relay nodes that employs an AF cooperative strategy as well as a DSTC scheme. The source node and the destination node have N antennas to transmit and receive data. An arbitrary number of antennas can be used at the relays which are denoted by B shown in Fig. 1. We consider only one user at the source node in our system that operates in a spatial multiplexing configuration. Let $\boldsymbol{s}[i]$ denote the transmitted information symbol vector at the source node, which contains N parameters, $\boldsymbol{s}[i]=[s_1[i], s_2[i], \ldots, s_N[i]$, and has a covariance matrix $E[\boldsymbol{s}[i]\boldsymbol{s}^{\mathrm{H}}[i]]=\sigma_s^2\boldsymbol{I}_N$, where σ_s^2 is the signal power which we assume to be equal to 1. The source node broadcasts $\boldsymbol{s}[i]$ from the source to n_r relay nodes as well as to the destination node in the first hop, which can be described by

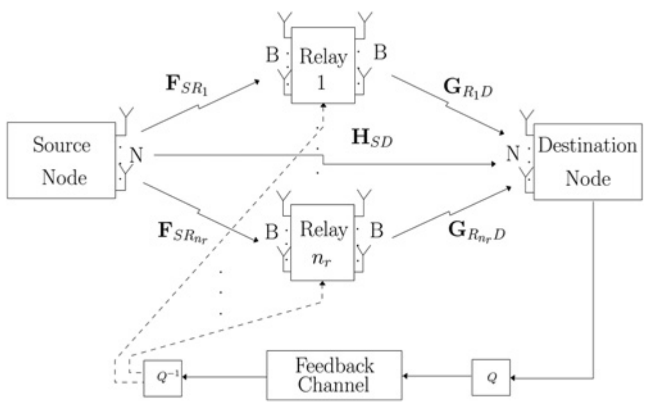

Fig. 1 *Cooperative MIMO system model with n_r relay nodes*

$$\begin{aligned}\boldsymbol{r}_{\mathrm{SD}}[i]&=\boldsymbol{H}_{\mathrm{SD}}[i]\boldsymbol{A}_S[i]\boldsymbol{s}[i]+\boldsymbol{n}_{\mathrm{SD}}[i]\\ \boldsymbol{r}_{\mathrm{SR}_k}[i]&=\boldsymbol{F}_{\mathrm{SR}_k}[i]\boldsymbol{A}_S[i]\boldsymbol{s}[i]+\boldsymbol{n}_{\mathrm{SR}_k}[i]\\ k&=1,2,\ldots,n_r,\quad i=1,2,\ldots\end{aligned}\tag{1}$$

where $\boldsymbol{A}_S[i]=\mathrm{diag}\left[a_{S_1}[i], a_{S_2}[i], \ldots, a_{S_N}[i]\right]$ denotes the diagonal $N\times N$ PA matrix assigned for the source node, and $\boldsymbol{r}_{\mathrm{SR}_k}[i]$ and $\boldsymbol{r}_{\mathrm{SD}}[i]$ denote the received symbol vectors at the kth relay node and at the destination node, respectively. The $B\times 1$ vectors $\boldsymbol{n}_{\mathrm{SR}_k}[i]$ and $\boldsymbol{n}_{\mathrm{SD}}[i]$ denote the zero-mean complex circular symmetric additive white Gaussian noise (AWGN) vector generated at the kth relay node and at the destination node with variance σ^2. The matrices $\boldsymbol{F}_{\mathrm{SR}_k}[i]$ and $\boldsymbol{H}_{\mathrm{SD}}[i]$ are the $B\times N$ channel coefficient matrices. It is worth to mention that an orthogonal transmission protocol is considered which requires that the source node does not transmit during the time period of the second hop.

The received symbols are amplified and re-encoded at each relay node prior to transmission to the destination node in the second hop. We assume that the synchronisation at each node is perfect. The received vector $\boldsymbol{r}_{\mathrm{SR}_k}[i]$ at the kth relay node is assigned a $B\times B$ diagonal PA matrix $\boldsymbol{A}_k[i]=\mathrm{diag}\left[a_{k_1}, a_{k_2}, \ldots, a_{k_B}\right]$ which leads to $\tilde{\boldsymbol{s}}_{\mathrm{SR}_k}[i]=\boldsymbol{A}_k[i]\boldsymbol{r}_{\mathrm{SR}_k}[i]$. The $B\times 1$ signal vector $\tilde{\boldsymbol{s}}_{\mathrm{SR}_k}[i]$ will be re-encoded by a $B\times T$ DSTC matrix $\boldsymbol{M}(\tilde{\boldsymbol{s}})$, and then forwarded to the destination node. The relationship between the kth relay and the destination node can be described as

$$\boldsymbol{R}_{R_kD}[i]=\boldsymbol{G}_{R_kD}[i]\boldsymbol{M}_{R_kD}[i]+\boldsymbol{N}_{\mathrm{RD}}[i]\tag{2}$$

The $N\times T$ received symbol matrix $\boldsymbol{R}_{R_kD}[i]$ in (2) can be written as an $NT\times 1$ vector $\boldsymbol{r}_{R_kD}[i]$ given by

$$\begin{aligned}\boldsymbol{r}_{R_kD}[i]&=\boldsymbol{G}_{\mathrm{eq}_k}[i]\tilde{\boldsymbol{s}}_{\mathrm{SR}_k}[i]+\boldsymbol{n}_{R_kD}[i]\\ &=\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{r}_{\mathrm{SR}_k}[i]+\boldsymbol{n}_{\mathrm{RD}}[i]\\ &=\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]\boldsymbol{A}_S[i]\boldsymbol{s}[i]+\boldsymbol{n}_{R_k}[i]+\boldsymbol{n}_{\mathrm{RD}}[i]\end{aligned}\tag{3}$$

where the $NT\times B$ matrix $\boldsymbol{G}_{\mathrm{eq}_k}[i]$ stands for the equivalent channel matrix which is the DSTC scheme $\boldsymbol{M}(\tilde{\boldsymbol{s}}[i])$ combined with the channel matrix $\boldsymbol{G}_{R_kD}[i]$. The second term $\boldsymbol{n}_{R_k}[i]=\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{n}_{\mathrm{SR}_k}[i]$ in (3) stands for the amplified noise received from the relay node, and the $NT\times 1$ equivalent noise vector $\boldsymbol{n}_{\mathrm{RD}}[i]$ generated at the destination node contains the noise parameters in $\boldsymbol{N}_{\mathrm{RD}}[i]$.

After rewriting $\boldsymbol{R}_{R_kD}[i]$, we can consider the received symbol vector at the destination node as a $(T+1)N\times 1$ vector with two parts, one is from the source node and another one is the superposition of the received vectors from each relay node. Therefore we can write the received

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symbol at the destination node as

$$\begin{aligned}\boldsymbol{r}[i] &= \begin{bmatrix}\boldsymbol{r}_{\mathrm{SD}}[i]\\ \boldsymbol{r}_{\mathrm{RD}}[i]\end{bmatrix}\\ &= \begin{bmatrix}\boldsymbol{H}_{\mathrm{SD}}[i]\boldsymbol{A}_S[i]\boldsymbol{s}[i]+\boldsymbol{n}_{\mathrm{SD}}[i]\\ \sum_{k=1}^{n_r}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]\boldsymbol{A}_S[i]\boldsymbol{s}[i]+n_{\mathrm{RD}}[i]\end{bmatrix}\\ &= \begin{bmatrix}\boldsymbol{H}_{\mathrm{eq_{SD}}}[i]\\ \sum_{k=1}^{n_r}\boldsymbol{H}_{\mathrm{eq}_k}[i]\end{bmatrix}\boldsymbol{s}[i]+\begin{bmatrix}\boldsymbol{n}_{\mathrm{SD}}[i]\\ \boldsymbol{n}_{\mathrm{RD}}[i]\end{bmatrix}=\boldsymbol{H}_D[i]\boldsymbol{s}[i]+\boldsymbol{n}_D[i]\end{aligned} \tag{4}$$

where the $(T+1)N\times N$ matrix $\boldsymbol{H}_{\mathrm{D}}[i]$ denotes the channel gain matrix with the PA of all the links in the network. The $N\times N$ channel matrix $\boldsymbol{H}_{\mathrm{eq_{SD}}}[i]=\boldsymbol{H}_{\mathrm{SD}}[i]\boldsymbol{A}_S[i]$, whereas the kth equivalent channel matrix $\boldsymbol{H}_{\mathrm{eq}_k}[i]=\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]$ $\boldsymbol{A}_S[i]$. We assume that the coefficients in all channel matrices are statistically independent and remain constant over the transmission. The $(T+1)N\times 1$ noise vector $\boldsymbol{n}_{\mathrm{D}}[i]$ contains the equivalent received noise vector at the destination node, which can be modelled as an AWGN with zero mean and covariance matrix $\sigma^2\big(1+\|\sum_{k=1}^{n_r}\boldsymbol{G}_{\mathrm{eq}_k}[i]$ $\boldsymbol{A}_k[i]\|_F^2\big)\boldsymbol{I}_{(T+1)N}$. It is worth to mention that the value in B is variable, and in this paper we focus on the PA optimisation algorithm designs in cooperative MIMO systems and, for simplicity, we consider the scenario in which $B=N$ antennas at the relays.

3 Adaptive power allocation matrix optimisation strategies

In this section, we consider the design of a two-phase adjustable PA matrix according to various criteria using a DSTC scheme in cooperative MIMO systems. The linear receive filter is determined jointly with the PA matrices. A feedback channel is considered in order to convey the information about the PA prior to transmission to the destination node.

3.1 Joint linear MMSE receiver design with power allocation

The linear MMSE receiver design with PA matrices is derived as follows. By defining the $(T+1)N\times 1$ parameter vector $\boldsymbol{w}_j[i]$ to determine the jth symbol $s_j[i]$ in the signal vector $\boldsymbol{s}$ $[i]$, we propose the MSE-based optimisation with a power constraint described by

$$\begin{aligned}&\big[\boldsymbol{w}_j[i],\boldsymbol{A}_S[i],\boldsymbol{A}_k[i]]\big]\\ &\quad=\underset{\boldsymbol{w}_j[i],\boldsymbol{A}_S[i],\boldsymbol{A}_k[i]}{\operatorname{argmin}} E\Big[\big\|s_j[i]-\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{r}[i]\big\|^2\Big]\\ &\text{s.t.}\quad \mathrm{Tr}\Bigg(\sum_{k=1}^{n_r}\boldsymbol{A}_k[i]\boldsymbol{A}_k^{\mathrm{H}}[i]\Bigg)\le P_R,\quad \mathrm{Tr}\big(\boldsymbol{A}_S[i]\boldsymbol{A}_S^{\mathrm{H}}[i]\big)\le P_T\end{aligned} \tag{5}$$

where P_{T} and P_{R} denote the transmit power assigned to all the relay nodes and to the source node, respectively. The values of the parameters in the PA matrices are restricted by P_{T} and P_{R}. By employing the Lagrange multipliers λ_1 and λ_2 we can obtain the Lagrangian function shown as

$$\begin{aligned}\mathcal{L}&=E\Big[\big\|s_j[i]-\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{r}[i]\big\|^2\Big]+\lambda_1\Bigg(\sum_{j=1}^{N}a_{S_j}[i]-P_{\mathrm{T}}\Bigg)\\ &\quad+\lambda_2\Bigg(\sum_{k=1}^{n_r}\sum_{j=1}^{N}a_{k_j}[i]-P_{\mathrm{R}}\Bigg)\end{aligned} \tag{6}$$

where $a_{S_j}[i]$ denotes the jth parameters in the diagonal of $\boldsymbol{A}_S[i]$, whereas $a_{k_j}[i]$ stands for the jth parameters in the diagonal of $\boldsymbol{A}_k[i]$.

By expanding the right-hand side of (6), taking the gradient with respect to $\boldsymbol{w}_j^*[i]$, $a_{S_j}^*[i]$ and $a_{k_j}^*[i]$, and equating the terms to zero, we can obtain

$$\begin{aligned}\boldsymbol{w}_j[i]&=\boldsymbol{R}^{-1}\boldsymbol{p}\\ a_{S_j}[i]&=\tilde{R}_S^{-1}\tilde{P}_S\\ a_{k_j}[i]&=\tilde{R}^{-1}\tilde{P}\end{aligned} \tag{7}$$

where

$$\begin{aligned}\boldsymbol{R}&=E\big[\boldsymbol{r}[i]\boldsymbol{r}^{\mathbf{H}[i]}\big],\quad \boldsymbol{p}=E\Big[\boldsymbol{r}[i]s_j^*[i]\Big]\\ \tilde{R}_S&=E\Big[\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{h}_{\mathrm{SDA}_j}[i]s_j[i]s_j^*[i]\boldsymbol{h}_{\mathrm{SDA}_j}^{\mathbf{H}}[i]\boldsymbol{w}_j[i]+\lambda_1 a_{S_j}[i]\Big]\\ \tilde{P}_S&=E\Big[\boldsymbol{h}_{\mathrm{SDA}_j}^{\mathbf{H}}[i]\boldsymbol{w}_j[i]s_j^*[i]s_j[i]\Big]\\ \tilde{P}&=E\Big[s_j[i]s_j^*[i]a_{S_j}^*[i]f_{k_j}^*[i]\boldsymbol{g}_{\mathrm{eq}_{k_j}}^{\mathbf{H}}[i]\boldsymbol{w}_j[i]\Big]\\ \tilde{R}&=E\Big[\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]f_{k_j}[i]a_{S_j}[i]s_j[i]s_j^*[i]a_{S_j}^*[i]f_{k_j}^*[i]\boldsymbol{g}_{\mathrm{eq}_{k_j}}^{\boldsymbol{H}}\\ &\quad\times[i]\boldsymbol{w}_j[i]\lambda_2 a_{k_j}[i]\Big]\end{aligned} \tag{8}$$

The vector $\boldsymbol{h}_{\mathrm{SDA}_j}$ denotes the channel vector assigned to the power parameter a_{S_j} and is the jth column of the equivalent channel matrix $\boldsymbol{H}_{\mathrm{SDA}}[i]=\Big[\boldsymbol{H}_{\mathrm{SD}}[i];\ \boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]\Big]$, and $f_{k_j}[i]$ and $\boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]$ denotes the jth parameter in $\boldsymbol{F}_k[i]$ and the jth column in $\boldsymbol{G}_{\mathrm{eq}_k}[i]$. The value of the Lagrange multipliers λ_1 and λ_2 can be determined by substituting $\boldsymbol{A}_S[i]$ and $\boldsymbol{A}_k[i]$ into $\mathrm{Tr}\big(\boldsymbol{A}_S[i]\boldsymbol{A}_S^{\mathrm{H}}[i]\big)\le P_{\mathrm{T}}$ and $\lambda\mathrm{Tr}\big(\sum_{k=1}^{n_r}\boldsymbol{A}_k[i]\boldsymbol{A}_k^{\mathrm{H}}[i]\big)=P_{\mathrm{R}}$, respectively, and then solving the power constraint equations. The problem is that a high computational complexity of $O(((T+1)N)^3)$ is required, and it will increase cubically with largening the number of antennas or the use of more complicated STC encoders.

3.2 Joint linear MBER receiver design with power allocation

The MBER receiver design with PA in the second phase is derived as follows. The binary phase shift keying (BPSK) modulation scheme is utilised for simplicity. According to the expression in (4), the desired information symbols at the destination node can be computed as

$$b_j[i]=\mathrm{sgn}\Big(\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{r}[i]\Big)=\mathrm{sgn}(\tilde{s}_j[i]) \tag{9}$$

where $\tilde{s}_j[i]$ denotes the detected symbol at the receiver which can be further written as

$$\begin{aligned}\tilde{s}_j[i] &= \Re\left[\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{r}[i]\right] = \Re\left[\boldsymbol{w}_j^{\mathbf{H}}[i](\boldsymbol{H}_{\mathrm{D}}[i]\boldsymbol{s}[i] + \boldsymbol{n}_{\mathrm{D}}[i])\right] \\ &= \Re\left[\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{H}_{\mathrm{D}}[i]\boldsymbol{s}[i] + \boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{n}_{\mathrm{D}}[i]\right] \\ &= \Re\left[\tilde{s}'_j[i] + e_j[i]\right]\end{aligned} \tag{10}$$

where $\tilde{s}'_j[i]$ is the noise-free detected symbol, and $e_j[i]$ denotes the error factor for the jth detected symbol. Define an $N \times N_b$ matrix $\bar{\boldsymbol{S}}$ which is constructed by a set of vectors $\bar{\boldsymbol{s}}_l = \left[s_{l_1}, s_{l_2}, \ldots, s_{l_N}\right]^{\mathrm{T}}$, $l = 1, 2, \ldots, N_b$ and $N_b = 2^N$, containing all the possible combinations of the transmitted symbol vector $\boldsymbol{s}[i]$ and we can obtain

$$\begin{aligned}\bar{s}_{l_j}[i] &= \Re\left[\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{H}_{\mathrm{D}}[i]\bar{\boldsymbol{s}}_l + \boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{n}_{\mathrm{D}}[i]\right] \\ &= \Re\left[\boldsymbol{w}_j^{\mathbf{H}}[i]\bar{\boldsymbol{r}}_l[i]] + e_{l_j}[i]\right] = \bar{s}'_{l_j}[i] + e_{l_j}[i]\end{aligned} \tag{11}$$

where $\bar{s}'_{l_j}[i]$ denotes the noise-free detected symbol in the lth column and the jth row of $\bar{\boldsymbol{S}}$. Since the probability density function (pdf) of $\bar{\boldsymbol{r}}[i]$ is given by

$$p_{\bar{r}[i]} = \frac{1}{N_b\sqrt{2\pi\sigma_n^2\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}}\sum_{l=1}^{N_b}\exp\left(-\frac{\left(\bar{s}_{l_j}[i] - \bar{s}'_{l_j}[i]\right)^2}{2\sigma_n^2\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}\right) \tag{12}$$

by utilising the Q function, we can obtain the BER expression of the cooperative MIMO network which is

$$P_{\mathrm{E}}\left(\boldsymbol{w}_j[i], a_{S_j}[i], a_{k_j}[i]\right) = \frac{1}{N_b}\sum_{l=1}^{N_b}\boldsymbol{Q}\left(c_{l_j}[i]\right) \tag{13}$$

where

$$c_{l_j}[i] = \frac{\mathrm{sgn}(s_{l_j})\bar{s}'_{l_j}[i]}{\sigma_n\sqrt{\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{w}_j[i]}} = \frac{\mathrm{sgn}(s_{l_j})\Re\left[\boldsymbol{w}_j^{\mathbf{H}}[i]\bar{\boldsymbol{r}}_l[i]\right]}{\sigma_n\sqrt{\boldsymbol{w}_j^{\mathbf{H}}[i]\boldsymbol{w}_j[i]}} \tag{14}$$

The joint PA with linear receiver design problem is given by

$$\begin{aligned}&[\boldsymbol{w}_j[i], a_{S_j}[i], a_{k_j}[i]] = \underset{\boldsymbol{w}_j[i], a_{S_j}[i], a_{k_j}[i]}{\arg\min} P_{\mathrm{E}}\left(\boldsymbol{w}_j[i], a_{S_j}[i], a_{k_j}[i]\right) \\ &\text{s.t.} \quad \sum_{j=1}^{N} a_{S_j}[i] \le P_{\mathrm{T}}, \quad \sum_{k=1}^{n_{\mathrm{r}}}\sum_{j=1}^{N} a_{k_j}[i] \le P_{\mathrm{R}}\end{aligned} \tag{15}$$

According to (13) and (14), the solution of the design problem in (15) with respect to $\boldsymbol{w}_j[i]$, $a_{S_j}[i]$ and $a_{k_j}[i]$ is not a closed-form one. Therefore we design an adaptive JAPA strategy according to the MBER criterion using the SG algorithm in order to update the parameters iteratively to achieve the optimal solution in the next section.

3.3 Joint linear MSR receiver design with power allocation

We will develop a joint PA strategy focusing on maximising the sum rate at the destination node. The expression of the sum rate after the detection is derived in [17] as

$$I = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}_{\mathrm{ins}}\right) \tag{16}$$

where

$$\mathrm{SNR}_{\mathrm{ins}} = \frac{E[\boldsymbol{s}^{\mathrm{H}}\boldsymbol{s}]\mathrm{Tr}\left(\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{H}_{\mathrm{D}}[i]\boldsymbol{H}_{\mathrm{D}}^{\mathrm{H}}[i]\boldsymbol{W}[i]\right)}{E\left[\boldsymbol{n}_{\mathrm{D}}^{\mathrm{H}}[i]\boldsymbol{n}_{\mathrm{D}}[i]\right]} \tag{17}$$

and $\boldsymbol{W}[i] = [\boldsymbol{w}_1[i], \boldsymbol{w}_2[i], \ldots, \boldsymbol{w}_N[i]]$ denotes the $N(T+1) \times N$ linear receive filter matrix, and $\boldsymbol{n}[i]$ denotes the received noise vector. By substituting (4) into (17), we can obtain (see (18))

Since the logarithm is an increasing function, to maximise the sum rate is equivalent to maximising the instantaneous SNR. The optimisation problem can be written as

$$\begin{aligned}&\left[\boldsymbol{W}[i], \boldsymbol{A}_S[i], \boldsymbol{A}_k[i]\right] = \underset{\boldsymbol{W}[i], \boldsymbol{A}_S[i], \boldsymbol{A}_k[i]}{\arg\min} \mathrm{SNR}_{\mathrm{ins}} \\ &\text{s.t.} \quad \mathrm{Tr}(\boldsymbol{A}_k[i]\boldsymbol{A}_k^{\mathrm{H}}[i]) \le P_{\mathrm{R}}, \ \mathrm{Tr}(\boldsymbol{A}_S[i]\boldsymbol{A}_S^{\mathrm{H}}[i]) \le P_{\mathrm{T}}\end{aligned} \tag{19}$$

where $\mathrm{SNR}_{\mathrm{ins}}$ is given by (18).

As expressed in (18), the solution of (19) with respect to the matrices $\boldsymbol{W}[i]$, $\boldsymbol{A}_S[i]$ and $\boldsymbol{A}_k[i]$ does not result in closed-form expressions. Therefore in the next section, we propose a JAPA SG algorithm to obtain the joint optimisation algorithm for determining the linear receiver filter parameter matrix and PA matrices to maximise the sum rate.

4 Low-complexity joint linear receiver design with power allocation

In this section, we jointly design an adjustable PA matrix and the linear receiver for the DSTC scheme in cooperative MIMO systems. Adaptive SG algorithms [16] with reduced complexity are devised.

4.1 Joint adaptive SG estimation for MMSE receive filter and power allocation

According to (5) and (6), the joint optimisation problem for PA matrices and receiver parameter vectors depend on each other. By computing the instantaneous gradient terms of (6)

$$\mathrm{SNR}_{\mathrm{ins}} = \frac{\sigma_s^2\mathrm{Tr}\left(\boldsymbol{W}^{\mathrm{H}}[i]\left(\sum_{k=1}^{n_{\mathrm{r}}}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]\boldsymbol{A}_S[i]\right)\left(\sum_{k=1}^{n_{\mathrm{r}}}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]\boldsymbol{A}_S[i]\right)^{\mathrm{H}}\boldsymbol{W}[i]\right)}{\sigma_n^2\mathrm{Tr}\left(\boldsymbol{W}^{\mathrm{H}}[i]\left(\boldsymbol{I}_{N(T+1)} + \left(\sum_{k=1}^{n_{\mathrm{r}}}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_{R_kD}[i]\right)\left(\sum_{k=1}^{n_{\mathrm{r}}}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_{R_kD}[i]\right)^{\mathrm{H}}\right)\boldsymbol{W}[i]\right)} \tag{18}$$

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with respect to $\boldsymbol{w}_j[i]$, $a_{S_j}[i]$ and $a_{k_j}[i]$, we can obtain

$$
\begin{aligned}
\nabla \mathcal{L}_{\boldsymbol{w}_j^*[i]} &= -\boldsymbol{r}[i]\left(s_j[i] - \boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{r}[i]\right)^* = -\boldsymbol{r}[i]e_j^*[i] \\
\nabla \mathcal{L}_{a_{S_j}^*[i]} &= -\nabla_{a_{S_j}^*[i]}\left(\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{h}_{\mathrm{SDA}_j}[i]a_{S_j}[i]s_j[i]\right)^{\mathrm{H}} \\
&\quad \left(s_j[i] - \boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{r}[i]\right) = \boldsymbol{h}_{\mathrm{SDA}_j}^{\mathrm{H}}[i]\boldsymbol{w}_j[i]s_j^*[i]e_j[i] \\
\nabla \mathcal{L}_{a_{k_j}^*[i]} &= -\nabla_{a_{k_j}^*[i]}\left(\boldsymbol{w}_{R_j}^{\mathrm{H}}[i]\boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]\boldsymbol{f}_{k_j}[i]a_{k_j}[i]\boldsymbol{s}[i]\right)^{\mathrm{H}} \\
&\quad \times \left(s_j[i] - \boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{r}[i]\right) = -\left(\boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]\boldsymbol{f}_{k_j}[i]\boldsymbol{s}[i]\right)^{\mathrm{H}}\boldsymbol{w}_{R_j}[i]e_j[i]
\end{aligned}
\tag{20}
$$

where $\boldsymbol{h}_{\mathrm{SDA}_j}[i]$ denotes the jth column with dimension $N(T+1)\times 1$ of the equivalent channel matrix $\boldsymbol{H}_{\mathrm{SDA}}[i]$, and $\boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]$ and $\boldsymbol{f}_{k_j}[i]$ denote the jth column and the jth row of the channel matrices $\boldsymbol{F}_k[i]$ and $\boldsymbol{G}_{\mathrm{eq}_{k_j}}[i]$, respectively. The $NT\times 1$ vector $\boldsymbol{w}_{\mathrm{R}_j}[i]$ is the parameter vector for the received symbols from the relay nodes. The error signal is denoted by $e_j[i] = s_j[i] - \boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{r}[i]$. We can devise an adaptive SG estimation algorithm by using the instantaneous gradient terms of the Lagrangian which were previously derived with the SG descent rules [16] as

$$
\begin{aligned}
\boldsymbol{w}_j[i+1] &= \boldsymbol{w}_j[i] - \mu\nabla\mathcal{L}_{\boldsymbol{w}_j^*[i]} \\
a_{S_j}[i+1] &= a_{S_j}[i] - \nu\nabla\mathcal{L}_{a_{S_j}^*[i]} \\
a_{k_j}[i+1] &= a_{k_j}[i] - \tau\nabla\mathcal{L}_{a_{k_j}^*[i]}
\end{aligned}
\tag{21}
$$

where μ, ν and τ are the step sizes of the recursions for the estimation procedure. The computational complexity of $\boldsymbol{w}_j[i]$, $a_{S_j}[i]$ and $a_{k_j}[i]$ in (21) is $(\boldsymbol{O}(NT))$, $(\boldsymbol{O}(3NT))$ and $(\boldsymbol{O}(N^2T^2))$, respectively, which is much less than that of the algorithm we described in Section 3.

It is worth to mention that instead of calculating the Lagrange multiplier λ, a normalisation of the PA matrices after the optimisation which ensures that the energy is not increased is required and implemented as

$$
\begin{aligned}
\boldsymbol{A}_S[i+1] &= \frac{\sqrt{P_{\mathrm{T}}}\boldsymbol{A}_S[i+1]}{\|\boldsymbol{A}_S[i+1]\|_F} \\
\boldsymbol{A}_k[i+1] &= \frac{\sqrt{P_{\mathrm{R}}}\boldsymbol{A}_k[i+1]}{\|\sum_{k=1}^{n_r}\mathrm{Tr}(\boldsymbol{A}_k[i+1]\|_F}
\end{aligned}
\tag{22}
$$

4.2 Joint adaptive MBER SG estimation and power allocation

The key to achieving the adaptive estimation algorithm described in (13) is to find out an efficient and reliable method to calculate the pdf of the received symbol vector $\boldsymbol{r}[i]$ at the destination node. According to the algorithms in [18], the kernel density estimation algorithm provides a solid method for the pdf estimate to guarantee the accuracy of the probability distribution.

By transmitting a block of M training samples $\hat{\boldsymbol{s}} = \mathrm{sgn}(\hat{\boldsymbol{b}})$, the kernel density estimated pdf of $\hat{\boldsymbol{s}}[i]$ is given by

$$
p_{\hat{s}} = \frac{1}{M\sqrt{2\pi}\rho_n\sqrt{\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}}\sum_{j=1}^{M}\exp\left(-\frac{\left(\tilde{s}_j - \hat{s}_j\right)^2}{2\rho_n^2\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}\right)
\tag{23}
$$

where ρ_{n} is related to the standard deviation of noise σ_{n} and it is suggested in [18] that a lower bound of $\rho_{\mathrm{n}} = (4/3M)^{(1/5)}\sigma_{\mathrm{n}}$ should be chosen. The symbol $\tilde{s}_j$ is calculated by (11), and $\hat{s}_j$ stands for the jth element in the $M\times 1$ training samples $\hat{\boldsymbol{s}}$. The expression of the BER can be derived as

$$
\hat{P}_{\mathrm{E}}\left(\boldsymbol{w}_j[i],\ a_{S_j}[i],\ a_{k_j}[i]\right) = \frac{1}{M}\sum_{j=1}^{M}\boldsymbol{Q}\left(c_j[i]\right)
\tag{24}
$$

where

$$
c_j[i] = \frac{\mathrm{sgn}\left(\hat{s}_j\right)\bar{s}_j}{\rho_{\mathrm{n}}\sqrt{\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}}
\tag{25}
$$

By substituting (25) into (24) and taking the gradient with respect to different arguments, we can obtain

$$
\nabla P_{E_{w_j}}[i] = \frac{1}{M\sqrt{2\pi}\sqrt{\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}} \sum_{j=1}^{M}\exp\left(-\frac{c_j^2[i]}{2}\right)\mathrm{sgn}(s_j)\frac{\bar{\boldsymbol{r}}[i] - (1/2)\bar{s}_j[i]\boldsymbol{w}_j[i]}{\sigma_n\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}
\tag{26}
$$

$$
\nabla P_{E_{a_{S_j}}}[i] = \frac{1}{M\sqrt{2\pi}\sigma_n\sqrt{\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}} \sum_{j=1}^{M}\exp\left(-\frac{c_j^2[i]}{2}\right)\mathrm{sgn}(s_j)\Re\left[\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{h}_{\mathrm{SDA}_j}[i]s_j\right]
\tag{27}
$$

$$
\nabla P_{E_{a_{k_j}}}[i] = \frac{1}{M\sqrt{2\pi}\sigma_n\sqrt{\boldsymbol{w}_{D_j}^{\mathrm{H}}[i]\boldsymbol{w}_{R_j}[i]}} \sum_{j=1}^{M}\exp\left(-\frac{c_j^2[i]}{2}\right)\mathrm{sgn}(s_j)\Re\left[\boldsymbol{w}_{R_j}^{\mathrm{H}}[i]\boldsymbol{h}_{k_j}[i]s_j\right]
\tag{28}
$$

where $\boldsymbol{h}_{k_j}[i] = \boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]f_{k_j}[i]$ denotes the equivalent channel vector assigned for s_{m_j}. By making use of an SG algorithm in [16], the updated $\boldsymbol{w}_j[i]$, $a_{S_j}[i]$ and $a_{k_j}[i]$ can be calculated by (21). The convergence property of the joint iterative optimisation problems has been tested and proved by Niesen *et al.* in [19]. In the designed problem, the receive filter parameter vectors and the PA parameters depend on each other, and the proposed JAPA algorithms provide an iterative update process and finally both of the desired items will achieve at least a local optimisation of the BER cost function.

4.3 Joint adaptive MSR SG algorithm for power allocation and receiver design

The proposed PA algorithm maximising the sum rate at the destination node is derived as follows. We consider the design problem in (19) and the instantaneous received $\mathrm{SNR}_{\mathrm{ins}}$ as given in (18). According to the property of the trace Tr(·), we can obtain

$$\begin{aligned}\mathrm{SNR}_{\mathrm{ins}} &= \frac{\sigma_s^2}{n_{\mathrm{eq}}[i]}\mathrm{Tr}\big(\boldsymbol{R}_{\mathrm{SDA}}[i]\boldsymbol{A}_S^{\mathrm{H}}[i]\big)\\ &= \frac{\sigma_s^2}{n_{\mathrm{eq}}[i]}\mathrm{Tr}\left(\sum_{k=1}^{n_r}\boldsymbol{R}_{\boldsymbol{G}_{\mathrm{eq}_k}}[i]\boldsymbol{A}_k^{\mathrm{H}}[i]\right)\end{aligned} \tag{29}$$

where (see equation at the bottom of the page)

Since the PA matrices $\boldsymbol{A}_S[i]$ and $\boldsymbol{A}_k[i]$ are diagonal, we just focus on the terms containing the conjugate of the jth parameter in order to simplify the derivation, and rewrite (29) as

$$\begin{aligned}\mathrm{SNR}_{\mathrm{ins}} &= \frac{\sigma_s^2\sum_{k=1}^{n_r}\sum_{j=1}^{N} r_{\mathrm{SDA}_j}[i]a_{S_j}^*[i]}{\sigma_n^2\sum_{j=1}^{N}\left(\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]+\sum_{k=1}^{n_r} r_{\boldsymbol{N}_{kj}}[i]a_{k_j}^*[i]\right)}\\ &= \frac{\sigma_s^2\sum_{k=1}^{n_r}\sum_{j=1}^{N} r_{\boldsymbol{G}_{\mathrm{eq}_{k_j}}}[i]a_{k_j}^*[i]}{\sigma_n^2\sum_{j=1}^{N}\left(\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]+\sum_{k=1}^{n_r} r_{\boldsymbol{N}_{kj}}[i]a_{k_j}^*[i]\right)}\end{aligned} \tag{30}$$

where $r_{\boldsymbol{G}_{\mathrm{eq}_{k_j}}}[i]$ and $r_{\boldsymbol{N}_{kj}}[i]$ denote the jth element in the diagonal of $\boldsymbol{R}_{\boldsymbol{G}_{\mathrm{eq}_k}}[i]$ and $\boldsymbol{R}_{\boldsymbol{N}_k}[i]$, respectively. $\boldsymbol{R}_{\boldsymbol{N}_k}[i] = \boldsymbol{G}_{\mathrm{eq}_k}^{\mathrm{H}}[i]\boldsymbol{W}[i]\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]$ denotes the equivalent matrix assigned for the noise at the kth relay node.

By taking the SG of (30) with respect to $a_{S_j}^*[i]$, $a_{k_j}^*[i]$ and $\boldsymbol{W}^{\mathrm{H}[i]}$, we can obtain

$$\begin{aligned}\nabla_{\boldsymbol{W}[i]} =& \frac{\sigma_s^2}{n_{\mathrm{eq}}[i]}\Big(\mathrm{Tr}\big(\|\boldsymbol{H}_{\mathrm{SDA}}[i]\boldsymbol{A}_S[i]\|_F^2\,\boldsymbol{W}[i]\big)n_{\mathrm{eq}}[i]\\ &-\|\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{H}_{\mathrm{SDA}}[i]\boldsymbol{A}_S[i]\|_F^2\\ &\times\mathrm{Tr}\Big(\boldsymbol{W}[i]+\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\big(\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\big)\boldsymbol{W}[i]\Big)\Big)\\ \nabla_{a_{S_j}[i]} =& \frac{\sigma_s^2}{n_{\mathrm{eq}}[i]} r_{\mathrm{SDA}_j}[i]\\ \nabla_{a_{R_k\mathrm{D}}[i]} =& \frac{\sigma_s^2}{n_{\mathrm{eq}}[i]}\left(r_{\boldsymbol{G}_{\mathrm{eq}}}[i]n_{\mathrm{eq}}[i]-\sigma_n^2 r_{\boldsymbol{N}_{kj}}[i]\sum_{j=1}^{N} r_{\boldsymbol{G}_{\mathrm{eq}_{k_j}}}[i]a_{k_j}^*[i]\right)\end{aligned} \tag{31}$$

By using (21) and (22), the proposed algorithm is achieved. Table 1 shows a summary of the JAPA SG algorithms with different criteria. A low-complexity channel estimation method derived in [14] can be also employed to obtain the channel matrices required in the proposed algorithms.

5 Analysis

The proposed JAPA SG algorithms according to three different criteria compute the PA matrices iteratively at the destination node and then send them back via a feedback channel. In this section, we will illustrate the low computational complexity required by the proposed JAPA SG algorithms compared with the existing PA optimisation algorithms using the same criteria and examining the requirement of the feedback.

5.1 Computational complexity analysis

In Table 2, we compute the number of additions and multiplications to compare the complexity of the proposed JAPA SG algorithms with the conventional PA strategies. The computational complexity of the proposed algorithms is calculated by summing the number of additions and multiplications, which is related to the number of antennas N, the number of relay nodes n_r, and the $N \times T$ STC scheme employed in the network. Note that the computational complexity in [12, 13] is high because the key parameters in the algorithms can only be obtained by eigenvalue decomposition, which requires a high-cost computing process when the matrices are large [19].

5.2 Feedback requirements

The proposed JAPA SG algorithms require communication between the relay nodes and the destination nodes according to different algorithms. The feedback channel we considered is modelled as an AWGN channel. A 4 bit quantisation scheme, which quantises the real part and the imaginary part by 4 bits, is utilised prior to the feedback channel.

For simplicity, we show how the feedback errors in PA matrices at the relay nodes affect the accuracy of the detection and only one relay node is employed. The $N \times N$ diagonal PA matrix with feedback errors at the kth relay node is derived as

$$\hat{\boldsymbol{A}}[i] = \boldsymbol{A}[i] + \boldsymbol{E}[i] \tag{32}$$

where $\boldsymbol{A}[i]$ denotes the accurate PA matrix and $\boldsymbol{E}[i]$ stands for the error matrix. We assume the parameters in $\boldsymbol{E}[i]$ are Gaussian with zero mean and variance σ_f. Then, the received symbol vector is given by

$$\begin{aligned}\hat{\boldsymbol{r}}[i] &= \boldsymbol{G}_{\mathrm{eq}}[i]\hat{\boldsymbol{A}}[i]\boldsymbol{F}[i]\boldsymbol{s}[i] + \boldsymbol{G}_{\mathrm{eq}}[i]\hat{\boldsymbol{A}}[i]\boldsymbol{n}_{\mathrm{SR}}[i] + \boldsymbol{n}_{\mathrm{RD}}[i]\\ &= \boldsymbol{G}_{\mathrm{eq}}[i]\hat{\boldsymbol{A}}[i]\boldsymbol{F}[i]\boldsymbol{s}[i] + \hat{\boldsymbol{n}}_{\mathrm{D}}[i]\end{aligned} \tag{33}$$

where $\hat{n}_{\mathrm{D}}[i]$ denotes the received noise with zero mean and

$$\boldsymbol{R}_{\mathrm{SDA}}[i] = \boldsymbol{H}_{\mathrm{SDA}}^{\mathrm{H}}[i]\boldsymbol{W}[i]\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{H}_{\mathrm{SDA}}[i]\boldsymbol{A}_S[i]$$

$$\boldsymbol{R}_{\boldsymbol{G}_{\mathrm{eq}_k}}[i] = \boldsymbol{G}_{\mathrm{eq}_k}^{\mathrm{H}}[i]\boldsymbol{W}[i]\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{H}_{\mathrm{SDA}}[i]\boldsymbol{A}_S[i]\boldsymbol{A}_S^{\mathrm{H}}[i]\boldsymbol{F}_{\mathrm{SR}_k}^{\mathrm{H}}[i]$$

$$n_{\mathrm{eq}}[i] = \sigma_n^2\mathrm{Tr}\left(\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{W}[i] + \boldsymbol{W}^{\mathrm{H}}[i]\left(\sum_{k=1}^{n_r}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\right)\left(\sum_{k=1}^{n_r}\boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\right)^{\mathrm{H}}\boldsymbol{W}[i]\right)$$

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Table 1 JAPA SG algorithms

1: Initialisation:
$\boldsymbol{W}[0] = \boldsymbol{I}_{(T+1)N\times 1}$,
$a_{S_j}[0] = 1, \quad a_{k_j}[0] = 1$,

$\boldsymbol{H}_{\mathrm{SDA}}[i] = \sum_{k=1}^{n_r} \boldsymbol{G}_{\mathrm{eq}_k}[i]\boldsymbol{A}_k[i]\boldsymbol{F}_{\mathrm{SR}_k}[i]$,
2: for $j = 1$ to N do
2-1: JAPA SG MMSE algorithm
$e_j[i] = s_j[i] - \boldsymbol{w}^{\mathrm{H}_j}[i]\boldsymbol{r}[i]$

$\nabla\mathcal{L}_{\boldsymbol{w}_j^{\mathrm{H}}[i]} = -\boldsymbol{r}[i]e_j^*[i]$

$\nabla\mathcal{L}_{a_{S_j}^*[i]} = \boldsymbol{h}_{\mathrm{SDA}_j}^{\mathrm{H}}[i]\boldsymbol{w}_j[i]s_j^*[i]e_j[i]$

$\nabla\mathcal{L}_{a_{k_j}^*[i]} = -\left(\boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]\boldsymbol{f}_{k_j}[i]\boldsymbol{s}[i]\right)^{\mathrm{H}}\boldsymbol{w}_{\mathrm{D}_j}[i]e_j[i]$

2-2: JAPA SG MBER algorithm

$c_{m_j}[i] = \frac{\mathrm{sgn}(\hat{s}_{m_j})\bar{\hat{s}}_{m_j}}{\rho_n\sqrt{\boldsymbol{w}_j^H[i]\boldsymbol{w}_j[i],}}\boldsymbol{h}_{k_j}[i] = \boldsymbol{g}_{\mathrm{eq}_{k_j}}[i]f_{k_j}[i]$
$\nabla P_{E_{\boldsymbol{w}_j}}[i] = \frac{1}{M\sqrt{2\pi}\sqrt{\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}}\sum_{j=1}^{M} exp\left(-\frac{c_j^2[i]}{2}\right)\mathrm{sgn}(s_j)\frac{\bar{\boldsymbol{r}}_j[i]-\frac{1}{2}\bar{s}_j[i]\boldsymbol{w}_j[i]}{\sigma_n\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}$
$\nabla P_{E_{a_{S_j}}}[i] = \frac{1}{M\sqrt{2\pi}\sigma_n\sqrt{\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{w}_j[i]}}\sum_{j=1}^{M}\exp\left(-\frac{c_j^2[i]}{2}\right)\mathrm{sgn}(s_j)\Re\left[\boldsymbol{w}_j^{\mathrm{H}}[i]\boldsymbol{h}_{\mathrm{SDA}_j}[i]s_j\right]$
$\nabla P_{E_{a_{k_j}}}[i] = \frac{1}{M\sqrt{2\pi}\sigma_n\sqrt{\boldsymbol{w}_{\mathrm{D}_j}^{\mathrm{H}}[i]\boldsymbol{w}_{\mathrm{D}_j}[i]}}\sum_{j=1}^{M}\exp\left(-\frac{c_j^2[i]}{2}\right)\mathrm{sgn}(s_j)\Re\left[\boldsymbol{w}_{\mathrm{D}_j}^{\mathrm{H}}[i]\boldsymbol{h}_{k_j}[i]s_j\right]$
2-3: JAPA SG MSR algorithm
$\boldsymbol{R}_{\mathrm{SDA}}[i] = \boldsymbol{H}_{\mathrm{SDA}}^{\mathrm{H}}[i]W[i]W^{\mathrm{H}}[i]H_{\mathrm{SDA}}[i]A_S[i]$

$\boldsymbol{R}_{\boldsymbol{G}_{\mathrm{eq}_k}}[i] = \boldsymbol{G}_{\mathrm{eq}_k}^{\mathrm{H}}[i]\boldsymbol{W}[i]\boldsymbol{W}^{\mathrm{H}}[i]H_{\mathrm{SDA}}[i]A_S[i]A_S^{\mathrm{H}}[i]F_{\mathrm{SR}K}^{\mathrm{H}}[i]$
$n_{\mathrm{eq}}[i] = \sigma_{\mathrm{n}}^2\mathrm{Tr}\left(\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{W}[i] + \boldsymbol{W}^{\mathrm{H}}[i]\left(\sum_{k=1}^{nr}\boldsymbol{G}_{\mathrm{eq}k}[i]\boldsymbol{A}k[i]\right)\left(\sum_{k=1}^{nr}\boldsymbol{G}_{\mathrm{eq}k}[i]\boldsymbol{A}k[i]\right)^{\mathrm{H}}\boldsymbol{W}[i]\right)$
$\nabla_{\boldsymbol{W}[i]} = \frac{\sigma_{\mathrm{s}}^2}{n_{\mathrm{eq}}[i]}\left(\mathrm{Tr}(\parallel \boldsymbol{H}_{\mathrm{SDA}}[i]\boldsymbol{A}_S[i]\parallel_F^2\ \boldsymbol{W}[i]\right)n_{\mathrm{eq}}[i]$
$-\left\|\boldsymbol{W}^{\mathrm{H}}[i]\boldsymbol{H}_{\mathrm{SDA}}[i]\boldsymbol{A}_{\mathrm{S}}[i]\right\|_F^2 T_{\mathrm{r}}\left(\boldsymbol{W}[i] + \boldsymbol{G}_{\mathrm{eq}k}[i]\boldsymbol{A}_k[i]\left(\boldsymbol{G}_{\mathrm{eq}k}[i]\boldsymbol{A}_k[i]\boldsymbol{W}[i]\right)\right)$
$\nabla_{a_{S_j}[i]} = \frac{\sigma_{\mathrm{s}}^2}{n_{\mathrm{eq}}[i]}r_{\mathrm{SDA}_j}[i]$
$\nabla_{a_{R_k\mathrm{D}}[i]} = \frac{\sigma_{\mathrm{s}}^2}{n_{\mathrm{eq}}[i]}\left(r_{\boldsymbol{G}_{\mathrm{eq}}}[i]n_{\mathrm{eq}}[i] - \sigma_n^2 r_{\boldsymbol{N}_{kj}}[i]\sum_{j=1}^{N} r_{\boldsymbol{G}_{\mathrm{eq}_{k_j}}}[i]a_{k_j}^*[i]\right)$
end for
3: Update:
$\boldsymbol{w}_j[i+1] = \boldsymbol{w}_j[i] - \mu\nabla_{\boldsymbol{w}_j^*[i]}$
$a_{S_j}[i+1] = a_{S_j}[i] - \nu\nabla_{a_{S_j}^*[i]}$

$a_{k_j}[i+1] = a_{k_j}[i] - \tau\nabla_{a_{k_j}^*[i]}$

4: Normalisation:

$\boldsymbol{A}_S[i+1] = \frac{\sqrt{\mathrm{P_T}}\boldsymbol{A}_S[i+1]}{\|\boldsymbol{A}_S[i+1]\|_F}$
$\boldsymbol{A}_k[i+1] = \frac{\sqrt{P_{\mathrm{R}}}\boldsymbol{A}_k[i+1]}{\left\|\sum_{k=1}^{n_r}\mathrm{Tr}(\boldsymbol{A}_k[i+1]\right\|_F}$

Table 2 Computational complexity of the algorithms

Algorithms	Number of operations per symbol Multiplications	Additions
PA MMSE(III-A)	$(T+1)^6N^6 + (T+1)N + 8(T+1)N$	$7(T+1)N + 2$
JAPA MMSE SG (IV-A)	$(7T+5)N$	$4(T+1)N$
JAPA MBER SG (IV-B)	$(M+1)(T+1)N + M$	$(2M+1)(T+1)N$
OPA* [13]	$N^4 + 2N^2 + N^2T^2$	$2NT - 1$
JAPA MSR SG (IV-C)	$7(T+1)N + N + 1$	$7(T+1)N + N + 2$
PO-PR-SIM** [12]	$N^4 + 2N^2$	$2NT$

* Optimal power allocation
** Power optimisation pure relay SIM

variance $\sigma_f\left(\boldsymbol{I} + \left\|G_{\mathrm{eq}}[i]\hat{\boldsymbol{A}}[i]\right\|_F\right)$. By defining $\hat{\boldsymbol{p}} = E\left[\hat{r}s^{\mathrm{H}}\right]$ and $\hat{\boldsymbol{R}}\boldsymbol{x} = E\left[\hat{r}\hat{r}^{\mathrm{H}}\right]$, we can obtain the MSE with the feedback errors as

$$
\begin{aligned}
m_{\mathrm{e}} &= \mathrm{Tr}\left(\hat{p}^{\mathrm{H}}\hat{\boldsymbol{R}}x^{-1}\hat{p}\right)\\
&= \mathrm{Tr}\Bigg(\left(\boldsymbol{G}_{\mathrm{eq}}[i](\boldsymbol{A}[i] + \boldsymbol{E}[i])\boldsymbol{F}[i]\sigma_{\mathrm{s}}\right)^{\mathrm{H}}\\
&\quad \times \left(\left\|\boldsymbol{G}_{\mathrm{eq}}[i](\boldsymbol{A}[i] + \boldsymbol{E}[i])\boldsymbol{F}[i]\right\|_{\mathrm{F}}^2\sigma_{\mathrm{s}}\right.\\
&\quad \left.+ \left(\boldsymbol{I} + \left\|\boldsymbol{G}_{\mathrm{eq}}[i](\boldsymbol{A}[i] + \boldsymbol{E}[i])\right\|_F^2\right)\sigma_f\right)^{-1}\\
&\quad \times \left(\boldsymbol{G}_{\mathrm{eq}}[i](\boldsymbol{A}[i] + \boldsymbol{E}[i])\boldsymbol{F}[i]\sigma_{\mathrm{s}}\right)\Bigg)
\end{aligned}
\tag{34}
$$

whereas the MSE expression of the system with accurate PA parameters is given by

$$m = \mathrm{Tr}\Big(\big(\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{A}[i]\boldsymbol{F}[i]\sigma_{\mathrm{s}}\big)^{\mathrm{H}} \times \Big(\big\|\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{A}[\mathrm{i}]\boldsymbol{F}[i]\big\|_F^2\sigma_{\mathrm{s}} + \Big(\boldsymbol{I} + \big\|\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{A}[i]\big\|_F^2\Big)\sigma_n\Big)^{-1} \times \big(\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{A}[\mathrm{i}]\boldsymbol{F}[i]\sigma_{\mathrm{s}}\big)\Big) \quad (35)$$

By substituting (35) into (34), we can obtain the difference between the MSE expressions with accurate and inaccurate PA matrices which is given by

$$m_{\mathrm{e}} = m + \mathrm{Tr}\Big(\big(\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{E}[i]\boldsymbol{F}[i]\sigma_{\mathrm{s}}\big)^{\mathrm{H}} \times \Big(\big\|\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{E}[i]\boldsymbol{F}[i]\big\|_F^2\sigma_{\mathrm{s}} + \Big(\boldsymbol{I} + \big\|\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{E}[i]\big\|_F^2\Big)\sigma_{\mathrm{n}}\Big)^{-1} \times \big(\boldsymbol{G}_{\mathrm{eq}}[i]\boldsymbol{E}[i]\boldsymbol{F}[i]\sigma_{\mathrm{s}}\big)\Big) = m + m_{\mathrm{eo}} \quad (36)$$

The received PA matrices are positive definite according to the power constraint, which indicates m_{eo} is a positive scalar. The expression in (36) denotes an analytical derivation of the MSE at the destination node, which indicates the impact of the limited feedback employed in the JAPA SG algorithms.

6 Simulations

The simulation results are provided in this section to assess the proposed JAPA SG algorithms. The equal PA (EPA) algorithm in [10] is employed in order to identify the benefits achieved by the proposed PA algorithms. The cooperative MIMO system considered employs an AF protocol with the Alamouti space-time block code (STBC) scheme in [12] using BPSK modulation in a quasi-static block fading channel with AWGN. The effect of the direct link (DL) is also considered. It is possible to employ the DF protocol or use a different number of antennas and relay nodes with a simple modification. The ML detection is considered at the destination node to indicate the achievement of full receive diversity. The system is equipped with $n_{\mathrm{r}} = 1$ relay node and $N = 2$ antennas at each node. In these simulations, we set the symbol power σ_{s}^2 to 1. The SNR in these simulations is the received SNR which is calculated by (29).

The proposed JAPA SG algorithms derived in Section 4 are compared with the EPA algorithm and the PA algorithms in [12, 21, 22] with and without the DL in Fig. 2. The results illustrate that the performance of the proposed JAPA SG algorithms is superior to the EPA algorithm by more than 3 dB. The performance of the PA algorithms in the literature is designed for AF systems without re-coding at the relays, and in order to obtain a fair comparison they have been adapted to the system considered in Fig. 1. However, as shown in the plot, the performance of the existing PA algorithms cannot achieve a BER performance

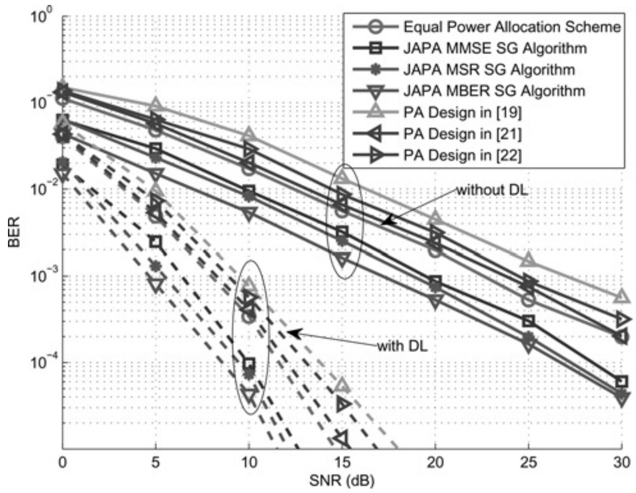

Fig. 2 *SNR versus BER for JAPA SG algorithms*

as good as the proposed algorithms can. In the low SNR scenario, the JAPA MSR SG algorithm can achieve a better BER performance compared with the JAPA MMSE SG algorithm, whereas with the increase of the SNR, the BER curves of the JAPA MSR and MMSE SG algorithms approach the BER performance of the JAPA MBER SG algorithm with enough Monte Carlo simulation numbers. The BER of the JAPA MBER SG algorithm achieves the best performance because of the received BER is minimised by the algorithm in Section 4. The performance improvement of the proposed JAPA SG algorithms is achieved with more relays employed in the system as an increased spatial diversity is provided by the relays.

The simulation results shown in Fig. 3 illustrate the influence of the feedback channel on the JAPA MBER SG algorithm. As mentioned in Section 5, the optimised PA matrices will be sent back to each relay node and the source node through an AWGN feedback channel. The quantisation and feedback errors are not considered in the simulation results in Fig. 2, so the optimised PA matrices are perfectly known at the relay node and the source node after the JAPA SG algorithm, whereas in Fig. 3, it indicates that the performance of the proposed algorithm will be affected by the accuracy of the feedback information. In this simulation, we use 2, 3 and 4 bits to quantise the real

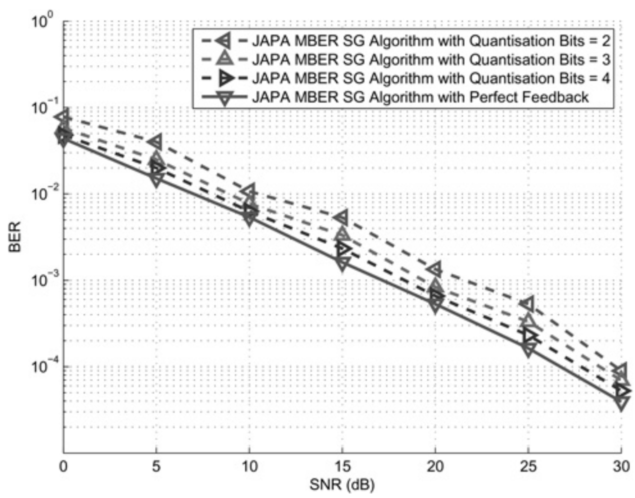

Fig. 3 *JAPA MBER SG algorithm SNR versus BER*

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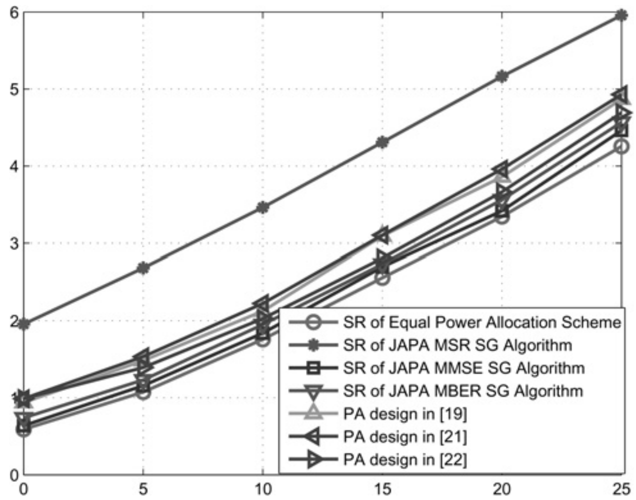

Fig. 4 *JAPA SG algorithms sum rate versus SNR*

part and the imaginary part of the element in $A_S[i]$ and $A_k[i]$, and the feedback channel is modelled as an AWGN channel. As we can see from Fig. 3, by increasing the number of the quantisation bits for the feedback, the BER performance approaches the performance with the perfect feedback, and by making use of four quantisation bits for the real and imaginary parts of each parameter in the matrices, the performance of the JAPA SG algorithm is about 1 dB worse.

The transmission rate of the cooperative MIMO network with EPA and PA schemes in [12, 21, 22] and the proposed JAPA SG algorithms in Section 4.3 is given by Fig. 4. The number of relay nodes is equal to 1 for all the algorithms. The proposed JAPA MSR SG optimisation algorithm adjusts the power allocated to each antenna in order to achieve the maximum of the sum rate in the system. From the simulation results, it is obvious that higher throughput can be achieved by the existing PA algorithms in [12, 21, 22] compared with the proposed JAPA MMSE and MBER SG algorithms. The reason for that lies in the design criterion in the existing algorithms and the proposed algorithms. However, the improvement in the sum rate by employing the JAPA MSR SG algorithm can be observed as well. The rate improvement of the JAPA MMSE and MBER SG algorithms is not as much as the JAPA MSR SG algorithm because the optimisation of the proposed JAPA MMSE and MBER optimisation algorithms is not suitable for the maximisation of the sum rate.

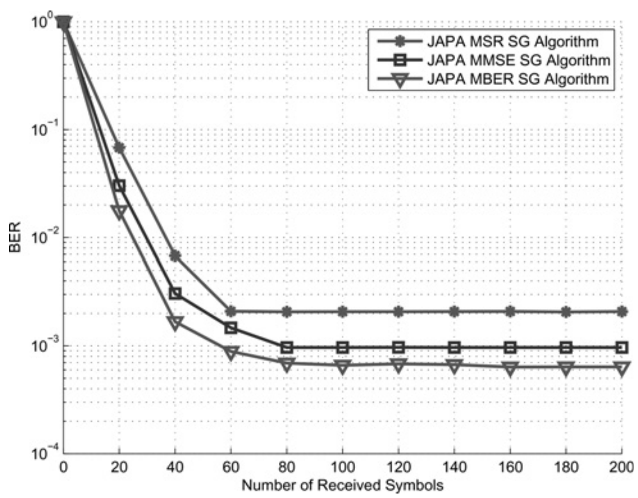

Fig. 5 *BER performance against number of symbols for JAPA SG algorithms*

The simulation results shown in Fig. 5 illustrate the convergence property of the proposed JAPA SG algorithm. All the schemes have an error probability of 0.5 at the beginning, and after the first 20 symbols are received and detected, the JAPA MMSE scheme achieves a better BER performance compared with the JAPA MSR scheme and the JAPA MBER scheme can reach a lower BER than the other algorithms. With the number of received symbols increasing, the BER curve of all the schemes are almost straight, whereas the BER performance of the JAPA MBER algorithm can be further improved and obtain a fast convergence after receiving 80 symbols.

7 Conclusion

We have proposed JAPA and receiver design algorithms according to different criteria with the power constraint between the source node and the relay node, and between relay nodes and the destination nodes to achieve low BER performance. Joint iterative estimation algorithms with low computational complexity for computing the PA parameters and the linear receive filter have been derived. The simulation results illustrated the advantage of the proposed PA algorithms by comparing it with the EPA algorithm. The proposed algorithm can be utilised with different DSTC schemes and a variety of detectors [23, 24] and estimation algorithms [25] in cooperative MIMO systems with AF strategy and can also be extended to the DF cooperation protocols.

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