# Intelligent Cruise Control and Reliable Communication of Mobile Stations

Rolf Hager, *Member, IEEE,* Rudolf Mathar, and Jürgen Mattfeldt

***Abstract*—The reliability of a short range packet radio network for mobile stations is investigated via the probability of a successful transmission. The instantaneous power of an interfering station's transmission is described by a Rayleigh distribution. We determine the distribution of cumulated interference power and the probability of a successful transmission, when the number of interfering stations is random and each station transmits only with probability $p$. Two lower bounds of increasing complexity are given. We evaluate these bounds for an intelligent cruise control system, when mobile stations are lined up in a traffic jam, and one approaching vehicle should be warned by the last member of the queue.**

## I. Introduction

The basic tool of intelligent cruise control is a reliable communication link between vehicles by radio technology. This will be used to automatically exchange relevant information concerning position, velocity, warning messages to approaching vehicles, etc. Particularly for the last case it is necessary to estimate the reliability of a radio channel via the probability of successfully transmitting a data packet.

In this paper we assume a decentralized system, based on the slotted ALOHA protocol, and a channel subject to Rayleigh fading. In order to analyze the probability of a successful transmission the following parameters have to be taken into account: 1) the number of involved stations, 2) their locations, 3) the probability of transmitting in a slot, 4) the distribution of cumulated power of interfering stations, and 5) the internal thermal noise at a receiver. We model the capture effect by defining a transmission as being successful, whenever the signal power of a transmitter is larger than a certain minimum strength threshold $\kappa$, and simultaneously the signal-to-noise ratio exceeds a certain capture ratio $\gamma$.

In this paper we deal with the above characteristics, and calculate for an important application like intelligent cruise control explicit lower bounds. The organization is as follows. Paying attention to the above quantities, in section 2 we derive the distribution of cumulated interference plus thermal noise power, which is needed to calculate the probability of a successful transmission. In order to gain explicit values of reliability, sharp upper bounds are obtained by stochastic preordering. These results are applied in section 3 when mobile stations are lined up in a traffic jam, and one approaching vehicle has to be warned by the last member of the queue via the communication link (see Fig. 1). We consider this (one-dimensional) model appropriate, e.g., for a highway scenario. Because of short range transmission lateral interferers from other roads may be neglected. Even in urban environments lateral interferences are shadowed by buildings along the streets.

Manuscript received April 15, 1994; revised August 7, 1994. This work was supported in part by the Deutsche Forschungsgemeinschaft under Grant Ma 1184/2-4.

The authors are with Aachen University of Technology, D-52056 Aachen, Germany.

IEEE Log Number 9413001.

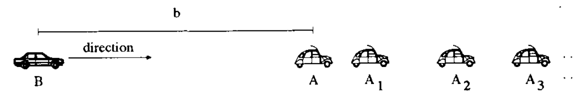



Fig. 1. Location of stations.

For this scenario the lower bounds are calculated numerically for certain parameters which allows an assessment of system reliability. The key point of our approach is to combine stochastic properties of the location model and the channel behavior in a joint realistic model, and to give an analysis of system reliability by a new mathematical setup.

## II. Probability of a Successful Transmissions

We consider a reference station $A$, going to transmit a data packet to a receiver $B$. We furthermore assume interfering stations $A_1, A_2, \dots$ whose number is a random variable $N$ with support $\boldsymbol{N}_0$ and discrete density

$$P(N = n) = a_n, \quad n \in \boldsymbol{N}_0.$$

The channel access protocol is slotted ALOHA. We suppose that each station transmits with probability $p$ in a slot, independently of all others, and remains silent with probability $q = 1-p$. Due to shadowing effects a Rayleigh fading channel is assumed such that the instantaneous interference power $S_i$ of station $A_i$ at $B$ has distribution function

$$F_{S_i}(x) = q + p\left(1 - e^{-d_i^2 x}\right), \quad x \geq 0 \tag{1}$$

with $d_i > 0$, $i \in \boldsymbol{N}$, proportional to the distance between $A_i$ and $B$. (1) is a mixture of an exponential distribution with parameter $d_i^2$ and the Dirac measure at 0 . The exponent 2 in $d_i^2$ is typical for radio wave propagation in a landscape scenario which we assume furtheron. $S_i$ has a representation as $S_i = U_i \cdot T_i$, where $U_i$, $T_i$ are independent, $T_i$ is exponentially distributed with parameter $d_i^2$, and $P(U_i = 1) = p$ , $P(U_i = 0) = 1 - p$.

We denote by

$$S^{(n)} = \sum_{i=1}^{n} S_i, \quad \text{and} \quad S = \sum_{i=1}^{N} S_i$$

the cumulated interference power when $n$ stations are present, and the cumulated interference power with a random number of stations, respectively.

The instantaneous signal power $X$ of reference station $A$ at $B$ is assumed to have a density $f_X(x)$, $x \geq 0$, which may correspond to a Rician or Rayleigh fading channel. Its explicit form will be specified later.

To be close to reality we take account of the internal thermal noise power of receiver $B$, described by the random variable $Z$, distributed according to an exponential distribution with parameter $\sigma^2 > 0$ and density

$$f_{\exp(\sigma^2)}(x) = \sigma^2 e^{-\sigma^2 x}, x \geq 0. \qquad (2)$$

In the following we suppose that the random variables $X$, $Z$, $S_i$, $i \in \boldsymbol{N}$ , and $N$ are all stochastically independent which seems to be a quite natural assumption.

According to [11] a signal is correctly received if

i) $X > \kappa$, with $\kappa$ the minimum signal power threshold to realize a correct reception, and

ii) $X/(S+Z) > \gamma$, with signal-to-noise ratio $X/(S+Z)$ and capture ratio $\gamma$.

Condition (ii) has been used for the special case $Z \equiv 0$ in [1], [3], [7] and [8]. The more realistic assumption of nonvanishing thermal noise was recently introduced by [2] and [5].

The probability of a successful transmission from $A$ to $B$ is now given by

$$\begin{aligned} p_{\text{suc}} &= p\sum_{n=0}^{\infty} a_n \int_0^{\infty} P\Big(X > \kappa, \frac{X}{S+Z} > \gamma \;\Big|\; X = x, N = n\Big) \\ &\quad \cdot f_X(x)dx \\ &= p\sum_{n=0}^{\infty} a_n \int_\kappa^{\infty} P\Big(S^{(n)} + Z \leq \frac{x}{\gamma}\Big) f_X(x)\, dx. \qquad (3) \end{aligned}$$

The key point is to evaluate $P(S^{(n)} + Z \leq x/\gamma)$ in this expression, i.e., determining the distribution function of $S^{(n)} + Z$. By (1) and (2) the Laplace transforms of $S_i$ and $Z$ are given by

$$L_{S_i}(s) = q + p\frac{d_i^2}{d_i^2 + s} \quad \text{and} \quad L_Z(s) = \frac{\sigma^2}{\sigma^2 + s}, s \geq 0.$$

Because of independence, the Laplace transform of $S^{(n)} + Z$ is obtained as

$$L_{S^{(n)}+Z}(s) = L_Z(s) \cdot \prod_{i=1}^{n} L_{S_i}(s), s \geq 0.$$

With $l_i(s) = d_i^2/(d_i^2 + s)$, $i = 1, \ldots, n$, and $l(s) = \sigma^2/(\sigma^2 + s)$ it follows that

$$\begin{aligned} L_{S^{(n)}+Z}(s) &= l(s) \cdot \prod_{i=1}^{n} \big(q + p\, l_i(s)\big) = l(s) \\ &\quad \Big(q^n + \sum_{i=1}^{n} q^{n-i} p^i \sum_{1 \leq k_1 < \cdots < k_i \leq n} l_{k_1}(s) \cdots l_{k_i}(s)\Big) \qquad (4) \end{aligned}$$

The product $l(s) l_{k_1}(s) \cdots l_{k_i}(s)$ is the Laplace transform of a hypoexponential distribution with density (see, e.g., [10], p. 202)

$$f_{\text{hypexp}(\lambda_0,\ldots,\lambda_i)}(x) = \sum_{m=0}^{i} \Big( \prod_{\substack{j=0 \\ j \neq m}}^{i} \frac{\lambda_j}{\lambda_j - \lambda_m} \Big) \lambda_m e^{-\lambda_m x}, x \geq 0,$$

where $\lambda_j = d_{k_j}^2$, $j = 1, \ldots, i$ , and $\lambda_0 = \sigma^2$, provided $\lambda_i \neq \lambda_j$ for all $i \neq j \in \{0, \ldots, i\}$. Obviously, $i = 0$ yields $f_{\exp(\lambda_0)}(x)$ as a corresponding density.

From the uniqueness theorem of Laplace transforms, by (4) it follows that

$$\begin{aligned} F_{S^{(n)}+Z}(x) &= q^n F_{\exp(\sigma^2)}(x) \\ &+ \sum_{i=1}^{n} q^{n-i} p^i \sum_{1 \leq k_1 < \cdots < k_i \leq n} F_{\text{hypexp}(\sigma^2, d_{k_1}^2, \ldots, d_{k_i}^2)}(x), x \geq 0, \qquad (5) \end{aligned}$$

is the distribution function of $S^{(n)} + Z$. In summary, $p_{\text{suc}}$ in (3) is given by

$$\begin{aligned} p_{\text{suc}} = p\sum_{n=0}^{\infty} a_n \int_\kappa^{\infty} f_X(x) \Big(q^n F_{\exp(\sigma^2)}(x/\gamma) + \sum_{i=1}^{n} q^{n-i} p^i \\ \sum_{1 \leq k_1 < \cdots < k_i \leq n} F_{\text{hypexp}(\sigma^2, d_{k_1}^2, \ldots, d_{k_i}^2)}(x/\gamma)\Big) dx \end{aligned} \qquad (6)$$

with the hypoexponential distribution function

$$F_{\text{hypexp}(\lambda_0,\ldots,\lambda_n)} = \sum_{i=0}^{n} \Big( \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{\lambda_j}{\lambda_j - \lambda_i} \Big) \big(1 - e^{-\lambda_i x}\big), x \geq 0. \qquad (7)$$

The structure of equation (6) is rather complicated since the number of summands in the second term is $\sum_{i=0}^{n} \binom{n}{i} = 2^n$, which increases exponentially with $n$. This means that for a heavy tailed distribution $(a_n)_{n \in \boldsymbol{N}_0}$ the total sum of (6) can be hardly approximated numerically. This leads to the objective to find good lower bounds for the desired probability $p_{\text{suc}}$ for certain scenarios. Such lower bounds may serve as estimates of the reliability of a system.

Let $m \in \boldsymbol{N}_0$ be fixed and $Z + S = Z + \sum_{i=1}^{N} U_i T_i$ with $Z, U_i, T_i$, and $N$ independent random variables with the distributions introduced above. We easily get lower bounds for $p_{\text{suc}}$ in (6) from the following chain of inequalities

$$\begin{aligned} Z + S &= Z + \Big(\sum_{i=1}^{N} U_i T_i\Big) 1\!\!1_{\{N \leq m\}} \\ &\quad + \Big(\sum_{i=1}^{m} U_i T_i + \sum_{i=m+1}^{N} U_i T_i\Big) 1\!\!1_{\{N > m\}} \overset{\text{st}}{\leq} Z \\ &\quad + \Big(\sum_{i=1}^{N} U_i T_i\Big) 1\!\!1_{\{N \leq m\}} + \Big(\sum_{i=1}^{m} U_i T_i\Big) 1\!\!1_{\{N > m\}} \\ &\quad + \Big(\sum_{i=m+1}^{\infty} T_i\Big) 1\!\!1_{\{N > m\}} \qquad (8) \\ &\overset{\text{st}}{\leq} Z + \Big(\sum_{i=1}^{N} U_i T_i\Big) 1\!\!1_{\{N \leq m\}} + \Big(\sum_{i=1}^{\infty} T_i\Big) 1\!\!1_{\{N > m\}}, \qquad (9) \end{aligned}$$

where '$\overset{\text{st}}{\leq}$' means stochastically larger (see, e.g., [6]). The distribution of $Z + S$, conditional on $N = n$ , is given by (5).

(8) means that in the case of more than $m$ stations present, infinite many with indices $m + 1, m + 2, \ldots$ transmit with probability $p = 1$. (9) reflects the fact that with more than $m$ stations present, infinite many with indices $1, 2, \ldots$ transmit with probability one. This shows the above inequalities on an intuitive basis.

## III. Traffic Scenarios and Lower Bounds

We start with a description of the location model. The transmitting station $A$ is located at the origin of the real line $\boldsymbol{R}^1$. $A$ is addressing a station $B$ at distance $b$ left of $A$. There is a random number $N$ of interfering stations $A_i$, located to the right of A with constant interspace $d > 0$ . Thus, station $A_i$ has the distance $d_i = b + di$ from $B$. Fig. 2 gives a comprehensive overview of positions.

This model is well suited to describe a traffic jam with vehicles $A$ and $A_1, A_2, \ldots, A_N$ lined up at an obstacle, and $B$ is a vehicle approaching the end of the queue formed by $A$. $A$ is willing to transmit a warning to $B$ via a corresponding data packet, and it is most important to estimate the probability of a successful transmission of this packet. We accept the same independence assumptions as in section 2. As has been pointed out there, a direct evaluation of (6) seems not to be feasable.

In the following we calculate lower bounds of $p_{\text{suc}}$ . The first one is derived from (9), where we need the distribution of $Z + \sum_{i=1}^{\infty} T_i$. This distribution is obtained by taking $p = 1$ and the limit in $n \to \infty$ in (5),

$$\lim_{n\to\infty} F_{\text{hypexp}(\lambda_0,\ldots,\lambda_n)} = \lim_{n\to\infty} \sum_{i=0}^{n} \Big( \prod_{\substack{j=0\\ j\neq i}}^{n} \frac{\lambda_j}{\lambda_j - \lambda_i} \Big) \big(1 - e^{-\lambda_i x}\big), x \geq 0. \quad (10)$$

with $\lambda_0 = \sigma^2$, $\lambda_i = d_i^2$ , and $\sigma^2 \neq d_i^2$ for all $i \geq 1$.

It remains to calculate $\lim_{n\to\infty} F_{\text{hypexp}(\sigma^2, d_1^2,\ldots,d_n^2)}$ , i.e., the limit distribution w.r.t. weak convergence. For this purpose, $\prod_{j=0, j\neq i}^{n} \lambda_j/(\lambda_j - \lambda_i)$ has to be determined as $n \to \infty$ in representation (10). To simplify calculations we assume that $b$ is a multiple of $d$, i.e.,

$$b = kd \text{ for some } k \in \boldsymbol{N}.$$

We furthermore assume that $\lambda_0 = \sigma^2 \neq \lambda_j = ((k + j)d)^2$ for all $j \geq 1$. The product is treated separately for $i = 0$ and $i > 0$. In the first case we have

$$\prod_{\substack{j=0\\ j\neq i}}^{n} \frac{\lambda_j}{\lambda_j - \lambda_i} = \prod_{j=k+1}^{n+k} \frac{1}{1 - \sigma^2/(jd)^2} \overset{(n\to\infty)}{\longrightarrow} \frac{\pi\sigma/d}{\sin(\pi\sigma/d)} \prod_{j=1}^{k} \Big(1 - \frac{\sigma^2}{(dj)^2}\Big),$$

where the limit is due to Euler's formula (see e.g. [4]),

$$\prod_{j=1}^{\infty} (1 - z^2/j^2) = \frac{\sin(\pi z)}{\pi z}, \quad z \in \boldsymbol{C}. \quad (11)$$

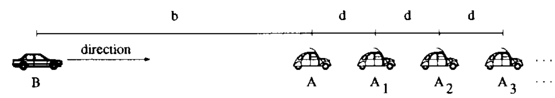



Fig. 2. Location of stations.

The case $i > 0$ is treated along similar lines. With $\ell = k + i$ we obtain

$$\prod_{\substack{j=0\\ j\neq i}}^{n} \frac{\lambda_j}{(\lambda_j - \lambda_i)} = \frac{1}{1 - (d\ell/\sigma)^2} \prod_{\substack{j=k+1\\ j\neq \ell}}^{n+k} \frac{1}{1 - (\ell/j)^2} \overset{n\to\infty}{\longrightarrow} \frac{1}{1 - (d\ell/\sigma)^2} \prod_{\substack{j=k+1\\ j\neq \ell}}^{\infty} \frac{1}{1 - (\ell/j)^2}.$$

The infinite product is now calculated using (11) and l'Hospital's rule as follows

$$\begin{aligned}\prod_{\substack{j=k+1\\ j\neq \ell}}^{\infty} (1 - \ell^2/j^2) &= \lim_{x\to\ell} \prod_{\substack{j=k+1\\ j\neq \ell}}^{\infty} (1 - x^2/j^2)\\ &= \lim_{x\to\ell} \frac{\sin(\pi x)}{\pi x(1 - x^2/\ell^2)\prod_{j=1}^{k}(1 - x^2/j^2)}\\ &= \Big(\prod_{j=1}^{k} (1 - \ell^2/j^2)\Big)^{-1}\\ &\quad \cdot \lim_{x\to\ell} \frac{\pi\cos(\pi x)}{\pi(1 - x^2/\ell^2) - \pi x(2x)/\ell^2}\\ &= \frac{(-1)^{\ell+1}}{2} \Big(\prod_{j=1}^{k} (1 - \ell^2/j^2)\Big)^{-1}.\end{aligned}$$

Combining both cases, the limit distribution $F_{\sigma^2,k}$ is obtained as

$$\begin{aligned}F_{\sigma^2,k}(x) &= \lim_{n\to\infty} F_{\text{hypexp}(\sigma^2, d_1^2,\ldots,d_n^2)}(x)\\ &= 1 - \lim_{n\to\infty} \sum_{i=0}^{n} \Big( \prod_{\substack{j=0\\ j\neq i}}^{n} \frac{\lambda_j}{\lambda_j - \lambda_i} \Big) \exp(-\lambda_i x)\\ &= 1 - \frac{\pi\sigma/d}{\sin(\pi\sigma/d)} \prod_{j=1}^{k} \Big(1 - \frac{\sigma^2}{(dj)^2}\Big) \exp(-\sigma^2 x)\\ &\quad + 2 \cdot \sum_{i=k+1}^{\infty} \frac{(-1)^i}{1 - (di/\sigma)^2}\\ &\quad \prod_{j=1}^{k} \Big(1 - \frac{i^2}{j^2}\Big) \exp(-(di)^2 x), \quad x > 0. \quad (12)\end{aligned}$$

and $F_{\sigma^2,k}(0) = 0$.

$\sigma \to \infty$ lets the thermal noise power at receiver $B$ vanish. This is a special case of (12), which together with $d = 1$ and $k = 0$ has been considered by [7]. The distribution function reduces to

$$F_{\infty,0}(x) = \begin{cases} 1 + 2 \cdot \sum_{i=1}^{\infty} (-1)^i \exp(-i^2 x), & x > 0\\ 0, & x = 0 \end{cases}.$$

where the empty product in (12) is set to 1.

A lower bound for the probability of a successful transmission is now easily combined from (3), (5), (9), and (12). For any $m \in \boldsymbol{N}_0$ it holds that

$$\begin{aligned}p_{\text{suc}} &\geq p\Big(\sum_{n=0}^{m} a_n \int_{\kappa}^{\infty} P\big(Z + S^{(n)} \leq \frac{x}{\gamma}\big) f_X(x)dx \\ &\quad + \sum_{n=m+1}^{\infty} a_n \int_{\kappa}^{\infty} P\big(Z + \textstyle\sum_{i=1}^{\infty} T_i \leq \frac{x}{\gamma}\big) f_X(x)dx\Big) \\ &= p\sum_{n=0}^{m} a_n \int_{\kappa}^{\infty} f_X(x)\Big(q^n F_{\exp(\sigma^2)}\big(\frac{x}{\gamma}\big) \\ &\quad + \sum_{i=1}^{n} q^{n-i}p^i \sum_{1\leq k_1<\cdots<k_i\leq n} F_{\text{hypexp}(\sigma^2, d_{k_1}^2,\ldots,d_{k_i}^2)}\big(\frac{x}{\gamma}\big)\Big)dx \\ &\quad + p\Big(1 - \sum_{n=0}^{m} a_n\Big)\int_{\kappa}^{\infty} f_X(x)F_{\sigma^2,k}\big(\frac{x}{\gamma}\big)dx = \hat{p}_1(m).\end{aligned} \tag{13}$$

The lower bound (13) can be easily calculated since the series in the representation of $F_{\sigma^2,k}$ converges very fast, due to squared negative exponents. The approximation is more accurate the larger $m$ is. The worst approximation holds for $m = 0$, the exact value is attained as $m \to \infty$. Note that for any $m$ this bound only depends on $a_0, \ldots, a_m$.

A superior bound of $p_{\text{suc}}$—but at the price of increased numerical effort—can be achieved from (8). The distribution of $Z + \sum_{i=m+1}^{\infty} T_i$ is clearly given by $F_{\sigma^2,k+m}$ from (12). The distribution function of $S^{(m)} = \sum_{i=1}^{m} U_i T_i$ is obtained along similar lines as (5)

$$F_{S^{(m)}}(x) = q^m + \sum_{i=1}^{m} q^{m-i}p^i \sum_{1\leq k_1<\cdots<k_i\leq m} F_{\text{hypexp}(d_{k_1}^2,\ldots,d_{k_i}^2)}(x), \quad x \geq 0.$$

Thus, the distribution function $G_{\sigma^2,k,m}$ of $Z + \sum_{i=1}^{m} U_i T_i + \sum_{i=m+1}^{\infty} T_i$ is given by

$$\begin{aligned}G_{\sigma^2,k,m}(x) &= \Big(F_{\sigma^2,k+m} * F_{S^{(m)}}\Big)(x) \\ &= \int_0^x f_{\sigma^2,k+m}(x-t)dF_{S^{(m)}}(t), x \geq 0,\end{aligned}$$

where $*$ denotes the convolution operator.

In summary, by conditioning on $N = n$ and exploiting (8) it follows that

$$\begin{aligned}p_{\text{suc}} &\geq p\sum_{n=0}^{m} a_n \int_{\kappa}^{\infty} f_X(x)\Big(q^n F_{\exp(\sigma^2)}\big(\frac{x}{\gamma}\big) \\ &\quad + \sum_{i=1}^{n} q^{n-i}p^i \sum_{1\leq k_1<\cdots<k_i\leq n} F_{\text{hypexp}(\sigma^2, d_{k_1}^2,\ldots,d_{k_i}^2)}\big(\frac{x}{\gamma}\big)\Big)dx \\ &\quad + p\Big(1 - \sum_{n=0}^{m} a_n\Big)\int_{\kappa}^{\infty} f_X(x)G_{\sigma^2,k,m}\big(\frac{x}{\gamma}\big)dx = \hat{p}_2(m).\end{aligned} \tag{14}$$

The lower bound (14) is better than (13). This is an easy consequence of inequalities (8) and (9).

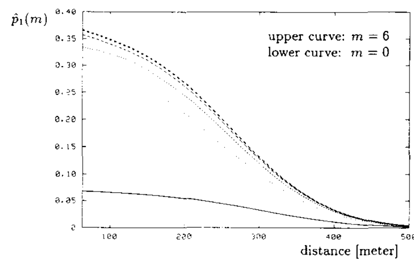



Fig. 3. Lower bounds $\hat{p}_1(m)$ with $N \sim$ Poi(2), $\gamma = 2$.

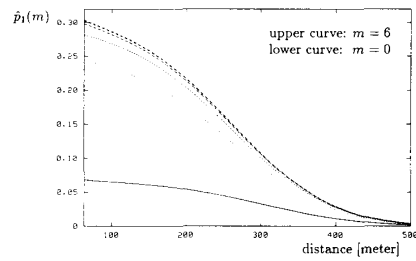



Fig. 4. Lower bounds $\hat{p}_1(m)$ with $N \sim$ Poi(2), $\gamma = 4$.

## IV. Numerical Evaluation

The lower bound $\hat{p}_1(m)$ of (13), which approximates the true value $p_{\text{suc}}$ at arbitrary precision as $m \to \infty$, has been calculated numerically for different parameters. We assume that the signal power $X$ is derived from a Rayleigh fading channel. This yields worst case bounds for $p_{\text{suc}}$ , even in the case of direct line-of-sight between stations $A$ and $B$. The bounds thus ensure the safety requirements of the system. The signal threshold is set to $\kappa = 17$dB [9] throughout this section. To get an impression, Fig. 3 depicts the curves of $\hat{p}_1(m)$ for $m = 0, \ldots, 6$. Distances are measured in meters.

The lower bound $\hat{p}_1(m)$ is represented versus transmitter-to-receiver distance $b$ for varying $m$. The traffic congestion length $N$ is assumed to be Poissonian distributed with expectation $\lambda = 2$, i.e., $N \sim$ Poi(2) . The channel access probability $p$, the distance $d$, and the capture ratio $\gamma$ are set to $p = 0.5$, $d = 7$ meter, and $\gamma = 2$, respectively. The curves are monotone decreasing due to an increasing distance of transmitter $A$ to receiver $B$. The decreasing probability of a successful transmission is due to the reduced signal power of vehicle A, that decreases proportional to $d^{-2}$. Depending on $m$, the curves increase obviously to a limiting curve, which is clear from (9). The limiting curve represents $p_{\text{suc}}$ versus transmitter-to-receiver distance itself.

The next diagram Fig. 4 shows curves of the same parameter configuration, except the capture ratio which is now set to $\gamma = 4$ .The curves exhibit a similar behavior compared to the above, but generally run below the corresponding curves of Fig. 3. This behavior was to be expected, because an increased capture ratio reduces the probability of a successful transmission, which may be easily seen from equation (3).

To get insight into the effect of bigger traffic jams, the random variable $N$ is now assumed to be discrete uniformly distributed on the set $\{i \in \boldsymbol{N}_0 \mid 0 \leq i \leq 50\}$, i.e.,

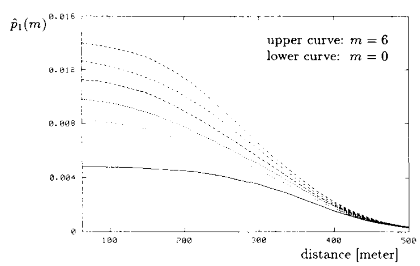



Fig. 5. Lower bounds $\hat{p}_1(m)$ with $N \sim U(0,\ldots,50)$, $\gamma = 2$.

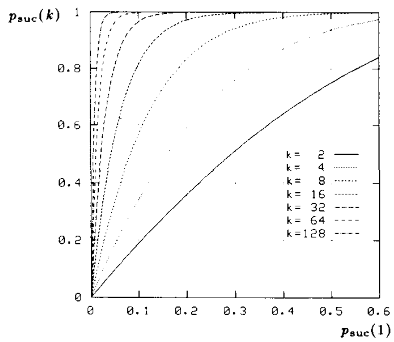



Fig. 6. $p_{suc}(k) = 1 - (1 - p_{suc}(1))^k$.

$N \sim U(0,\ldots,50)$. In addition, the parameters capture ratio $\gamma = 2$, distance $d = 7$ meter, and channel access probability $p = 0.1$ apply. The probability $p$ has been reduced in order to achieve a more realistic application behavior. Fig. 5 shows the results.This model induces significantly reduced success probabilities due to an increased number of interferers with expectation $\mathrm{E}[N] = 25$ , and a reduced channel access probability $p$. Evidently, $p_{suc}$ cannot exceed $p = 0.1$, and $\hat{p}_1(50)$ would represent the exact values $p_{suc}$ for all $p$.

Figs. 3, 4 and especially Fig. 5 show that the success probability of a single warning, $p_{suc}(1)$ say, is rather low. But with $k$ independent attempts it increases to $p_{suc}(k) = 1 - (1 - p_{suc}(1))^k$, where $p_{suc}(k)$ denotes the probability of at least one succesful warning with $k$ attempts. This method decisively increases the success probability (see Fig. 6). For instance, if $p_{suc} \geq 0.3$ for a single warning, with 8 attempts we get an overall success probability larger than 0.92, and with 16 attempts $p_{suc}$ is nearly 1.

The question arises, which channel access probability $p$ is optimal for one slot. The dependence of the estimated success probability $\hat{p}_1(m)$ on the channel access probability $p$ is shown in Fig. 7. The curves have been calculated for $m = 6$ and intervehicle distance $d = 7$ meter, the number of interferers is assumed to be Poissonian distributed with expectation $\lambda = 2$. Observe that for $m = 6$ the lower bound $\hat{p}_1(m)$ is very close to the true value $p_{suc}$ (cp. Fig. 3 and Fig. 4).

Fig. 7 illustrates two sets of curves for a transmitter-to-receiver distance 200 and 300 meter, respectively, each calculated for the capture ratios $\gamma = 1, 2, 4$. It turns out that the success probability bound $\hat{p}_1(6)$ decreases with increasing capture ratio. Interestingly, for a capture ratio $\gamma = 4$, $\hat{p}_1(6)$

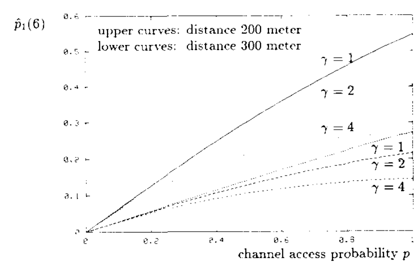



Fig. 7. Lower bounds $\hat{p}_1(6)$ versus $p$ with $N \sim \mathrm{Poi}(2)$, $\gamma = 1, 2, 4$ .

hardly changes between $p = 0.6$ and $p = 1$ . This means that a channel access probability of $p = 0.6$ yields nearly the same success probability as $p = 1$. In other words, reducing $p$ from the optimal value 1 to 0.6, thus saving 40% of channel capacity, causes nearly no loss in $p_{suc}$ . Observe that this effect depends on the specific scenario, here particularly a Poisson random number of interfering stations with expected value 2.

## REFERENCES

[1] J. C. Arnbak and W. v. Blitterswijk, "Capacity of slotted ALOHA in Rayleigh fading channels," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 2, pp. 261–269, Feb. 1987.

[2] A. B. Carlson, *Communication Systems*. New York: McGraw-Hill, 3rd Ed., 1986.

[3] T. C. Hou and V. O. K. Li, "Routing strategies in multihop packet radio networks with a more realistic packet reception probability," in *IEEE INFOCOM*, pp. 303–309, 1985.

[4] K. Knopp, *Theorie und Anwendungen der unendlichen Reihen*. Berlin: Springer, 1964.

[5] H. D. Lüke, *Signalübertragung – Grundlagen der digitalen und analogen Nachrichtenübertragungssysteme*. Berlin: Springer-Verlag, 1988.

[6] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications* . New York: Academic, 1979.

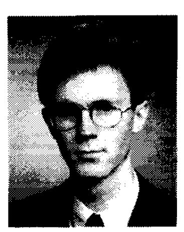
[7] R. Mathar and J. Mattfeldt, "On the distribution of cumulated interference power in Rayleigh fading channels," *Wireless Networks*, vol. 1, no. 1, pp. 31–36, Feb. 1995.

[8] R. Mathar, J. Mattfeldt, and R. Hager, "Analysis of mobile packet radio networks in Rayleigh fading environments," *IEEE Trans. Veh. Technol.*, vol. 43, no. 4, Nov. 1994.

[9] F. Reichert, "Dezentrale Koordination mobiler Stationen in Datenfunknetzen," Ph.D. Thesis, RWTH Aachen, 1991.

[10] K.S. Trivedi, *Probability & Statistics with Reliability, Queueing and Computer Science Applications*. Englewood Cliffs, NJ: Prentice-Hall, 1982.

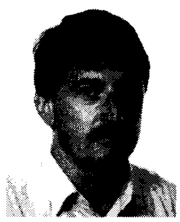
[11] E. Zainal and R. Garcia, "The effects of Rayleigh fading on capture phenomenon in ALOHA channels," in *IEEE INFOCOM*, p. 888–893, 1987.



**Rolf Hager** (M'94) was born in 1965 in Dormagen, Germany. He studied computer science and finished his M.Sc. degree in 1991 at Aachen University of Technology, Germany. He received his Ph.D. degree in 1994 with his thesis on management of traffic information networks. He was project manager within the European projects PROMETHEUS and DRIVE. He then joined the Royal University of Stockholm, Sweden, as a guest researcher on mobile cluster management and contributed to Ph.D. courses and seminars on data communications, specifically network management.

Since 1995, he has been project manager within AEG Mobile Communication GmbH in Ulm, Germany, where he coordinates the development of mobile handheld terminals in the GSM, DCS, and PCS area. His research interests are management and tuning of mobile radio networks.

Dr. Hager is a member of the German Society of Computer Science (GI) and the German Institute for Standardization (DIN).



**Rudolf Mathar** was born in Kalterherberg, Germany, in 1952. He received his Dipl.Math. and Dr.rer.nat. degrees in mathematics from Aachen University of Technology in 1978 and 1981, respectively.

In 1986 and 1987, he worked at the European Business School as a lecturer in computer science, and in 1988 and 1989, he joined a research group in applied optimization at the University of Augsburg. In October 1989, he joined the faculty at Aachen University of Technology, where he is currently a Professor of Stochastics. He is especially interested in applications to computer science. His research interests include mobile communication systems, performance analysis and optimization of networks, and applied probability.

**Jürgen Mattfeldt** was born in Stade, Germany, in 1965. He received his Dipl.Math. and Dr.rer.nat. degrees in mathematics from Aachen University of Technology in 1991 and 1994, respectively.

In October 1994, he joined Thyssen Telecom AG, Düsseldorf. His research interests include mobile communication systems, queueing theory, applied probability, and optimization.