Cooperative Protocols for Random Access Networks

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***Abstract*—Cooperative communications have emerged as a significant concept to improve reliability and throughput in wireless systems. On the other hand, WLANs based on random access mechanism have become popular due to ease of deployment and low cost. Since cooperation introduces extra transmissions among the cooperating nodes and therefore increases the number of packet collisions, it is not clear whether there is any benefit from using physical layer cooperation under random access. In this paper, we develop new low complexity cooperative protocols for random access that outperform the conventional non cooperative scheme for a large range of signal-to-noise ratios.**

I. Introduction

Cooperative communications have emerged as a significant concept to improve reliability and throughput in wireless systems [1]–[5]. In cooperative communications, the resources of distributed nodes are effectively pooled for the collective benefit of all nodes. The broadcast nature of the wireless medium is the key property that allows for cooperation among the nodes: transmitted signals can, in principle, be received and processed by any number of nodes. Although these extra observations of the transmitted signals are available for free (except, possibly, for the cost of additional energy consumption for sensing operation), wireless network protocols often ignore or discard them. The main reason for this is that additional transmissions among the cooperating nodes are needed in order to efficiently pool their resources. In large random access networks without centralized scheduler like in IEEE 802.11 DCF systems [6], these extra transmissions will increase the number of packet collisions and it is not clear whether there is any benefit of using physical layer cooperation in this case. In the case of random access, cooperative strategies, if handled poorly, can even cause performance degradation and a non cooperative scheme, which consists in transmitting the messages of all nodes directly to the access point, might be preferrable.

In this paper, we take the first steps in understanding the issues in designing practical cooperative communication systems for random access networks. Specifically, we closely model the interaction between the physical and medium access channel (MAC) layers in case of physical layer cooperation by a finite state machine. Our model is quite generic since it includes any cooperative or non cooperative multihop transmission scheme. Based on this model, we develop and analyze three new protocols that take full advantage of the node cooperation at the physical layer. We focus on Decode-and-Forward protocols where the intermediate node N decodes the full message sent by the source and forwards only the information missing from the original transmission needed by the destination (here, the access point) to decode the original packet. The Decode-and-Forward protocol was shown to considerably increase the throughput [7].

The remainder of this paper is organized as follows. In Section II, we model the medium access channel by taking the specifications of the physical layer cooperation into account. In Section III, we develop two new simple cooperative protocols that outperform the conventional approach. The throughput analysis for these protocols is elaborated in Section IV and performance results are discussed in Section V. Concluding remarks are presented in Section VI.

II. System Model

We consider the network topology shown in Fig. 1 where nodes F and N send data to the access point A, and in doing so, both nodes are susceptible to mutually help each other. In this study, we consider half-duplex relay channels [7], i.e., the nodes cannot transmit and receive simultaneously.

A. Medium Access

Throughout the paper, the nodes F and N transmit their messages to node A using the distributed coordination function (DCF) mechanism as in IEEE 802.11 standard [6]. In principle, other random access schemes such as Slotted Aloha [8] can be analyzed in a similar way. Under this assumption, no packet/sample synchronization between the nodes is expected, which greatly simplifies the implementation of the communication protocols. Collisions may occur between F and N at the access point. In order to avoid collisions, DCF adopts an exponential backoff scheme with a discrete time backoff scale, in which a contention window initiated with a minimum size can be adapted exponentially up to a maximum size in case

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of collision. The length of a discrete timeslot depends on the PHY specifications, a typical value being 50μs [6].

In the model shown in Fig. 1, we assume that the nodes operate in saturation conditions, i.e., they are backlogged and we do not need to consider packet arrival processes in our derivations. Since all three nodes share the same wireless channel, the state of the network can be described by the current channel state. We distinguish between three phases: first, when node F or node N *successfully transmits* a packet; second, when a *collision* between F and N occurs, and third, when the channel is *idle*. Note that different phases can have different durations. There are three types of transmission: F transmitting its own packet during the amount of time t_{f}, N transmitting its own packet during t_{n}, and N relaying a packet from F during t_{r}. In our notations, the subscript $_{\mathrm{sc}}$ indicates that a transmission was successful. Similarly, we denote t_{c} as the amount of time collisions occur and t_{i} as the amount of time the channel is in idle state. The duration t of the observation time interval can thus be expressed as

$$t = t_{\mathrm{sc}} + t_{\mathrm{c}} + t_{\mathrm{i}} = t_{\mathrm{sc,f}} + t_{\mathrm{sc,n}} + t_{\mathrm{sc,r}} + t_{\mathrm{c}} + t_{\mathrm{i}}. \quad (1)$$

By normalizing the duration of each phase by the observation time interval t, we can express the normalized time division parameters as follows

$$\mathcal{S}_{\mathrm{f}} = \frac{t_{\mathrm{sc,f}}}{t}, \quad \mathcal{S}_{\mathrm{n}} = \frac{t_{\mathrm{sc,n}}}{t}, \quad \mathcal{S}_{\mathrm{r}} = \frac{t_{\mathrm{sc,r}}}{t},$$
$$\mathcal{T}_i = \frac{t_{\mathrm{i}}}{t}, \quad \mathcal{T}_c = \frac{t_{\mathrm{c}}}{t}, \quad \mathcal{T}_{\mathrm{F}} = \frac{t_{\mathrm{f}}}{t}, \quad \mathcal{T}_{\mathrm{N}} = \frac{t_{\mathrm{n}}}{t}. \quad (2)$$

The fractions of time $\mathcal{T}_{\mathrm{F}}, \mathcal{T}_{\mathrm{N}}$ refer to the time F respectively N is transmitting, F is successfully transmitting during $\mathcal{S}_{\mathrm{f}}$ and N is successfully transmitting its own packets during $\mathcal{S}_{\mathrm{n}}$ and successfully relaying during $\mathcal{S}_{\mathrm{r}}$. Clearly, $\mathcal{S}_{\mathrm{f}} \leq \mathcal{T}_{\mathrm{F}}$, $\mathcal{S}_{\mathrm{n}} \leq \mathcal{T}_{\mathrm{N}}$, and $\mathcal{S}_{\mathrm{r}} \leq \mathcal{T}_{\mathrm{N}}$ due to the collisions. For sake of simplicity, we assume that F and N are either idle or transmit with constant power, e.g., F transmits either with power zero or with power $\mathcal{P}/\mathcal{T}_{\mathrm{F}}$. It can easily be verified that $\mathcal{T}_i + \mathcal{T}_c + \mathcal{S}_{\mathrm{f}} + \mathcal{S}_{\mathrm{n}} + \mathcal{S}_{\mathrm{r}} = 1$.

B. Physical layer considerations

Under the above orthogonality between the channel states, we can now conveniently, and without loss of generality, characterize our channel models using a time-division notation. We assume free-space path loss, i.e., the power of the propagating signal is attenuated with the source-destination distance to the power of γ. The coefficient γ denotes the pathloss exponent [9, Chap. 2] with a typical range of $1.5 \leq \gamma \leq 4$. We utilize a baseband-equivalent, discrete-time channel model for the continuous-time channel. The distance between F and A is normalized to the unit. Denote $\overline{\beta} = 1 - \beta$ as the distance between nodes N and A. When F is transmitting (under our assumptions, meanwhile N and A are listening),

$$y_{\mathrm{N}}[k] = \overline{\beta}^{-\gamma/2} x_{\mathrm{F}}[k] + z_{\mathrm{N}}[k] \quad (3)$$
$$y_{\mathrm{A}}[k] = x_{\mathrm{F}}[k] + z_{\mathrm{A}}[k], \quad (4)$$

where x_{F} is the signal transmitted by node F. The sequences y_{N} and y_{A} represent the signals received at node N and A,

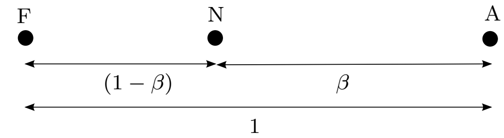


Fig. 1. The 3-node relay channel. Node N serves as a relay for node F as in [1]. However, we assume here that the relay node N additionally has its own data to transmit. For sake of simplicity, F, N and A are assumed to be aligned. The distance between F and A is normalized to the unit. Nodes N and A are separated by distance β.

respectively. The signals z_{N} and z_{A} capture the effects of receiver noise and other forms of interference in the system. We model them as zero-mean mutually independent, circular symmetric, complex Gaussian random sequences with variance 1. When N is transmitting and A is listening, we model the channel as

$$y_{\mathrm{A}}[k] = \beta^{-\gamma/2} x_{\mathrm{N}}[k] + z_{\mathrm{A}}[k]. \quad (5)$$

During the remaining time, both nodes F and N can simultaneously transmit (collision) or remain idle. In the case of collision, we assume that the access point cannot detect none of the messages and discards the received signal. Therefore, there is no need to model the channel in this case.

Assuming that the transmitted signals x_{F} and x_{N} are subject to the average power constraints

$$\lim_{m\to\infty} \frac{1}{2m+1} \sum_{k=-m}^{m} |x_{\mathrm{F}}[k]|^2 \leq \mathcal{P},$$
$$\lim_{m\to\infty} \frac{1}{2m+1} \sum_{k=-m}^{m} |x_{\mathrm{N}}[k]|^2 \leq \mathcal{P}, \quad (6)$$

we parameterize the channel model by the signal-to-noise-ratios $\mathcal{P}/(1-\beta)^\gamma$ between F and N, $\mathcal{P}/\beta^\gamma$ between N and A and $\mathcal{P}$ between F and A.

III. Cooperative Protocols

In this section, we describe three low-complexity cooperative protocols that can be utilized in the network of Fig. 1. All three protocols are subject to the same power constraint (6).

In our study, we are interested in protocols that optimize resource allocation such that the flow with lowest rate is maximized. We define the achievable minimum rate C as the minimum rate granted over all flows. In the transmission model in Fig. 1, there are two flows, one initiated by node F and one initiated by node N. The maximum achievable minimum rate is determined by the flow with lowest rate:

$$C = \max_{T} \min \{C_{\mathrm{F}}, C_{\mathrm{N}}\}, \quad (7)$$

where the maximum is taken over all possible time division configurations of the network parameterized by the set

$$T = (\mathcal{T}_{\mathrm{F}}, \mathcal{T}_{\mathrm{N}}, \mathcal{S}_{\mathrm{f}}, \mathcal{S}_{\mathrm{n}}, \mathcal{S}_{\mathrm{r}}). \quad (8)$$

A. Benchmark for cooperative schemes

In order to evaluate the benefit of cooperation among the nodes F and N, we first determine the maximum achievable minimum rate for non cooperative schemes. We consider two basic non cooperative schemes: the Direct-Link and the Two-Hop schemes.

1) Direct-Link: The Direct-Link scheme has been successfully adopted by the standard IEEE 802.11, in which each node communicates directly with the access point. The max-min capacity (7) for the Direct-Link transmission scheme is readily given by the capacity formula for the additive white Gaussian noise (AWGN) channel [10] with the corresponding SNR values for F and N as stated in Section II:

$$\mathsf{C}_{\mathrm{dir}} = \max_{T} \min \left\{ \mathcal{S}_{\mathrm{n}} \log\left(1 + \frac{\mathcal{P}}{\beta^{\gamma} \mathcal{T}_{\mathrm{N}}}\right), \mathcal{S}_{\mathrm{f}} \log\left(1 + \frac{\mathcal{P}}{\mathcal{T}_{\mathrm{F}}}\right) \right\}, \tag{9}$$

where the first and second terms correspond to the achievable rate for node N and node F, respectively. Since both F and N are transmitting their data directly to A, no relaying is needed and we have $\mathcal{S}_{\mathrm{r}} = 0$.

When node F is very far from the access point A, the rate of the link between nodes F and A becomes the bottleneck of the achievable minimum rate. In this case, it might be preferable to consider the Two-Hop solution, which consists of first transmitting the message from F to N and second forwarding it from N to A.

2) Two-Hop: By applying the capacity formula for AWGN channels with the corresponding SNR values, the achievable rate for the Two-Hop scheme can be expressed as:

$$\mathsf{C}_{\mathrm{2h}} = \max_{T} \min \left\{ \mathcal{S}_{\mathrm{n}} \log\left(1 + \frac{\mathcal{P}}{\beta^{\gamma} \mathcal{T}_{\mathrm{N}}}\right), \mathcal{S}_{\mathrm{f}} \log\left(1 + \frac{\mathcal{P}}{\overline{\beta}^{\gamma} \mathcal{T}_{\mathrm{F}}}\right), \mathcal{S}_{\mathrm{r}} \log\left(1 + \frac{\mathcal{P}}{\beta^{\gamma} \mathcal{T}_{\mathrm{N}}}\right) \right\} \tag{10}$$

where the first and second terms correspond to the achievable rate for the transmission of the own data of nodes N and F to their respective one-hop neighbors A and N. The last term represents the achievable rate for the flow of node F forwarded by N.

Remark 1 (MAC Considerations for the Two-Hop scheme): The main challenge of designing a MAC protocol for the Two-Hop scheme resides in the *coordination* strategy for F and N. In order to complete the transmission initiated by F, N needs to forward the received packet to A. We propose here a very simple policy as follows. Nodes F and N initially compete for the channel. If N gains the channel access, it transmits its packet. Once the transmission has been acknowledged by the access point, both nodes F and N compete again for the channel. If F gains channel access, it transmits its packet to N. Under our policy, the node N is *obliged* to tentatively decode the packet and, if it succeeds, to put it first in its packet queue. Next time N gains the channel access, it forwards the packet to A. In order to keep F from flooding N with packets, N keeps only one packet from F at a time (in first position of its queue). Consequently, if F gains channel access and transmits its packet whereas N has still a packet to forward, N will ignore the transmission of F. This principle is illustrated in Fig. 2(b).

TABLE I
SUMMARY OF THE PHASES FOR DIRECT-LINK, TWO-HOP AND DECODE-AND-FORWARD PROTOCOLS FROM A PHYSICAL LAYER PERSPECTIVE.

	Direct-Link	Two-Hop	Decode-and-Forward
Phase 1	$\mathrm{N} \to \mathrm{A}$		
Phase 2	$\mathrm{F} \to \mathrm{N}$	$\mathrm{F} \to \mathrm{N}$	$\mathrm{F} \to \mathrm{N}, \mathrm{A}$
Phase 3		$\mathrm{N} \to \mathrm{A}$	$\mathrm{N} \to \mathrm{A}$

Assuming that the optimal decision of selecting Direct-Link or Two-Hop scheme is taken by a routing protocol (AODV for instance), the maximum achievable minimum rate for the non-cooperative case can be expressed as

$$C_{\text{no coop}} = \max\left\{\mathsf{C}_{\mathrm{dir}}, \mathsf{C}_{\mathrm{2h}}\right\}. \tag{11}$$

In the sequel, the performance gain of cooperative protocols will be evaluated against (11). The main idea behind the three cooperative protocols is to consider the Two-Hop scheme without discarding the signal that has been sent by F at the access point (4).

B. Naive Decode-and-Forward protocol

We first consider the basic Decode-and-Forward scheme in which both nodes F and N have to send their own data to the access point A. For sake of clarity, we first expose the strategy from the physical layer point of view. We can distinguish the three phases in Table I. In Phase 1, N directly sends its message to node A. In this phase, F cannot help. In Phase 2, F sends its message to the intermediate node N such that N can decode the message. Node A receives the message but cannot decode it due to the larger distance between F and N. However, contrary to the Two-Hop scheme, A stores the received signal for the next phase. In Phase 3, N transmits only the missing information to A such that together with the message previously received in Phase 2, A can completely decode the message from F. During Phase 3, we assume that F remains idle for two reasons: first, the throughput gain by allowing F to transmit together with N is rather little especially if its distance to A is large; second, simultaneous transmissions of F and N require time synchronization at the sample level, which is costly in practice. Comparison of the different phases of the Decode-and-Forward protocol with Direct-Link and Two-Hop schemes from a physical layer perspective is summarized in Table I. We can define the achievable rate for this protocol as:

$$\mathsf{C}_{\mathrm{df}} = \max_{T} \min \left\{ \mathcal{S}_{\mathrm{n}} \log\left(1 + \frac{\mathcal{P}}{\beta^{\gamma} \mathcal{T}_{\mathrm{N}}}\right), \mathcal{S}_{\mathrm{f}} \log\left(1 + \frac{\mathcal{P}}{\overline{\beta}^{\gamma} \mathcal{T}_{\mathrm{F}}}\right), \mathcal{S}_{\mathrm{f}} \log\left(1 + \frac{\mathcal{P}}{\mathcal{T}_{\mathrm{F}}}\right) + \mathcal{S}_{\mathrm{r}} \log\left(1 + \frac{\mathcal{P}}{\beta^{\gamma} \mathcal{T}_{\mathrm{N}}}\right) \right\}. \tag{12}$$

The first term in (12) corresponds to N transmitting its own packet to A during $\mathcal{S}_{\mathrm{n}}$. The second and third terms correspond to the packet transmission of F using Decode-and-Forward protocol during $\mathcal{S}_{\rm f}$ (Phase 2 in Table I) and $\mathcal{S}_{\rm r}$ (Phase 3). There is a simple interpretation of this two-phase transmission. In $\mathcal{S}_{\rm f}$, the packet is completely transmitted to N. This is guaranteed by the second term in (12). Then, the transmission F–A during $\mathcal{S}_{\rm f}$ and the transmission N–A during $\mathcal{S}_{\rm r}$ can be interpreted as the transmission of data over two parallel AWGN channels [10]. The sum in the third term of (12) then follows immediately as the maximum mutual information between (x_F, x_N) and y_A from Eqs. (3)-(5). Note that the last two terms can be seen as a special case of [7, Prop. 2], but differ from the SIMO interpretation of the corresponding protocols II in [11] and Decode-and-Forward as defined in [3].

From a MAC perspective, we adopt the same coordination strategy as in the Two-Hop case, which is described in Remark 1. In each term in (12), the transmission time $\mathcal{T}$ can be strictly larger than $\mathcal{S}$ because of protocol overhead such as acknowledgments (ACK) and packet headers or because of collision when node F and node N are transmitting at the same time, which can lead to interference between the transmissions that cannot be resolved by the receiving node. Acknowledgement signals can resolve collisions such that in each phase, the receiving node transmits an ACK if it can successfully decode the message. After some timeout, if the source node did not receive ACK, the packet is considered lost and the source node retransmits the packet. In our analysis of Section IV, collisions of ACK transmissions are neglected. The reason for this assumption is that the duration of ACK messages is very short compared to the transmission duration of payload packets.

C. Decode-Idle-Forward

As we shall see in Section IV, the naive (basic) Decode-and-Forward protocol suffers significantly from the contention between nodes F and N. A simple but efficient strategy consists of using at node F the ACK signal sent by A to N right after Phase 3. Once F receives ACK from N after Phase 2, F stays idle until receiving ACK from the access point A. Note that the protocol has to ensure that A sends ACK packet to N at a rate sufficiently low such that F can decode it. Once F gets ACK from A, F starts to compete again for the channel access.

D. Decode-Straightforward

The Protocol Decode-Idle-Forward can be further improved by noting that when F is idle, N does not need to compete for the channel access but can directly forward the message (Phase 3). Clearly, this strategy is only valid in the network model of Fig. 1. For larger networks, N has still to compete for the channel access with all other nodes except F.

IV. Throughput Analysis

The purpose of this section is to calculate the max-min throughput (9), (10), and (12) for the different MAC protocols that we proposed in Section III. Maximization over the time division parameters (2) cannot be performed directly because of the interdependency between transmission times $\mathcal{T}$ (which include collisions) and the successful transmission times $\mathcal{S}$ (which exclude collisions). We resolve this interdependency along the lines of [6]: First, we describe the network communication system in terms of the independent parameters *packetsize* and *transmission probability*. We then express the time division variables (2) as functions of these parameters and maximize (9), (10), and (12) over these parameters. In the low and high SNR regimes, this maximization can be performed analytically by using asymptotic approximations for (9), (10), and (12); in the medium SNR range solutions can be found numerically. We start by defining the aforementioned parameters.

1) Packetsizes $\mathtt{t}_{\rm f}, \mathtt{t}_{\rm n}, \mathtt{t}_{\rm r}$: The transmitters can adjust the size of transmitted packets. For F and N transmitting their own packets and N relaying, we denote the corresponding packetsizes by $\mathtt{t}_{\rm f}$, $\mathtt{t}_{\rm n}$, and $\mathtt{t}_{\rm r}$, respectively. We arbitrary normalize the packetsizes such that $\mathtt{t}_{\rm f} + \mathtt{t}_{\rm n} + \mathtt{t}_{\rm r} = 1$ for sake of simplicity. As previously mentioned, DCF adopts an exponential backoff scheme with a discrete time backoff scale. Since we normalize the packet sizes, the corresponding timeslot duration has to be normalized accordingly. We denote the normalized timeslot duration by σ. A typical value would be $\sigma = 50\mu{\rm s}/(3 \cdot 8184\mu{\rm s}) \approx 0.002$ [6], where the value $8184\mu{\rm s}$ reflects the average packetsize for the three types of transmission.

2) Probability of transmission τ: Following [6], the key modelling step is to assume that the network is in steady state and that in any arbitrary phase, each node is transmitting with a probability of τ. For DCF, τ was calculated in [6] in terms of minimum contention window size, number of backoff stages, and number of nodes competing for the channel. For simplicity, we directly use τ as a protocol parameter over which throughput is maximized. When both F and N are competing for the channel, the probabilities of success, collision, and idle state can be calculated as

$$p_s = \tau(1-\tau), \quad p_{\rm c} = \tau^2, \quad p_{\rm i} = (1-\tau)^2. \tag{13}$$

A collision occurs when both F and N are transmitting at the same time. Since both cannot send and receive simultaneously, they have to finish their transmission before being able to detect collision. Therefore, the duration of collision $\mathtt{t}_{\rm c}$ is given by $\max\{\mathtt{t}_{\rm f}, \mathtt{t}_{\rm n}\}$.

A. Calculation of Throughput

In the following, we express the time division variables (2) as a function of packetsize and transmission probability for Direct-Link and the three cooperative protocols proposed in Section III. The three cooperative protocols can readily be used for Two-Hop, with the only difference that for Two-Hop, A will discard what it receives from F. Therefore, the derived formulas for the time division variables (2) can directly be used for the corresponding Two-Hop schemes.

1) Random Access Direct-Link: Both F and N are constantly competing for channel access. As illustrated in Fig. 2(a), there are four different transition phases: successful

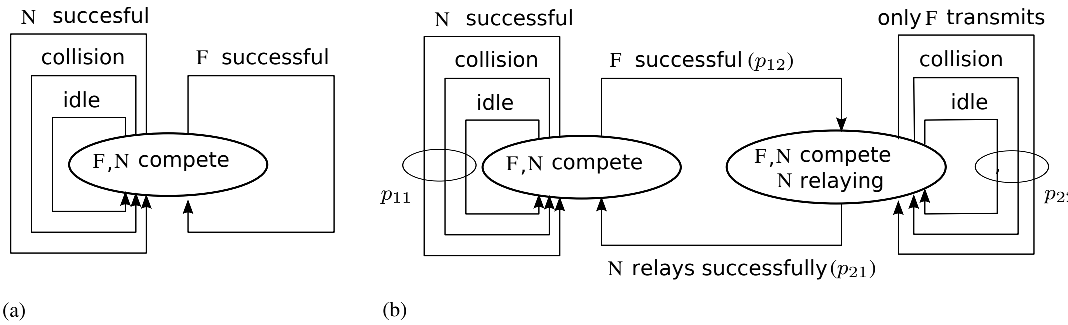


Fig. 2. Channel state diagram for Direct-Link (a) and Naive Decode-and-Forward (b). When the transition phases "only F transmits" and "collision" are removed from state 2 in (b), the diagram illustrates Decode-Idle-Forward. If in addition the transition phase "idle" is removed from state 2, the resulting diagram illustrates Decode-Straightforward. Note that the transition probabilities have to be adapted appropriately.

transmission of N, successful transmission of F, collision, and idle mode. For Direct-Link, N never relays, so $\mathtt{t}_\mathrm{r} = \mathcal{S}_\mathrm{r} = 0$ and $\mathtt{t}_\mathrm{f} + \mathtt{t}_\mathrm{n} = 1$. By using the probabilities from (13), the expected duration t of a transition phase is given by

$$t = p_s\mathtt{t}_\mathrm{n} + p_s\mathtt{t}_\mathrm{f} + p_\mathrm{c}\mathtt{t}_\mathrm{c} + p_\mathrm{i}\sigma \tag{14}$$

$$= \tau(1-\tau) + \tau^2\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\} + (1-\tau)^2\sigma. \tag{15}$$

The average time F successfully transmits in a transition phase is

$$t_{\mathrm{sc,f}} = \tau(1-\tau)\mathtt{t}_\mathrm{f}. \tag{16}$$

Using (15) and (16) in (2), the fraction of time $\mathcal{S}_\mathrm{f}$ when F is successfully transmitting can be expressed as

$$\mathcal{S}_\mathrm{f} = \frac{t_{\mathrm{sc,f}}}{t} = \frac{\tau(1-\tau)\mathtt{t}_\mathrm{f}}{\tau(1-\tau) + \tau^2\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\} + (1-\tau)^2\sigma} \tag{17}$$

which is completely defined by the new set of parameters packetsize and transition probability as introduced at the beginning of this section. Similarly

$$\mathcal{T}_\mathrm{F} = \frac{\tau\mathtt{t}_\mathrm{f}}{t}, \quad \mathcal{S}_\mathrm{n} = \frac{\tau(1-\tau)\mathtt{t}_\mathrm{n}}{t}, \quad \mathcal{T}_\mathrm{N} = \frac{\tau\mathtt{t}_\mathrm{n}}{t}. \tag{18}$$

We can use (17) and (18) to express the time division variables in (9). The maximization problem over $\{\mathcal{T}_\mathrm{F}, \mathcal{T}_\mathrm{N}, \mathcal{S}_\mathrm{f}, \mathcal{S}_\mathrm{N}\}$ has been turned into a maximization problem over $\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}, \tau\}$ subject to the constraints $\mathtt{t}_\mathrm{f} + \mathtt{t}_\mathrm{n} = 1$ and $0 \leq \tau \leq 1$. It can now easily be solved numerically.

Since the calculations are quite similar, we will only calculate $\mathcal{S}_\mathrm{F}$ for the remaining protocols. The other corresponding time division variables can be expressed by packetsize and transmission probability in an analogous way.

2) Naive Decode-and-Forward Approach: The network can be in the two states "F, N compete" and "F, N compete, N relaying", with which we associate the state probabilities π_1 and π_2, respectively. See Fig. 2(b) for an illustration. The transition probabilities between the two states are

$$p_{12} = p_{21} = p_s, \quad p_{11} = p_{22} = 1 - p_s \tag{19}$$

which implies $\pi_1 = \pi_2 = 1/2$. The expected transition phase duration is $t = t_\mathrm{sc} + t_\mathrm{c} + t_\mathrm{i}$ with

$$t_\mathrm{sc} = \pi_1(p_s\mathtt{t}_\mathrm{n} + p_s\mathtt{t}_\mathrm{f}) + \pi_2 p_s\mathtt{t}_\mathrm{r} \tag{20}$$

$$t_\mathrm{c} = \pi_1 p_\mathrm{c}\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\} + \pi_2(p_\mathrm{c}\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{r}\} + p_s\mathtt{t}_\mathrm{f}) \tag{21}$$

$$t_\mathrm{i} = \pi_1 p_\mathrm{i}\sigma + \pi_2 p_\mathrm{i}\sigma. \tag{22}$$

By using $t_{\mathrm{sc,f}} = \pi_1\tau(1-\tau)\mathtt{t}_\mathrm{f}$, $\mathcal{S}_\mathrm{f}$ in (2) becomes

$$\mathcal{S}_\mathrm{f} = \frac{1}{2}\tau(1-\tau)\mathtt{t}_\mathrm{f}\left[\tau(1-\tau) + \frac{1}{2}\tau^2\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\} + \frac{1}{2}\tau^2\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{r}\} + \frac{1}{2}\tau(1-\tau)\mathtt{t}_\mathrm{f} + (1-\tau)^2\sigma\right]^{-1}. \tag{23}$$

If we assume $\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{r}\} \approx \max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\}$, the main differences between $\mathcal{S}_\mathrm{f}$ for Direct-Link (17) and $\mathcal{S}_\mathrm{f}$ for Naive Decode-and-Forward consists in the factor of $1/2$ in the numerator and the term $\tau(1-\tau)\mathtt{t}_\mathrm{f}$ in the denominator, which both result from the "queue collision" in state 2. It occurs when F successfully gains channel access, but N is ignoring the transmission since it is still trying to forward the previous packet of F.

3) Decode-Idle-Forward: For Decode-Idle-Forward, the node F remains idle in state 2 in Fig 2(b) and the state transition probabilities for state 2 are given by $p_{21} = \tau$ and $p_{22} = 1 - \tau$. Consequently $(\pi_1, \pi_2) \propto (1, 1-\tau)$ and the parameters for the expected phase duration $t = t_\mathrm{sc} + t_\mathrm{c} + t_\mathrm{i}$ are given by

$$t_\mathrm{sc} = \pi_1(p_s\mathtt{t}_\mathrm{n} + p_s\mathtt{t}_\mathrm{f}) + \pi_2\tau\mathtt{t}_\mathrm{r} \tag{24}$$

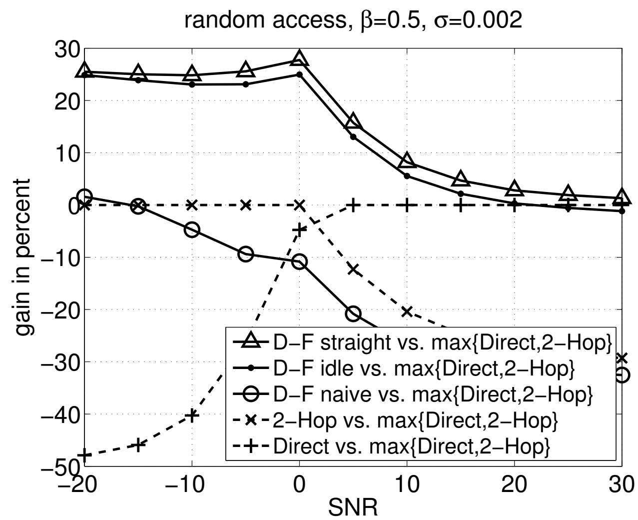
$$t_\mathrm{c} = \pi_1 p_\mathrm{c}\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\} \tag{25}$$

$$t_\mathrm{i} = \pi_1 p_\mathrm{i}\sigma + \pi_2(1-\tau)\sigma. \tag{26}$$

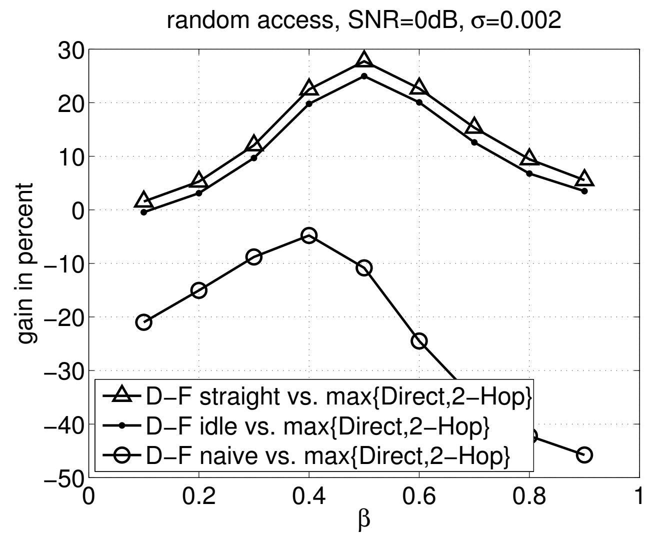
Because $t_{\mathrm{sc,f}} = \pi_1\tau(1-\tau)\mathtt{t}_\mathrm{f}$,

$$\mathcal{S}_\mathrm{f} = \frac{\tau(1-\tau)\mathtt{t}_\mathrm{f}}{\tau(1-\tau) + \tau^2\max\{\mathtt{t}_\mathrm{f}, \mathtt{t}_\mathrm{n}\} + 2(1-\tau)^2\sigma}. \tag{27}$$

Compared to Direct-Link, there is an additional factor of two for the idle timeslot σ in the denominator.



(a) Throughput improvement for the three Decode-Forward protocols. Throughput $\mathtt{C}_{\mathtt{df}}$ (12) developed in Section III is compared to the conventional approach which consists of selecting the scheme Direct-Link or Two-Hop with highest throughput (11). The values are numerically calculated for $\beta = 0.5$, $\sigma = 0.002$, and $\gamma = 2$ following the procedure of Section IV.



(b) Throughput improvement for the three Decode-and-Forward protocols. Throughput $\mathtt{C}_{\mathtt{df}}$ (12) developed in Section III is compared to the conventional approach which consists of selecting the scheme Direct-Link or Two-Hop with highest throughput (11). The values are numerically calculated for SNR= 0.5 decibels, $\sigma = 0.002$, and $\gamma = 2$ following the procedure of Section IV.

4) Decode-Straightforward: For Decode-Straightforward, when F successfully transmits its packet to N, N knows that F will remain idle until N successfully forwards the packet to A. Therefore, it forwards the packet directly with probability one to A. If we identify the effective packet duration for F by $\mathtt{t}_{\mathrm{f}} + \mathtt{t}_{\mathrm{r}}$, the Decode-Straightforward protocol is equivalent to the Direct-Link protocol from the MAC layer perspective. Consequently, $\mathcal{S}_{\mathrm{f}}$ is given by (17) and the remaining time division variables in (2) can easily be determined.

V. Discussion

In this section, we illustrate the performance of the three cooperative protocols Naive Decode-and-Forward, Decode-Idle-Forward, and Decode-Straightforward developed in Section III as a function of the distance β between the relaying node N and the access point A, and the average signal-to-noise ratio of the link F-A defined in Section II-B. The performance of these protocols is compared to the conventional Direct-Link and Two-Hop schemes

Fig. 3a shows the throughput improvement for the three cooperative protocols compared to a conventional approach (11), which consists of selecting the scheme Direct-Link or Two-Hop with highest throughput as in (11). We favour Two-Hop in our comparison by always using it with the MAC protocol of Decode-Straightforward, which leads to least collisions. Node N is assumed to be exactly in the middle of F and A ($\beta = 0.5$). We use the value of 0.002 for the normalized timeslot σ as in Section IV. The throughput improvement is shown for SNR ranging from -20 decibels to 30 decibels. Concerning the conventional approaches, Direct-Link scheme outperforms the Two-Hop scheme in high SNR regime (SNR$>$ 5 decibels) whereas the Two-Hop scheme outperforms the Direct-Link scheme in low SNR regime. Therefore, selecting the conventional scheme with highest throughput is essential to be robust when operating over a very large SNR range. Note that for the Two-Hop scheme, we assume here that F remains idle as long as N has to forward a packet from F. It is interesting that the Naive Decode-and-Forward protocol performs slightly worse (approximatively 10%) than the Two-Hop scheme at any SNR (except for very low values). The degradation comes from the "queue collision" in state 2 in Fig. 2(b) and cannot be compensated by exploiting the information received by A when F is transmitting. Queue collision occurs at the MAC layer when F successfully gains channel access, but N ignores the transmission since it is still trying to forward the previous packet of F. For the cooperative protocols, the strategy that consists in maintaining F idle as long as N has to forward the missing information, provides significant throughput gain at moderate and low SNR values (more than 20%). In high SNR regime, the throughput gain versus the Direct-Link scheme becomes less substantial. In this case, node A receives most of the information directly from F reducing the importance of the relay node.

Fig. 3b shows the throughput improvement for the three cooperative protocols compared to a conventional approach (11) as a function of the position of the intermediate node N in low SNR regime (SNR $=$ 0 decibel). For the cooperative protocols that avoid the "queue collision" at Node N (Decode-Idle-Forward and Decode-Straightforward), the throughput gain over the conventional approach is maximal when N is located in the middle between F and A. Interestingly, this throughput gain is equal to or greater than 20% for β ranging from 0.4 to 0.6. This is important in larger networks where the selection of a relay is not trivial. For these cooperative protocols, large throughput gains are observed even when the selected relay is not precisely in the middle between F and A. As in the previous setup, the degradation for the protocol Naive Decode-and-Forward comes from the “queue collisions” and cannot be compensate by exploiting the information received by A when F is transmitting.

VI. Conclusions

We proposed three cooperative protocols and compared them to the conventional schemes Direct-Link and Two-Hop with respect to max-min throughput. The key property of the proposed protocols is low complexity achieved by random access. The first proposed protocol suffers from collision and is outperformed by the conventional schemes. The second and third protocol solve this problem in a distributed manner and outperform the conventional schemes in the low SNR regime around 0 decibel for a wide range of network topologies. A natural application would be to increase the coverage of an access point while maintaining the current max-min rate by using a cooperative protocol.

We immediately note that our work has only scratched the surface in exploring the issues in implementing cooperative systems. Natural next steps are to investigate how the proposed protocols scale with an increasing number of nodes in the network and what impact the relay selection problem has on the achievable throughput.

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