Proportional QoS Adjustment for Achieving Feasible Power Allocation in CDMA Systems

Rudolf Mathar and Anke Schmeink

Abstract— **Resource management is the general topic of the present paper, particularly, we deal with capacity sharing for interference limited wireless networks by power control. Proportional reduction of the signal-to-interference ratio (SIR) requirements is suggested as the control mechanism to accommodate users in the case of overload. For this purpose, we carefully describe the geometrical structure and the asymptotic behavior of the set of feasible power vectors as a proportionality factor tends to its boundaries. In the case that there is no feasible power adjustment, the minimum proportional SIR reduction is determined under general power constraints. We conclude with developing a locally quadratic convergent algorithm for numerical computation of the optimum power assignment. The investigations provide both insight into the theoretical structure of optimum power allocation as well as a practical method for call admission control.**

Index Terms— **Cellular networks, code division multiple access, resource management, optimal power control, power region, call admission control.**

I. INTRODUCTION

POWER control is one of the major ingredients for code division multiple access (CDMA) mobile networks to achieve the potential capacity. The quality-of-service (QoS) performance of users depends on the power assignment in the whole network and usually becomes better with increasing sum power (see [1]). However, in order to save sparse energy for handhold devices, and to keep interference to other stations low, it is desirable that stations transmit with the minimum power such that a required QoS level is just guarantied.

The existence of some feasible power allocation for a community of transmitters and related problems have been extensively investigated over the last years. The sheer existence of a solution, assuming unlimited power is clarified by Perron-Frobenius theory, as we briefly outline in Section II, and has been used, e.g., in [2]. If the power budget is limited, additional constraints arise.

Three important questions are directly connected to power control. First, for practical applications individual power settings must be computed, favorably in a decentral manner using only local information. In [2] a convergent algorithm is presented which solves this task and simultaneously allocates mobiles to base stations. In an elegant setup, the author [3] develops a general framework for proving convergence of a whole class of power assignment algorithms. For improving efficiency different approaches to dimensionality reduction are used in [4], [5], [6]. Probabilistic aspects of the channel are included in [4], [7], [8].

The second class of problems is concerned with the set of QoS requirements which can be supported by a feasible power assignment. This leads to the concept of user capacity which is investigated by [9], [10], [11] by considering the optimum linear receiver jointly with signature sequences. Convexity, monotonicity and asymptotic properties of the capacity region are themes of the works [12], [13], [14], [15].

Access control by power adjustment is the third type of problem to be solved when operating CDMA mobile radio. In [16], active links are protected when new users are admitted to the network. A fast algorithm to decide if new users can be accommodated while maintaining the required QoS is given in [17]. A novel game theoretic approach to admission control is used in [18]. How this approach relates to the point of least power adjustment via monotonic functionals is shown in [19].

In this work, we approach the problem of admitting new users by proportionally reducing the QoS parameters of all users whenever there is need for. The idea behind this concept is that each user sacrifices a proportional part of his transmission capacity to admit further subscribers to the network. To apply this strategy a graceful degradation of service quality in terms of higher bit error or lower transmission rates must be acceptable to the involved users.

After introducing the system model and some basic preliminaries in Section II we deal with the geometry of the power region. It turns out that the shifted power region is a closed convex cone containing a componentwise minimum power assignment. This element increases monotonically as the proportionality factor does. In Section III, we investigate the orbit of the optimum power assignment by determining derivatives, and also the direction of divergence as the proportionality factor approaches the boundary of the interval where a feasible power allocation exists.

For practical applications power restrictions must be taken into account. In Section IV, we consider the case that power constraints can be described by a certain functional. We present a convergent algorithm for determining the largest proportional QoS vector which allows for a feasible power adjustment. The most common cases such as total and componentwise power constraints are contained as special cases. We conclude with a short summary and possible future extensions in Section V.

Paper approved by S. Ulukus, the Editor for Wireless Communication Theory of the IEEE Communications Society. Manuscript received April 13, 2005; revised May 22, 2006.

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Digital Object Identifier 10.1109/TCOMM.2008.041173.

II. System Model and Preliminaries

In a synchronous multiuser CDMA communication system with K users and processing gain N let $\boldsymbol{s}_i \in \mathbb{R}^N$, $i = 1, \dots, K$, denote the N-dimensional signature sequence of user i. Let G_{ij} denote the fixed path gain from user j to the assigned base station of user i. Usually G_{ij} is subject to slow fading effects which are assumed to be known to the transmitter. Suppose the symbol of user i is decoded using a linear receiver represented by some vector $\boldsymbol{c}_i \in \mathbb{R}^N$. The signal-to-interference ratio of user i is then given as

$$\mathrm{SIR}_i(\boldsymbol{p}) = \frac{G_{ii}(\boldsymbol{c}_i^\mathsf{T}\boldsymbol{s}_i)^2 p_i}{\sum_{j\neq i} G_{ij}(\boldsymbol{c}_i^\mathsf{T}\boldsymbol{s}_j)^2 p_j + \sigma^2(\boldsymbol{c}_i^\mathsf{T}\boldsymbol{c}_i)^2},$$

where σ^2 denotes the variance of the additive Gaussian noise and $\boldsymbol{p} = (p_1, \dots, p_K)^\mathsf{T}$ the vector of transmit powers. In the following we assume that the receiver sequences $\boldsymbol{c}_i$ are fixed. Combining the known channel and receiver effects into $A_{ij} = G_{ij}(\boldsymbol{c}_i^\mathsf{T}\boldsymbol{s}_j)^2$ we obtain $\mathrm{SIR}_i(\boldsymbol{p})$ of user i as

$$\mathrm{SIR}_i(\boldsymbol{p}) = \frac{A_{ii}p_i}{\sum_{j\neq i} A_{ij}p_j + C_{ii}\sigma^2},$$

with $C_{ii} = (\boldsymbol{c}_i^\mathsf{T}\boldsymbol{c}_i)^2$. Now given QoS requirements $\gamma_1, \dots, \gamma_K$ for each user, we define the *power region* $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ as the set of power settings $\boldsymbol{p} \in \mathbb{R}^K$ such that each user i meets his SIR requirement γ_i, i.e.,

$$\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma}) = \{\boldsymbol{p} > \boldsymbol{0} \mid \mathrm{SIR}_i(\boldsymbol{p}) \geq \gamma_i,\ i = 1, \dots, K\}. \quad (1)$$

Here and in the following orderings '$<$' and '$\leq$' between vectors are always meant componentwise. Obviously it may happen that not all requirements γ_i can be simultaneously satisfied, in which case $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ is empty.

For convenience of notation we quote the following result from [15]. It deals with solutions of the equation

$$[\boldsymbol{I} - \boldsymbol{A}]\boldsymbol{x} = \boldsymbol{c} \quad (2)$$

when $\boldsymbol{A}$ is a nonnegative but not necessarily irreducible matrix. The proof given in [15] is direct and self-contained, and does not rely on the Perron-Frobenius theory. In the irreducible case the result is well known from [20]. Let $\rho(\boldsymbol{A})$ denote the spectral radius of a square matrix $\boldsymbol{A}$, defined as $\rho(\boldsymbol{A}) = \max\{|\lambda_i(\boldsymbol{A})|\}$, where $\lambda_i(\boldsymbol{A})$ denotes the complex eigenvalues of $\boldsymbol{A}$.

Proposition 1: Let $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ be non-negative.

a) If there are $\boldsymbol{x} > \boldsymbol{0}$, $\boldsymbol{c} > \boldsymbol{0}$ satisfying (2), then $\rho(\boldsymbol{A}) < 1$.
b) If $\rho(\boldsymbol{A}) < 1$, then $\boldsymbol{I} - \boldsymbol{A}$ is non-singular and for every $\boldsymbol{c} > \boldsymbol{0}$, the unique solution $\boldsymbol{x} \in \mathbb{R}^n$ of (2) is positive.
c) If $\rho(\boldsymbol{A}) < 1$, then for every $\boldsymbol{c} \geq \boldsymbol{0}$, the unique solution $\boldsymbol{x} \in \mathbb{R}^n$ of (2) is non-negative.
d) If $\boldsymbol{c} > \boldsymbol{0}$ and there exists $\boldsymbol{y} > \boldsymbol{0}$ such that $[\boldsymbol{I} - \boldsymbol{A}]\boldsymbol{y} \geq \boldsymbol{c}$, then (2) has a unique solution $\boldsymbol{x}$ and $\boldsymbol{0} < \boldsymbol{x} \leq \boldsymbol{y}$.

The above is now applied to $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$. The inequalities defining (1) can be rewritten as a system of linear inequalities. For this purpose write $\boldsymbol{B} = (b_{ij})_{i,j=1}^K$, with

$$b_{ij} = \begin{cases} A_{ij}/A_{ii}, & i \neq j, \\ 0, & i = j, \end{cases}$$

and $\boldsymbol{\tau} = (\tau_1, \dots, \tau_K)^\mathsf{T}$, where $\tau_i = C_{ii}\sigma^2/A_{ii}$. Then for every $\boldsymbol{p} > \boldsymbol{0}$ it holds that $\boldsymbol{p} \in \mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ if and only if

$$[\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]\,\boldsymbol{p} \geq \boldsymbol{\Gamma}\boldsymbol{\tau}, \quad (3)$$

where $\boldsymbol{\Gamma} = \mathrm{diag}(\boldsymbol{\gamma})$ denotes the matrix with diagonal entries γ_i and nondiagonal entries equal to zero.

If $\boldsymbol{\Gamma}\boldsymbol{\tau} > \boldsymbol{0}$ and system (3) has a solution $\boldsymbol{p} > \boldsymbol{0}$, then there is a unique solution $\boldsymbol{p}^* \leq \boldsymbol{p}$ satisfying

$$[\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]\,\boldsymbol{p}^* = \boldsymbol{\Gamma}\boldsymbol{\tau},$$

as follows from Proposition 1. Moreover, for any given $\boldsymbol{\gamma} > \boldsymbol{0}$, the equation $[\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]\,\boldsymbol{p} = \boldsymbol{\Gamma}\boldsymbol{\tau}$ has a positive solution $\boldsymbol{p}$ if and only if the spectral radius $\rho(\boldsymbol{\Gamma}\boldsymbol{B}) < 1$, and in that case, the solution is unique. Denote it by $\boldsymbol{p}^*(\boldsymbol{\gamma}) = (p_1^*(\boldsymbol{\gamma}), \dots, p_K^*(\boldsymbol{\gamma}))^\mathsf{T}$. Thus

$$\boldsymbol{p}^*(\boldsymbol{\gamma}) = [\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]^{-1}\,\boldsymbol{\Gamma}\boldsymbol{\tau} \quad (4)$$

with all components positive.

Summarizing the above arguments, we see that there is a unique componentwise minimum power allocation in $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$, provided the power region is nonempty, see also Theorem 2 in [2].

Proposition 2: If $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma}) \neq \emptyset$, then there is a unique power allocation $\boldsymbol{p}^* = \boldsymbol{p}^*(\boldsymbol{\gamma})$ such that $\mathrm{SIR}_i(\boldsymbol{p}^*) = \gamma_i$ for all $i = 1, \dots, K$ and $\boldsymbol{p}^* \leq \boldsymbol{p}$ for all $\boldsymbol{p} \in \mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$.

The shifted power region $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma}) - \boldsymbol{p}^*(\boldsymbol{\gamma})$ has a nice geometrical structure which is important for finding, e.g., the projection of inadmissible points onto $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$. For completeness we recall the following definition. A set $\mathcal{C}$ in a linear vector space is said to be a cone if $\boldsymbol{c} \in \mathcal{C}$ implies that $\alpha\boldsymbol{c} \in \mathcal{C}$ for all $\alpha \geq 0$ (see, e.g., [21]).

Proposition 3: $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma}) - \boldsymbol{p}^*(\boldsymbol{\gamma})$ is a closed convex cone.

Consider the sets (cp. [19])

$$\mathcal{P}_i = \{\boldsymbol{p} \mid A_{ii}p_i - \gamma_i \sum_{j\neq i} A_{ij}p_j \geq \gamma_i C_{ii}\sigma^2\},\ i = 1, \dots, K,$$

which are closed convex affine halfspaces in $\mathbb{R}^K$. Obviously, $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma}) = \bigcap_{i=1}^K \mathcal{P}_i$, and from Theorem C in Section III of [22] it follows that $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ is a closed and convex polytope.

To prove that $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma}) - \boldsymbol{p}^*(\boldsymbol{\gamma})$ is a cone, we show that $\mathrm{SIR}_i(\boldsymbol{p}^*(\boldsymbol{\gamma}) + \alpha[\boldsymbol{p} - \boldsymbol{p}^*(\boldsymbol{\gamma})]) \geq \gamma_i$, and hence $\boldsymbol{p}^*(\boldsymbol{\gamma}) + \alpha[\boldsymbol{p} - \boldsymbol{p}^*(\boldsymbol{\gamma})] \in \mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ for any $\boldsymbol{p} \in \mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ and $\alpha \geq 0$. By assumption $\mathrm{SIR}_i(\boldsymbol{p}) \geq \gamma_i$ for all $i = 1, \dots, K$, in detail,

$$A_{ii}p_i - \gamma_i \sum_{j\neq i} A_{ij}p_j \geq \gamma_i C_{ii}\sigma^2,\ i = 1, \dots, K.$$

Denote by $\boldsymbol{a}_{(i)} = (-\gamma_i A_{i1}, \dots, A_{ii}, \dots, -\gamma_i A_{iK})^\mathsf{T}$ and $\zeta_i = \gamma_i C_{ii}\sigma^2$. Then $\boldsymbol{a}_{(i)}^\mathsf{T}\boldsymbol{p} \geq \zeta_i$ and $\boldsymbol{a}_{(i)}^\mathsf{T}\boldsymbol{p}^*(\gamma) = \zeta_i$ for all $i = 1, \dots, K$. It follows that

$$\boldsymbol{a}_{(i)}^\mathsf{T}\boldsymbol{p}^*(\gamma) + \alpha\big[\boldsymbol{a}_{(i)}^\mathsf{T}\boldsymbol{p} - \boldsymbol{a}_{(i)}^\mathsf{T}\boldsymbol{p}^*(\gamma)\big] \geq \boldsymbol{a}_{(i)}^\mathsf{T}\boldsymbol{p}^*(\gamma) = \zeta_i,$$

for all $i = 1, \dots, K$ and all $\alpha \geq 0$, and hence $\boldsymbol{p}^*(\gamma) + \alpha[\boldsymbol{p} - \boldsymbol{p}^*(\gamma)] \in \mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ for all $\alpha \geq 0$.

A related result, however, not including convexity and without shifting $\mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ to the origin is derived in [23].

The uniformly minimal point $\boldsymbol{p}^*(\boldsymbol{\gamma}) \in \mathcal{P}_{\mathrm{SIR}}(\boldsymbol{\gamma})$ is of particular interest since it requires minimal power while maintaining the SIR demands $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_n)^\mathsf{T}$ of all users. In the following we deal with the behavior of $\boldsymbol{p}^*(\boldsymbol{\gamma})$ as a function of $\boldsymbol{\gamma}$.

Proposition 4: The function $\boldsymbol{p}^*(\boldsymbol{\gamma})$ is monotonically increasing, i.e., if $\mathcal{P}_{\text{SIR}}(\boldsymbol{\gamma}^{(2)}) \neq \emptyset$ and $\boldsymbol{\gamma}^{(1)} \leq \boldsymbol{\gamma}^{(2)}$, then $\boldsymbol{p}^*(\boldsymbol{\gamma}^{(1)}) \leq \boldsymbol{p}^*(\boldsymbol{\gamma}^{(2)})$. Furthermore, $\boldsymbol{p}^*(\boldsymbol{\gamma}) \to \mathbf{0}$ as $\boldsymbol{\gamma} \to \mathbf{0}$.

From Proposition 1 a) it follows that $\rho(\boldsymbol{\Gamma}^{(2)}\boldsymbol{B}) < 1$. Hence, expanding representation (4) in a von Neumann series gives

$$\begin{aligned}\boldsymbol{p}^*(\boldsymbol{\gamma}^{(1)}) &= [\boldsymbol{I} - \boldsymbol{\Gamma}^{(1)}\boldsymbol{B}]^{-1}\boldsymbol{\Gamma}^{(1)}\boldsymbol{\tau} \\ &= \sum_{l=0}^{\infty}(\boldsymbol{\Gamma}^{(1)}\boldsymbol{B})^l\boldsymbol{\Gamma}^{(1)}\boldsymbol{\tau} \\ &\leq \sum_{l=0}^{\infty}(\boldsymbol{\Gamma}^{(2)}\boldsymbol{B})^l\boldsymbol{\Gamma}^{(2)}\boldsymbol{\tau} \\ &= [\boldsymbol{I} - \boldsymbol{\Gamma}^{(2)}\boldsymbol{B}]^{-1}\boldsymbol{\Gamma}^{(2)}\boldsymbol{\tau} \\ &= \boldsymbol{p}^*(\boldsymbol{\gamma}^{(2)}),\end{aligned}$$

where $\boldsymbol{\Gamma}^{(i)} = \text{diag}(\boldsymbol{\gamma}^{(i)})$.

It is immediate from (4) that $\boldsymbol{p}^*(\boldsymbol{\gamma}) \to \mathbf{0}$ as $\boldsymbol{\gamma} \to \mathbf{0}$. Observe that $[\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]^{-1}$ exists in a sufficiently small neighborhood of $\mathbf{0}$.

Further conditions for strict monotonicity of $\boldsymbol{p}^*(\boldsymbol{\gamma})$ are given in [15]. It should be mentioned that for a different, but related model the generic concept of $\boldsymbol{p}^*(\boldsymbol{\gamma})$ as a one-dimensional manifold and its monotonicity are also considered in [24].

III. Proportional Power Adjustment

As we have seen in the previous section, $\mathcal{P}_{\text{SIR}}(\boldsymbol{\gamma}) \neq \emptyset$ whenever $\boldsymbol{\gamma}$ is sufficiently small. If the requirements $\boldsymbol{\gamma}$ of a community of users are such that there is no power allocation, i.e., $\mathcal{P}_{\text{SIR}}(\boldsymbol{\gamma}) = \emptyset$, access control becomes inevitable. A rational concept is to request that each user sets aside a proportional fraction of his requirement until a feasible power allocation can be found. This concept quantifies to a certain extent the notion of graceful degradation of CDMA as discussed in [25] and leads to investigating the behavior of

$$\boldsymbol{p}^*(\alpha\boldsymbol{\gamma}) = [\boldsymbol{I} - \alpha\boldsymbol{\Gamma}\boldsymbol{B}]^{-1}\alpha\boldsymbol{\Gamma}\boldsymbol{\tau},\ \alpha \geq 0.$$

By Proposition 1, the point $\boldsymbol{p}^*(\alpha\boldsymbol{\gamma})$ exists whenever $\alpha < 1/\rho(\boldsymbol{\Gamma}\boldsymbol{B})$. This breakpoint can be described as the solution of the following max-min problem.

Proposition 5: Let $\boldsymbol{\gamma} > 0$ be fixed and $\boldsymbol{B}$ be irreducible. A proportional SIR requirement vector $\alpha\boldsymbol{\gamma}$, $\alpha > 0$, allows for a feasible power assignment if and only if

$$\alpha < \sup_{\boldsymbol{p}>0}\ \min_{i=1,\dots,K} \frac{\text{SIR}_i(\boldsymbol{p})}{\gamma_i}. \tag{5}$$

The only point remaining is to show that the right hand side of (5) coincides with $1/\rho(\boldsymbol{\Gamma}\boldsymbol{B})$. For this purpose we use Corollary 8.1.31 in [26], stating that for any irreducible nonnegative matrix $\boldsymbol{C} = (c_{ij})_{i,j=1,\dots,K}$ the spectral radius is given by

$$\rho(\boldsymbol{C}) = \min_{\boldsymbol{p}>\mathbf{0}}\ \max_{i=1,\dots,K} \frac{1}{p_i}\sum_{j=1}^{K} c_{ij}p_j.$$

Some elementary algebra gives that

$$\frac{\gamma_i}{\text{SIR}_i(\boldsymbol{p})} = \frac{\sum_j c_{ij}p_j + \gamma_i\tau_i}{p_i}$$

with $\boldsymbol{C} = \boldsymbol{\Gamma}\boldsymbol{B}$. Hence,

$$\begin{aligned}\inf_{\boldsymbol{p}>0}\ \max_i \frac{\gamma_i}{\text{SIR}_i(\boldsymbol{p})} &= \min_{\boldsymbol{p}>0}\ \max_i \frac{1}{p_i}\sum_{j=1}^{K} c_{ij}p_j \\ &= \rho(\boldsymbol{C}) = \rho(\boldsymbol{\Gamma}\boldsymbol{B}),\end{aligned}$$

which yields the assertion by considering the reciprocal value.

To describe the set $\{\boldsymbol{p}^*(\alpha\boldsymbol{\gamma}) \mid 0 < \alpha < 1/\rho(\boldsymbol{\Gamma}\boldsymbol{B})\}$ the componentwise derivative with respect to α is important.

Proposition 6: For any $0 < \alpha < 1/\rho(\boldsymbol{\Gamma}\boldsymbol{B})$, the derivative of $\boldsymbol{p}^*(\alpha\boldsymbol{\gamma}) = [\boldsymbol{I} - \alpha\boldsymbol{\Gamma}\boldsymbol{B}]^{-1}\alpha\boldsymbol{\Gamma}\boldsymbol{\tau}$ is given by

$$\frac{d}{d\alpha}\boldsymbol{p}^*(\alpha\boldsymbol{\gamma}) = [\boldsymbol{I} - \alpha\boldsymbol{\Gamma}\boldsymbol{B}]^{-2}\boldsymbol{\Gamma}\boldsymbol{\tau}. \tag{6}$$

First write $\boldsymbol{p}^*(\alpha\boldsymbol{\gamma}) = [\frac{1}{\alpha}\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]^{-1}\boldsymbol{\Gamma}\boldsymbol{\tau}$. The derivative of the inverse is given by (see, e.g., [27])

$$\frac{d}{d\alpha}\big[\frac{1}{\alpha}\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}\big]^{-1} = \frac{1}{\alpha^2}\big[\frac{1}{\alpha}\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}\big]^{-2} = \big[\boldsymbol{I} - \alpha\boldsymbol{\Gamma}\boldsymbol{B}\big]^{-2}.$$

By linearity, (6) follows from multiplication by $\boldsymbol{\Gamma}\boldsymbol{\tau}$.

Since $[\boldsymbol{I} - \alpha\boldsymbol{\Gamma}\boldsymbol{B}]^{-1}$ consists of nonnegative entries, $\frac{d}{d\alpha}\boldsymbol{p}^*(\alpha\boldsymbol{\gamma})$ in (6) is nonnegative for each component, in accordance with the componentwise monotonicity property in Proposition 4.

If power is not the limiting factor, as may be assumed approximately true for the downlink in cellular networks, it is relevant to determine how the power adjustment $\boldsymbol{p}^*(\alpha\boldsymbol{\gamma})$ diverges as the quality profile $\alpha\boldsymbol{\gamma}$ tends to its limit at $\alpha = 1/\rho(\boldsymbol{\Gamma}\boldsymbol{B})$. This is important if power consumption is of no concern and the main objective is to achieve best possible performance.

Since $\boldsymbol{\gamma}$ is fixed in the sequel we may reparametrize $\beta = 1/\alpha$ and write

$$\boldsymbol{p}^*(\beta) = [\beta\boldsymbol{I} - \boldsymbol{\Gamma}\boldsymbol{B}]^{-1}\boldsymbol{\Gamma}\boldsymbol{\tau},$$

provided that $\beta > \rho(\boldsymbol{\Gamma}\boldsymbol{B})$.

If $\boldsymbol{B}$ is irreducible, in [28], p. 315, and [29], (2.3), the following spectral representation of $[\beta\boldsymbol{I} - \boldsymbol{A}]^{-1}$ for the nonnegative matrix $\boldsymbol{A} = \boldsymbol{\Gamma}\boldsymbol{B}$ is shown. Let $\{\lambda_1 = \rho(\boldsymbol{A}), \lambda_2, \dots, \lambda_K\}$ denote the spectrum of $\boldsymbol{A}$ and $m(\lambda) = (\lambda - \rho(\boldsymbol{A}))\prod_{k=2}^{K}(\lambda - \lambda_k)^{m_k}$ the minimal polynomial of $\boldsymbol{A}$. Then for $\beta > \rho(\boldsymbol{A})$ it holds that

$$\begin{aligned}[\beta\boldsymbol{I} - \boldsymbol{A}]^{-1} &= \frac{1}{\beta - \rho(\boldsymbol{A})}\boldsymbol{x}\boldsymbol{y}^{\mathsf{T}} + \sum_{k=2}^{K}\sum_{j=1}^{m_k}\frac{(j-1)!}{(\beta - \lambda_k)^j}\boldsymbol{Z}_{kj} \\ &= \frac{1}{\beta - \rho(\boldsymbol{A})}\boldsymbol{x}\boldsymbol{y}^{\mathsf{T}} + \boldsymbol{R}(\beta),\end{aligned} \tag{7}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the right and left Perron vectors of $\boldsymbol{A}$, respectively, and $\boldsymbol{Z}_{kj}$ are fixed matrices, known as principal component matrices. The right and left Perron vector are defined as the the right and left eigenvectors of $\boldsymbol{A}$ with positive components corresponding to eigenvalue $\rho(A)$, the spectral radius.

From representation (7) the order and direction of divergence can be easily deduced. As $\beta \to \rho(\boldsymbol{A})$ the first term $\frac{1}{\beta - \rho(\boldsymbol{A})}\boldsymbol{x}\boldsymbol{y}^{\mathsf{T}}$ tends componentwise to infinity while $\boldsymbol{R}(\beta)$ tends to a fixed matrix $\boldsymbol{R}\big(\rho(\boldsymbol{C})\big)$. Note that $\rho(\boldsymbol{A}) > \lambda_k$ for

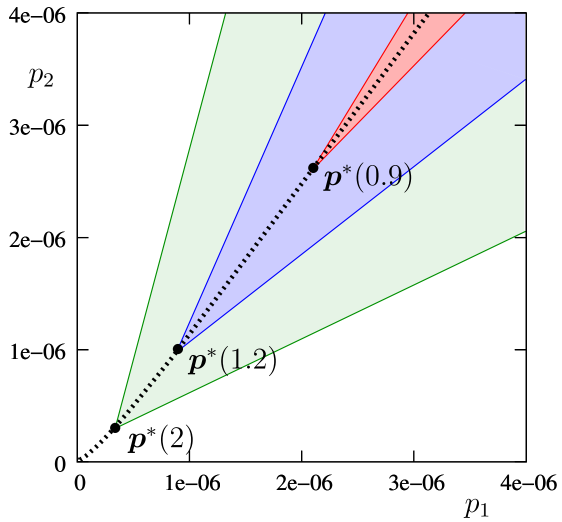

Fig. 1. The dotted line is the orbit $\boldsymbol{p}^*(\beta)$ for $\beta \in (\rho(\boldsymbol{\Gamma B}), \infty)$, the cones $\mathcal{P}_{\text{SIR}}(\frac{1}{\beta}\boldsymbol{\gamma})$ are depicted for $\beta = 0.9,\ 1.2,\ 2$.

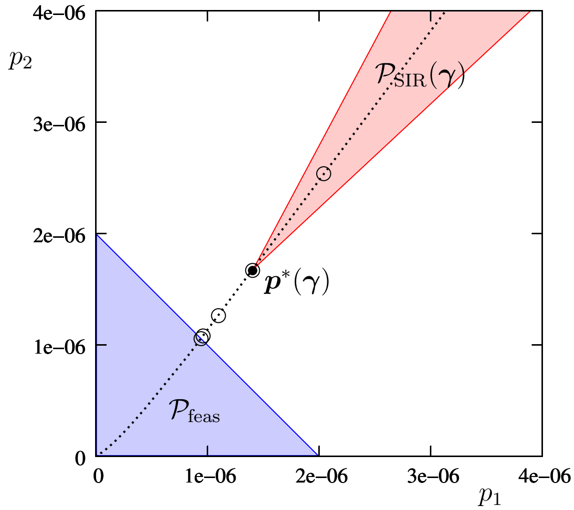

Fig. 2. An example where $\boldsymbol{p}^*(\boldsymbol{\gamma}) \notin \mathcal{P}_{\text{feas}}$. The dotted line is the orbit $\boldsymbol{p}^*(\beta)$ for $\beta \in (\rho(\boldsymbol{\Gamma B}), \infty)$. Circles indicate iteration steps of the algorithm in section IV starting with $\beta_0 = 0.9$.

all $k = 2, \dots, K$. Hence, multiplication by $\boldsymbol{\Gamma\tau}$ yields in the limit as $\beta \to \rho(\boldsymbol{A})$ that

$$\boldsymbol{p}^*(\beta) - \frac{\boldsymbol{y}^{\mathsf{T}}\boldsymbol{\Gamma\tau}}{\beta - \rho(\boldsymbol{A})}\boldsymbol{x} - \boldsymbol{R}\big(\rho(\boldsymbol{A})\big)\boldsymbol{\Gamma\tau} \to \boldsymbol{0}.$$

The interpretation is that $\boldsymbol{p}^*(\beta)$ diverges to infinity along direction $\boldsymbol{x}$ up to the constant shift vector $\boldsymbol{R}\big(\rho(\boldsymbol{A})\big)\boldsymbol{\Gamma\tau}$. In summary we obtain the following result.

Proposition 7: It holds that

$$\lim_{\beta \to \rho(\boldsymbol{\Gamma B})} \Big(\boldsymbol{p}^*(\beta) - \frac{\boldsymbol{y}^{\mathsf{T}}\boldsymbol{\Gamma\tau}}{\beta - \rho(\boldsymbol{\Gamma B})}\boldsymbol{x}\Big) = \boldsymbol{R}\big(\rho(\boldsymbol{\Gamma B})\big)\boldsymbol{\Gamma\tau}.$$

Hence, $\boldsymbol{p}^*(\beta)$ diverges as $\beta \to \rho(\boldsymbol{\Gamma B})$. The factor $\frac{\boldsymbol{y}^{\mathsf{T}}\boldsymbol{\Gamma\tau}}{\beta - \rho(\boldsymbol{\Gamma B})}$ represents the order of divergence, the right Perron vector $\boldsymbol{x}$ is the direction of divergence, and $\boldsymbol{R}\big(\rho(\boldsymbol{\Gamma B})\big)\boldsymbol{\Gamma\tau}$ is a constant deviation between $\boldsymbol{p}^*(\beta)$ and the limiting direction.

Spectral representation (7) is also employed in [12] for analyzing the asymptotic behavior of the minimum total power.

Our results so far are visualized in Figure 1 for $K = 2, A_{11} = 0.6, A_{22} = 1, A_{12} = 0.1, A_{21} = 0.3, \gamma_i = 3.16W, C_{ii}\sigma^2 = 10^{-7}W, i = 1, 2$. The orbit $\boldsymbol{p}^*(\beta)$ is shown as a parametric plot of $\beta \in (\rho(\boldsymbol{\Gamma B}), \infty)$. The convex cones $\mathcal{P}_{\text{SIR}}(\frac{1}{\beta}\gamma)$ are depicted for three cases $\beta = 0.9, 1.2, 2$. Divergence along a fixed direction can be clearly recognized as $\beta \to \rho(\boldsymbol{\Gamma B})$.

IV. Access Control with Limited Power Budget

In practice, power is limited, particularly for small handsets as are used for the uplink. Hence, mobiles may select their power adjustment only from a bounded set $\mathcal{P}$, say. To be quite general, we describe power constraints by help of a convex function $g : \mathbb{R}^K \to \mathbb{R}$ satisfying $g(\boldsymbol{0}) = 0$. The set of feasible power allocations is then defined as

$$\mathcal{P}_{\text{feas}} = \{\boldsymbol{p} \geq \boldsymbol{0} \mid g(\boldsymbol{p}) \leq m\}$$

for some positive threshold $m > 0$. It is clear that $\mathcal{P}_{\text{feas}}$ is a convex set which contains with each power assignment $\boldsymbol{p} > \boldsymbol{0}$ any componentwise smaller $\boldsymbol{q} > \boldsymbol{0}$.

Important special cases are covered by this approach. Individual peak power constraints $p_i \leq m_i$ for positive bounds m_i, $i = 1, \dots, K$, evolve from choosing $g(\boldsymbol{p}) = \max_i \frac{p_i}{m_i}$ and $m = 1$. Total power restrictions like $\sum_i p_i \leq m$ follow by setting $g(\boldsymbol{p}) = \sum_i p_i$. Both are special cases of general restrictions of the form $g(\boldsymbol{p}) = \|\boldsymbol{p}\|$ for some norm $\|\cdot\|$. We will deal with this case later under numerical aspects. The intersection of peak and sum power constraints is also covered by the present general approach.

Let $\boldsymbol{\gamma} > \boldsymbol{0}$ be given and assume that $\boldsymbol{p}^*(\boldsymbol{\gamma}) \notin \mathcal{P}_{\text{feas}}$, as visualized in Figure 2 for $K = 2$ and parameters according to Figure 1. We search for a minimum proportional reduction of $\boldsymbol{\gamma}$ by some $0 \leq \alpha \leq 1$ such that $\boldsymbol{p}^*(\alpha\boldsymbol{\gamma}) \in \mathcal{P}_{\text{feas}}$. With $\beta = 1/\alpha$ it is clear from the above that a solution of the system

$$\begin{aligned}(\beta\boldsymbol{I} - \boldsymbol{\Gamma B})\boldsymbol{p} &= \boldsymbol{\Gamma\tau}\\ g(\boldsymbol{p}) &= m\end{aligned} \tag{8}$$

with variables $\boldsymbol{p}$ and β is sought. Observe that a solution exists and is unique, however, in general it is hard to determine. For this purpose rewrite (8) as

$$F : \big(\rho(\boldsymbol{\Gamma B}), \infty\big) \to \mathbb{R} : \beta \mapsto g\big([\beta\boldsymbol{I} - \boldsymbol{\Gamma B}]^{-1}\boldsymbol{\Gamma\tau}\big) - m. \tag{9}$$

Seeking the roots of F yields the solution β^* of(8).

In the case that g is continuously differentiable Newton's Method is a favorite candidate for finding a solution β^* of (9) as follows,

$$\beta_{n+1} = \beta_n + \Delta\beta_n, \quad \Delta\beta_n = -\frac{F(\beta_n)}{F'(\beta_n)}. \tag{10}$$

Algorithm (10) converges locally quadratic, see, e.g., [30].

Using the chain rule for multivariate functions and setting

$\boldsymbol{p}(\beta) = [\beta \boldsymbol{I} - \boldsymbol{\Gamma B}]^{-1}\boldsymbol{\Gamma\tau}$ gives

$$\begin{aligned}\frac{d}{d\beta}F(\beta) &= \frac{d}{d\boldsymbol{u}}g(\boldsymbol{u})\big|_{\boldsymbol{u}=\boldsymbol{p}(\beta)}\,\frac{d}{d\beta}\boldsymbol{p}(\beta)\\ &= -\frac{d}{d\boldsymbol{u}}g(\boldsymbol{u})\big|_{\boldsymbol{u}=\boldsymbol{p}(\beta)}\,[\beta\boldsymbol{I}-\boldsymbol{\Gamma B}]^{-2}\boldsymbol{\Gamma\tau},\end{aligned} \tag{11}$$

cp. Proposition 6.

We evaluate (11) for the ℓ_q-norms, $1 \le q < \infty$, i.e., $g(\boldsymbol{u}) = \|\boldsymbol{u}\|_q = \left(\sum_{i=1}^K u_i^q\right)^{1/q}$, $\boldsymbol{u} > \boldsymbol{0}$. The derivative of $g(\boldsymbol{u}) = \|\boldsymbol{u}\|_q$ is given by

$$\frac{d}{d\boldsymbol{u}}g(\boldsymbol{u}) = \frac{d}{d\boldsymbol{u}}\|\boldsymbol{u}\|_q = \|\boldsymbol{u}\|_q^{1-q}\left(u_1^{q-1},\dots,u_K^{q-1}\right).$$

Abbreviating componentwise exponentiation as $\left(u_1^{q-1},\dots,u_K^{q-1}\right) = \left(\boldsymbol{u}^{q-1}\right)^{\mathsf{T}}$, Newton's iteration (10) becomes

$$\begin{aligned}\beta_{n+1} &= \beta_n + \frac{\|[\beta_n\boldsymbol{I}-\boldsymbol{\Gamma B}]^{-1}\boldsymbol{\Gamma\tau}\|_q - m}{\|\boldsymbol{p}(\beta_n)\|_q^{1-q}(\boldsymbol{p}^{q-1}(\beta_n))^{\mathsf{T}}[\beta_n\boldsymbol{I}-\boldsymbol{\Gamma B}]^{-2}\boldsymbol{\Gamma\tau}}\\ &= \beta_n - \frac{\|\boldsymbol{p}(\beta_n)\|_q - m}{\|\boldsymbol{p}(\beta_n)\|_q^{1-q}(\boldsymbol{p}^{q-1}(\beta_n))^{\mathsf{T}}\boldsymbol{p}'(\beta_n)},\end{aligned} \tag{12}$$

where $\boldsymbol{p}'(\beta) = \frac{d}{d\beta}\boldsymbol{p}(\beta)$ means the column vector of componentwise derivatives w.r.t. β.

Important special cases are $q = 1$, describing a total limited power budget (cf. [12]), and $q = 2$. In the case $q = 1$ iteration (12) reads as

$$\beta_{n+1} = \beta_n - \frac{\|\boldsymbol{p}(\beta_n)\|_1 - m}{(1,\dots,1)^{\mathsf{T}}\boldsymbol{p}'(\beta_n)} = \beta_n + \frac{\|\boldsymbol{p}(\beta_n)\|_1 - m}{\|\boldsymbol{p}'(\beta_n)\|_1},$$

since $\boldsymbol{p}'(\beta_n) \le 0$ by Proposition 4. For $q = 2$ we obtain from (12)

$$\beta_{n+1} = \beta_n - \frac{\|\boldsymbol{p}(\beta_n)\|_2 - m}{\|\boldsymbol{p}'(\beta_n)\|_2^{-1}\boldsymbol{p}(\beta_n)^{\mathsf{T}}\boldsymbol{p}'(\beta_n)}.$$

A numerical problem in implementing algorithms (10), or (12), respectively, lies in computing $\boldsymbol{p}(\beta_n) = [\beta_n\boldsymbol{I} - \boldsymbol{\Gamma B}]^{-1}\boldsymbol{\Gamma\tau}$ and $\boldsymbol{p}'(\beta_n) = -[\beta_n\boldsymbol{I}-\boldsymbol{\Gamma B}]^{-2}\boldsymbol{\Gamma\tau}$.

The following algorithm converges towards $\boldsymbol{p}(\beta_n)$. The i-th component in the iteration $\boldsymbol{p}_n(k)$ is given by

$$p_{n,i}(k) = \frac{\gamma_i}{\beta_n}\big(\tau_i + \sum_{j\ne i}\frac{A_{ij}}{A_{ii}}p_{n,j}(k-1)\big), \tag{13}$$

$i = 1,\dots,K$, $k \in \mathbb{N}$. If $\beta_n > \rho(\boldsymbol{\Gamma B})$, then it holds that $\lim_{k\to\infty}\boldsymbol{p}_n(k) = \boldsymbol{p}(\beta_n)$ for any initial value $\boldsymbol{p}_n(0)$, as is shown in the appendix.

Similarly, the following sequence $\boldsymbol{p}'_n(k)$ converges towards $\boldsymbol{p}'(\beta_n)$,

$$\boldsymbol{p}'_n(k) = \big(\frac{2}{\beta_n}\boldsymbol{\Gamma B} - \frac{1}{\beta_n^2}(\boldsymbol{\Gamma B})^2\big)\boldsymbol{p}'_n(k-1) + \frac{1}{\beta_n^2}\boldsymbol{\Gamma\tau}, \tag{14}$$

$k \in \mathbb{N}$. If $\rho(\frac{2}{\beta_n}\boldsymbol{\Gamma B} - \frac{1}{\beta_n^2}(\boldsymbol{\Gamma B})^2) < 1$, then $\lim_{k\to\infty}\boldsymbol{p}'_n(k) = \boldsymbol{p}'(\beta_n)$ holds for any n and any initial value $\boldsymbol{p}'_n(0)$.

We conclude this section by evaluating the proposed algorithm on the example depicted in Figure 1 and 2 for sum power constraints. Table I shows the results, convergence is fast giving an optimum value $\beta^* = 1.1469$ with corresponding power allocation $\boldsymbol{p}^*(\beta^*) = 10^{-6}(0.944, 1.056)^{\mathsf{T}}$ after 6 iterations with a relative error of 10^{-6}. The results $\boldsymbol{p}(\beta_n)$ of the iterations $n = 0, 1, \dots, 5$ are also indicated by open circles in Figure 2.

TABLE I

NUMERICAL CONVERGENCE FOR THE VALUES USED IN FIGURE 1 AND 2, CONVERGENCE UP TO 4 DIGITS ACHIEVED AFTER 5 ITERATIONS.

n	0	1	2	3	4
β_n	0.9	1.0084	1.1032	1.1426	1.1468
$10^6\boldsymbol{p}(\beta_n)$	$\begin{pmatrix}2.061\\2.522\end{pmatrix}$	$\begin{pmatrix}1.348\\1.580\end{pmatrix}$	$\begin{pmatrix}1.041\\1.181\end{pmatrix}$	$\begin{pmatrix}0.953\\1.067\end{pmatrix}$	$\begin{pmatrix}0.944\\1.056\end{pmatrix}$

V. CONCLUSIONS

This paper has introduced the concept of proportional SIR reduction for a community of users if there exists no feasible power adjustment for given transmission rate requirements. We have derived an algorithm to determine the point of minimal reduction numerically for a rather general class of power restrictions, including the case of limited total power. To achieve these results we have investigated the geometrical structure of the set of admissible power assignments, and furthermore, monotonicity and asymptotic properties when the proportionality factor tends to its boundaries.

Interesting open problems are the development of algorithms for a nonsmooth boundary of the power restrictions, and decentralizing the computation such that power assignments can be determined locally with only small global information exchange.

APPENDIX

We complement the convergence proofs of (13) and (14). As $\beta_n\boldsymbol{I} - \boldsymbol{\Gamma B}$ is an M-matrix, we can apply Jakobi's method which is convergent for all initial vectors, see [31]. Equation (13) is the i-th component of

$$\boldsymbol{p}_n(k) = \frac{1}{\beta_n}\boldsymbol{\Gamma B}\boldsymbol{p}_n(k-1) + \frac{1}{\beta_n}\boldsymbol{\Gamma\tau}.$$

Successive application yields

$$\boldsymbol{p}_n(k) = \Big(\frac{1}{\beta_n}\boldsymbol{\Gamma B}\Big)^k\boldsymbol{p}_n(0) + \Big[\sum_{i=0}^{k-1}\Big(\frac{1}{\beta_n}\boldsymbol{\Gamma B}\Big)^i\Big]\frac{1}{\beta_n}\boldsymbol{\Gamma\tau}. \tag{15}$$

As $\beta_n > \rho(\boldsymbol{\Gamma B})$ by assumption, using the von Neumann Series yields $(\boldsymbol{I} - \frac{1}{\beta_n}\boldsymbol{\Gamma B})^{-1} = \sum_{i=0}^{\infty}\left(\frac{1}{\beta_n}\boldsymbol{\Gamma B}\right)^i$. In particular, it holds that

$$\lim_{k\to\infty}\Big(\frac{1}{\beta_n}\boldsymbol{\Gamma B}\Big)^k = 0 \text{ and } \lim_{k\to\infty}\Big[\sum_{i=0}^{k-1}\Big(\frac{1}{\beta_n}\boldsymbol{\Gamma B}\Big)^i\Big]\boldsymbol{\Gamma\tau} = \boldsymbol{p}_n. \tag{16}$$

Thus, we get $\lim_{k\to\infty}\boldsymbol{p}_n(k) = \boldsymbol{p}(\beta_n)$ which concludes the proof of (13). Equation (14) is shown in a similar way noting that $(\beta_n\boldsymbol{I} - \boldsymbol{\Gamma B})^{-2} = (\beta_n^2\boldsymbol{I} - (2\beta_n\boldsymbol{\Gamma B} - (\boldsymbol{\Gamma B})^2))^{-1}$.

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