Preamble-based SNR Estimation in Frequency Selective Channels for Wireless OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) offers high data rates and robust performance in frequency selective channels by link adaptation utilizing information about the channel quality. A crucial parameter required for adaptive transmission is the signal-to-noise ratio (SNR). In this paper, we propose a novel SNR estimation algorithm for wireless OFDM systems based on the reuse of the synchronization preamble. The periodic structure of the preamble is utilized for the computationally efficient SNR estimation algorithm, based on the second-order moments of received preamble samples. The performance of the proposed algorithm is compared with the MMSE algorithm and two preamble-based algorithms found in the literature. It is shown that the proposed algorithm is robust against frequency selectivity and may therefore be used for subchannel SNR estimation.**

I. INTRODUCTION

OFDM is a multicarrier modulation scheme that provides strong robustness against intersymbol interference (ISI) by dividing the broadband channel into many narrowband subchannels in such a way that attenuation across each subchannel stays flat. Orthogonalization of subchannels is performed with low complexity by using the fast Fourier transform (FFT). The serial high-rate data stream is converted into multiple parallel low-rate streams, each modulated on a different subcarrier.

An important task in the design of future OFDM system is to exploit frequency selective channels by adaptable transmission parameters (bandwidth, coding/data rate, power) to preserve power and bandwidth efficiency according to channel conditions at the receiver. In order to achieve such improvements, efficient and exact signal-to-noise ratio (SNR) estimation algorithm is requisite. The SNR is defined as the ratio of the desired signal power to the noise power and is widely used as a standard measure of signal quality for communication systems. SNR estimators derive estimate by averaging the observable properties of the received signal over a number of symbols. Prior to SNR per subcarrier estimation for adaptive transmission, the average SNR and channel frequency response have to be estimated.

There are two general categories of average SNR estimators. *Data-aided* (DA) estimators are based on either perfect or estimated knowledge of the transmitted data. However, a certain portion of data is needed for estimation purposes, which reduces bandwidth efficiency. Blind or *in-service* estimators derive SNR estimate from an unknown information-bearing portion of the received signal preserving efficiency at the cost of decreased performance. For packet based communications, block of information data is usually preceded by several training symbols (preambles) of known data used for synchronization and equalization purposes. Therefore, DA SNR estimators can utilize preambles without additional throughput reduction.

Most of the SNR estimators proposed in the literature so far are related to single carrier transmission. In [1], a detailed comparison of various algorithms is presented, together with the derivation of the Cramer-Rao bound (CRB). Most of these algorithms can be directly applied to OFDM systems in additive white Gaussian noise (AWGN) [2], while the SNR estimation in frequency selective channels additionally requires efficient estimation of channel state information (CSI).

In this paper, we propose an efficient algorithm for the average SNR estimation in wireless OFDM systems. The SNR per subcarrier can be additionally estimated using channel estimates and the estimated average SNR. The proposed estimator utilizes preamble structure, proposed by Morelli and Mengali in [3]. Compared to Schmidl and Cox synchronization method [4], it allows synchronization over a wider frequency offset range with only one preamble, hence reducing the training symbol overhead. Since the proposed estimation algorithm relies on the signal samples at the output of the FFT, its performance depends strongly on the given preamble structure.

The remainder of this paper is organized as follows. Section II provides the system model and specifies the SNR estimation problem. In Section III, Minimum Mean Square Error (MMSE), Boumard's and Ren's estimators are briefly described and according SNR estimates are given. The novel SNR estimator is described in Section IV. Its performance is analyzed by computer simulations in Section V. Finally, some concluding remarks are given in Section VI.

II. SYSTEM MODEL

In many wireless OFDM systems, transmission is normally organized in frames. Typical frame structure is shown in Fig.

This work has been partially supported by the UMIC Research Center, RWTH Aachen University.

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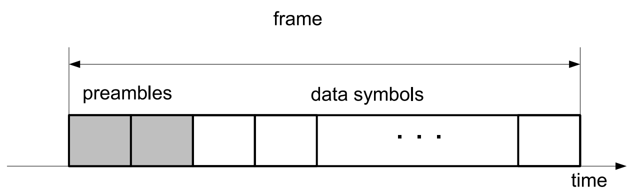

Fig. 1. Frame structure

1 where sequence of data symbols is preceded by several preambles of known data used for the synchronization and/or channel estimation purposes. We consider general model of frame structure composed of K preambles where each preamble contains N modulated subcarriers. Let $C(k,n)$ denote the complex data symbol on nth subcarrier in kth preamble, where $k = 0,\ldots,K-1$ and $n = 0,\ldots,N-1$. It is assumed that modulated subcarrier has unit magnitude, i.e. $|C(k,n)|^2 = 1$, which is a regular assumption since present OFDM standards usually contain preambles composed of QPSK and/or BPSK modulated subcarriers. Since we consider SNR estimation performed in frequency domain, given model contains only frequency domain characterization of received signal in frequency-selective AWGN channels. At the receiver, perfect synchronization is assumed, hence after FFT, received signal on ntth subcarrier in kth preamble can be expressed as

$$Y(k,n) = \sqrt{S}C(k,n)H(k,n) + \sqrt{W}\eta(k,n), \qquad (1)$$

where $\eta(k,n)$ is sampled complex zero-mean AWGN of unit variance, S and W are transmitted signal power and noise power on each subcarrier, respectively, and $H(k,n)$ is the channel frequency response given by

$$H(k,n) = \sum_{l=1}^{L} h_l(kT_s)\cdot e^{-j2\pi\frac{n\tau_l}{NT_s}}, \qquad (2)$$

where $h_l(kT_s)$ and τ_l denote the channel lth path gain and delay during the kth preamble, respectively, T_s is the duration of the OFDM preamble and L is the length of the channel memory. The channel path gains $h_l(kT_s)$ in each OFDM symbol independently experience Rayleigh fading, while $\sum_{l=1}^{L}|h_l(kT_s)|^2 = 1$ is satisfied. Our initial assumption is that channel is constant during the whole frame, since we consider SNR estimation algorithms for the purposes of adaptive transmission. Therefore, time index k is omitted during the estimation procedure, i.e. $H(k,n)$ is replaced by $H(n)$. It is also assumed that average SNR and SNR per subcarrier estimates are valid for all information data bearing OFDM symbols within the frame. As it is shown in [5], the average SNR of the kth received OFDM preamble can be expressed as

$$\begin{aligned}\rho_{av}(k) &= \frac{E\{\frac{1}{N}\sum_{n=0}^{N-1}|\sqrt{S}C(k,n)H(n)|^2\}}{E\{\frac{1}{N}\sum_{n=0}^{N-1}|\sqrt{W}\eta(k,n)|^2\}}\\ &= \frac{S}{W},\end{aligned} \qquad (3)$$

where $\sum_{n=0}^{N-1}|H(n)|^2 = N$ is satisfied, while the SNR of the nth subcarrier is given by

$$\begin{aligned}\rho(k,n) &= \frac{E\{|\sqrt{S}C(k,n)H(n)|^2\}}{E\{|\sqrt{W}\eta(k,n)|^2\}}\\ &= \frac{S|H(n)|^2}{W} = \rho_{av}\cdot|H(n)|^2.\end{aligned} \qquad (4)$$

III. SNR Estimators

A. MMSE Estimator

MMSE algorithm for SNR estimation in OFDM system is based on the orthogonality between the estimation error and the estimate of the channel frequency response expressed as

$$(Y(n) - \hat{H}(n)C(n))(\hat{H}(n)C(n))^* = 0, \quad n = 1,\ldots,N,$$

where $\hat{H}(n)$ denotes the estimate of $H(n)$ and $(\cdot)^*$ refers to the conjugation operation. The MMSE average SNR estimate is given by [2]

$$\hat{\rho}_{av,MMSE} = \frac{\hat{S}_{MMSE}}{\hat{W}_{MMSE}}, \qquad (5)$$

where

$$\hat{S}_{MMSE} = \left|\frac{1}{N}\sum_{n=0}^{N-1} Y(n)C(n)^*\right|^2$$

and

$$\hat{W}_{MMSE} = \frac{1}{N}\sum_{n=0}^{N-1}|Y(n)|^2 - \hat{S}_{MMSE}$$

are the MMSE estimates of S and W, respectively.

B. Boumard's Estimator

In [6], Boumard proposed a second-order moment-based SNR estimator for 2×2 MIMO OFDM system in slow varying channel in both time and frequency domain. In [5], Ren et al. derived its corresponding SISO version keeping the presumption that the channel is time-invariant and that two identical preambles are used for SNR estimation, i.e. $k = 0, 1$ and $C(0,n) = C(1,n) = C(n)$, for $n = 1,\ldots,N$. Average SNR estimate can be expressed as

$$\hat{\rho}_{av,Bou} = \frac{\hat{S}_{Bou}}{\hat{W}_{Bou}}, \qquad (6)$$

where

$$\hat{S}_{Bou} = \frac{1}{N}\sum_{n=0}^{N-1}\left|\hat{H}(n)\right|^2$$

and

$$\begin{aligned}\hat{W}_{Bou} = \frac{1}{4N}\sum_{n=1}^{N-1}&|C(n-1)(Y(0,n)+Y(1,n))\\ &- C(n)(Y(0,n-1)+Y(1,n-1))|^2\end{aligned}$$

are the estimates of S and W, respectively, and

$$\hat{H}(n) = \frac{C^*(n)}{2}(Y(0,n)+Y(1,n)) \qquad (7)$$

is the least squares (LS) estimate of $H(n)$ averaged over two preamble symbols. Using $\hat{H}(n)$, SNR on nth subcarrier is estimated as

$$\hat{\rho}(n) = \frac{|\hat{H}(n)|^2}{\hat{W}_{Bou}}. \tag{8}$$

C. Ren's Estimator

The main disadvantage of Boumard's estimator is high sensitivity to frequency selectivity. In [5], Ren et al. proposed more accurate second-order moment-based SNR estimator robust to the frequency selectivity, employed the presumed preamble arrangement from Boumard's estimator. Derived average SNR estimate can be expressed as

$$\hat{\rho}_{av,Ren} = \frac{\hat{S}_{Ren}}{\hat{W}_{Ren}}, \tag{9}$$

where

$$\hat{W}_{Ren} = \frac{4}{N}\sum_{n=0}^{N-1}\left\{Im\left[Y(0,n)C^*(0,n)\hat{H}^*(n)/|\hat{H}^*(n)|\right]\right\}^2$$

and

$$\hat{S}_{Ren} = \frac{1}{N}\sum_{n=0}^{N-1}|Y(0,n)|^2 - \hat{W}_{Ren}$$

are the estimates of W and S, respectively, and $\hat{H}(n)$ is defined in (7). It is shown that the performance is independent of the channel frequency response estimation although the estimated channel states are used in average SNR estimation. Additionally, SNR on nth subcarrier is estimated as in (8) using the noise power estimate from (9).

IV. Proposed Estimator

A new estimator based on periodically used subcarriers is explored in this section, named PS estimator in the following. The key idea rests upon the time domain periodic preamble structure for time and frequency synchronization in [4]. In order to cover a wider frequency range, in [3] a preamble of Q identical parts, each containing N/Q samples is proposed as depicted in Fig. 2a. The corresponding frequency domain representation is shown in Fig. 2b. In the sequel we assume that Q divides N, so that $N_p = N/Q$ is integer.

Starting from the 0th, each Qth subcarrier is modulated with a QPSK signal $C_p(m)$, $m = 0, 1, \ldots, N_p - 1$ with $|C_p(m)| = 1$. The remainder of $N_z = N - N_p = \frac{(Q-1)}{Q}N$ subcarriers is not used (nulled). In order to maintain the total energy level over all symbols within the preamble, the power is scaled by factor Q yielding a total transmit power of SQ in the loaded subcarriers.

Write $n = mQ + q$, $m = 0, \ldots, N_p - 1$, $q = 0, \ldots, Q - 1$. The transmitted signal on the nth subcarrier is written as

$$C(n) = C(mQ+q) = \begin{cases} C_p(m), & q = 0 \\ 0, & q = 1, \ldots, Q-1 \end{cases}. \tag{10}$$

By (1) the nth received signal is given by

$$Y(n) = Y(mQ+q) = \begin{cases} Y_p(m), & q = 0 \\ Y_z(mQ+q), & q = 1, \ldots, Q-1 \end{cases},$$

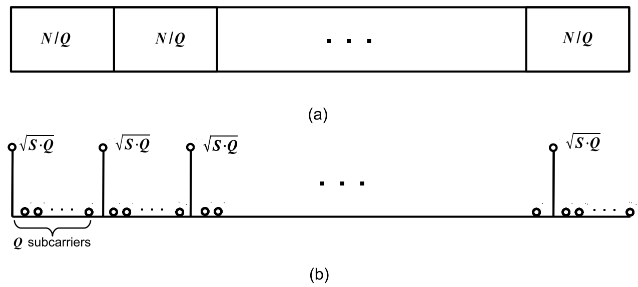

Fig. 2. Preamble structure in (a) time and (b) frequency domain

where

$$Y_p(m) = \sqrt{SQ}\,C_p(m)H(m) + \sqrt{W}\,\eta(m) \tag{11}$$

denotes the received signal on loaded subcarriers, and

$$Y_z(mQ+q) = \sqrt{W}\,\eta(mQ+q) \tag{12}$$

the received signal on nulled subcarriers consisting only of noise. The empirical second-order moment of loaded subcarriers is

$$\hat{M}_{2,p} = \frac{1}{N_p}\sum_{m=0}^{N_p-1}|Y_p(m)|^2 \tag{13}$$

Its expected value is given as

$$\begin{aligned} E\left\{\hat{M}_{2,p}\right\} &= \frac{1}{N_p}\sum_{m=0}^{N_p-1}E\left\{|Y_p(m)|^2\right\} \\ &= \frac{QS}{N_p}\sum_{m=0}^{N_p-1}E\left\{|H(m)|^2\right\} + \frac{W}{N_p}\sum_{m=0}^{N_p-1}E\left\{|\eta(m)|^2\right\} \\ &= QS + W. \end{aligned}$$

Similarly, the empirical second moment of the received signal in nulled subcarriers,

$$\hat{M}_{2,z} = \frac{1}{N_p(Q-1)}\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}|Y_z(mQ+q)|^2, \tag{14}$$

has expectation

$$\begin{aligned} E\left\{\hat{M}_{2,z}\right\} &= \frac{1}{N_p(Q-1)}\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}E\left\{|Y_z(mQ+q)|^2\right\} \\ &= \frac{W}{N_p(Q-1)}\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}E\left\{|\eta(mQ+q)|^2\right\} \\ &= W. \end{aligned}$$

In summary, the average SNR ρ_{av} can be estimated by forming

$$\begin{aligned} \hat{\rho}_{av} &= \frac{1}{Q}\frac{\hat{M}_{2,p} - \hat{M}_{2,z}}{\hat{M}_{2,z}} \\ &= \frac{1}{Q}\Big((Q-1)\frac{\sum_{m=0}^{N_p-1}|Y_p(m)|^2}{\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}|Y_z(mQ+q)|^2} - 1\Big), \end{aligned} \tag{15}$$

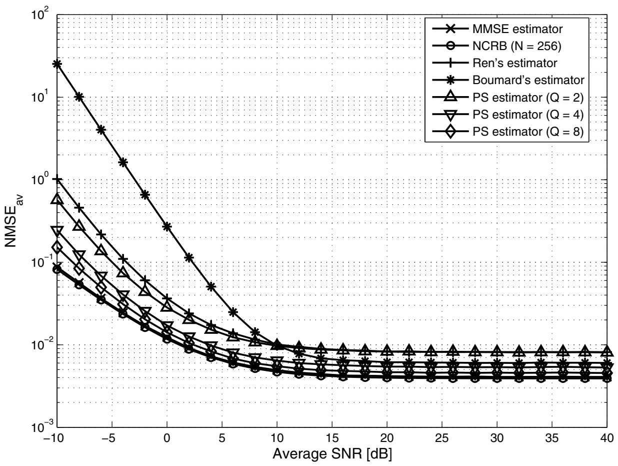

Fig. 3. NMSE of the average SNR in AWGN channel

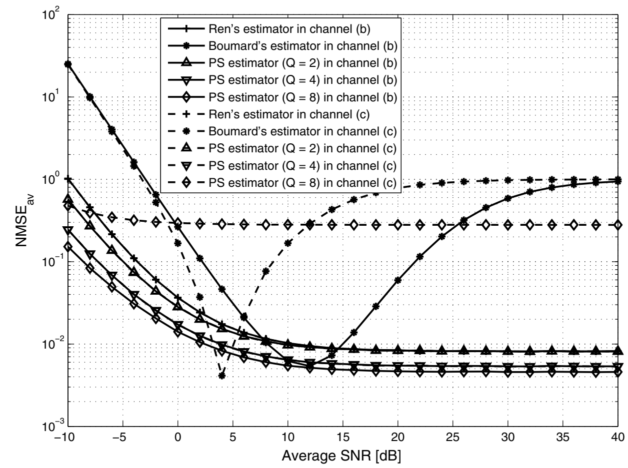

Fig. 4. NMSE of the average SNR in channel (b) and (c)

where, by the strong law of large numbers, $\hat{M}_{2,p}$ and $\hat{M}_{2,z}$ are strongly consistent unbiased estimators of $QS + W$ and average noise power W, respectively.

Note that in contrast to the previously described estimators the PS estimator does not need any knowledge of the transmitted symbols on loaded subcarriers. Only the arrangement of loaded and nulled subcarriers must be known to the receiver. The channel estimates $\hat{H}(m)$, $m = 0, 1, \ldots, N_p - 1$, are available only for the loaded subcarriers. However, they are more accurate since the transmitted power on each loaded subcarriers is increased by factor Q. Channel estimates for nulled subcarriers $\hat{H}(mQ+q)$, $m = 0, \ldots, N_p-1$, $q = 1, \ldots, Q-1$, can be obtained by linear or DFT based interpolation, see [7].

To estimate the SNR on the nth subcarrier formula (8) is used with the noise power estimate from (14). Finally, increasing the number of parts N_p improves the accuracy of the noise power estimation and increases sensitivity of SNR per subcarrier estimates to frequency selectivity due to performed interpolation on nulled subcarriers during the channel estimation.

From an implementation point of view the PS estimator has less complexity than Boumard's and Ren's estimator. For average SNR estimation Boumard's estimator (6) requires $5N$ and $2N$ multiplications and additions per estimate, respectively. Ren's estimator (9) needs $4N$ and $3N$ multiplications and additions, respectively. The PS algorithm (15) requires only N multiplications and N additions per estimate. Moreover, the PS estimator is of higher bandwidth efficiency since only one preamble is needed unlike Ren's and Boumard's estimator.

V. Simulation Results

The performance of PS estimator is evaluated and compared with the performance of MMSE, Boumard's and Ren's estimator using Monte-Carlo simulation. OFDM system parameters used in the simulation are taken from WiMAX specifications giving $N = 256$ subcarriers and cyclic prefix length of 32 samples [8]. Performance is evaluated for three different channels: (a) AWGN channel, (b) a 3-tap time-invariant fading channel with a root mean square delay spread $\tau_{rms} = 2$ samples and (c) a 3-tap time-invariant fading channel with a $\tau_{rms} = 10$ samples. Parameters for considered channels are taken from [6]. The number of independent trials is set to $N_t = 100000$ assuring the high confidence interval of the estimates. The evaluation of the performance is done in terms of normalized MSE (NMSE) of the estimated average SNR values following

$$\text{NMSE}_{\text{av}} = \frac{1}{N_t} \sum_{i=i}^{N_t} \left(\frac{\hat{\rho}_{av,i} - \rho_{av}}{\rho_{av}} \right)^2, \quad (16)$$

where $\hat{\rho}_{av,i}$ is the estimate of the average SNR in the ith trial, and ρ_{av} is the true value. Second considered performance measure is the NMSE of the estimated SNR per subcarrier given by

$$\text{NMSE}_{\text{sc}} = \frac{1}{NN_t} \sum_{i=i}^{N_t} \sum_{n=0}^{N} \left(\frac{\hat{\rho}(n)_i - \rho(n)}{\rho(n)} \right)^2, \quad (17)$$

where $\hat{\rho}(n)_i$ is the estimate of the $\rho(n)$ in the ith trial. During the simulation, MMSE and proposed algorithm are evaluated with only one preamble used for estimation procedure, while Boumard's and Ren's estimators inherently use two preambles. Proposed method is evaluated for 3 different cases of preamble's repeated parts, i.e. $Q = 2, 4$ and 8.

A. AWGN channel

Fig. 3 shows the NMSE$_{\text{av}}$ of considered estimators in AWGN channel. In order to assess the absolute performances of the estimators, they are compared with the Cramer-Rao bound (CRB) which is the lower bound for the variance of any unbiased estimator, see [9]. Normalized CRB (NCRB) for OFDM signal with N QPSK modulated subcarriers in AWGN channel can be expressed as

$$\text{NCRB} = \frac{1}{N} \left(\frac{2}{\rho_{av}} + 1 \right). \quad (18)$$

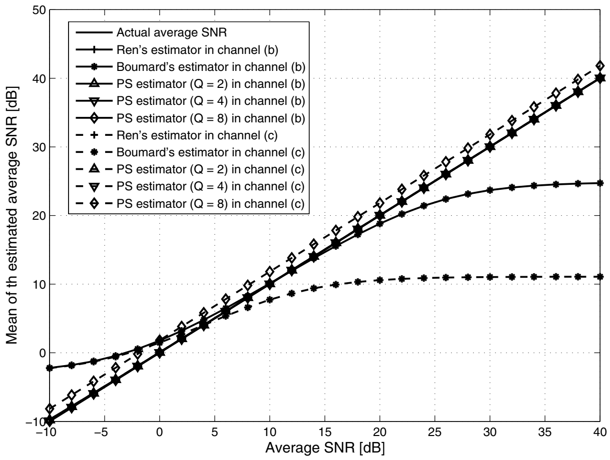

Fig. 5. Mean of the estimated average SNR in channel (b) and (c)

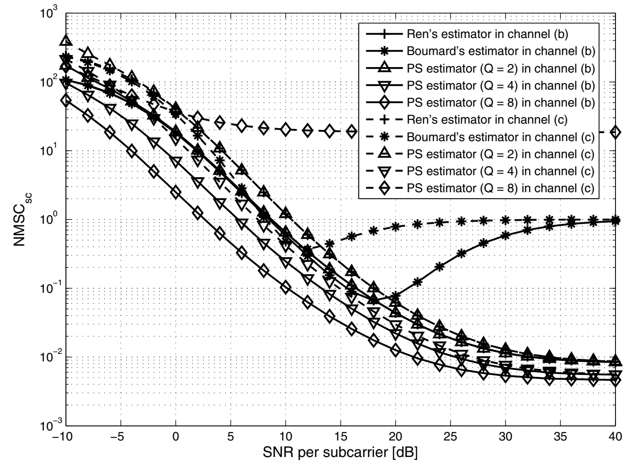

Fig. 6. NMSE of the average SNR per subcarrier in channel (b) and (c)

MMSE estimator shows the best performance with the NMSE_{av} curve undistinguishable from the NCRB defined in (18). Boumard's estimator for average SNR values smaller than 10 dB performs worse than Ren's and PS estimator. For average SNR values greater than 10 dB it outperforms both the Ren's and PS estimator for $Q = 2$. Note that the increase of the number of repeated parts in the preamble ($Q = 4, 8$), brings its performance closer to the NCRB. It can be explained with the notion that more subcarriers are used for the average noise power estimation (14) while at the same time transmitted signals on loaded subcarriers are getting more power due to the scaling by Q, giving the more accurate estimate in (13).

B. Time-Invariant Frequency Selective Channel

Fig. 4 compares the NMSE_{av} of considered estimators in time-invariant frequency selective channels (b) and (c). It is shown that the performance of Ren's estimator does not depend on frequency selectivity, while Boumard's estimator performs highly sensitive to channel selectivity. PS estimator in channel (b), which corresponds to moderate selectivity, performs the same as in AWGN channel for all considered values of Q, outperforming Ren's and Boumard's estimators. In channel (c), characterized with the strong selectivity, PS estimator stops to benefit from the increase of Q. Fig. 5 shows that PS estimator with $Q = 8$ becomes biased in the channel with the strong selectivity, with he constant bias value of 1.82 dB over the whole estimated SNR range. NMSE_{sc} performance of considered estimators is shown in Fig. 6. Since all considered estimators depend on channel estimates, bad performance in the region of low values of SNR is expected. Performance can be further improved by combining estimated average noise power with more sophisticated channel estimation algorithms using pilot subcarriers within the data symbols. It can be noticed that in the region of high values of SNR, channel estimates stop to act as deteriorating factor and NMSE_{sc} approaches the NMSE_{av}. PS estimator outperforms Ren's estimator, except for $Q = 8$ in channel (c), although its performance depends on channel selectivity, which is expected behaviour due to the interpolation performed during the channel estimation.

VI. Conclusion

In this paper, a novel preamble-based SNR estimator for wireless OFDM systems has been proposed. Reuse of the synchronization preamble for the SNR estimation purposes by exploiting its time domain periodic structure puts no additional overhead on transmitted OFDM frame. Increasing the number of repeated parts by nulling the subcarriers on specified positions improves the performance of considered estimator, but also increases its sensitivity to frequency selectivity. Low complexity and robustness to frequency selectivity combined with the bandwidth efficiency favors the proposed estimator compared to the considered preamble-based estimators given in the literature.

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