Accelerating Resource Allocation for OFDMA Downlink with CNR Variation Over Users

Chunhui Liu, Georg Böcherer and Rudolf Mathar
Institute for Theoretical Information Technology,
RWTH Aachen University D-52056 Aachen, Germany
Email: {liu,boecherer,mathar}@ti.rwth-aachen.de

Abstract—**This paper addresses the problem of resource allocation for orthogonal frequency division multiple access (OFDMA) downlink that aims at minimizing the total transmission power under data transmission constraints. To accelerate this multiuser resource allocation with small performance loss, an efficient technique is introduced that provides the power variation of single-user water-filling when the subcarrier assignment is changed. Based on this technique, an intelligent resource allocation method for OFDMA downlink is designed. First, a good starting point is determined by estimating the cardinality of the set of subcarriers that will be assigned to each user. Second, the convergence of the resource allocation is accelerated by reassigning those subcarriers first, which have the greatest channel-to-noise ratio (CNR) variation over the users. Compared to previous works, simulations show that the presented method provides an improved balance between performance and computational complexity.**

I. INTRODUCTION

Depending on channel characteristics, OFDMA can allocate power and rate optimally on subcarriers to take advantage of the spatial diversity among users in a fading environment. Concerning the problem of minimizing the total transmission power while satisfying individual transmission requirements of users, the method given in [1] provides the optimal solution and the methods proposed in [2], [3] achieve near-optimal performance, but they are computationally intensive and difficult to implement for a large number of users. Heuristic approaches with low complexity are suggested in [4]–[9] at the expense of performance loss in terms of allocated power.

In this paper, we inherit the basic structure of the method suggested in [4], which consists of three consecutive algorithms. We introduce a new technique to efficiently update the transmission power after varying subcarrier assignments. The technique works best when subcarrier assignments are already close to good. Therefore, we suggest an intelligent initialization that chooses a good starting point by considering the estimated cardinalities of the sets of subcarriers that will be assigned to users. After that, the subcarriers are sorted according to their CNR variations over the users, such that subcarriers with higher CNR variations are reassigned first. In this way, our algorithm converges faster to the optimal solution. Simulations validate that little performance loss and much lower computing time is observed compared to SUSI [2]. In addition, our method performs better than RCG [4] at the expense of a tolerable increased computing time.

The remainder of this paper is organized as follows. In Section II, the problem is stated. In Section III, we provide an efficient technique to calculate the power variation for single-user water-filling. Based on the feature of this technique, we develop a heuristic method for multiuser resource allocation in Section IV. We analyze its complexity and compare it to previous works in Section V. Simulation results are given in Section VI. Finally, the content of this paper is concluded.

II. PRELIMINARIES

We consider OFDMA downlink with K users over N subcarriers. Perfect channel knowledge is available at both the transmitter and the receiver. The transmissions of different users are subject to independent frequency selective fading. Each user k requires an individual data rate R_k and a bit error rate BER_k. The modulation scheme of M-ary quadrature amplitude modulation (M-QAM) is employed.

Let $G_k[n]$ denote the CNR of subcarrier n multiplied with $3\left[Q^{-1}(\frac{\mathrm{BER}_k}{4})\right]^{-2}$ for user k [10]. The power and rate allocated on subcarrier n for user k are denoted by $P_k[n]$ and $r_k[n]$, respectively. As shown in [11] this relation holds: $r_k[n] = \log_2(1 + P_k[n]G_k[n])$.

The maximum allowed rate on each subcarrier is M bits per OFDM symbol. Each subcarrier is allowed to be used by at most one user at the same time.

The aim is to minimize the total transmission power while satisfying the data rate and BER requirements of the users. The optimization problem can be stated as

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} P_k[n] && (1) \\ \text{subject to} \quad & \sum_{n=1}^{N} r_k[n] \geq R_k, \quad \forall\, k, \\ & 0 \leq r_k[n] \leq M, \quad \forall\, k, \forall\, n, \\ & \sum_{n=1}^{N} r_k[n] r_l[n] = 0, \ \forall\, k, l, k \neq l. \end{aligned}$$

The subcarrier assignment of user k is denoted by the set $\mathcal{S}_k = \{n \mid r_k[n] > 0\}$ with cardinality $|\mathcal{S}_k| = s_k$. The transmission power for user k is $P_k = \sum_{n=1}^{N} P_k[n]$.

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III. Power Variation by Modifying A Subcarrier Assignment

After excluding a subcarrier from or adding a subcarrier to the subcarrier assignment of user k, the transmission power for user k may change and water-filling can be used to update the transmission power and obtain the power variation [11]. However, the existing water-filling implementations have complexity of $\mathcal{O}(s_k)$ or even higher, see [11]–[14]. Therefore, multiuser resource allocation becomes very computationally complex, when water-filling is utilized iteratively, see [2], [5], [8]. Under certain conditions, the power variation can be calculated much more efficiently.

Assume a subcarriers assignment $\mathcal{S}_k$, that is, the rate and power allocated on any subcarrier $n \in \mathcal{S}_k$

$$\begin{aligned} r_k[n] &= \log_2(\lambda_k) + \log_2(G_k[n]), & (2) \\ P_k[n] &= \lambda_k - \frac{1}{G_k[n]} & (3) \end{aligned}$$

are positive. They both are monotonically increasing in the water level λ_k, which can be determined by

$$\lambda_k = 2^{\frac{R_k}{s_k}} \left(\prod_{n \in \mathcal{S}_k} \frac{1}{G_k[n]} \right)^{\frac{1}{s_k}}. \quad (4)$$

After excluding subcarrier m from $\mathcal{S}_k$, the subcarrier assignment becomes $\mathcal{S}_k \setminus \{m\}$. Consequently, the water level increases to

$$\lambda_k^{(r)}(m) = \lambda_k (\lambda_k G_k[m])^{\frac{1}{s_k - 1}}. \quad (5)$$

The resulting power variation can be calculated in two steps. First, the transmission power decreases by not allocating any power on subcarrier m for user k. After that, the rate, previously allocated on subcarrier m, is equally distributed on the remaining active subcarriers, which can be seen from (2). From (3), we see that the resulting power increments on the remaining active subcarriers are the same and equal to the increment of the water level. The transmission power increases because of the increased water level.

Considering both steps together, excluding one subcarrier leads to a higher transmission power, since water-filling over a smaller number of available subcarriers can only degrade the power efficiency. We denote the additional required power resulting from excluding m by $\Delta P_k^{(r)}(m)$.

However, by using this updating approach, performance loss may happen, when the rates on some of the remaining active subcarriers increase beyond the rate limit M on one subcarrier.

In the other case, when adding subcarrier $m \notin \mathcal{S}_k$ to $\mathcal{S}_k$, the water level decreases to

$$\lambda_k^{(a)}(m) = \lambda_k \left(\frac{1}{\lambda_k G_k[m]} \right)^{\frac{1}{s_k + 1}}, \quad (6)$$

if $\lambda_k > 1/G_k[m]$. We first allocate the rate $r_k[m]$ also on subcarrier m given the previous water level. By taking (6) to (2), it follows that the rates on all subcarriers in the new subcarrier assignment decrease by the same amount of rate $\frac{r_k[m]}{s_k + 1}$. The power variation is the difference of the power allocated on subcarrier m given the previous water level and the sum of the power decrements on all subcarriers in the new subcarrier assignment, denoted by $\Delta P_k^{(a)}(m)$. This updating approach may also provide non-optimal solutions when the CNR on subcarrier m is very large and the rates on some subcarriers become negative.

As we have seen, it is very cheap to obtain the power variation and update the water level after excluding a subcarrier from or adding a subcarrier to a subcarrier assignment. Only one exponential operation is needed to derive the pair of $(\lambda_k^{(r)}(m), \Delta P_k^{(r)}(m))$ or the pair of $(\lambda_k^{(a)}(m), \Delta P_k^{(a)}(m))$, from which other values of interest can be calculated easily. This efficient updating technique is used in the following. As earlier mentioned, both excluding and adding subcarriers from and to a subcarrier assignment by this technique can lead to sub-optimal solutions for some settings. It is therefore important to determine good subcarrier assignments to start with. We will discuss this in the following.

IV. Resource Allocation for OFDMA Downlink

To utilize the efficient updating technique introduced in the previous section, we must have intelligent methods for multiuser resource allocation. These methods must keep the number of subcarriers assigned to each user large enough such that the rate limit on one subcarrier is hardly broken. Furthermore, they must let subcarriers with relatively large CNRs remain in the subcarrier assignment of the corresponding user so that rates rarely take negative values after subcarrier reassignment. To satisfy these requirements, the following three consecutive algorithms are designed.

A. Cardinality estimation

When an user k requires a higher data rate or has worse channel conditions (smaller CNRs) than other users, it normally needs more subcarriers. Both parameters are combined to the water level in (4) and the transmission power in (3). They can be used to estimate the cardinality of each subcarrier assignment in order to assign enough subcarriers to each user.

Algorithm 1 returns the estimated cardinality of each subcarrier assignment. The set $\mathcal{K}$ always contains all users. Instead

Algorithm 1 Cardinality Estimation

```
$\mathcal{K} \leftarrow \{1, \ldots, K\}$
$N_k^{\min} \leftarrow \lceil R_k / M \rceil, \quad \forall k$
$g_k \leftarrow (\prod_{n=1}^{K} G_k[n])^{\frac{1}{N}} \ \forall k$
$\lambda_k \leftarrow 2^{\frac{R_k}{N}} g_k^{-1}, \quad \forall k$
$p_k \leftarrow \lambda_k - g_k, \quad \forall k$
$L_k \leftarrow N_k^{\min} + \left[ \lceil (p_k / \sum_{k=1}^{K} p_k)(N - \sum_{k=1}^{K} N_k^{\min}) \rceil \right]^+, \ \forall k$
repeat
  $\Delta P_k \leftarrow (7), \forall k$
  $\Delta P_k \leftarrow \infty, \ \forall k \in \{k \mid L_k = N_k^{\min}\}$
  $\tilde{k} \leftarrow \mathrm{argmin}_{k \in \mathcal{K}} \Delta P_k$
  $L_{\tilde{k}} \leftarrow L_{\tilde{k}} - 1$
until $\sum_{k=1}^{K} L_k = N$
```

Algorithm 2 Initialization of Subcarrier Assignments

```
$\mathcal{N} \leftarrow \{1, \ldots, N\}$
$\mathcal{S}_k \leftarrow \emptyset$
$L \leftarrow (\prod_{k=1}^{K} L_k)^{\frac{1}{K}}$
$l_k \leftarrow \lceil L_k / L \rceil, \ \forall\, k$
repeat
  for each $k \in \mathcal{K}$ do
    if $|\mathcal{S}_k| < L_k$ then
      $\hat{l}_k \leftarrow \min(L_k - |\mathcal{S}_k|, l_k)$
      $\mathcal{T} \leftarrow \{\hat{l}_k$ subcarriers with the largest $G_k[n], n \in \mathcal{N}\}$
      $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \mathcal{T}$
      $\mathcal{N} \leftarrow \mathcal{N} \setminus \mathcal{T}$
    end if
  end for
until $\mathcal{N} = \emptyset$
$(\lambda_k, \{r_k[n]\}) \leftarrow \mathrm{SUWF}_k(\mathcal{S}_k, R_k), \ \forall\, k, \ \forall\, n$
$\mathcal{S}_k \leftarrow \{n \mid r_k[n] > 0\}, \forall\, k$
```

of the arithmetic average of CNRs over subcarriers used in [4], the geometric average g_k is adopted. The water level λ_k is deduced while $\mathcal{S}_k$ contains all subcarriers. The average power p_k over all subcarriers for user k is obtained then, while it may be negative. The cardinalities of subcarrier assignments, referred to as L_k, are first set equal to the minimal numbers of subcarriers needed by the corresponding users, denoted by $N_k^{\min}$. Then they proportionally increase according to users' average powers p_k, where $[x]^+ = \max(x, 0)$.

Due to the ceiling function in last step, the sum of L_k may be larger than N. To solve this problem, a procedure similar to BABS in [4] is applied. In each iteration, the cardinality of the subcarrier assignment decreases by one only for the user, who has the smallest power increment while reducing L_k by one, determined by

$$\Delta P_k = \frac{L_k}{g_k}(2^{\frac{R_k}{L_k}} - 1) - \frac{L_k - 1}{g_k}(2^{\frac{R_k}{L_k - 1}} - 1). \qquad (7)$$

When $L_k = N_k^{\min}$ holds, ΔP_k must be set to ∞. This procedure finishes when the sum of cardinalities is equal to N. Then performance loss is now kept small, when the efficient updating approach is used after excluding a subcarrier from a subcarrier assignment.

B. Initialization of subcarrier assignments

In Algorithm 2, subcarriers with large CNRs are assigned to users by taking the estimated cardinalities into account. This reduces the performance impairment that can result from using the efficient updating approach after adding a subcarrier to a subcarrier assignment. With the geometric mean of the estimated cardinalities L, the number of subcarriers assigned to users in each iteration is determined and denoted by l_k. In each iteration, each user gets the l_k subcarriers with the largest CNRs from the set $\mathcal{N}$ of unassigned subcarriers. This step can be implemented by the order statistic algorithm [15] with the complexity of $\mathcal{O}(|\mathcal{N}|)$. However, in the last iteration for user k, the remaining number of subcarriers needed by user k, $L_k - |\mathcal{S}_k|$, may be smaller than l_k. Therefore, in such an iteration, only $L_k - |\mathcal{S}_k|$ subcarriers are selected from $\mathcal{N}$. This iterative process ends when set $\mathcal{N}$ is empty. Based on the obtained subcarrier assignments, the water levels and rates on subcarriers are determined by single-user water-filling (SUWF), see [11]. At last, the subcarrier assignments are updated to contain only the subcarriers with positive rates.

Algorithm 3 Successive Subcarrier Assignment

```
$\mathcal{N} \leftarrow \{1, \ldots, N\}$
repeat
  $V[n] \leftarrow (9), \ \forall\, n \in \mathcal{N}$
  $m \leftarrow \mathrm{argmin}_{n \in \mathcal{N}}\, V[n]$
  $\mathcal{N} \leftarrow \mathcal{N} \setminus \{m\}$
  $\mathcal{B} \leftarrow \{k \mid m \in \mathcal{S}_k\}$
  if $\mathcal{B} = \emptyset$ then
    $\Delta P^{(r)}(m) \leftarrow 0$
  else if $|\mathcal{S}_k| = N_k^{\min}, k \in \mathcal{B}$ then $\{\mathcal{B} = \{k\}\}$
    $\Delta P^{(r)}(m) \leftarrow \infty$
  else
    $\Delta P^{(r)}(m) \leftarrow \Delta P_k^{(r)}(m), k \in \mathcal{B}$
  end if
  $\mathcal{U} \leftarrow \{k \mid \lambda_k G_k[m] > 1\} \setminus \mathcal{B}$
  $\Delta P_k(m) \leftarrow P_k^{(a)}(m) + \Delta P^{(r)}(m), k \in \mathcal{U}$
  $\hat{k} \leftarrow \mathrm{argmin}_{k \in \mathcal{U}}\, \Delta P_k(m)$
  if $\Delta P_{\hat{k}}(m) < 0$ then
    $\lambda_k \leftarrow \lambda_k^{(r)}(m), k \in \mathcal{B}$
    $\mathcal{S}_k \leftarrow \mathcal{S}_k \setminus \{m\}, k \in \mathcal{B}$
    $\lambda_{\hat{k}} \leftarrow \lambda_{\hat{k}}^{(a)}(m)$
    $\mathcal{S}_{\hat{k}} \leftarrow \mathcal{S}_{\hat{k}} \cup \{m\}$
  end if
until $\mathcal{N} = \emptyset$
$P_k \leftarrow \mathrm{SWF}(\mathcal{S}_k, R_k), \ \forall\, k$
```

C. Successive subcarrier assignment

The above two algorithms provide a good starting point for utilizing the efficient updating technique given in Section III leading to small performance loss from reassigning subcarriers. Before doing so, let us consider how the CNR of one subcarrier varies over different users. When a CNR variation is small, it means that users have similar CNRs on this subcarrier. Improper assigning this subcarrier may probably happen. On the other hand, when some users or even only one user has relatively large CNR on one subcarrier, the CNR variation is very large. This subcarrier will be assigned to an inappropriate user with lower probability. Hence, subcarriers with large CNR variations over users should be considered first while reassigning subcarriers.

To evaluate the CNR variation over users on one subcarrier n, the variation coefficient of CNRs is defined in the following. With the water levels obtained from Algorithm 2, for each subcarrier n, we first define the subcarrier index $c_k[n]$ to be 1 if $\lambda_k G_k[n] > 1$, and $c_k[n] = 0$, otherwise. We call the users with subcarrier indices equal to 1 the active users on one subcarrier. The arithmetic average of CNRs on subcarrier n over active users is defined as

$$a[n] = \frac{1}{\sum_{l=1}^{K} c_l[n]} \sum_{k=1}^{K} c_k[n] \lambda_k G_k[n], \tag{8}$$

where the CNR $G_k[n]$ is normalized by the water level λ_k. Consequently, the variation coefficient of CNRs on subcarrier n over users is given by

$$V[n] = \frac{1}{a[n]} \sqrt{\sum_{k=1}^{K} c_k[n] |\lambda_k G_k[n] - a[n]|^2}, \tag{9}$$

which may change after one iteration in Algorithm 3.

In each iteration, the subcarrier with the largest variation coefficient, indexed by m, is processed. Excluding m from a subcarrier assignment results in a power increment, denoted by $\Delta P^{(r)}(m)$, see Section III. It may happen that subcarrier m is not used by any user currently, then $\Delta P^{(r)}(m)$ is equal to 0. If there exists user k using subcarrier m but it has only N_k^{min} subcarriers, $\Delta P^{(r)}(m)$ is set to ∞. The set $\mathcal{U}$ contains the active users not using subcarrier m. The power decrement, by adding m to the subcarrier assignments of those users, is referred to as $\Delta P_k^{(a)}(m)$ in Section III.

For user $k \in \mathcal{U}$, the power difference, by moving m from the subcarrier assignment of the user in $\mathcal{B}$ to $\mathcal{S}_k$, denoted by $\Delta P_k(m)$, is the sum of the power increment $\Delta P^{(r)}(m)$ and the power decrement $\Delta P_k^{(a)}(m)$. If the smallest power difference $\Delta P_{\hat{k}}(m)$ is negative, such a move is actually performed for user $\hat{k}$. At last given the newly derived $\mathcal{S}_k$, the transmission power is determined by the strict water-filling (SWF) [14], where the rate limit on one subcarrier is met.

V. Comparison and Complexity Analysis

In [6] the cardinalities are estimated only according to the required data rates. In Algorithm 1, the channel conditions and the required BERs are additionally considered and the iterative process dominates the complexity, which is not related to N. Its complexity is $\mathcal{O}(K^2)$ unlike the similar step in [4], which has complexity of $\mathcal{O}(KN)$, where $K \ll N$ normally holds.

In the initialization step given in [4] and some other works, a subcarrier is assigned to the user having the largest CNR. In our method, users with larger estimated cardinalities of subcarrier assignments are privileged to select out more subcarriers in each iteration, which implies that more subcarriers with higher CNRs may be initially assigned to those users. This more intelligent initialization guarantees tolerable performance loss while using the efficient updating technique given in Section III in the latter process. Therefore, it makes a better compromise between the data transmission requirements and the channel conditions. Since the order statistic algorithm can be applied, Algorithm 2 has complexity of $\mathcal{O}(KN)$.

Algorithm 3 successively adjusts the subcarrier assignments along subcarriers to reduce the total transmission power. The resulting power increments and decrements are obtained by using the technique from Section III, which may be also adopted to simplify other resource allocation methods, e.g., [2], [9]. Obviously, Algorithm 3 has complexity of $\mathcal{O}(KN)$. Moreover, in [9], subcarriers are randomly adjusted to be assigned to users over and over again untill no further improvement can be made. To avoid such a complex process, iterative arranging subcarriers in a descending order of their variation coefficients of CNRs makes one adjustment more effective. This is an important feature of our method. We therefore call the proposed method *iterative sorting and subcarrier assignment* (ISSA). The complexity of ISSA is $\mathcal{O}(KN)$.

TABLE I
Users in the simulation system

User type	Proportion	Rate (bits/OFDM symbol)	BER
Video user	10%	32	8.00E-5
Audio user	40%	8	3.67E-6
Data user	50%	16 (mean)	4.66E-7

VI. Simulation Results

In this section, our method is compared to two previous works by simulations. One is SUSI from [2], which has better performance than the methods from [5], [8], [9]. The other is RCG suggested in [4], which is well-known since it is fast.

The frequency selective channel is modeled as consisting of $Q = 16$ taps with an exponentially decaying profile, given by the impulse response $h(t) = \sum_{q=0}^{Q-1} e^{-q\beta} h_q \delta(t - qT)$. The taps h_q are jointly independent, circular symmetric, complex Gaussian distributed with zero mean and variance 1, T is the tap interval, q is the tap index, β is the decaying exponent, and t is the time index. The expected CNR on each subcarrier is set to 5 dB. We consider an OFDMA system with 64 subcarriers and 2 to 12 users for simulations, which can serve three types of users, see Table I. The rate of a data user is exponentially distributed with a maximum rate of 32 bits per OFDM symbol. The maximum sum rate of the system, 384 bits per OFDM symbol, may be achieved at $K = 12$. The simulation system

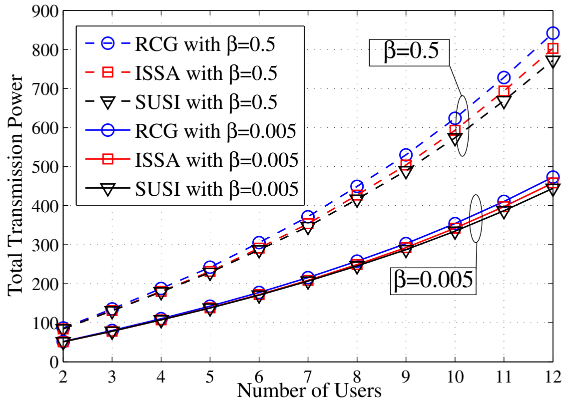

Fig. 1. Total transmission power vs. number of users for $\beta = 0.005$ and $\beta = 0.005$.

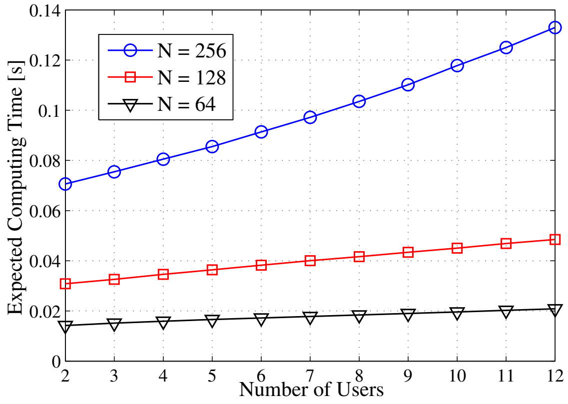

Fig. 2. Expected computing time vs. number of users with $\beta = 0.5$.

is implemented in MATLAB. A number of $500\,000$ channel samples are generated for each simulation.

In [5] it has been shown that the intensity of frequency selectivity over transmission band can impact the performance of resource allocation methods. In our channel model, the frequency selectivity becomes higher as β decreases. Fig. 1 shows the total transmission power against different numbers of users with different values of β. The decrement of total transmission power by using ISSA

$$\frac{\text{power by RCG} - \text{power by ISSA}}{\text{power by RCG}} \times 100\%$$

is up to 3.30% in the case of $\beta = 0.005$. When $\beta = 0.5$, the frequency selectivity becomes lower, this decrement is up to 4.65%. This means that ISSA always has better performance than RCG, unlike the method given in [5], which has worse performance than RCG when the frequency selectivity is low. Compared to SUSI, the increment of total transmission power by using ISSA is limited to 3.12% and 3.91% for $\beta = 0.005$ and $\beta = 0.5$, respectively. In our implementation, ISSA is almost 100% faster than SUSI. Compared to RCG, the increased computing time for ISSA decreases from 340% to 130% as K increases.

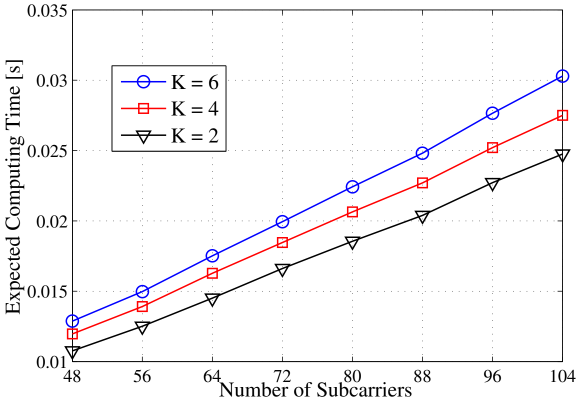

Fig. 3. Expected computing time vs. number of subcarriers with $\beta = 0.5$.

Fig. 2 and Fig. 3 show the expected computing time for ISSA against different combinations of N and K. It can be seen that the expected computing time is nearly linearly increasing in K and N. This validates the complexity analysis in Section V.

VII. CONCLUSION

In fast time-varying environments, efficient methods are required to perform power and rate allocation. To accelerate the resource allocation with small performance loss, we proposed a computationally efficient resource allocation method for OFDMA downlink. We introduced an efficient technique to calculate the power variations resulting from subcarrier reassignments. To benefit from this technique, a good starting point is first obtained in our initialization procedure, where subcarriers are initially assigned by taking into account the estimated cardinalities of subcarrier assignments for users. After that, CNR variation of subcarriers over users are used to accelerate the convergence behavior of the iterative subcarrier assignments. Simulations have shown that the proposed methods achieve an improved balance between performance and computing time compared to previous works.

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