This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE ICC 2009 proceedings

A generalised multi-receiver radio network and its decomposition into independent transmitter-receiver pairs: Simple feasibility condition and power levels in closed form

Virgilio Rodriguez, Rudolf Mathar
Institute for Theoretical Information Technology
RWTH Aachen
Aachen, Germany
email: vr@ieee.org, mathar@ti.rwth-aachen.de

***Abstract*—We consider a generalised multi-receiver radio network under a quality-of-service (QoS) constraint that involves generalised carrier-to-interference ratios. This model includes as special cases many well-known schemes, such as those discussed by Yates (JSAC, 13(7):1341-1348, 1995). A simple feasibility condition for the QoS targets, and a power-vector that yields those targets are given in closed form for the case in which additive noise is negligible and the key functions are homogeneous of degree one. The condition has the simple form, k_i <= q_i, where "i" identifies a terminal, "k_i" its desired QoS level, and "q_i" its quality of service when all power levels equal unity. If the feasibility condition is satisfied, the power levels P_i = k_i / q_i yield or exceed the desired levels of quality. The generalised multi-receiver network can be *conservatively* represented by an equivalent set of independent transmitter-receiver pairs, with q_i equal to the channel gain of the pair that represents "i". Macro-diversity and multiple-connection reception are some of the specific models discussed as examples.**

I. Introduction

In a telecommunication network, terminals often need a minimum quality-of-service (QoS) level, or there is one such level that is "optimal" in some sense. In the radio environment, QoS often depends on carrier-to-interference ratios (CIR), which are determined by the power levels of all transmitters. A fundamental question is whether there are allowable power levels that can produce the desired QoS targets, i.e. the targets are "feasible". If this question has an affirmative answer, another fundamental question immediately arises: which power levels can yield the desired QoS vector? All the feasible QoS vectors form the "capacity region". This information is relevant to the important "admission control" question: can the system accommodate a terminal wishing service at a certain QoS level, while continuing to satisfy the QoS requirements of the "incumbent" terminals?

The answers to these questions depend, in principle, on the details of the system. For a example, for a CDMA wireless communication system in which base stations "cooperate" (macro-diversity) and utilise maximal-ratio combining, [1] shows under certain approximation that if the sum of the target CIR's is less than the number of receivers these targets are feasible, and can be achieved through a decentralised power-adjustment algorithm. More recently, [2] presents a still simple but more sophisticated condition that — through a dependence on *relative* channel gains — sensibly adjusts itself to special channel states, such as when terminals are in range of only a subset of the receivers. However, neither [1] nor [2] characterises the power levels that can produce a (feasible) CIR vector.

A relevant strand of the scientific literature *indirectly* addresses the question of the power levels, by providing conditions for the convergence of a "greedy" power adjustment process in which terminals take turns, each "greedily" choosing a power level in order to achieve its desired QoS while taking the other power levels as fixed. This literature focuses on general models in which *all that is known* about the communication system is that certain mathematical functions have certain simple properties. This approach is important because its results apply to all practical systems that can be shown to posses the assumed properties. In particular, [3] shows that if the "interference functions" are non-negative, non-decreasing, and — in certain sense — (sub)homogeneous, greedy power adjustment converges to a unique vector, *provided* that the underlying QoS targets are *feasible*. Related work includes [4], which extends [5] to a "canonical class" of algorithms that can handle discrete power levels; [6] which focuses on opportunistic power adjustment for delay-tolerant traffic; and [7], which considers interference functions whose properties are similar but non-identical to those in [3]. However, none of these works provides explicit feasibility conditions for the QoS levels.

Most recently, through Banach's contraction mapping theory (an approach also pursued by [8]) [9] shows that if the function f_i that gives i's QoS-achieving power level when the other power levels are known and fixed — in addition to having properties similar to those of [3] and [7] — is sub-additive, then $f_i(\mathbf{1}) < 1$ for each i guarantees the convergence of the greedy power adjustment ($\mathbf{1}$ denotes the "all ones" vector). Since the f_i's inherently depend on the QoS targets, $f_i(\mathbf{1}) < 1$

978-1-4244-3435-0/09/$25.00 ©2009 IEEE

constitute a set of conditions for the feasibility of these targets (indeed, these conditions lead to the *new* results reported in [2]). The most significant difference between [9] and the related work, [10], is their degree of "abstraction". Following the approach of [3], [9] explores the convergence of a power adjustment process in which all the details of the system are "hidden" inside the adjusting functions f_i. By contrast, [10] explicitly considers such details as channel gains, number of receivers and QoS constraints. In principle, both of these models are equivalent, but the "lower-level view" of [10] provides certain details and insights not available under the "high-level-view" of [9]. However, none of the references mentioned so far characterise the actual power levels that yield a QoS vector known to be feasible.

The present work studies a general model of a multi-receiver radio network in which *all that is known* about the key mathematical functions is that they satisfy (besides non-negativity) two properties: (i) non-decreasing monotonicity and (ii) (a form of) homogeneity. The present model is similar to [10]'s, but [10] makes stronger assumptions on the interference functions (they are additionally "sub-additive"), utilises fixed-point theory, and does not characterise the limiting power vector. This model is sufficiently rich to include as special cases a plethora of models, such as all those discussed in [3]. Yet, in spite of its generality — and without utilising fixed-point theory or any advanced mathematics — the present work contributes *in closed form:* (i) a simple feasibility condition for the QoS targets, as well as (ii) a power vector that yields or exceeds those targets, when they are feasible. Furthermore, these results can be interpreted as a (conservative) decomposition of the generalised multi-receiver radio network into an equivalent set of independent transmitter-receiver pairs. In the next section, we state and further discuss our main results. Subsequently, we provide the formal definitions, derivations, and specific examples (which include the challenging scenarios of macro-diversity and multiple-connection reception [3]).

II. Discussion of main result

In this section, we *informally* state our main result, discuss and interpret it. More formal definitions, statements and derivations are given in subsequent sections[1].

A. Main result

There are N transmitters and K receivers. Associated with each transmitter, there is a K-vector of (generalised) carrier-to-interference ratios (gCIR). The gCIR for transmitter i at receiver k equals the quotient $P_i h_{i,k}/(\sigma_k + Y_{i,k})$ where $P_i h_{i,k}$ is the power received at k in the signal from i, σ_k denotes random noise and $Y_{i,k}$ denotes interference. $Y_{i,k} = \mathcal{Y}_{i,k}(\mathbf{P})$ with $\mathcal{Y}_{i,k}$ a general function of the power vector $\mathbf{P}$. A general function Q_i takes as input the vector of i's gCIR and returns i's QoS.

For example, under macro-diversity, i's QoS target is given by $P_i h_{i,1}/(Y_{i,1} + \sigma_1) + \cdots + P_i h_{i,K}/(Y_{i,K} + \sigma_K)$ with $Y_{i,k} = \sum_{n \neq i} P_n h_{n,k}$ [1]. Thus, $\mathcal{Y}_{i,k}(\mathbf{P}) = \sum_{n \neq i} P_n h_{n,k}$ and $Q_i(\mathbf{x}) = Q^{\text{MD}}(\mathbf{x}) = x_1 + \cdots + x_K$.

Let κ_i denote i's QoS target, and $q_i = Q_i(h_{i,1}/\mathcal{Y}_{i,1}(\mathbf{1}), \cdots, h_{i,K}/\mathcal{Y}_{i,K}(\mathbf{1}))$ with $\mathbf{1}$ denoting the appropriate "all ones" vector (notice that q_i only depends on *known* parameters and functions). Our main results are as follows:

> If the functions Q_i and $\mathcal{Y}_{i,k}$ are non-negative and non-decreasing, and additionally each $\mathcal{Y}_{i,k}$ is sub-homogeneous (i.e., $\mathcal{Y}_{i,k}(\lambda\mathbf{x}) \leq \lambda\mathcal{Y}_{i,k}(\mathbf{x})$), and each Q_i is super-homogeneous (i.e., $Q_i(\lambda\mathbf{x}) \geq \lambda Q_i(\mathbf{x})$), and random noise is negligible, then $\kappa_i \leq q_i \,\forall i$ implies that (i) each QoS target can be achieved, in particular, (ii) with the power levels $P_i^* = \kappa_i/q_i$.

B. Methodology

Our results are obtained by neglecting additive noise ("high SNR regime"), and replacing each interference function $\mathcal{Y}_{i,k}$ with a function that dominates it: $\mathcal{Y}_{i,k}(\mathbf{1})\,\|\mathbf{P}\|$ where $\|\cdot\|$ denotes the largest component of $\mathbf{P}$ (its "max-norm"). While our approach is *not* Shannon-theoretic, it is consistent with the deterministic model introduced by [11].

C. Core argument

- Since each Q_i is super-homogeneous, $Q_i(P_i h_{i,1}/(\mathcal{Y}_{i,1}(\mathbf{1})\,\|\mathbf{P}\|), \cdots, P_i h_{i,K}/(\mathcal{Y}_{i,K}(\mathbf{1})\,\|\mathbf{P}\|)) \geq (P_i/\|\mathbf{P}\|)) Q_i(h_{i,1}/\mathcal{Y}_{i,1}(\mathbf{1}), \cdots, h_{i,K}/\mathcal{Y}_{i,K}(\mathbf{1})) \equiv (P_i/\|\mathbf{P}\|)\, q_i$
- Therefore, $(P_i/\|\mathbf{P}\|)\, q_i \geq \kappa_i \implies Q_i(P_i h_{i,1}/(\mathcal{Y}_{i,1}(\mathbf{1})\,\|\mathbf{P}\|), \cdots, P_i h_{i,K}/(\mathcal{Y}_{i,K}(\mathbf{1})\,\|\mathbf{P}\|)) \geq \kappa_i$
- Lemma 1 establishes that any function f that is non-decreasing and sub-homogeneous (as defined below) satisfies $f(\mathbf{x}) \leq f(\|\mathbf{x}\|\mathbf{1}) \leq \|\mathbf{x}\| f(\mathbf{1})$; hence, each $\mathcal{Y}_{i,k}$ satisfies $\mathcal{Y}_{i,k}(\mathbf{P}) \leq \|\mathbf{P}\|\mathcal{Y}_{i,k}(\mathbf{1})$.
- Thus, since each Q_i is non-decreasing, $Q_i(P_i h_{i,1}/(\mathcal{Y}_{i,1}(\mathbf{1})\,\|\mathbf{P}\|), \cdots, P_i h_{i,K}/(\mathcal{Y}_{i,K}(\mathbf{1})\,\|\mathbf{P}\|)) \geq \kappa_i \implies Q_i(P_i h_{i,1}/\mathcal{Y}_{i,1}(\mathbf{P}), \cdots, P_i h_{i,K}/\mathcal{Y}_{i,K}(\mathbf{P})) \geq \kappa_i$ (i.e., the "true" QoS level exceeds κ_i)
- $\therefore$ $(P_i/\|\mathbf{P}\|) q_i \geq \kappa_i \implies Q_i(P_i h_{i,1}/(\mathcal{Y}_{i,1}(\mathbf{1})\,\|\mathbf{P}\|), \cdots, P_i h_{i,K}/(\mathcal{Y}_{i,K}(\mathbf{1})\,\|\mathbf{P}\|)) \geq \kappa_i \implies Q_i(P_i h_{i,1}/\mathcal{Y}_{i,1}(\mathbf{P}), \cdots, P_i h_{i,K}/\mathcal{Y}_{i,K}(\mathbf{P})) \geq \kappa_i$
- In conclusion, if $\forall i$, $P_i/\|\mathbf{P}\| \geq \kappa_i/q_i$, each κ_i is reached or exceeded
- But for any $\mathbf{P}$, $P_i \leq \|\mathbf{P}\| \,\forall i$, by definition. Therefore, *no* power vector can satisfy $P_j/\|\mathbf{P}\| \geq \kappa_j/q_j > 1$for some j
- With $\hat{\kappa} := (\kappa_1/q_1, \cdots, \kappa_N/q_N) := (\hat{\kappa}_1, \cdots, \hat{\kappa}_K)$, $\hat{\kappa}_i = \kappa_i/q_i \leq 1 \,\forall i \implies \|\hat{\kappa}\| \leq 1 \implies \hat{\kappa}_i/\|\hat{\kappa}\| \geq \hat{\kappa}_i \,\forall i \therefore \mathbf{P}^* = \hat{\kappa}$ satisfies $P_i/\|\mathbf{P}\| \geq \kappa_i/q_i \,\forall i$ and yields or exceeds the desired QoS levels

D. Meaning of key quantity

Each $\mathcal{Y}_{i,k}(\mathbf{1})$ is the unit-power interference; that is, a constant equal to the amount of interference experienced by i at receiver k when $\mathbf{P} = \mathbf{1}$. Since $h_{i,k}$ equals the power from i received at k when $P_i = 1$, then $h_{i,k}/\mathcal{Y}_{i,k}(\mathbf{1})$ can be viewed as a unit-power carrier-to-interference ratio, and q_i the corresponding unit-power QoS. Alternatively, ratios of the

[1]Some may prefer to (re-)read this section *after* reading all others.

form $h_{i,k}/\mathcal{Y}_{i,k}(\mathbf{1})$ can be viewed as "scaled" channel gains, and q_i as an 'effective' channel gain.

E. Network decomposition

Our results admit the following interpretation. Consider the (virtual) "network" in which there are N independent (orthogonal) transmitter-receiver pairs. Each transmitter has a power limit $\bar{P}_i$ that equals the (average) noise power at its receiver, σ_i which can be set to one choosing the right units. Let the channel gain of transmitter i be $h_i := q_i$. The maximal signal-to-noise ratio (SNR) that i can achieve is $\bar{P}_i h_i/\sigma_i = h_i = q_i$. Thus, i's desired SNR, κ_i is achievable provided it does not exceed q_i. Furthermore, if $\kappa_i/q_i \leq 1$ then $P_i = \kappa_i/q_i$ is feasible ($\leq \bar{P}_i = 1$), and yields an SNR exactly equal to κ_i.

Thus, each generalised multi-receiver network with the assumed properties can be *conservatively* associated with a much simpler "dual" network consisting of independent transmitter-receiver pairs.

III. QoS constraint for a Generalised multi-receiver Radio network

A. Function Properties

1) Definitions:

Definition 1: A non-negative function $f : \Re^M \to \Re$ is *weakly-non-decreasing* if it satisfies (1). f is positively quasi-sub-homogeneous (of degree one) if it satisfies (2); and f is positively sub-homogeneous (of degree one) if it satisfies (3). If the inequalities (2) and (3) are reversed, the prefix "super" replaces "sub".

$$f(\mathbf{x}) \leq f(\|\mathbf{x}\|_\infty \mathbf{1}_M) \qquad \forall \mathbf{x} \in \Re^M \tag{1}$$
$$f(\lambda \mathbf{1}) \leq \lambda f(\mathbf{1}) \qquad \forall \lambda \in \Re_+ \tag{2}$$
$$f(\lambda \mathbf{x}) \leq \lambda f(\mathbf{x}) \qquad \forall \mathbf{x} \in \Re^M\ \lambda \in \Re_+ \tag{3}$$

Remark 1: For $\mathbf{x} \in \Re^N$, $\|\mathbf{x}\|_\infty$ denotes the "max-norm" of $\mathbf{x}$; that is, the largest absolute value of $\mathbf{x}$'s components. In (1), $\|\mathbf{x}\|_\infty \mathbf{1}_M$ is obtained by replacing each component of $\mathbf{x}$ with $\|\mathbf{x}\|_\infty$. Thus, $f(\mathbf{x}) \leq f(\|\mathbf{x}\|_\infty \mathbf{1}_M)$ is a very mild form of monotonicity.
Below, unless otherwise indicated, homogeneity is positive of degree one.

2) Key Lemma:

Lemma 1: Let $f : \Re^M \to \Re$, and $\mathbf{1}_M$ denote the "all ones" M-vector (the sub-index may be omitted). If f satisfies both conditions (1) and (2) then $f(\mathbf{x}) \leq f(\|\mathbf{x}\|_\infty \mathbf{1}) \leq \|\mathbf{x}\|_\infty f(\mathbf{1})$.

Proof:

By (1), $f(\mathbf{x}) \leq f(\|\mathbf{x}\|_\infty \mathbf{1}_M)$.
By (2), $f(\|\mathbf{x}\|_\infty \mathbf{1}_M) \leq \|\mathbf{x}\|_\infty f(\mathbf{1}_M)$. ■

B. General model

Consider a system with N transmitters and K receivers, such as in the reverse-link of a cellular system. A generalised quality-of-service (QoS) index for terminal i is defined as:

$$Q_i(\rho_{i,1}, \cdots, \rho_{i,K}) \geq \kappa_i \tag{4}$$

where $Q_i : \Re_+^K \to \Re$ is a non-negative positively super-homogeneous function (i.e., $Q_i(\lambda \mathbf{x}) \geq \lambda Q_i(\mathbf{x})$ for all $\mathbf{x} \in \Re^K$ and $\lambda \in \Re_+$), and

$$\rho_{i,k} := \frac{P_i h_{i,k}}{Y_{i,k} + \sigma_k} \tag{5}$$

where

- P_i is the transmission power level of terminal i,
- $h_{i,k}$ is the channel gain in the signal from terminal i arriving at receiver k,
- σ_k is the (average) power of the additive random noise at receiver k, and
- $Y_{i,k} := \mathcal{Y}_{i,k}(\mathbf{P})$ where $\mathbf{P}$ is the vector of the transmission power levels of each terminal, and $\mathcal{Y}_{i,k}$ denotes a weakly non-decreasing (1), positively quasi-sub-homogeneous (2) function that yields the interfering power experienced by transmitter i at receiver k.

C. Examples

The model of subsection III-B encompasses numerous interesting scenarios as special cases, including all the examples cited by [3].

For all the examples below,

$$\mathcal{Y}_{i,k}(\mathbf{P}) := \sum_{\substack{n=1 \\ n \neq i}}^{N} P_n h_{n,k} \tag{6}$$

(6) is obviously homogeneous.

1) Macro-diversity (MD): Under macro-diversity, the relevant QoS constraint is [1] :

$$\frac{P_i h_{i,1}}{Y_{i,1} + \sigma_1} + \cdots + \frac{P_i h_{i,K}}{Y_{i,K} + \sigma_K} \geq \kappa_i \tag{7}$$

Thus, $Q_i^{\mathrm{MD}}(x_1, \cdots, x_K) = Q^{\mathrm{MD}}(\mathbf{x}) = x_1 + \cdots + x_K$ which is obviously homogeneous.

2) Multiple-connection reception (MC): Under MC, user i must maintain an acceptable SIR κ_i at d_i distinct receivers. The system "assigns" i to the d_i "best" receivers. For $\mathbf{x} \in \Re^N$ and $M \in \mathbb{N}$, $M < N$, let $\max(x; M)$ denote the Mth largest *absolute value* of the components of $\mathbf{x}$. The QoS requirements of i can be written as [3]:

$$\max\left(\left(\frac{P_i h_{i,1}}{Y_{i,1} + \sigma_1}, \cdots, \frac{P_i h_{i,K}}{Y_{i,K} + \sigma_K}\right); d_i\right) \geq \kappa_i \tag{8}$$

Thus, $Q_i^{\mathrm{MC}}(x_1, \cdots, x_K) = Q^{\mathrm{MC}}(\mathbf{x}) = \max((x_1, \cdots, x_K); d_i)$, which is evidently homogeneous.

With $d_i = 1$, the MC scheme becomes the minimum power assignment (MPA) [3].

3) Fixed base-station assignment (FA): Let k_i denote the index of the base station receiver to which i has been assigned. The corresponding QoS constraint can be written as

$$\rho_{i,k_i} = \frac{P_i h_{i,k_i}}{Y_{i,k_i} + \sigma_{k_i}} \geq \kappa_i \tag{9}$$

Thus, $Q_i^{\mathrm{FA}}(x_1, \cdots, x_K) = x_{k_i}$. This function simply "picks" the component of σ that corresponds to k_i. With $e^i \in \Re^K$

denoting the unit vector whose only non-zero component is the k_ith component, then $Q_i^{\text{FA}}(x_1,\cdots,x_K)$ can be written in scalar product form as : $\sum_{k=1}^{K} e_k^i x_k$. Q_i^{FA} is evidently homogeneous in $\mathbf{x}$.

IV. Capacity results

A. System simplification

Lemma 2: Let $q_i := Q_i(h_{i,1}/\mathcal{Y}_{i,1}(\mathbf{1}),\cdots,h_{i,K}/\mathcal{Y}_{i,K}(\mathbf{1}))$. Suppose that (i) $\sigma_k \approx 0\ \forall k$, (ii) the functions Q_i and $\mathcal{Y}_{i,k}$ are non-negative and weakly non-decreasing, and, additionally, (iii) each $\mathcal{Y}_{i,k}$ is quasi-sub-homogeneous, and (iv) each Q_i is super-homogeneous. If a power vector $\mathbf{P}$ satisfies $(P_i/\|\mathbf{P}\|)q_i \geq \kappa_i\ \forall i$ then $Q_i(P_ih_{i,1}/\mathcal{Y}_{i,1}(\mathbf{P}),\cdots,P_ih_{i,K}/\mathcal{Y}_{i,K}(\mathbf{P})) \geq \kappa_i$.

Proof:

$Q_i(P_ih_{i,1}/(\|\mathbf{P}\|\,\mathcal{Y}_{i,1}(\mathbf{1})),\cdots,P_ih_{i,K}/(\|\mathbf{P}\|\,\mathcal{Y}_{i,K}(\mathbf{1}))) \geq (P_i/\|\mathbf{P}\|)q_i$ (super-homogeneity); thus, $(P_i/\|\mathbf{P}\|)q_i \geq \kappa_i \implies Q_i(P_ih_{i,1}/(\|\mathbf{P}\|\,\mathcal{Y}_{i,1}(\mathbf{1})),\cdots,P_ih_{i,K}/(\|\mathbf{P}\|\,\mathcal{Y}_{i,K}(\mathbf{1}))) \geq \kappa_i$.

By Lemma 1, $\mathcal{Y}_{i,k}(\mathbf{P}) \leq \|\mathbf{P}\|\mathcal{Y}_{i,k}(\mathbf{1})$; thus, by monotonicity of Q_i, $Q_i(P_ih_{i,1}/(\|\mathbf{P}\|\,\mathcal{Y}_{i,1}(\mathbf{1})),\cdots,P_ih_{i,K}/(\|\mathbf{P}\|\,\mathcal{Y}_{i,K}(\mathbf{1}))) \geq \kappa_i \implies Q_i(P_ih_{i,1}/\mathcal{Y}_{i,1}(\mathbf{P}),\cdots,P_ih_{i,K}/\mathcal{Y}_{i,K}(\mathbf{P})) \geq \kappa_i$. ■

Thus, by Lemma 2, one can reach all the QoS targets through a power vector $\mathbf{P}$ that satisfies the system of inequalities:

$$\frac{1}{\|\mathbf{P}\|}\begin{bmatrix} P_1 \\ \vdots \\ P_N \end{bmatrix} \geq \begin{bmatrix} \kappa_1/q_1 \\ \vdots \\ \kappa_N/q_N \end{bmatrix} := \begin{bmatrix} \hat{\kappa}_1 \\ \vdots \\ \hat{\kappa}_N \end{bmatrix} \tag{10}$$

or, in vector notation, $\mathbf{P}/\|\mathbf{P}\| \geq \hat{\kappa}$.

B. A feasibility condition and a closed-form solution

Lemma 3: Let $a \in \Re_+^N, \mathbf{x} \in \Re^N$ and $|\mathbf{x}| = (|x_1|,\cdots,|x_N|)$. (i) The set of inequalities $|x_i|/\|\mathbf{x}\|_\infty \geq a_i$ has a solution if and only if $\|a\|_\infty \leq 1$. (ii) If $\|a\|_\infty \leq 1$ then $\mathbf{x}^* = \mathbf{a}$ satisfies $|x_i|/\|\mathbf{x}\|_\infty \geq a_i\ \forall i$.

Proof:

(i) For any $\mathbf{y} \in \Re^N$, $\|\mathbf{y}\|_\infty$ is the largest absolute value of the components of $\mathbf{y}$. $\therefore \|a\|_\infty > 1 \implies \exists j$ such that $a_j > 1$. But $|x_i| \leq \|\mathbf{x}\|_\infty\ \forall i$. Therefore, no $\mathbf{x} \in \Re^N$ can satisfy $|x_j|/\|\mathbf{x}\|_\infty \geq a_j > 1$.

(ii) $\|a\|_\infty \leq 1 \implies a_i/\|\mathbf{a}\|_\infty \geq a_i\ \forall i$. Thus, $\mathbf{x}^* = \mathbf{a}$ does satisfy $|x_i|/\|\mathbf{x}\|_\infty \geq a_i$. ■

Proposition 1: Let $\hat{\kappa} = (\hat{\kappa}_1,\cdots,\hat{\kappa}_N) \equiv (\kappa_1/q_1,\cdots,\kappa_N/q_N)$. (10) has a solution if and only if $\|\hat{\kappa}\| \leq 1$. If $\|\hat{\kappa}\| \leq 1$ then $\mathbf{P} := \hat{\kappa}$ satisfies (10).

Proof: See Lemma 3. ■

Remark 2: If $\|\hat{\kappa}\| = 1$ then setting $\mathbf{P}$ equal to (any multiple of) $\hat{\kappa}$ evidently satisfies (10) as an equality.

C. Examples

In all the examples of subsection III-C, $\mathcal{Y}_{i,k}$ is given by (6). Thus,

$$\mathcal{Y}_{i,k}(\mathbf{1}) = \sum_{\substack{n=1\\n\neq i}}^{N} h_{n,k} \equiv \sum_{n=1}^{N} h_{n,k} - h_{i,k} := H_k - h_{i,k} \tag{11}$$

with $H_k := \sum_{n=1}^{N} h_{n,k}$.

Thus,

$$q_i = Q_i\left(\frac{h_{i,1}}{H_1 - h_{i,1}},\cdots,\frac{h_{i,K}}{H_K - h_{i,K}}\right) \tag{12}$$

For macro-diversity

$$q_i^{\text{MD}} = \sum_{k=1}^{K} \frac{h_{i,k}}{H_k - h_{i,k}} \tag{13}$$

For multiple-connection reception, the condition becomes

$$q_i^{\text{MC}} = \max\left(\left(\frac{h_{i,1}}{H_1 - h_{i,1}},\cdots,\frac{h_{i,K}}{H_K - h_{i,K}}\right);d_i\right) \tag{14}$$

and for fixed assignment, the condition reduces to

$$q_i^{\text{FA}} = \frac{h_{i,k_i}}{H_{k_i} - h_{i,k_i}} \tag{15}$$

From (13), (14) and (15) one can directly obtain the feasibility condition ($\kappa_i \leq q_i$), and corresponding power vector ($P_i = \kappa_i/q_i$).

D. Macro-diversity discussed further

1) Conditions obtained in other works: The analysis of [2] yields the feasibility condition:

$$\sum_{\substack{n=1\\n\neq i}}^{N} \kappa_n g_{n,k} < 1\,\forall i,k \tag{16}$$

with $g_{i,k} = h_{i,k}/h_i$ and $h_i = \sum_k h_{i,k}$. The left side of (16) is simply a weighted sum of $N-1$ QoS targets, and the weights are relative channel gains.

Reference [1] provides for all cases the condition

$$\sum_{n=1}^{N} \kappa_n < K \tag{17}$$

2) Macro-diversity formulae compared: If all $h_{i,k}$ are of the same order of magnitude $q_i^{\text{MD}} \approx \sum_{k=1}^{K} 1/(N-1) = K/(N-1)$, which leads to the feasibility condition:

$$\kappa_i \leq K/(N-1) \tag{18}$$

This implies that

$$\sum_{k=1}^{N} \kappa_i \leq \frac{N}{N-1}K \tag{19}$$

Condition (16) also simplifies under symmetry: if $h_{i,k} \approx h_{i,m}$ for each i,k,m (each transmitter is "equidistant" to the receivers) then $g_{i,k} \approx 1/K$, and (16) yields

$$\sum_{\substack{n=1\\n\neq i}}^{N} \kappa_n < K\ \forall i,k \tag{20}$$

which — assuming that $\kappa_N \leq \kappa_n \forall n$ — reduces to:

$$\sum_{n=1}^{N-1} \kappa_n < K \tag{21}$$

The 3 feasibility conditions above need *not* coincide because they have been derived under different sets of (simplifying)

Table I
MACRO-DIVERSITY FORMULAE UNDER SYMMETRY

Herein	Rodriguez08	Hanly96
$\sum_{k=1}^{N} \kappa_i \leq KN/(N-1)$	$\sum_{n=1}^{N-1} \kappa_n < K$	$\sum_{n=1}^{N} \kappa_n < K$

assumptions. Yet, as shown in Table I, the 3 conditions seem mutually consistent, at least under certain symmetry assumptions. However, as extensively discussed in [2], condition (16) has some major advantages over (17), including an ability to "adjust itself" to the *realistic* situation in which a transmitter can only be "heard" by a subset of the receivers, for which (17) is inappropriate. In principle, (16), which limits the sum of the QoS targets, seems more flexible than $\kappa_i \leq q_i^{\text{MD}}$ (q_i^{MD} given by (13)), which imposes an individual limit on each target. This fact is unsurprising, because, (16) exploits the sub-additivity of the macro-diversity interference function, whereas (13) is obtained by viewing macro-diversity as as a special case of a general network model in which interference functions need *not* be sub-additive (i.e., *without* considering some of the special structure of the macro-diversity model).

3) A numerical example: Suppose that transmitters 1, 2 and 3 are located at (0,0), (-1,0) and (1,0) respectively; and receivers 1 and 2 are at (0,-1), and (0,1) respectively. Thus, with $d_{i,k}$ denoting the distance between transmitter i and receiver k, $d_{1,k} = 1$ and $d_{2,k} = d_{3,k} = \sqrt{2}$ for $k \in \{1,2\}$. Further, assume that $h_{i,k} \propto d_{i,k}^{-2}$ so that, for $k \in \{1,2\}$, $h_{1,k} \propto 1$ and $h_{2,k} = h_{3,k} \propto 1/2$.

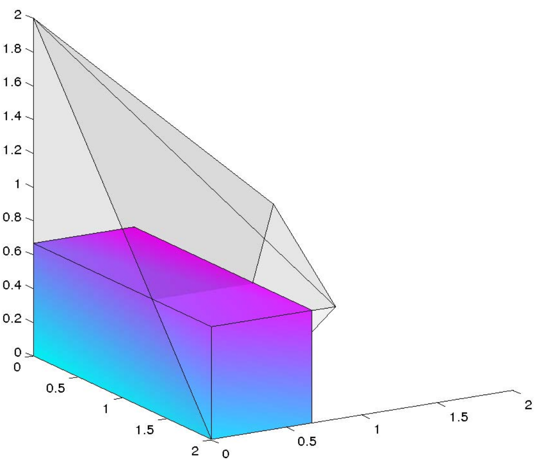

Figure 1. QoS achievable region for a simple macro-diversity system

Applying (13), $q_1 = 2(1)/(1/2+1/2) = 2$ and $q_2 = q_3 = 2(1/2)/(1+1/2) = 2/3$. This leads to achievable region: $\kappa_1 \leq 2$, $\kappa_2 \leq 2/3$, $\kappa_3 \leq 2/3$, represented by the "cuboid" shown in figure 1. Also shown is the shadow of the region obtained by applying (20) with $K = 2$, which is discussed further in [2]. This latter region seems "bigger", although it does *not* strictly contain the cuboid (recall the discussion in subsection IV-D2).

ACKNOWLEDGEMENT

Financial support from the UMIC project is gratefully acknowledged.

REFERENCES

[1] S. V. Hanly, "Capacity and power control in spread spectrum macrodiversity radio networks," *Communications, IEEE Transactions on*, vol. 44, no. 2, pp. 247–256, Feb 1996.
[2] V. Rodriguez, R. Mathar, and A. Schmeink, "Capacity and power control in spread spectrum macro-diversity radio networks revisited," in *IEEE Australasian Telecommunications Networking and Application Conference*, 2008.
[3] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Communications*, vol. 13, pp. 1341–1347, Sept. 1995.
[4] K. K. Leung, C. W. Sung, W. S. Wong, and T. Lok, "Convergence theorem for a general class of power-control algorithms," *Communications, IEEE Transactions on*, vol. 52, no. 9, pp. 1566–1574, Sept. 2004.
[5] C. W. Sung and W. S. Wong, "A distributed fixed-step power control algorithm with quantization and active link quality protection," *Vehicular Technology, IEEE Transactions on*, vol. 48, pp. 553–562, Mar 1999.
[6] C. W. Sung and K.-K. Leung, "A generalized framework for distributed power control in wireless networks," *Information Theory, IEEE Transactions on*, vol. 51, no. 7, pp. 2625–2635, July 2005.
[7] M. Schubert and H. Boche, *QoS-Based Resource Allocation and Transceiver Optimization*, vol. 2 of *Foundations and Trends in Communications and Information Theory*. Hanover, MA, USA: Now Publishers Inc., 2005.
[8] C. J. Nuzman, "Contraction approach to power control, with non-monotonic applications," in *Global Telecommunications Conference, IEEE*, pp. 5283–5287, 26-30 Nov. 2007.
[9] V. Rodriguez, R. Mathar, and A. Schmeink, "Asymptotic stability and capacity results for a broad family of power adjustment rules: Expanded discussion," tech. rep., January 2009. arXiv:0901.0573.
[10] V. Rodriguez, R. Mathar, and A. Schmeink, "Generalised multi-receiver radio network: Capacity and asymptotic stability of power control through Banach's fixed-point theorem," in *Wireless Communications and Networking, IEEE*, 2009. To appear.
[11] A. El Gamal and M. Costa, "The capacity region of a class of deterministic interference channels (corresp.)," *Information Theory, IEEE Transactions on*, vol. 28, pp. 343–346, Mar 1982.