Power Allocation for Broadcasting in Multiuser OFDM Systems with Sublinear Complexity

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***Abstract*—It has been shown that adaptive power and rate allocation for multiuser orthogonal frequency multiplexing (OFDM) improves the system performance significantly. In this paper, the resource allocation that aims at minimizing the total transmission power under certain data transmission constraints is considered. First, the power variation for single-user water-filling while changing the subcarrier assignment is derived. Based on this, a class of methods for multiuser resource allocation is proposed. The presented methods, consisting of tactical processes, can achieve a good balance of computational complexity and performance. Compared to previous works, simulations show that our methods have comparable or better performance and that the computing time for the proposed methods is approximately sublinearly increasing in the number of users K and the number of subcarriers N.**

I. Introduction

OFDM is a very promising effective multicarrier solution for the broadband wireless transmission [1]. Adapting to time-varying channel characteristics, multiuser OFDM can employ different modulation schemes over subcarriers to take the advantage of channel diversity among users in different locations.

One resource allocation problem for multiuser OFDM is to maximize the performance margin while satisfying individual data transmission requirements of users [2]. The optimal solution is provided in [3] and near-optimal solutions are given in [4], [5] at the expense of high complexity. Methods suggested in [6], [7], [8], [9], [10], [11] have low complexity while performance loss cannot be avoided.

By inheriting the basic scheme in [9], the method in [12] assigns subcarriers group-wise to reduce the complexity while the performance loss is enlarged. In this paper, we use a similar idea of initialization as in [9]. Meanwhile, better criteria have been developed to sort subcarriers, more efficient approaches have been investigated to update the transmission power while varying subcarrier assignments and more tactical processes have been devised to assign subcarriers. Such improvements lead to much lower complexity and even better performance than the method in [9] when K/N is relatively large.

The remainder of this paper is organized as follows. Section II presents the system model and the problem formulation. In Section III, we analyze and quantify the power variation for single-user water-filling while changing the subcarrier assignment for one user. Based on this analysis, heuristic methods for multiuser resource allocation are addressed in Section IV. The performance and computing time of the proposed methods are thoroughly measured in Section V. Finally, the content of this paper is concluded.

This work was supported by the UMIC research centre at RWTH Aachen University in Germany.

II. System Model And Problem Formulation

We consider wireless broadcasting transmission in multiuser OFDM systems with K users and N subcarriers. Perfect channel knowledge is available at both the transmitter and the receiver. The transmissions of different users are subject to independent frequency selective fading and resource allocation is performed at the base station and the users. Each user requires individual data rate R_k and bit error rate BER_k. M-ary QAM is employed in the considered system.

Let $G_k[n]$ denote the channel-to-noise ratio (CNR) of subcarrier n multiplied with $3\left[Q^{-1}(\frac{\mathrm{BER}_k}{4})\right]^{-2}$ for user k [13]. The power and rate allocated on subcarrier n for user k are denoted by $P_k[n]$ and $r_k[n]$, respectively. As shown in [1] the following relation holds

$$r_k[n] = \log_2(1 + P_k[n]G_k[n]). \tag{1}$$

The maximal allowed rate on each subcarrier is M bits per OFDM symbol. Each subcarrier is allowed to be used by at most one user at a specific time.

The aim is to minimize the total transmission power while satisfying the data rate and BER requirement of users. In mathematical terms it is represented by

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} P_k[n] \qquad (2)\\
\text{subject to} \quad & \sum_{n=1}^{N} r_k[n] \geq R_k, \quad \forall\, k,\\
& 0 \leq r_k[n] \leq M, \quad \forall\, k, \forall\, n,\\
& \sum_{n=1}^{N} r_k[n] r_l[n] = 0, \ \forall\, k, l, k \neq l.
\end{aligned}$$

The assigned index $c_k[n]$ is 1 if $r_k[n] > 0$ holds, otherwise $c_k[n] = 0$. The subcarrier assignment for user k is denoted by the set $\mathcal{S}_k = \{n \mid c_k[n] = 1\}$ with cardinality $s_k = \sum_{n=1}^{N} c_k[n]$. The transmission power for user k is $P_k = \sum_{n=1}^{N} P_k[n]$.

III. Power Variation by Changing A Subcarrier Assignment

The transmission power for an user k may increase or decrease by varying its subcarrier assignment. In this section, we investigate the power variation after removing a subcarrier from the subcarrier assignment or substituting a subcarrier in the subcarrier assignment with another one for user k.

Given the subcarrier assignment $\mathcal{S}_k$, the optimal power and rate allocation for user k is derived by water-filling [1]. It is assumed that all subcarriers in $\mathcal{S}_k$ can be used, which implies that the rate and power allocated on any subcarrier $n \in \mathcal{S}_k$

$$r_k[n] = \log_2(\lambda_k G_k[n]), \quad (3)$$

$$P_k[n] = \lambda_k - \frac{1}{G_k[n]} \quad (4)$$

are positive, where λ_k is the water level determined by

$$\lambda_k = 2^{\frac{R_k}{s_k}} \left(\prod_{n \in \mathcal{S}_k} \frac{1}{G_k[n]} \right)^{\frac{1}{s_k}}.$$

The transmission power for user k is

$$P_k = s_k \lambda_k - \sum_{n \in \mathcal{S}_k} \frac{1}{G_k[n]}.$$

After removing subcarrier m from $\mathcal{S}_k$, the subcarrier assignment is $\mathcal{S}_k \setminus \{m\}$. Then the water level becomes

$$\lambda_k^{(r)}(m) = \lambda_k (\lambda_k G_k[m])^{\frac{1}{s_k - 1}} \quad (5)$$

and the new transmission power is

$$P_k^{(r)}(m) = (s_k - 1)\lambda_k^{(r)}(m) - \sum_{n \in \mathcal{S}_k \setminus \{m\}} \frac{1}{G_k[n]}.$$

The power variation after such removing is the difference between the power increment on the still activated subcarriers and the power on the removed subcarrier, shown as

$$\begin{aligned} \Delta P_k^{(r)}(m) &= P_k^{(r)}(m) - P_k \\ &= (s_k - 1)(\lambda_k^{s_k} G_k[m])^{\frac{1}{s_k - 1}} - s_k \lambda_k + \frac{1}{G_k[m]} \\ &= (s_k - 1) \underbrace{\lambda_k (2^{\frac{r_k[m]}{s_k - 1}} - 1)}_{\Delta P_k[n]} - \underbrace{(\lambda_k - \frac{1}{G_k[m]})}_{P_k[m]}, \end{aligned} \quad (6)$$

where $P_k[m]$ and $r_k[m]$ are the power and rate previously allocated on subcarrier m and $\Delta P_k[n]$ is the power increment on subcarrier $n \in \mathcal{S}_k \setminus \{m\}$ after increasing $r_k[n]$ by $\Delta r_k[n] = \frac{r_k[m]}{s_k - 1}$. This power increment transforms to

$$\begin{aligned} \Delta P_k[n] &= \frac{1}{G_k[n]}(2^{r_k[n] + \Delta r_k[n]} - 1) - \frac{1}{G_k[n]}(2^{r_k[n]} - 1) \\ &= \lambda_k (2^{\Delta r_k[n]} - 1), \end{aligned}$$

where (3) is used in the last line.

After removing m from $\mathcal{S}_k$, it can be seen from (6) that the rate previously allocated on subcarrier m is equally distributed on subcarriers in $\mathcal{S}_k \setminus \{m\}$. Obviously, the induced power differences on these subcarriers are the same and positive. The sum power increment $\Delta P_k^{(r)}(m)$ can therefore be computed efficiently, where only one exponential operation is required.

With the same deduction as for (6), we can obtain the power difference by substituting subcarrier $m \in \mathcal{S}_k$ with subcarrier $j \notin \mathcal{S}_k$, denoted by

$$\begin{aligned} \Delta P_k^{(s)}(m,j) &= s_k \lambda_k \left(\frac{G_k[m]}{G_k[j]} \right)^{\frac{1}{s_k}} - s_k \lambda_k - \frac{1}{G_k[j]} + \frac{1}{G_k[m]} \\ &= s_k \underbrace{\lambda_k (2^{\frac{r_k[j] - r_k[m]}{s_k}} - 1)}_{\Delta P_k[n]} + \underbrace{(\lambda_k - \frac{1}{G_k[j]})}_{P_k[j]} - \underbrace{(\lambda_k - \frac{1}{G_k[m]})}_{P_k[m]}, \end{aligned} \quad (7)$$

where $P_k[m]$ and $r_k[m]$ are the power and rate previously allocated on subcarrier m. Given λ_k, $P_k[j]$ and $r_k[j]$ can be viewed as the power and rate virtually allocated on subcarrier j. The differential rate between $r_k[m]$ and $r_k[j]$ is equally distributed on the s_k subcarriers in $\mathcal{S}_k \setminus \{m\} \cup \{j\}$. After such a substitution the water level becomes

$$\lambda_k^{(s)}(m,j) = \lambda_k \left(\frac{G_k[m]}{G_k[j]} \right)^{\frac{1}{s_k}}. \quad (8)$$

As shown by (5), (6), (7) and (8), it is very cheap to derive the power difference and update the water level after removing or substituting some subcarrier in a subcarrier assignment. Only one exponential operation is needed to derive the pair of $(\lambda_k^{(r)}(m), \Delta P_k^{(r)}(m))$ or the pair of $(\lambda_k^{(s)}(m,j), \Delta P_k^{(s)}(m,j))$ and others are just simple operations. This computationally efficient updating approach is used in the following.

IV. Tactical Multiuser Resource Allocation

In multiuser OFDM systems, resource allocation needs to be updated, even when the channel of only one user changes. A resource allocation scheme therefore often becomes ineffective after a short period of time. The computational efficiency becomes crucial in this case. In this section, resource allocation methods for broadcasting in multiuser OFDM systems are devised, which inherit the idea of initialization in [9].

A. Initialization

In Algorithm 1, all subcarriers are first set to be available for every user. Each user then greedily performs single-user water-filling (SUWF) [1], whose inputs are the set of available subcarriers and the data rate requirement. It returns the water level and the set of used subcarriers for each user. The minimal number of subcarriers needed for each user, indicated by N_k^{min}, is calculated.

Algorithm 1 User-Independent Initialization

$(\lambda_k, \mathcal{S}_k) \leftarrow \text{SUWF}_k(\{1, \ldots, N\}, R_k), \ \forall k$
$c_k[n] \leftarrow 1, \forall n \in \mathcal{S}_k, \ \forall k$
$c_k[n] \leftarrow 0, \forall n \notin \mathcal{S}_k, \ \forall k$
$N_k^{min} \leftarrow \lceil R_k / M \rceil, \ \ \forall k$

Algorithm 2 Tactical Conflict Resolution

$c[n] = \sum_{k=1}^{K} c_k[n],\ \forall\, n$
$\mathcal{B} \leftarrow \{n \mid c_k[n] > 1\}$
$\mathcal{F} \leftarrow \emptyset$
for each $n \in \mathcal{B}$ **do**
  $\mathcal{K} \leftarrow \{k \mid c_k[n] = 1\}$
  $c_k \leftarrow \sum_{n=1}^{N} c_k[n],\ \forall\, k \in \mathcal{K}$
  $\mathcal{T} \leftarrow \left\{k \in \mathcal{K} \mid c_k = N_k^{min}\right\}$
  $t \leftarrow |\mathcal{T}|$
  if $t = 0$ **then**
    $l \leftarrow \text{argmax}_{k\in\mathcal{K}}\ P_k^{(r)}(n)$
    $c_l[n] \leftarrow 0,\ \forall\, k \in \mathcal{K} \setminus \{l\}$
    $\lambda_k \leftarrow \lambda_k^{(r)}(n),\ \forall\, k \in \mathcal{K} \setminus \{l\}$
  else
    $c_k[n] \leftarrow 0,\ \forall\, k \in \mathcal{K} \setminus \mathcal{T}$
    $\lambda_k \leftarrow \lambda_k^{(r)}(n),\ \forall\, k \in \mathcal{K} \setminus \mathcal{T}$
    if $t \neq 1$ **then**
      $\mathcal{F} \leftarrow \mathcal{F} \cup \{n\}$
    end if
  end if
end for

B. Conflict resolution

For a subcarrier n, the number of users sharing it, denoted by $c[n]$, is obtained first. If $c[n] \leq 1$ holds for all subcarriers, the subcarrier assignment for each user is optimal. When conflict happens on subcarrier n, i.e. $c[n] > 1$, we call subcarrier n the *conflicting subcarrier* and add it to set $\mathcal{B}$. To resolve such conflicts two consecutive algorithms are designed, shown as Algorithm 2 and Algorithm 3. Removing subcarrier n from the subcarrier assignment $\mathcal{S}_k$ results in a power increment, denoted by $\Delta P_k^{(r)}(m)$ in (6). In Algorithm 2, a conflicting subcarrier remains to be used only by the user, who has the largest power increment, while the water levels of other users, previously sharing it, are updated by (5).

However, for a conflicting subcarrier n, it may happen that t users share it, who have only N_k^{min} subcarriers, we call these users the *marginal users*. Subcarrier n is removed only from the subcarrier assignments for the non-marginal users, whose water levels are updated also by (5). When there are $t > 1$ marginal users, subcarrier n is included in set $\mathcal{F}$ and such a conflict is resolved in Algorithm 3.

In Algorithm 3, for each subcarrier $m \in \mathcal{F}$, set $\mathcal{K}$ contains the users sharing subcarrier m, set $\mathcal{V}$ contains the users having more than N_k^{min} subcarriers, set $\mathcal{A}$ contains the subcarriers used by the users in $\mathcal{V}$, and $\mathcal{U}$ contains the subcarriers not used by any user. We iteratively find for user $k \in \mathcal{K}$ the subcarrier in $\mathcal{A} \cup \mathcal{U}$ that can substitute subcarrier m while resulting in the smallest power increment. This power increment is derived by (6) and (7). This loop finishes while only one element remains in $\mathcal{K}$. At last given the newly derived $\mathcal{S}_k$, the transmission power for each user is determined by the strict water-filling (SWF) [14], where the rate limit on one subcarrier is met.

Algorithm 3 Greedy Conflict Resolution

for each $m \in \mathcal{F}$ **do**
  $\mathcal{K} \leftarrow \{k \mid c_k[m] = 1\}$
  repeat
    $\mathcal{V} \leftarrow \{k \mid \sum_{n=1}^{N} c_k[n] > N_k^{min}\}$
    $\mathcal{A} \leftarrow \{n \mid c_k[n] = 1, l \in \mathcal{V}\}$
    $\mathcal{U} \leftarrow \{n \mid \sum_{k=1}^{K} c_k[n] = 0\}$
    $\Delta P(j) \leftarrow \Delta P_l^{(r)}(j),\ \forall\, j \in \mathcal{A},\ \forall\, l \in \mathcal{V}$
    $\Delta P(j) \leftarrow 0,\ \forall\, j \in \mathcal{U}$
    $\mathcal{K} \leftarrow \mathcal{K} \setminus \{k\},\ k \in \mathcal{K}$
    $i \leftarrow \text{argmin}_{j\in\mathcal{A}\cup\mathcal{U}}\ \Delta P_k^{(s)}(m, j) + \Delta P(j)$
    $c_k[i] \leftarrow 1$
    if $\exists\, l, c_l[i] = 1$ **then**
      $c_l[i] \leftarrow 0$
      $\lambda_l \leftarrow \lambda_l^{(r)}(i)$
    end if
  until $|\mathcal{U}| = 1$
end for
$\mathcal{S}_k \leftarrow \{n \mid c_k[n] = 1\},\ \forall\, k$
$P_k \leftarrow \text{SWF}(\mathcal{S}_k, R_k),\ \forall\, k$

C. Sorting conflicting subcarriers

We simply name the group of the above three algorithms the **c**onflicting **s**ubcarrier **a**ssignment (CSA). Conflicting subcarriers are arbitrated to users following a random order with respect to their CNRs and data rate requirements in Algorithm 2. Consider the extreme case that only one user can use the nth subcarrier and others have too low CNRs on this subcarrier to use it. Improper assigning this subcarrier may hardly happen. On the contrary, users might have similar CNRs on one subcarrier, which may be assigned to an inappropriate user with higher probability. Meanwhile, different users have individual rate and BER requirements. Such kinds of information are combined to the water levels.

Hence, we first arrange the conflicting subcarriers in a descending order of their normalized CNR (NCNR) variabilities, which are defined as

$$V[n] = \sum_{k=1}^{K} c_k[n] |g[n] - \overline{G_k[n]}| \tag{9}$$

with the average NCNR of a conflicting subcarrier over users

$$g[n] = \frac{1}{\sum_{k=1}^{K} c_k[n]} \sum_{k=1}^{K} c_k[n] \overline{G_k[n]}, \tag{10}$$

where the NCNR is the CNR on subcarrier n for user k normalized by λ_k and is denoted by $\overline{G_k[n]} = \lambda_k G_k[n]$. Following this order conflicts are resolved by Algorithm 2. We name the revised CSA **s**equential **s**orting and **a**ssigning **c**onflicting subcarriers with NCNR variabilities (SSAC-NCNR). Note that this order and set $\mathcal{B}$ are kept while resolving conflicts in Algorithm 2.

Furthermore, (3) and (4) show that the power and rate are monotonically increasing in CNR, which inspires us that better performance may be achieved by replacing $\overline{G_k[n]}$ in (9) and (10) with $P_k[n]$ or $r_k[n]$. After such replacement, we call $V[n]$ in (9) the power or rate variability and the revised CSA, where conflicting subcarriers are sequentially sorted and assigned according to their power or rate variabilities, is denoted by SSAC-P or SSAC-R.

Obviously, some water levels increase in each iteration in Algorithm 2, which implies that the power, rate or NCNR variabilities may change after resolving one conflict. Therefore, we can iteratively perform sorting and assigning conflicting subcarriers. This means in each iteration the conflict with the largest power, rate or NCNR variability is resolved and then the variabilities are updated consequently. Accordingly, the revised CSA is denoted as **i**terative **s**orting and **a**ssigning **c**onflicting subcarriers (ISAC-P), ISAC-R or ISAC-NCNR.

D. Comparison and Complexity Analysis

In Algorithm 1, K subcarrier assignments are initialized by SUWF, which has the complexity of $\mathcal{O}(N)$, see [1]. Algorithm 1 therefore has the complexity of $\mathcal{O}(KN)$.

Zhang's method in [9] also uses the same initialization, but it does not consider the rate limit on each subcarrier, which can be met by Algorithm 3 in our methods. In SSAC-P, SSAC-R and SSAC-NCNR, the set of conflicting subcarriers $\mathcal{B}$ and the order of conflicting subcarriers are kept while executing Algorithm 2 unlike Zhang's method, where both may change.

Moreover, in ISAC-P, ISAC-R and ISAC-NCNR sorting and assigning conflicting subcarriers are iteratively performed according to different criteria similarly to but different from Zhang's method. For a conflicting subcarrier n shared by user k, it has been proved in Section III that the rate on subcarrier n may be equally distributed on other subcarriers used by user k. The induced power increment can be efficiently obtained by (6), where only one exponential operation and several simple operations, like addition, subtraction and multiplication, are needed. This efficient updating approach can also be used in other resource allocation methods to reduce their complexities, e.g., [2], [5], [11], where a power increment is derived by water-filling as in [1]. However, in Zhang's method, conflicting subcarriers are sorted according to the sum powers over users on subcarriers. Further, the subcarriers, previously not used by user k, are also considered in one conflict resolution. In such a case new conflicts may happen, while the induced power increment can be obtained only by water-filling and the efficient approaches in Section III cannot be used any more.

In the worst case that there are N conflicting subcarriers and each of them is shared by K users, KN power increments must be calculated with (6). Hence, the complexity of Algorithm 2 is $\mathcal{O}(KN)$. It is assumed that on average qN subcarriers are in set $\mathcal{F}$ with $q \in (0,1)$ and each of them are shared by $K/2$ marginal users, then $qN(K/2-1)(N-qN)$ power increments must be calculated in Algorithm 3. Although the complexity of Algorithm 3 is bounded by $\mathcal{O}(KN^2)$, it is rarely called, shown in the later simulation results. Therefore, the complexity of our methods is approximately $\mathcal{O}(KN)$.

TABLE I
USERS IN THE SIMULATION SYSTEM

User type	Proportion	Rate (bits/OFDM symbol)	BER
Video user	10%	32	8.00E-5
Audio user	40%	8	3.67E-6
Data user	50%	16 (mean)	4.66E-7

V. SIMULATION RESULTS

In this section, simulation results are given to quantify the performance loss and complexity reduction by our methods compared to another two methods besides Zhang's method. One is SUSI from [5], which has better performance than other heuristic methods in [2], [9], [11]. The other is RCG suggested in [6], which is well known since it can achieve a good balance between performance and complexity. We combine these two methods as RCG-SUSI, where the output of RCG is put into SUSI. For fair comparison Zhang's method is revised with Algorithm 3 attached.

The frequency selective channels of different users are independent from each other and each of them is modeled as consisting of 16 independently Rayleigh distributed multipaths with an exponentially decaying profile. The maximal expected CNR on each subcarrier is set to be 5 dB, which fades with the distance from the transmitter to the receiver. We consider a multiuser OFDM system with 64 subcarriers and 2 to 12 users for simulations, which can serve three types of users, as shown in Table I. The rate of a data user is exponentially distributed with a maximal rate of 32 bits per OFDM symbol. The maximal sum rate of the system, 384 bits per OFDM symbol, can be possibly achieved when 12 users are served. The simulation system is implemented in MATLAB and the computing time is measured by the pair of commands (tic, toc), which is recommended by the MATLAB help. A number of 750000 channel samples are generated for each simulation.

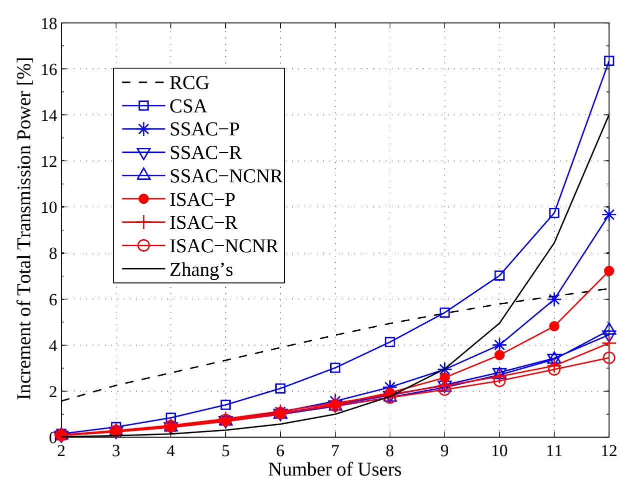

Fig. 1. Increment of total transmission power by using RCG and the proposed methods compared to RCG-SUSI, see (11), for different K with $N = 64$.

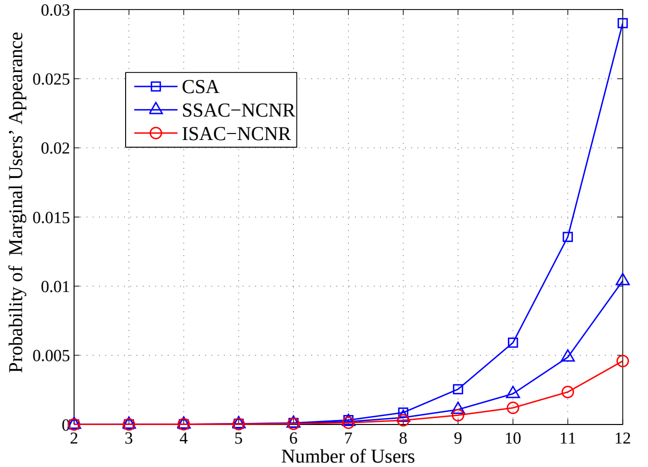

Fig. 2. Probability of marginal users' appearance vs. number of users with $N = 64$.

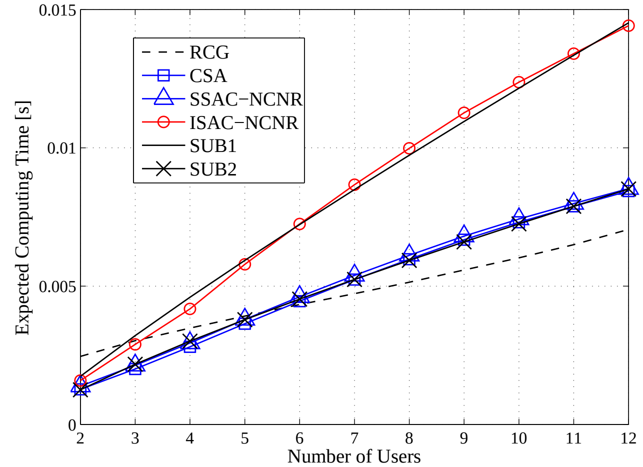

Fig. 3. Computing time vs. number of users with $N = 64$.

The curves representing the real values of the total transmission powers allocated by different methods would intersect. To clearly illustrate the difference, Fig. 1 shows the increments of total transmission power allocated by using other methods compared to that by using RCG-SUSI, equal to

$$\frac{\text{power by other methods} - \text{power by RCG-SUSI}}{\text{power by RCG-SUSI}} \times 100\%. \tag{11}$$

If a small number of users are accommodated in the considered system, our methods can achieve better performance than RCG with small performance loss compared to RCG-SUSI. Zhang's method has the least performance loss. When the number of users is very large, SSAC-R and SSAC-NCNR have similar performance better than those of RCG, CSA, SSAC-P and ISAC-P, while ISAC-NCNR achieves the best performance. The power increment by using Zhang's method grows rapidly as K increases even faster than the one by using SSAC-P, because the worse criterion is used to sort conflicting subcarriers and new conflicting subcarriers, emerging during the conflict resolution, deteriorate the order of conflicting subcarriers.

Sorting conflicting subcarriers according to their NCNR variabilities improves the performance by about 6% on average and iterative performing sorting and assigning conflicting subcarriers makes the performance loss limited to 3.5% compared to RCG-SUSI. Hence, we only focus on SSAC-NCNR and ISAC-NCNR in the later simulation results.

As mentioned in Section IV-D, the complexity of Algorithm 3 is bounded by $\mathcal{O}(KN^2)$. However, it can be neglected while considering the expected computing time as explained by Fig. 2. Marginal users almost do not appear when the number of users is small. Apparently, sorting conflicting subcarriers according to the descending order of their NCNR variabilities makes the probability of marginal users' appearance reduce significantly.

The expected computing time for RCG-SUSI and Zhang's method is far above that for RCG, as shown in Table II. Fig. 3 plots the computing time for RCG, CSA, SSAC-NCNR and ISAC-NCNR against different numbers of users. SSAC-NCNR has much better performance than CSA, but they need almost the same computing time. When the number of users is small, the computing time for the proposed methods is even shorter than that for RCG. To give an intuitive view to the curves, sublinear functions

$$\text{SUB1} = \frac{(K-1)^{0.885}}{575} \quad \text{and} \quad \text{SUB2} = \frac{(K-1)^{0.8}}{800}$$

are plotted and show that the computing time for the proposed methods approximately sublinearly increases with increasing K. The gap between the computing times, needed for SSAC-NCNR and ISAC-NCNR, is induced by iterative sorting conflicting subcarriers as the water-levels increase.

We still use the above simulation system but fix the number of users to 6. The number of subcarriers varies from 40 to 128. Fig. 4 gives the performance loss of the proposed methods against an increasing number of subcarriers. SSAC-NCNR and ISAC-NCNR have almost the same performance loss compared to RCG-SUSI, which is limited to 3.5% and is much lower than that of RCG. In the range of large numbers of subcarriers, Zhang's method achieves better performance than the proposed methods, while such difference may be ignored, since the performance loss of the proposed methods is also very small compared to RCG-SUSI.

TABLE II
EXPECTED COMPUTING TIME FOR THE REFERENCE METHODS [S]

	$K=4$	$K=6$	$K=8$	$K=10$	$K=12$
RCG-SUSI	3.77E-2	9.04E-2	1.63E-1	2.47E-1	3.31E-1
Zhang's	1.32E-2	2.63E-2	3.92E-2	5.04E-2	6.08E-2
RCG	3.45E-3	4.26E-3	5.09E-3	6.02E-3	7.02E-3

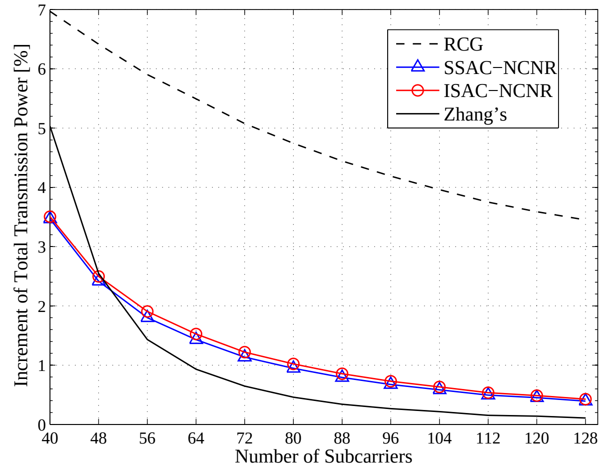

Fig. 4. Increment of total transmission power by using RCG and the proposed methods compared to RCG-SUSI, see (11), for different N with $K = 6$.

Fig. 5 displays the expected computing time for the proposed methods against different numbers of subcarriers with constant $K = 6$. The expected computing time for RCG does not linearly increase with N, shown by

$$\mathrm{EXP} = \frac{(N/8)^{1.3}}{3450}.$$

Sublinear functions

$$\mathrm{SUB3} = \frac{(N/8)^{0.2}}{200} \quad \text{and} \quad \mathrm{SUB4} = \frac{(N/8)^{0.12}}{260}$$

are plotted and demonstrate that the computing times for SSAC-NCNR and ISAC-NCNR are also approximately sublinearly increasing in N and they vary along the horizontal axis in a very small range.

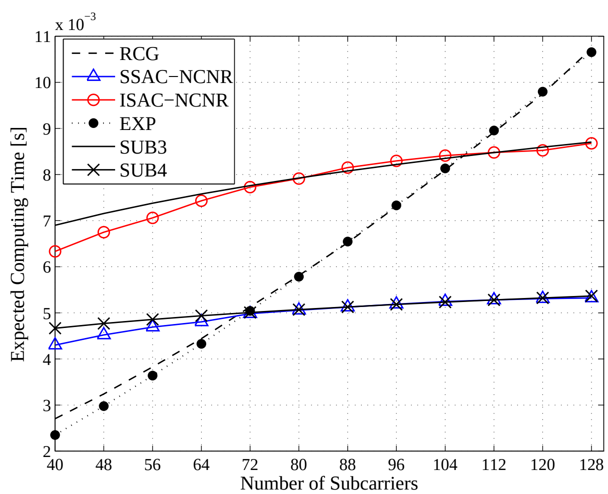

Fig. 5. Computing time vs. number of subcarriers with $K = 6$.

From the above simulation results SSAC-NCNR and ISAC-NCNR are proposed. They have small performance loss with significant complexity reduction compared to RCG-SUSI. When K/N is large, they have better performance and need much less computing time than Zhang's method. When K/N is small, they perform slightly worse than Zhang's method and even need less computing time than RCG, whose performance is always much worse than that of the proposed methods.

VI. Conclusion

In future communication networks efficient resource allocation methods for multiuser OFDM have to adapt to the fast time-varying environment effectively. In this paper, computationally efficient methods for multiuser resource allocation have been proposed. We have investigated and quantified the power variations after removing a subcarrier from a subcarrier assignment and after substituting a subcarrier in a subcarrier assignment, which can be computed very efficiently. To utilize such efficient updating approaches, tactical algorithms have been designed. Furthermore, several effective criteria are proposed to sort the conflicting subcarriers in order to improve the performance and reduce the complexity. Simulations have shown that the proposed methods can achieve a better balance between performance and complexity compared to previous works. The expected computing times for the proposed methods are approximately increasing sublinear functions of the number of users K and the number of subcarriers N.

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