Routing Path Selection and Power Allocation for Distributed Detection in Wireless Sensor Networks

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***Abstract*—The design of wireless sensor networks for signal processing applications has to consider the limited battery power of the sensor nodes. In this paper, a combined routing path selection and power allocation strategy is presented that is especially designed for distributed detection in sensor networks with a serial topology. The objective is to minimize the global probability of detection error of the serial network under a total network power constraint. The cross-layer approach for the selection of a routing path through the network is based on the local observation SNR of the sensors as well as fading conditions and is efficiently implemented by a backward greedy algorithm. After the routing path is established, a subsequently performed power allocation algorithm aims to optimally distribute a total power budget with respect to the detection performance of the sensor network.**

I. Introduction

Distributed detection of signal sources in a region of interest is one of the primary applications of wireless sensor networks [1], [2]. In distributed detection, the sensor nodes process their observations locally and make preliminary decisions about the state of the monitored environment, e.g., absence or presence of a target. In serial distributed detection, the sensors successively form decisions that depend on the received decision of a preceding sensor as well as the own local observation until a final detection result is reached [3], [4]. As the communication channels between sensor nodes are subject to noise and interference, it becomes necessary to take wireless channel conditions into account in order to optimally design the distributed detection system [5]. Yet, the channel quality can be controlled, e.g., by appropriate assignment of transmission power levels. In wireless sensor networks deployed for distributed detection, the power assignment eventually should be adapted to optimize application-specific metrics, thus exploiting dependencies between signal processing and wireless networking [6]. The main objective in the design of sensor networks for distributed detection applications is the minimization of the global probability of detection error.

To setup serial distributed detection in a practical sensor network a routing path is required, which determines the order in which the local decisions are routed in the network from sensor to sensor until the final node is reached. The final node delivers the final detection result to the external observer.

In [7] signal detection and energy-efficient routing are formulated as a combinatorial optimization problem. The focus of the work however is on distributed detection under the Neyman-Pearson criterion and radar-like sensor nodes which aim to detect a reflecting point in space.

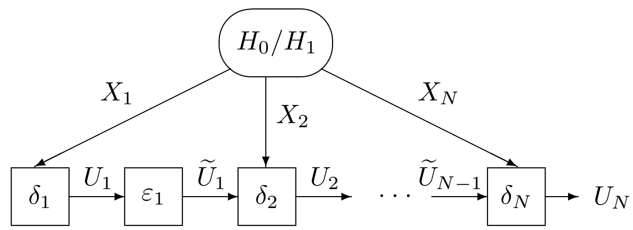

Fig. 1. Serial network with noisy channels.

In this paper, we present a combined routing path selection and power allocation strategy for distributed detection in wireless sensor networks in the serial topology. The objective is to establish a routing path through the network and subsequently allocate transmission power to the sensor nodes in order to minimize the global probability of detection error under a total network power constraint. The strategy for the selection of a routing path is based on the local observation SNR of the sensors as well as fading conditions and is implemented by a backward greedy algorithm. After the routing path is established, a subset of sensors is selected which provides the dominant contribution to the detection result. Then a subsequently performed power allocation algorithm aims to optimally distribute a total power budget among the selected nodes with respect to the detection performance of the sensor network. Numerical results show the effectiveness of the approach.

The remainder of this paper is organized as follows. In Section II, the problem of distributed detection in serial networks with noisy channels is stated. The routing path selection strategy, the sensor selection and the allocation of transmission power is introduced in Section III. Finally, we present numerical results and conclusions in Section IV.

II. Distributed Detection in Serial Networks

The problem of distributed detection in serial networks can be stated as follows (see Fig. 1). We consider a binary

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hypothesis testing problem with hypotheses H_0, H_1 indicating the state of the observed environment and associated prior probabilities $\pi_0 = P(H_0)$, $\pi_1 = P(H_1)$. In order to detect the true state of nature, a network of N sensors $S_1, \ldots, S_N \in \mathcal{N}$ collects random observations $X_1, \ldots, X_N$, which are assumed to be conditionally independent across sensors given the underlying hypothesis, i.e., the joint conditional probability density function (pdf) of the observations factorizes to

$$f(x_1, \ldots, x_N | H_k) = \prod_{i=1}^{N} f_i(x_i|H_k), \quad k = 0, 1.$$

In the serial network, the first sensor makes a decision $U_1 = \delta_1(X_1)$ which only depends on its own observation and subsequently transmits it to its neighbor. The succeeding sensors form decisions

$$U_j = \delta_j(\widetilde{U}_{j-1}, X_j), \quad j = 2, \ldots, N, \tag{1}$$

which depend on the received and potentially corrupted decision $\widetilde{U}_{j-1}$ of the preceding sensor as well as the own observation X_j. In the case that every wireless sensor is allowed to transmit only one bit per observation, the sensor decisions are binary-valued random variables $U_j \in \{0, 1\}$, $j = 1, \ldots, N$. The local detection error probabilities for each sensor are given by the local probability of false alarm P_{f_j} and the local probability of miss P_{m_j} according to

$$P_{f_j} = P(U_j = 1|H_0), \quad P_{m_j} = P(U_j = 0|H_1) \tag{2}$$

for $j = 1, \ldots, N$. Due to noisy channels, the received decisions $\widetilde{U}_1, \ldots, \widetilde{U}_N$ are potentially corrupted. We model the communication link of the jth sensor by a binary symmetric channel with bit-error probability ε_j, i.e.

$$\varepsilon_j = P(\widetilde{U}_j = 1|U_j = 0) = P(\widetilde{U}_j = 0|U_j = 1) \tag{3}$$

for $j = 1, \ldots, N$. The resulting modified error probabilities $\widetilde{P}_{f_j} = P(\widetilde{U}_j = 1|H_0)$ and $\widetilde{P}_{m_j} = P(\widetilde{U}_j = 0|H_1)$ can be calculated as

$$\begin{aligned} \widetilde{P}_{f_j} &= P_{f_j} + \varepsilon_j(1 - 2P_{f_j}), \\ \widetilde{P}_{m_j} &= P_{m_j} + \varepsilon_j(1 - 2P_{m_j}). \end{aligned} \tag{4}$$

The overall performance metric for the serial network is the probability of error P_{e_N} of the Nth sensor according to

$$P_{e_N} = \pi_0 P_{f_N} + \pi_1 P_{m_N}. \tag{5}$$

Applying locally optimal detection at each sensor, the probability of error P_{e_N} of the last node can be calculated iteratively. Performing optimal detection at the Nth sensor, we obtain

$$\begin{aligned} P_{e_N} = &\pi_0 \big[(1 - \widetilde{P}_{f_{N-1}})(1 - F_{L_N}(\tau_N^{(0)}|H_0)) \\ &\qquad + \widetilde{P}_{f_{N-1}}(1 - F_{L_N}(\tau_N^{(1)}|H_0))\big] \\ &+ \pi_1 \big[\widetilde{P}_{m_{N-1}} F_{L_N}(\tau_N^{(0)}|H_1)) \\ &\qquad + (1 - \widetilde{P}_{m_{N-1}}) F_{L_N}(\tau_N^{(1)}|H_1)\big], \end{aligned} \tag{6}$$

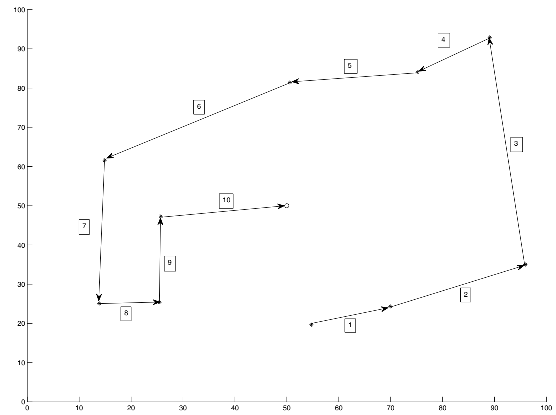

Fig. 2. Illustration of a routing path established by the proposed algorithm. The nodes of this exemplary scenario were randomly deployed in the region of interest.

where $F_{L_N}(\cdot|H_k)$ is the conditional cumulative distribution function (cdf) of the log-likelihood ratio L_N under H_k and

$$\tau_N^{(0)} = \log \frac{\pi_0(1 - \widetilde{P}_{f_{N-1}})}{\pi_1 \widetilde{P}_{m_{N-1}}}, \quad \tau_N^{(1)} = \log \frac{\pi_0 \widetilde{P}_{f_{N-1}}}{\pi_1(1 - \widetilde{P}_{m_{N-1}})}. \tag{7}$$

III. Routing Path Selection and Power Allocation

In this section, we propose a cross-layer routing path selection strategy. It determines the order in which the detection results $U_1, \ldots, U_N$ are transmitted from sensor to sensor by choosing a routing path through the network. The process is illustrated for an exemplary scenario in Fig. 2.

After the routing path is established, a subset of sensors is selected which provides the dominant contribution to the global detection result to increase the energy efficiency of the detection system.

Finally, the detection performance can be further improved by a subsequently performed application-specific power assignment to the selected nodes.

A. Cross-Layer Sensor Ordering

The strategy for the determination of the routing path is based on the local observation SNR of the sensors. Furthermore, fading conditions in terms of path gain are used as weighting factor. The intention of the approach is to order the sensors in the serial network from lowest weighted observation SNR to highest weighted observation SNR. The rationale behind this ordering is based on the observation, that for serial networks with perfect communication channels it is advantageous to sort the nodes with an ascending observation SNR. The simulation results in Fig. 3 illustrate that in case of perfect communication channels, the detection performance can be significantly increased compared to random ordering of the nodes. Hence, nodes with high local detection performance according to high observation SNR should be used in the later part of the route. The path gain as a weighting factor accounts

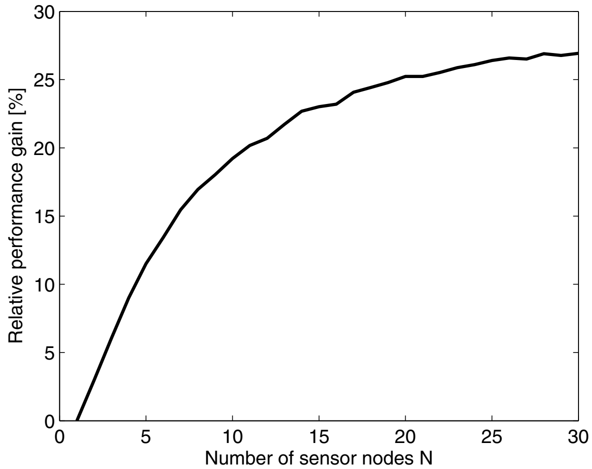

Fig. 3. Relative performance gain in terms of a reduction of global probability of detection error achieved by ordering the sensor nodes with ascending observation SNR compared to random ordering for perfect communication channels.

for the fact that in case of noisy communication channels a transmission with a high path gain results in a low bit-error probability ε_j of the channel. Especially in the later part of the serial network the bit-error probabilities of the communication channels should be low, such that the probability to corrupt high quality decisions during the transmission is also low.

While a sensor ordering based only on the observation SNR could simply be accomplished by a sorting algorithm, the problem becomes combinatorial by additionally considering fading conditions.

As an efficient heuristic approach we propose to implement the routing path selection strategy by a backward greedy algorithm. The determination of the routing path starts with the final node S_N and then iteratively each node S_k determines its predecessor in the route. The decision is based on the application-specific node measure μ calculated for all nodes that are not yet included in the route. It is defined by

$$\mu(S_j) = g_{kj} \cdot \mathrm{SNR}_j, \qquad (8)$$

where SNR_j is the observation SNR of node S_j and g_{kj} is the path gain between S_j and its potential successor S_k in the route. In our backward greedy algorithm the nodes iteratively choose among all available nodes the one with the maximum node measure μ as predecessor until all nodes are included in the routing path. Note, that this algorithm results in a sensor ordering with ascending observation SNR for perfect communication channels. A formal description of the routing algorithm including the sensor selection strategy as described in the following subsection is given in Algorithm 1. In the algorithm it is assumed that every node is identified by a unique number $j = 1, \ldots, N$. The routing path which is described by a permutation of these numbers is stored in the array *Route*. Set $\mathcal{D}$, which includes all nodes not yet included in the routing path and the counter c are auxiliary values.

Algorithm 1 Algorithm for determination of the routing path

```
Initialize:
D ← N \ S_N;
c ← |N|;
Route ← zeros(|N|, 1);
Route[c] ← N;
μ(S_j) ← g_Nj · SNR_j;    S_j ∈ D
while D ≠ ∅ do
   c ← c − 1;
   if c < κ then
      break;
   end if
   S_k = arg max_{S_j ∈ D} μ(S_j);
   Order[c] ← k;
   D ← D \ S_k;
   μ(S_j) ← g_kj · SNR_j;    S_j ∈ D
end while
```

B. Sensor Selection Strategy

The routing algorithm from the previous subsection aims to establish a routing path with ascending weighted observation SNRs of the nodes. This means that the first nodes in the route might only provide a limited contribution on the quality of the final detection result while still consuming transmission power. For a given budget of total transmission power p_{tot} it might be advantageous to distribute this power only among a subset of all nodes to achieve a lower global probability of detection error. Therefore, we propose to terminate the backward establishment of the routing path and cut off the remaining κ sensor nodes. After the sensor selection, only sensors $S_{\kappa+1}, \ldots, S_N$ are used in the detection process. The remaining sensors do not get any transmission power. This sensor selection strategy realizes a simple and efficiently implementable power control strategy where the sensor nodes either send with full power or do not get any power at all. A similar approach for the parallel topology is proposed in [8].

C. Power Control Based on Marginal Analysis

The sensor selection strategy in the previous subsection can be combined with a more sophisticated determination of the transmission power levels for the selected nodes. The objective of the opportunistic power assignment strategy is to optimize the sensor network detection performance in terms of the global probability of error (5) given a budget of total transmission power.

Based on the modified error probabilities (4), we define the effective sensor weights

$$\widetilde{\lambda}_j = \log\left(\frac{(1-\widetilde{P}_{f_j})(1-\widetilde{P}_{m_j})}{\widetilde{P}_{f_j}\widetilde{P}_{m_j}}\right) \qquad (9)$$

for $j = \kappa+1, \ldots, N$. Note that for $P_{f_j}, P_{m_j} \in [0, \frac{1}{2}]$, and an arbitrary bit-error rate $\varepsilon_j \in [0, 1]$, the effective sensor weight

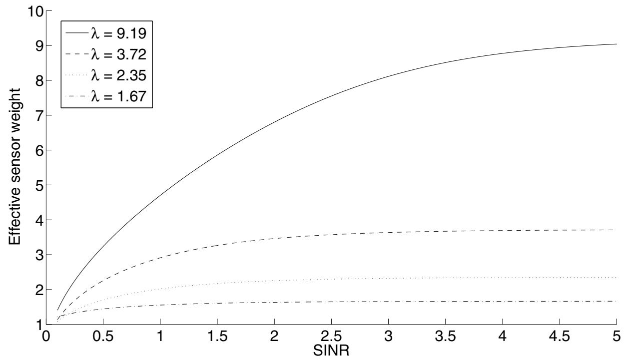

Fig. 4. Effective sensor weight $\widetilde{\lambda}$ as a function of the SINR γ for different values of the initial sensor weight λ.

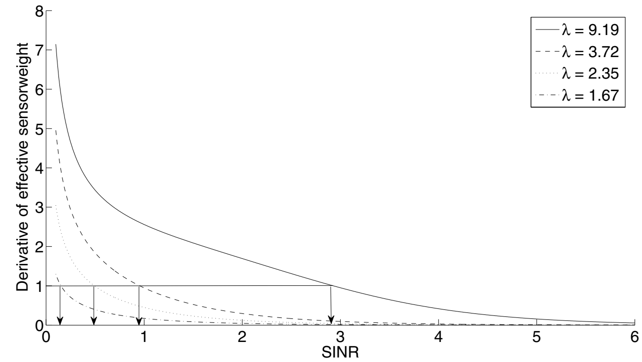

Fig. 5. Derivative $\partial\widetilde{\lambda}/\partial\gamma$ of the effective sensor weight $\widetilde{\lambda}$ with respect to the SINR γ. The threshold ϱ is chosen to be equal to 1.

$\widetilde{\lambda}_j$ is always less than or equal to the initial sensor weight λ_j, which is given as

$$\lambda_j = \log\left(\frac{(1-P_{f_j})(1-P_{m_j})}{P_{f_j}P_{m_j}}\right). \tag{10}$$

Fig. 4 shows the effective sensor weight $\widetilde{\lambda}$ dependent on the channel SINR γ for different initial sensor weights λ. It can be observed that for high values of γ, the effective sensor quality approaches the initial sensor quality. In this case, increasing γ does not result in an improved effective sensor quality. The value of γ, from which on the effective sensor quality $\widetilde{\lambda}$ is not further improved significantly, increases with the initial sensor quality λ. It is therefore advantageous to assign higher values of SINR to sensors with high initial quality than to ones with low initial quality. We employ a marginal analysis approach and assign the SINR for which the slope of the effective sensor weight $\widetilde{\lambda}$ with respect to γ falls under a predetermined threshold ϱ. Fig. 5 illustrates this procedure. The threshold value ϱ can be used as a trade-off parameter to balance total transmission power $p_{\text{tot}} = \sum_{j=1}^{N} p_j$ and global probability of error P_e. Eventually, we determine the designated SINR γ_j of S_j according to

$$\gamma_j = \left(\frac{\partial\widetilde{\lambda}_j}{\partial\gamma}\right)^{-1}(\varrho). \tag{11}$$

IV. Numerical Results and Conclusions

In this section, we investigate the performance of the proposed strategies from Section III. The scenario is generated by randomly deploying sensor nodes uniformly in a rectangular area of size A. The final node is supposed to be located in the middle of the scenario. The considered observation model is the problem of detecting the presence or absence of a deterministic signal in Gaussian noise [9]. The observation SNRs of the sensors are independent and uniformly distributed between 0 and 10 dB. As transmission technology we consider impulse radio ultra-wideband with binary pulse-position modulation, since it is considered to be an enabling technology for wireless sensor networks. For this technology the bit-error rate ε_j which corresponds to the assigned SINR γ_j from (11) is given by

$$\varepsilon_j = \frac{1}{2}\operatorname{erfc}(\sqrt{\gamma_j}). \tag{12}$$

It is equivalent to the bit-error probability in (3). The transmission power to achieve the SINR γ_j for an IR-UWB node is given by

$$p_j = \frac{\gamma_j \eta}{g_j N_j T_f}, \tag{13}$$

where in each time frame of length T_f exactly one ultra-short impulse is transmitted. The information bits are transmitted by a number of N_j equally modulated impulses and η denotes the energy of the additional noise. More details can be found in [10]. In the simulation, we assume path-loss according to $d^{-\beta}$.

The involved parameters for both the scenario and for the employed IR-UWB transceivers are summarized in Table I.

The simulation results are summarized in Fig. 6 to Fig. 8. Fig. 6 shows the absolute performance of the distributed detection system in terms of the probability of detection error at the final sensor of the serial network. In the figure, κ denotes the number of nodes that have been cut off by the sensor selection strategy. The total transmission power which is distributed among the selected nodes is denoted by p_{tot}. Of course, the highest detection performance is achieved by using all sensors ($\kappa = 0$) and maximum total transmission power. However, a more power-efficient operating point can be achieved by using, e.g., only the last 10 sensor nodes. At this point, a similar detection performance as before can be

TABLE I
Parameters used in the simulation

parameter	value
N	20
A	$100 \text{ m} \times 100 \text{ m}$
β	2
σ^2	$1.9966 \cdot 10^{-3}$
N_j	10
T_f	100 ns
η	10^{-11} J

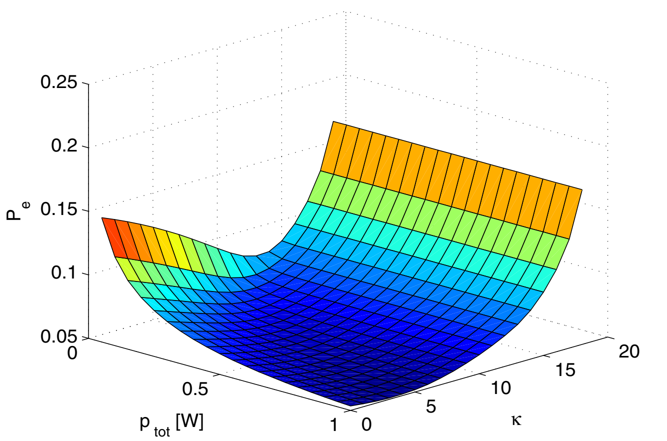

Fig. 6. Probability of detection error at the final node depending on the sensor selection parameter κ and the total transmission power p_{tot} for 20 sensor nodes.

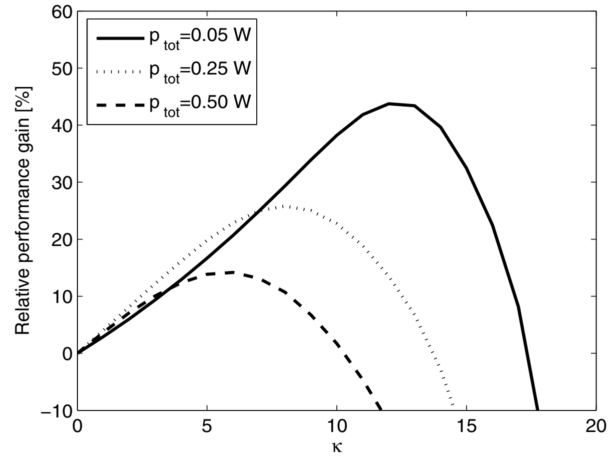

Fig. 7. Relative reduction of the probability of detection error at the final node achieved by using the sensor selection strategy without power control compared to using all sensors and no power control.

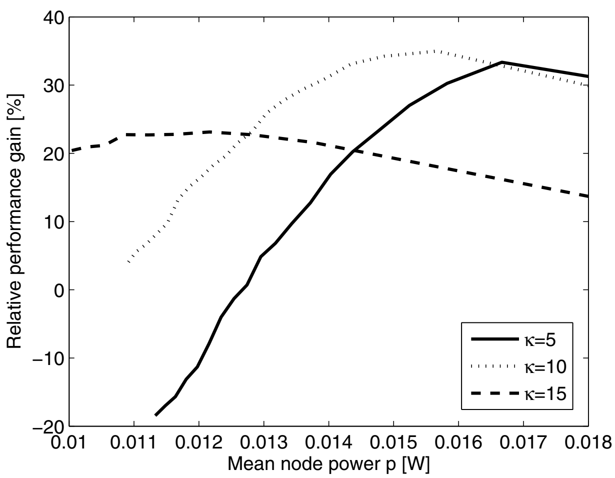

Fig. 8. Relative reduction of the probability of detection error at the final node achieved by using the sensor selection strategy combined with power control compared to using the sensor selection strategy without power control.

achieved with significantly reduced transmission power. Fig. 7 shows that for a given limited budget of total transmission power the performance of the system can even be increased if not all sensors in the network are used. Especially for the practically relevant case of very low transmission power the gain in terms of a relative reduction of the probability of detection error can be as high as almost 45% compared to case where all nodes are included. For applications which require a higher reliability, the performance can be further improved by additionally employing the more sophisticated power assignment for the selected nodes. Fig. 8 states this additional gain. It depends on the the total transmission power and the number of sensors that have been cut off and can increase the detection performance up to additional 35%.

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