Abstract—Generally, planning and configuring cellular radio networks lead to multi-objective optimization problems with conflicting objectives, e.g., coverage and cost. In this paper, we present an approach to combine those opponents in a closed-form objective for maximization of operator profit by means of joint base station and relay station placement in 4G multi-hop relay networks. The corresponding optimization model is formulated as mixed-integer linear program and particularly considers allocation of limited bandwidth for downlink data transmission in non-cooperative relaying mode. We suggest two economically motivated options how to choose appropriate weights for combining the conflicting objectives linearly. Furthermore, we apply the proposed optimization model to an exemplary planning scenario to analyze sensitivity to weight modifications numerically.

Index Terms—Radio network planning, multi-hop relay networks, multi-objective optimization, resource allocation.

I. INTRODUCTION

Fourth generation (4G) radio networks utilize sophisticated systems and techniques such as orthogonal frequency division multiplexing access (OFDMA), adaptive coding, higher dynamic modulation schemes, and multi-antenna transmission to cope with traffic-intensive user demand that comprises a mix of services such as telephony, video streaming, web browsing, and sheer data transfer [1]. Furthermore, relaying is a designated key component of advanced fourth generation cellular radio networks, e.g., IEEE 802.16j (WiMAX) and LTE Advanced networks. Adequate network planning, i.e., placement of base stations and relay stations as well as efficient resource allocation are prerequisites to achieve high capacity and coverage, respectively [2], [3].

Although advanced 4G wireless systems provide novel technical opportunities, the network operator’s overall aim has not altered and ultimately focuses on profit maximization, e.g., see [4] and [5]. Here, profit is defined as the difference of accumulated user revenue and cost according to operator’s capital expenditures (CAPEX) and operational expenditures (OPEX), respectively. Hence, the common planning and optimization models for former wireless network generations are still of relevance in terms of objective and problem structure. Particularly, the tradeoff between coverage and cost for deploying and operating network stations, as discussed in [6], remains as crucial issue to profit-oriented optimization of 4G systems. Typical strategies dealing with such conflicting objectives are to combine these into a closed-form objective, hierarchical optimization of objectives, and exploring Pareto optimal solutions.

In this paper, we aim at maximization of operator profit utilizing a closed-form objective. We consider joint placement of base stations (BS) and relay stations (RS) in bandwidth limited multi-hop 4G relay networks that operate in non-cooperative relaying mode. Our optimization model allows for planning a cellular network ab initio as well as for configuring existing ones.

The rest of this paper is organized as follows. In Section II, we discuss related work to the field of multi-objective network optimization and optimal planning of relaying enabled networks. Afterwards, we present our system model and formulate the joint BS and RS placement problem as mixed-integer linear program (MILP) in Section III. In Section IV, we discuss how to choose appropriate weights to achieve a satisfying closed-form objective and numerically analyze the impact of weight modifications. Finally, we conclude this paper in Section V.

II. RELATED WORK

Common optimization models for network deployment as well as for power control, admission control, and handover control in 3G cellular networks are discussed in [7], [8], and [9]. The articles in [10] particularly tackle network planning problems from an economical point of view as do [4] and [5] addressing 4G cellular networks. Sophisticated algorithms and heuristics have been developed to cope with complexity of relevant problems that turn out to be NP-hard in most cases, see [8], [11], and [12]. As the Maximal Covering Location Problem (MCLP) from [11] remains as subproblem to all relevant planning problems for 4G wireless networks, the problems discussed and presented in this paper are NP-hard.

If the limited number of deployed stations is not induced by a constraint but controlled by a penalty term for station deployment in the objective function, the MCLP becomes a multi-objective optimization problem. An approach to resolve...
the corresponding conflict between coverage and penalty for station deployment is discussed in [6]. In [13], Weicker et al. present heuristics to minimize both network deployment cost and co-channel interference in a multi-objective manner. An economically motivated multi-objective approach to minimize cost while system throughput and carrier-to-interference plus noise ratio are maximized, is proposed in [14] considering planning and optimization of LTE access networks. Here, a tabu search heuristic chooses BS configurations from a set of candidates to find non-dominated solutions on the Pareto front.

In [15], Niyato et al. utilize a hierarchical optimization framework to maximize uplink transmission rate while allocated bandwidth is minimized over a long-term period. In a first step, bandwidth allocation and admission control are performed at potentially deployed RSs with respect to Markov modelled user demand. Based on the computed results, RS placement and BS assignment for deployed RSs take place in a superordinate optimization stage afterwards. Further investigations on joint optimization of RS placement and bandwidth allocation are carried out for example in [16].

Joint BS and RS placement for IEEE 802.16j networks is presented in [17]. While that work focuses exclusively on capacity maximization, in [18], Yu et al. aim at joint BS and RS placement in two-hop relay networks using a multi-objective optimization criterion. However, this work does not consider resource constraints and uses specific weights for the conflicting objectives without further explanation. In [19], the same authors add capacity constraints to a modified model that supports transparent relaying, i.e., each user served by a RS additionally requires coverage by a BS. For that specific case, a heuristic is proposed that decomposes BS and RS placement in a two-stage procedure.

### III. System Model and Problem Formulation

First, we introduce our system model considering a relaying enhanced wireless network and downlink (DL) data transmission. Associated input parameters and variables are listed in Table I. BSs and RSs can be chosen from candidate sets to cover and serve users that are located in a geographical area. We utilize the approach from [11] to model user service request as traffic nodes (TN) \( t \) with demand \( c^t \). Here, demand is perceived as throughput and measured in kbps. The deployment of a BS \( s \) or RS \( r \) comes at cost \( c^s \) and \( c^r \), respectively.

Non-cooperative relaying with at most two hops from BS to TN is assumed, i.e., each covered TN is served exclusively by either one BS or one RS. A RS is connected to one BS via a radio link. As depicted in Figure 1, each RS forwards data by either one BS or one RS. A RS is connected to one BS via cabled leased lines. Hence, RSs are characterized by a simpler structure and less cost than BSs. Because fourth generation wireless networks utilize OFDMA for DL transmission they do ideally not suffer from intra-cell interference. BSs and RSs transmit in different frequency bands and do not interfere each other. Each transmission link occupies a certain amount of available DL bandwidth at BS or RS depending on throughput demand and spectral efficiency. Spectral efficiency itself is determined by modulation and coding scheme that the associated signal-to-noise ratio (SNR) supports. If spectral efficiency \( e \) and

<table>
<thead>
<tr>
<th>Symbol &amp; domain</th>
<th>Parameter description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S = {1, \ldots, N_S} )</td>
<td>Index set of BS candidates</td>
</tr>
<tr>
<td>( R = {1, \ldots, N_R} )</td>
<td>Index set of RS candidates</td>
</tr>
<tr>
<td>( T = {1, \ldots, N_T} )</td>
<td>Index set of TNs</td>
</tr>
<tr>
<td>( s \in S ), ( r \in R ), ( t \in T )</td>
<td>Representative set indices</td>
</tr>
<tr>
<td>( c^s ), ( c^r ) ( w^T \in \mathbb{R}_{&gt; 0} )</td>
<td>Deployment cost of BS, RS</td>
</tr>
<tr>
<td>( b^s ), ( b^r ) ( e^T \in \mathbb{R}_{&gt; 0} )</td>
<td>Throughput demand of TN</td>
</tr>
<tr>
<td>( d^B ), ( d^R ) ( e^l_{\min} \in \mathbb{R}_{&gt; 0} )</td>
<td>Total DL bandwidth of BS, RS</td>
</tr>
<tr>
<td>( e^l ), ( e^r ) ( e^T \in \mathbb{R}_{&gt; 0} )</td>
<td>DL spectral efficiencies</td>
</tr>
<tr>
<td>( x^s ), ( x^r \in {0, 1} )</td>
<td>Required minimum spectral efficiency to establish a link ( l \in {B \cup T, B \cup R, R \cup T} )</td>
</tr>
<tr>
<td>( x^s ), ( x^r \in {0, 1} )</td>
<td>Distance between BSs ( i, j \in S )</td>
</tr>
<tr>
<td>( x^s ), ( x^r \in {0, 1} )</td>
<td>Required minimum distance between any deployed BS pair</td>
</tr>
<tr>
<td>( d^B ), ( d^R )</td>
<td>Variable description</td>
</tr>
<tr>
<td>( x^s ), ( x^r \in {0, 1} )</td>
<td>BS, RS deployment indicators</td>
</tr>
<tr>
<td>( x^s ), ( x^r \in {0, 1} )</td>
<td>Coverage indicators</td>
</tr>
<tr>
<td>( d^B ), ( d^R )</td>
<td>DL bandwidth allocations</td>
</tr>
</tbody>
</table>
allocated bandwidth $b^t_l$ to an exemplary link $l$ are known, effective throughput $h^t_l$ on this link is given by
\[ h^t_l = c^t_l b^t_l. \] (2)

Consequently, from (1) and (2) it follows that allocated bandwidth at a RS as well as allocated bandwidth at its supplying BS have to be sufficiently high to enable the RS for serving throughput demand of covered TNs. Bandwidth is shared and limited for BS $s$ and RS $r$ by $b^s_s$ and $b^r_r$, respectively. For instance, in Figure 1 this leads to constraints
\[ \sum_{t=1}^{n} b^T_{st} + b^R_{sr} \leq b^s_s \quad \text{and} \quad \sum_{t=n+1}^{m} b^R_{rt} \leq b^r_r. \] (3)

We do not explicitly consider cell interdependencies or network topology aspects, e.g., inter-cell interference, handover, and cooperation. Nevertheless, station deployment with an acceptable level of interference is provided by demanding a minimum distance $d_{\text{min}}^B$ between any pair of deployed BSs.

The presented framework allows for configuring existing networks when location and number of deployed stations are fixed and candidates represent potential BS or RS configurations. In that case, additional minimum distance constraints for RSs ensure that each station selects at most one configuration. However, in the following we focus on the problem of network planning ab initio.

A. Bandwidth Limited Operator Profit Maximization

Using the introduced system model, we define the problem of joint BS and RS placement in 4G two-hop relay networks aiming at Bandwidth Limited Operator Profit maximization (BLOPmax) as
\[
\text{maximize} \quad \lambda \left( \sum_{s \in S, t \in T} c^T_{st} b^T_{st} + \sum_{r \in R, t \in T} c^R_{rt} b^R_{rt} \right) \] (4)
\[
- \left( \sum_{s \in S} B^B_{s} B^s_s + \sum_{r \in R} B^R_{r} R^r_r \right) \] (5)
subject to
\[
z^B_{st} \leq \left( \sum_{s \in S, t \in T} b^T_{st} x^B_{st} \right), \quad \forall s \in S, t \in T \] (6)
\[
z^R_{rt} \leq \left( \sum_{r \in R, t \in T} b^R_{rt} x^R_{rt} \right), \quad \forall r \in R, t \in T \] (7)
\[
z^B_{sr} \leq \left( \sum_{s \in S} b^B_{sr} x^B_{sr} \right), \quad \forall s \in S, r \in R \] (8)
\[
\sum_{s \in S} z^B_{st} + \sum_{r \in R} z^R_{rt} \leq 1, \quad \forall t \in T \] (9)
\[
\sum_{s \in S} z^B_{sr} \leq 1, \quad \forall r \in R \] (10)
\[
e^T_{st} - b^T_{st} x^T_{st} \leq w^T_{t} z^B_{st} x^B_{st}, \quad \forall s \in S, t \in T \] (11)
\[
e^R_{rt} - b^R_{rt} x^R_{rt} \leq w^R_{t} z^R_{rt} x^R_{rt}, \quad \forall r \in R, t \in T \] (12)
\[
\sum_{s \in S} e^B_{sr} z^B_{sr} x^B_{sr} \leq b^B_{sr} x^B_{sr}, \quad \forall s \in S, r \in R \] (13)
\[
b^B_{sr} \leq b^B_{sr}, \quad \forall s \in S, r \in R \] (14)
\[
\sum_{t \in T} b^T_{st} + \sum_{r \in R} b^R_{sr} \leq b^s_s, \quad \forall s \in S \] (15)
\[
\sum_{t \in T} b^R_{rt} \leq b^r_r, \quad \forall r \in R \] (16)
\[
(\forall j \in S) \left( x^B_{ji} + x^B_{j+1} \right) \leq 3 d_{\text{min}}^B, \quad \forall j > i \in S \] (17)

In this MILP formulated multi-objective optimization problem, the deployment and coverage indicators from Table I serve as binary decision variables and the bandwidth allocations as variables from continuous space, respectively. Neglecting the cost term (5) in the objective function, the BLOPmax problem is a relaying extended variant of the Maximal Covering Location Problem from [11], adding bandwidth limitation constraints and counting served user throughput demand instead of weights.

The parameter $\lambda > 0$ in (4) defines the relation between served throughput demand and network deployment cost. As there are only two contrary objectives, the weight factor for cost (5) is set to one without loss of generality. In that sense, $\lambda$ may be interpreted as throughput utilization factor for the network operator. Consequently, the objective function describes the network operator profit in a closed-form formulation for an appropriately chosen $\lambda$.

Constraints (6) to (8) allow a station to cover and serve a potential recipient only if the station is deployed and spectral efficiency on the corresponding link exceeds a minimum threshold. While (9) ensures non-cooperative relaying, deployment of a RS requires a radio link connection from exact one BS modelled by (10). Constraints (11) and (12) bound the served throughput to TNs to at most the requested demand. Particularly, at this point the interrelationship between served throughput demand and bandwidth consumption becomes obvious. Generalizing the throughput bound described by (1) and (2) leads to constraints (13) and (14). Here, the coverage indicator variables $z^B_{sr}$ in (14) allow at most one summand on the right hand side of (13) to stay above zero. Constraints (15) and (16) ensure bandwidth limitations at deployed BSs and RSs in analogy to (3). Finally, (17) is a smart linear formulation to ensure minimum distance $d_{\text{min}}^B$ between deployed BSs. Properness of (17) can easily be verified by considering all four possible constellations of $x^B_{ji}$ and $x^B_{j+1}$.

So far, a BS or RS can cover any TN that meets the minimum link quality requirements (6) and (7), respectively. Each deployed station allocates bandwidth for transmission to its covered TNs bounded by total available bandwidth and TN demand. Therefore, effective throughput to a covered TN $t$ is in the intervall $[0, w^T_{t}]$. To support only solutions that guarantee a certain quality of service, the optimization model can easily be extended by a further constraint that enforces a minimum served throughput demand for each covered TN.

B. Reasonable Choice for Throughput Utilization Factor $\lambda$

As the throughput utilization factor $\lambda$ in (4) defines the relation between served user throughput demand and network
deployment cost, that parameter strongly influences the number of deployed stations in optimal solutions. Basically, there exist two reasonable options how to choose an appropriate utilization factor $\lambda$.

First, from business experience the operator typically knows the number of users $n$ – demanding a certain wireless service with throughput requirement $w$ – that is needed to reach the break-even point with respect to cost for station deployment and operating a station, respectively. Hence, aiming at a break-even guarantee the utilization factor can be calculated as quotient of overall cost $c$ and cumulated throughput, i.e., $\lambda = c/nw$ [EUR/kbps]. If the operator introduces a safety margin for $n$, the optimal objective value will not represent the de facto profit.

Second, choosing the utilization factor as average revenue per unit (ARPU) allows BLOPmax solutions for predicting realistic profit values and evaluation of business cases in advance. Typically, the ARPU is calculated as average quotient of user revenue and cumulated amount of served user throughput over a fixed time period. The user revenue considers basic fee, data fee, and additional services fees and, therefore, may vary for different services. In that case, the throughput term (4) has to be split up linearly, i.e.,

$$\sum_{k \in \mathcal{K}} \lambda_k \left( \sum_{s \in \mathcal{S}} \sum_{t \in T_k} e_{st}^{B1T} b_{st}^{B1T} + \sum_{r \in \mathcal{R}} \sum_{t \in T_k} e_{st}^{R1T} b_{st}^{R1T} \right)$$

for a service set $\mathcal{K}$, $\{T_k\}_{k \in \mathcal{K}}$ a partition of $T$, and service-specific throughput utilization factors $\lambda_k$.

IV. NUMERICAL RESULTS

We consider the BLOPmax problem for an exemplary planning scenario and particularly analyze its behaviour to modifications of the comprehensive throughput utilization factor $\lambda = \lambda_1 = \ldots = \lambda_k$. The planning scenario is generated randomly based on the parameter list in Table II and is depicted in Figure 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area size</td>
<td>1750 m × 1750 m</td>
</tr>
<tr>
<td>Number of BS / RS candidates</td>
<td>15 / 45, uniformly distributed</td>
</tr>
<tr>
<td>Number of TNs</td>
<td>650, uniformly distributed</td>
</tr>
<tr>
<td>BS / RS carrier frequency</td>
<td>2 GHz / 2.5 GHz</td>
</tr>
<tr>
<td>BS / RS channel bandwidth</td>
<td>20 MHz / 10 MHz</td>
</tr>
<tr>
<td>BS / RS monthly cost</td>
<td>2500 EUR / 500 EUR (OPEX)</td>
</tr>
<tr>
<td>BS / RS / TN height</td>
<td>25 m / 5 m / 1.5 m</td>
</tr>
<tr>
<td>BS / RS Tx power</td>
<td>46 dBm / 24.38 dBm</td>
</tr>
<tr>
<td>BS / RS / TN antenna gain</td>
<td>14 dB / 9 dB / 0 dB</td>
</tr>
<tr>
<td>BS / RS / TN noise figure</td>
<td>5 dB / 7 dB / 7 dB</td>
</tr>
<tr>
<td>Path loss B ↓ 1 T / B ↓ R / R ↓ T</td>
<td>WINNER II / C2 NLOS / C1 NLOS / B1 (omnidirectional) [20]</td>
</tr>
<tr>
<td>Link penetration (worst case)</td>
<td>40 dB (shadowing, fast fading, etc.)</td>
</tr>
<tr>
<td>TN link penetration</td>
<td>10 db (indoor user)</td>
</tr>
<tr>
<td>SNR-related spectral efficiencies</td>
<td>WiMAX link budget [21]</td>
</tr>
</tbody>
</table>

BS candidates are visualized by a red dot in the center position and concentric circles colored in different grey tones that correspond to local spectral efficiencies due to available SNR and link budget, respectively. RS candidates are visualized analogously by a blue dot and provide only one high spectral efficiency region due to the RS transmission profile. TNs appear as green dots in Figure 2. Each TN demands one randomly assigned wireless service according to service probabilities from Table III. The corresponding throughput demand is uniformly distributed over the service-specific throughput interval given in Table III.

The MILP formulation of BLOPmax and the moderate problem size allow for utilizing CPLEX as solving engine to find optimal solutions. We choose the throughput utilization factor as ARPU that is assumed to be around 0.05 EUR/MB and to be constant for all offered services. As cost is given on a monthly basis, the ARPU is adapted by multiplication with a suitable coefficient assuming TNs to have active demand on twenty days per month and five hours each day. Hence, on a monthly basis the throughput utilization factor is supposed to be around $\lambda = 2.25$ EUR/kbps. Cost considers OPEX including site rental, leased line rental, air conditioning, and maintenance.

Figure 3 shows the number of deployed BSs and RSs in
optimal solutions to the applied BLOPmax problem for a stepwise increasing utilization factor $\lambda$ from 0.5 to 8 EUR/kbps. Even for increasing throughput utilization the number of deployed BSs constantly remains at a value of seven. This is the upper bound for that scenario due to the required minimum distance of $d_{\text{min}} = 500$ m between any pair of deployed BSs. For instance, $d_{\text{min}} = 250$ m leads to a constant number of ten deployed BSs in optimal solutions to the same scenario.

However, the number of deployed RSs rises with increasing throughput utilization and attains a maximum of 29 for the utilization $\lambda = 8$ EUR/kbps. Applying this utilization factor, a RS gains positive profit for serving a single TN that demands VoIP service. Generally, a RS candidate potentially gaining positive profit is deployed if that improves coverage at BS cell border or enhances BS capacity due to better link quality. According to Figure 3, moderate variation of presumable realistic throughput utilization around $\lambda = 2.25$ EUR/kbps may lead to considerable changes in the number of deployed RSs and associated cost, respectively. This effect particularly points out the optimization model sensitivity to parameter variation.

V. CONCLUSIONS

Utilizing the proposed BLOPmax problem for planning and configuration of modern 4G multi-hop relay networks enables the network operator for deploying and operating a profit gaining network. For a suitable choice of optimization parameters our model resolves the problem of conflicting objectives by a closed-form formulation and provides economical reasonable and realistic objective values. Therefore, our optimization framework is applicable to consider and evaluate business cases in advance.

The joint BS and RS placement problem is formulated as mixed-integer linear program and particularly considers technical system characteristics and resource restrictions such as limited bandwidth. As a side effect, each solution to the BLOPmax problem provides a feasible bandwidth allocation scheme with respect to the considered system setup.

According to the presented numerical evaluations, the optimization model is sensitive to parameter modifications and particularly the throughput utilization factor strongly influences potential solutions. Hence, accurate parameter calibration is a prerequisite to achieve sustainable results with our optimization model.

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