Factor Graph based Detection and Channel Estimation for MIMO-OFDM Systems in Doubly Selective Channel

Xiang Xu and Rudolf Mathar

Institute for Theoretical Information Technology, RWTH Aachen University

Aachen, Germany 52074

Email: {xu,mathar}@ti.rwth-aachen.de

Abstract—**Based on the factor graph, a joint channel estimation and detection method is proposed for orthogonal frequency division multiplexing (OFDM) systems employing multiple transmit and receive antennas (MIMO). Using belief propagation, statistics of the fast varying MIMO multipath channel are utilized in estimating channel coefficients. By proper message scheduling and passing, the proposed receiver is effective for MIMO-OFDM system in fast fading frequency-selective channel.**

Index Terms—**MIMO, OFDM, channel estimation, iterative receiver, factor graph.**

I. Introduction

High data-rate wireless communication is demanded by many applications. Novel techniques such as MIMO and OFDM are promising choice of future high data rate systems.

MIMO employs multiple antennas at transmitter side and receiver side to establish parallel channels over the same time and frequency, high data rate can be achieved without extra bandwidth. However, in MIMO multi-path channel, complex equalization algorithms are usually required to avoid inter-symbol interference(ISI). Another way to overcome ISI is OFDM, which converts broad band communication channel into several narrow band subchannels and employs a cyclic prefix to remove ISI. By employing orthogonal subcarriers, OFDM can be efficiently implemented using fast Fourier transformation (FFT). The combination of MIMO and OFDM is thus attractive for broadband wireless communications.

Channel estimation is a challenging task in wireless systems because of dynamic property of the radio channel. In a typical radio propagation environment, transmitted signals are only able to reach the destination through multiple paths of reflection or scattering. The multipath propagation leads to variation of channel response over frequencies. Moreover,any motion of the transmitters, receivers or scatterers will cause change of channel impulse response over time. These random variations make the channel difficult to track. Nevertheless, the variation over time or frequency follows certain statistics. This paper will show, by reasonably exploiting these statistics,

the performance of channel estimation could be improved significantly.

In recent years, joint channel estimation and data detection are gaining more and more attention. In joint receivers, the data decision obtained from the detector, either hard or soft, is used as additional training to reinforce the channel estimation [1]. Many detection algorithms have been developed, including sphere detection [2], joint Gaussian detection [3], etc. A very important issue in designing such a joint receiver is the complexity. To reduce complexity, the multiple antenna interference (MAI) can be regarded as an additive noise [4]. Making the approximation that MAI on different antennas are independent, the detection procedure can be greatly simplified.

In this paper, the message updating algorithm for joint detection and channel estimation is represented by the factor graph [5]. We consider a MIMO-OFDM system in time varying frequency-selective channel, where the statistics of the channel is known. The temporal correlated channel can be approximated by a autoregressive (AR) channel model. And the frequency correlation is described by the power-delay profile (PDP).

This paper is organized as follows. The system and channel models are described in Section II. Section III contains the detail about the iterative data detection and channel estimation algorithm. Simulation results are provided in Section IV for evaluation. Finally, conclusions are given in Section V. For notation, we use $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ to denote matrix transpose and transpose conjugate.

II. System Model

A. MIMO-OFDM Channel Model

A baseband equivalent model for MIMO-OFDM system with N_{T} and N_{R} antennas for transmitter and receiver, respectively, is depicted in Fig. 1. On each antenna, the bit streams are parallelized after encoding and interleaving. Baseband mapping is performed afterwards. With properly inserted training symbols, the modulated symbol stream are then modulated by orthogonal subcarriers via inverse fast Fourier transform (IFFT), and a cyclic prefix is added on the tail of an OFDM symbol. The parallel signals are converted to serial form and
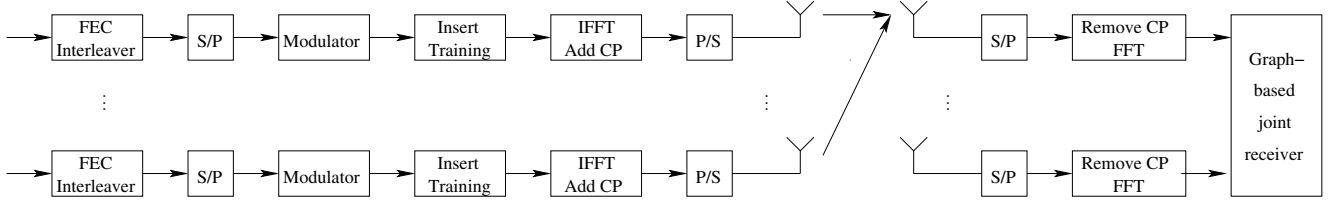
Fig. 1.   System model for MIMO-OFDM system

transmitted through different antennas. With suitable cyclic prefix insertion and sampling, the MIMO-OFDM system with K subcarriers decouples frequency-selective channel into K correlated flat-fading channel. The OFDM signal on sth transmit antenna is modulated by QPSK (quadrature phase shift keying) symbol $X_s[n,k]$ at the kth tone of the nth OFDM block. The received signal on each receive antenna at the kth tone of the nth OFDM block is superposition of N_T distorted transmitted signal, thus, can be expressed as

$$Y_u[n,k] = \sum_{s=1}^{N_T} H_{u,s}[n,k]X_s[n,k] + W_u[n,k], \qquad (1)$$

where $s = 1, 2, \cdots, N_T$ and $u = 1, 2, \cdots, N_R$ are transmit and receive antenna indices, respectively. $W_u[n,k]$ denotes additive white Gaussian noise (AWGN) with zero mean and variance σ_W^2, while the symbol power is normalized to 1. $H_{u,s}[n,k]$ denotes the channel frequency response (CFR) between the sth transmit and the uth receive antenna, the average power of the CFR in each subchannel is normalized to 1. On receiver side, all symbols from one frame is processed simultaneously, for blockwise transmission is adopted.

B. Statistics of doubly selective channel

We assume that there is no spatial correlation for different antenna pairs, i.e. $H_{u,s}[n,k]$ are independent for different s's and u's. Thus, the antenna indices can be omitted while investigating statistics of doubly selective channel.

Consider a wide-sense stationary uncorrelated scattering (WSSUS) model for the time varying frequency-selective mobile radio channel. The baseband impulse response of the channel can be described by

$$h(t,\tau) = \sum_{l=1}^{L} \gamma_l(t)\delta(\tau - \tau_l), \qquad (2)$$

where L is the total number of paths, τ_l is the delay of the lth path, and $\gamma_l(t)$ is the corresponding complex amplitude. Moreover, $\gamma_l(t)$'s are wide-sense stationary (WSS) complex Gaussian processes and independent for different paths with average power σ_l^2. The channel is normalized such that $\sum_{l=1}^{L} \sigma_l^2 = 1$ Furthermore, $\gamma_l(t)$'s are assumed to have the same correlation function $r_t(\Delta t)$. Hence

$$r_{\gamma_l}(\Delta t) \triangleq \mathrm{E}\{\gamma_l(t+\Delta t)\gamma_l^*(t)\} = \sigma_l^2 r_t(\Delta t). \qquad (3)$$

The frequency response at the kth tone of the nth OFDM block can be obtained as

$$H[n,k] = \sum_{l=1}^{L} \gamma_l(nT)W_K^{kl}, \qquad (4)$$

where $W_K = \exp(-\jmath 2\pi/K)$.

For OFDM systems with total bandwidth B and K subcarriers, the OFDM block length $T = K/B$. The correlation function of the CFR for different times and frequencies is

$$\mathrm{E}\{H[n+\Delta n, k+\Delta k]H^*[n,k]\}$$
$$= \mathrm{E}\{\sum_{l=1}^{L}\sum_{m=1}^{L} \gamma_l((n+\Delta n)T)\gamma_m^*(nT)W_K^{(k+\Delta k)l}W_K^{km}\}$$
$$= r_t(\Delta nT)\sum_{l=1}^{L}\sigma_l^2 W_K^{\Delta kl}$$
$$= r_t[\Delta n]r_f[\Delta k] \qquad (5)$$

where

$$r_t[\Delta n] \triangleq r_t(\Delta nT), \quad r_f[\Delta k] \triangleq \sum_{l=1}^{L}\sigma_l^2 W_K^{\Delta kl}. \qquad (6)$$

From (5), the correlation function can be separated as multiplication of the time-domain correlation $r_t[\Delta n]$ (depending on Doppler frequency) and frequency-domain correlation $r_f[\Delta k]$ (depending on power delay profile).

The time correlation of channel coefficients follows Jakes' isotropic scattering model [6]:

$$r_t[\Delta n] = J_0(2\pi f_d T \Delta n), \qquad (7)$$

where J_0 is the zeroth order Bessel function of the first kind and f_d is the maximum Doppler frequency.

It is known that dynamic systems can be well modeled by AR processes [7]. The Dth order AR model of the CFR is

$$H[n,k] = \sum_{d=1}^{D} a_d H[n-d,k] + \xi[n,k] \qquad (8)$$

where a_d is coefficient of the AR process and $\xi[n]$ is a complex white Gaussian noise process with uncorrelated real and imaginary components. These parameters can be obtained by solving the Yule-Walker equation using (7).

C. Factor Graph

Without knowledge of channel state information, the CFR should also be regarded as variable nodes in the factor graph. The factor graph of a single antenna system with single carrier modulation is shown in Fig. 2, with filled squares indicating function nodes and open circles indicating variable nodes.

According to the separability of time and frequency correlations in OFDM systems, the doubly selective channel represented by the factor graph depicted in Fig. 3 shows a 2-dimensional structure, where the time correlation and frequency correlation are considered individually.
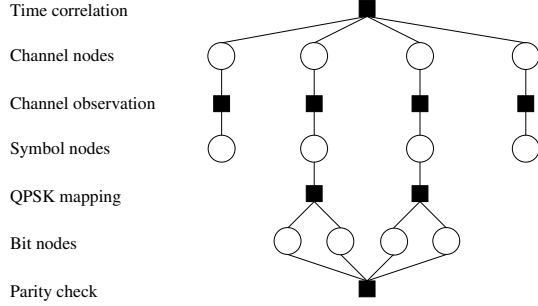

Fig. 2. General factor graph of a coded single antenna system with single carrier. The first and last symbol nodes correspond to pilot symbols.
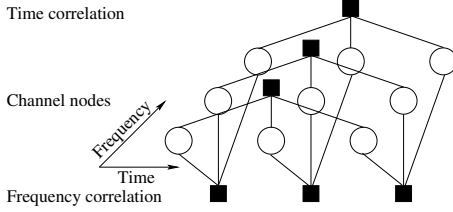

Fig. 3. Factor graph of a doubly selective channel.

III. ITERATIVE RECEIVER USING FACTOR GRAPH

A. Receiver Structure

On the receiver side, after removal of cyclic prefix and FFT, received signals are processed by a joint iterative receiver. The sum-product algorithm for channel estimation and data detection can be summarized as the following:

1) Step 1: Initialization: Before the first iteration, a raw estimation of the channel is obtained by training symbols.

2) Step 2: Data detection: Use soft information of estimated CFR to determine transmitted bits according to channel observation. Messages containing reliability of the detected symbols are combined to get log likelihood ratios (LLR) of transmitted bits. In case of coded transmission, soft decoding is executed based on LLRs.

3) Step 3: Channel estimation using channel statistics: With the help of successfully detected symbols, soft channel estimation is performed. The soft information are further refined with known channel statistics.

4) Step 4: Repeat until done: Step 2 and 3 are repeated until certain stopping criterion is fulfilled. Thereafter, hard decision is made as output.

B. Data Detection with Soft Channel Information

To reduce the complexity of MIMO detection, Gaussian approximation is made [4]. Consider an arbitrary symbol X_s (time and subcarrier indices are omitted for simplicity) and its observations Y_u, all the signal components contributed by symbols other than X_s can be regarded as effective noise:

$$
\begin{aligned}
Y_u &= \sum_{\varsigma=1}^{N_T} H_{u,\varsigma} X_\varsigma + W_u \\
&= H_{u,s} X_s + \sum_{\varsigma=1,\varsigma\neq s}^{N_T} H_{u,\varsigma} X_\varsigma + W_u \\
&= H_{u,s} X_s + V_{u,s},
\end{aligned} \tag{9}
$$

where the effective noise sample defined by

$$
V_{u,s} \triangleq Y_u - H_{u,s} X_s \tag{10}
$$

is approximated to have a complex Gaussian distribution $\mathcal{CN}(\mu_{V_{u,s}}, \sigma^2_{V_{u,s}})$.

Due to the inaccuracy of the channel estimation, the noisy estimate of CFR can be written as:

$$
\widehat{H}_{u,s} = H_{u,s} + e_{u,s}, \tag{11}
$$

where the estimation error $e_{u,s}$ can be assumed to have a Gaussian distribution $\mathcal{CN}(0, \sigma^2_{H_{u,s}})$ [8]. Hence, $H_{u,s}$ is also Gaussian distributed with mean $\widehat{H}_{u,s}$ and variance $\sigma^2_{H_{u,s}}$. According to (9), the received symbol can be written as

$$
Y_u = \widehat{H}_{u,s} X_s - e_{u,s} X_s + V_{u,s}. \tag{12}
$$

Therefore, the likelihood function can is given by

$$
\begin{aligned}
p(Y_u|X_s) &= \frac{1}{\pi(|X_s|^2 \sigma^2_{H_{u,s}} + \sigma^2_{V_{u,s}})} \cdot \\
&\quad \exp\left(-\frac{|Y_u - \widehat{H}_{u,s} X_s - \mu_{V_{u,s}}|^2}{|X_s|^2 \sigma^2_{H_{u,s}} + \sigma^2_{V_{u,s}}}\right). \tag{13}
\end{aligned}
$$

Since QPSK mapping is adopted, transmitted symbol X_s consists two bits c_s^i, $i \in \{1, 2\}$. Define $\mathscr{X}_b^i$ as the set of constellation points corresponding to $c_s^i = b$, the output LLRs of the data detector is given as

$$
\begin{aligned}
\mathrm{LLR}(c_s^i) &= \log \frac{P(c_s^i = 0|Y_u)}{P(c_s^i = 1|Y_u)} \\
&= \max_{X_s \in \mathscr{X}_0^i} \log p(Y_u|X_s) - \max_{X_s \in \mathscr{X}_1^i} \log p(Y_u|X_s) \\
&= \max_{X_s \in \mathscr{X}_0^i} \left(-\frac{|Y_u - \widehat{H}_{u,s} X_s - \mu_{V_{u,s}}|^2}{|X_s|^2 \sigma^2_{H_{u,s}} + \sigma^2_{V_{u,s}}}\right) \\
&\quad - \max_{X_s \in \mathscr{X}_1^i} \left(-\frac{|Y_u - \widehat{H}_{u,s} X_s - \mu_{V_{u,s}}|^2}{|X_s|^2 \sigma^2_{H_{u,s}} + \sigma^2_{V_{u,s}}}\right) \tag{14}
\end{aligned}
$$

LLR messages from different receive antennas are first combined and then sent to bit nodes via symbol demapping.

To find the mean value and variance of the effective noise sample $V_{u,s}$, we start from incoming LLR messages from bit nodes. The probability that one bit equal to 0 or 1 is

$$P(c_s^i = 0) = \frac{\exp(\text{LLR}(c_s^i))}{1 + \exp(\text{LLR}(c_s^i))}$$
$$P(c_s^i = 1) = \frac{1}{1 + \exp(\text{LLR}(c_s^i))}. \tag{15}$$

A symbol by symbol detection is to take the mean value over all the constellation points

$$\widehat{X}_s = \mu_{X_s} = \sum_{i=1}^{4} P(X_s = q_i) q_i \tag{16}$$

as the soft decision, where q_i, $i = 1, 2, 3, 4$ are QPSK constellation points. The variance of symbol X_s is

$$\sigma_{X_s}^2 = 1 - |\mu_{X_s}|^2, \tag{17}$$

as the symbol energy is normalized to 1.

Therefore, mean and variance of $V_{u,s}$ can be calculated via

$$\mu_{V_{u,s}} = \sum_{\varsigma=1, \varsigma \neq s}^{N_{\text{T}}} \widehat{H}_{u,\varsigma} \mu_{X_\varsigma}$$
$$\sigma_{V_{u,s}}^2 = \sum_{\varsigma=1, \varsigma \neq s}^{N_{\text{T}}} \sigma_{H_{u,\varsigma}}^2 + \sigma_{X_\varsigma}^2 |\widehat{H}_{u,\varsigma}|^2 + \sigma_W^2. \tag{18}$$

In case of coded transmission, soft information about transmitted bits are further processed by a channel decoder.

C. MIMO Channel Estimation with Soft Data Detection

In channel estimation stage, observation nodes send messages to channel nodes with the information about CFRs. These massages are then forwarded to time and frequency correlation nodes and refined with statistics of the channel.

The information about channel coefficient $H_{u,s}$ are fully contained in the conditional probability

$$p(Y_u | H_{u,s}) = \sum_{X_s \in \mathscr{X}} p(Y_u | H_{u,s}, X_s) P(X_s) \tag{19}$$

where $\mathscr{X}$ is the set contains all the constellation points. With messages from the data detector, this conditional pdf is mixed Gaussian distributed. Since there is no algorithm is fast and accurate in computing mixed Gaussian distribution, proper approximation has to be made. Inspiring by the factor that when the transmitted symbol is successfully detected, $P(X_s = q_i) \gg P(X_s = q_\nu), \nu \neq i$, $H_{u,s}$ can be approximated by a Gaussian distribution $\mathcal{CN}(\mu_{H_{u,s}}, \sigma_{H_{u,s}}^2)$, the messages of mean and variance are calculated using soft information of transmitted symbol X_s from (16) and (17) as follows:

$$\mu_{H_{u,s}} = (Y - \mu_{V_{u,s}}) \widehat{X}_s^*$$
$$\sigma_{H_{u,s}}^2 = \sigma_{V_{u,s}}^2 + \sigma_{X_s}^2 |Y - \mu_{V_{u,s}}|^2 \tag{20}$$

where $\widehat{X}_s^*$ is the complex conjugate of the estimated symbol.

D. Refined OFDM Channel Estimation for Doubly Selective Channel

To improve the performance, messages from different times or frequencies are combined in correlation nodes. During this stage, the distribution of an arbitrary CFR H is desired given information about multiple estimates from other time of frequency indices. Define column vector $\widehat{\mathbf{H}}$ as the vector of available channel estimates. Assuming that both H and $\widehat{\mathbf{H}}$ are Gaussian distributed with zero mean, the joint distribution of H and $\widehat{\mathbf{H}}$ is multivariate Gaussian with covariance matrix [9]

$$\mathbf{\Sigma} = \text{E}\left\{ \begin{pmatrix} H \\ \widehat{\mathbf{H}} \end{pmatrix} (H^* \widehat{\mathbf{H}}^{\text{H}}) \right\} = \begin{pmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{pmatrix}, \tag{21}$$

where

$$\mathbf{\Sigma}_{11} = \text{E}(HH^*) = 1 \quad \mathbf{\Sigma}_{12} = \text{E}\left(H \widehat{\mathbf{H}}^{\text{H}} \right)$$
$$\mathbf{\Sigma}_{21} = \text{E}\left(\widehat{\mathbf{H}} H^* \right) \quad \mathbf{\Sigma}_{22} = \text{E}\left(\widehat{\mathbf{H}} \widehat{\mathbf{H}}^{\text{H}} \right) \tag{22}$$

Then it can be shown $p(H | \widehat{\mathbf{H}})$ is Gaussian distributed with mean and variance

$$\mu_H = \mathbf{\Sigma}_{12} \mathbf{\Sigma}_{22}^{-1} \widehat{\mathbf{H}} \tag{23}$$
$$\sigma_H^2 = 1 - \mathbf{\Sigma}_{12} \mathbf{\Sigma}_{22}^{-1} \mathbf{\Sigma}_{21}. \tag{24}$$

The covariance matrix is obtained with known channel statistics. As described in (5), time and frequency correlation can be considered separately.

1) Message update in time correlation nodes: Suppose we want to estimate $H[n]$ while $\widehat{H}[n+1], \widehat{H}[n+2], \cdots, \widehat{H}[n+\eta]$ are available. Recall (11), being aware of the fact that CFR is independent of the estimation error σ_H^2, the covariance vectors and matrix can be written as

$$\mathbf{\Sigma}_{12} = \begin{pmatrix} r_t[1] & r_t[2] & \cdots & r_t[\eta] \end{pmatrix}$$
$$\mathbf{\Sigma}_{21} = \begin{pmatrix} r_t[1] & r_t[2] & \cdots & r_t[\eta] \end{pmatrix}^{\text{T}}, \tag{25}$$

$$\mathbf{\Sigma}_{22} = \begin{pmatrix} 1 + \sigma_{H[n+1]}^2 & r_t[1] & \cdots & r_t[\eta-1] \\ r_t[1] & 1 + \sigma_{H[n+2]}^2 & \cdots & r_t[\eta-2] \\ \vdots & \vdots & \ddots & \vdots \\ r_t[\eta-1] & r_t[\eta-2] & \cdots & 1 + \sigma_{H[n+\eta]}^2 \end{pmatrix},$$

respectively. $\sigma_{H[n+\eta]}^2$ is the variance of the noisy channel estimate $\widehat{H}[n+\eta]$.

Unfortunately, the inversion of correlation matrix $\mathbf{\Sigma}_{22}$, which is changing during the iterative process, is required. That leads to a very high complexity while processing the whole data frame at once. One approach to reduce the complexity is to divide the data frame into several small blocks and perform the message updating with in these blocks. As a matter of fact, if consider only the correlation of adjacent symbols, no matrix operation is needed. Therefore, the additional computational complexity is acceptable.

2) Message update in frequency correlation nodes: The message updating in frequency correlation nodes is similar to time correlation node, with substituting $r_f[\Delta k]$ for $r_t[\Delta n]$. Due to the periodic behavior of the Fourier transform, the subcarriers at the beginning and end of an OFDM symbol are correlated [10]. This property can be well utilized to improve the estimation of subcarriers on the edge.

After message update in time and frequency correlation nodes, the refined estimated CFR are sent back to observation nodes for the next iteration.

IV. SIMULATION RESULT

The performance of the proposed iterative receiver is investigated by simulation. For the simulation setup, 2 transmit antenna and 2 receive antennas are used. The MIMO channel coefficients for each antenna pair are independently generated by a first order AR model with the normalized Doppler frequency shift $f_d T = 0.01$. The total number of subcarrier is 64. The exponential power delay profile is applied with

$$\sigma_l^2 = \alpha_l \exp(-\frac{l}{\tau_{\text{RMS}}/T_{\text{S}}}), \tag{26}$$

where τ_{RMS} is the root mean square (RMS) delay, and T_{S} is the system sampling frequency. The total power of channel taps is normalized to 1 by α_l. τ_{RMS} is set to $0.25T_{\text{S}}$ in the simulation. The channel memory length is set to 5. QPSK with Gray labeling is adopted as baseband mapping. A regular low density parity check (LDPC) code [11] cross-subcarrier is applied to mitigate frequency selectivity. The coding rate is $1/2$ and column weight is 3. Each frame contains 10 percent training overhead, training symbols are inserted into the center of a frame. Hard decisions are made after 3 iterations.

Three systems are compared in Fig. 4 with bit error ratio (BER) and in Fig. 5 with mean square error (MSE), namely, system consider neither time correlation nor frequency correlation, system consider time correlation only and system consider both time and frequency correlation. Only the correlation of neighboring symbols are considered. Evidently, even only time correlation can make the performance better. With the help of frequency correlation, the performance can be further improved.

V. CONCLUSION

A factor-graph based iterative receiver is proposed for MIMO-OFDM systems in time varying frequency selective channel. Numerical results indicate that the utilization of time and frequency correlation can improve both BER and MSE performance with moderate increase of complexity.

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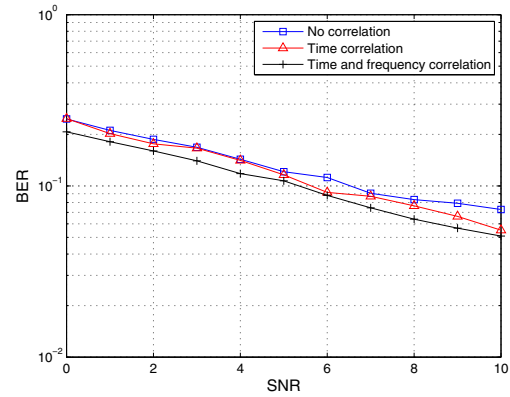
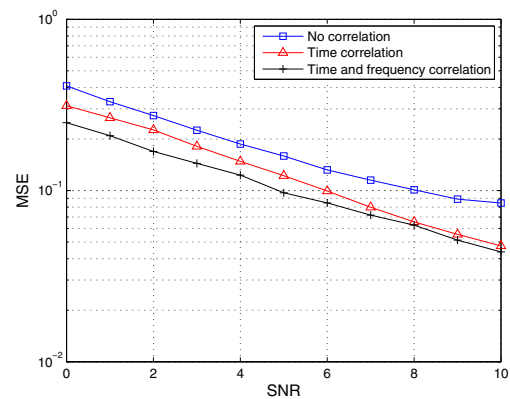Fig. 4. Bit error probability v.s. signal to noise ratio.

Fig. 5. Mean square error of estimated channel response.

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