Resource Allocation in OFDM Systems - Efficiency as Basis for Allocation and Initialization

Michael Reyer, Alexander Schäper, and Rudolf Mathar
Institute of Theoretical Information Technology
RWTH Aachen University, Germany
Email: {reyer, schaeper, mathar}@ti.rwth-aachen.de

***Abstract*—Rate and power allocation in OFDM systems is discussed extensively in literature. These approaches are either based on a finite set of realizable rates or make use of a continuous rate-power function. The motivation of this correspondence was to investigate how to transfer efficiently continuous rate allocations to realizable ones. To increase performance, we suggest to skip some realizable rates. Then, our proposed algorithm for single-user resource allocation on the set of realizable rates produces the same allocation independent of the initialization. This algorithm achieves the optimal solution when the given lower and upper bound for the optimum are identical. Furthermore, we suggest an initialization procedure which comes very close to this allocation. Finally, we present simulation results to assess some quantity measures of our approach.**

***Index Terms*—approximation error, OFDM, rate-power function, resource allocation**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is expected to be the transmission technology of next generation mobile networks, e.g., WiMAX or LTE Advanced. The advantages of OFDM are flexibility of allocating subcarriers to users, adaptive power allocation, high spectral efficiency, low receiver complexity and simple implementation by the inverse fast Fourier transform (IFFT) and FFT, see [1], [2]. OFDM can also be integrated with multiple-input multiple-output (MIMO) techniques to raise the diversity gain and increase capacity, see [1], [3]. Additionally, the OFDM network architecture is a qualified candidate for utilization of assigned spectrum by means of dynamic spectrum access (DSA), see [4].

One of the crucial problems in OFDM systems is rate and power allocation of (bunched) subcarriers in the available frequency band. Various studies have devised resource allocation algorithms. In [5], [6], [7], only integer bit steps are considered. This restriction allows for a prove of optimality of their proposed single-user allocation algorithms. [5] suggests to use the water-pouring distribution from [8] to achieve an optimal solution for the same problem where the rates are real numbers. By rounding the real-valued solution a good initialization for the integer problem is attained. This approach motivates to relax the multi-user optimization problem allowing for continuous rates following that distribution. Fast heuristics exploiting that approach for sub-optimal multi-user allocation are suggested in [9], [10], [11]. However, in those heuristics the transfer of the continuous rate allocation to the system-specific finite set of achievable rates is neglected. Additionally, it is not mentioned how the approximated function or its parameters are obtained.

In this paper, we propose an algorithm for single-user resource allocation on the set of realizable rates producing the same allocation independent on the initialization. This algorithm achieves the optimal solution when the given lower and upper bound for the optimum are identical. Furthermore, we suggest an initialization procedure exploiting the continuous rate-power function which comes very close to this allocation. We discuss two candidates of continuous rate-power functions. For assessing quantitative measures of our approach we present some simulative results.

The rest of the paper is organized as follows. In Section II, we present discrete, i.e., realistic, rate-power functions and deduce main properties which are transfered to continuous rate-power functions. Resource allocation in OFDM systems in general is presented in Section III. For our aim it is sufficient to investigate single-user resource allocation which is introduced and discussed in Section IV. In particular, aspects of optimality and initialization based on continuous rate-power functions are observed. Afterwards, the proposed methods are evaluated and discussed on simulative basis in Section V. Finally, we conclude this paper in Section VI.

II. Rate-Power Functions

In a given OFDM system a fixed and finite set of feasible transmission rates $\mathcal{R} = \{\eta_0, \dots, \eta_S\}$ is given where $S \in \mathbb{N}$ denotes the number of nonzero rates. In this work the rates are sorted in ascending order, i.e., $\eta_k < \eta_l,\ \forall\, 0 \le k < l \le S$. Furthermore, the rate $\eta_0 = 0$ is included to indicate unused subcarriers. This set depends on the available combinations of modulation schemes, channel coding rate, and MIMO mode and is normalized to the subcarrier bandwidth, thus, representing spectral efficiency. In the system specification the minimal required signal-to-noise ratio (SNR) according to some bit-error rates (BER) for each feasible transmission rate is usually tabulated. Such a table is shown for the WiMAX Standard IEEE 802.16e-2005 given a BER of 10^{-6} for the multi antenna case

978-1-61284-402-2/11/$26.00 ©2011 IEEE

(MIMO-STBC, multiple-input multiple-output - space-time block code) in Table I, see [12], [13]. Overall, the rate-

Table I
MINIMAL REQUIRED SNRS FOR BER=10^{-6}, IEEE 802.16E-2005, MIMO-STBC, SEE [13].

η	0.5	1.0	1.5	2.0	3.0
SNR [dB]	1.0	3.0	7.0	9.0	14.0
η	4.0	4.5	6.0	8.0	9.0
SNR [dB]	17.0	19.0	25.0	27.0	30.0

power function $\psi^{\mathrm{d}} : \mathcal{R} \to \mathbb{R}_+$ maps a rate $\eta \in \mathcal{R}$ on the minimal received SNR to realize this rate. If a subcarrier is unused ($\eta = 0$), the corresponding minimal SNR demand should be zero, i.e., $\psi^{\mathrm{d}}(0) = 0$.

The inverse rate-power function

$$(\psi^{\mathrm{d}})^{-1}(\rho) = \max\{\eta \in \mathcal{R} \mid \rho \geq \psi^{\mathrm{d}}(\eta)\}$$

is a step-function indicating the maximal rate for a given SNR ρ. For optimization it is much more convenient to have a continuously differentiable, convex rate-power function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$. Such a function with $\psi(0) = 0$ is a candidate for a *continuous rate-power function*. A natural candidate for rate r and parameters $b, c > 0$ is

$$\psi^{\mathrm{pot}}(r) = b\,(c^r - 1)$$

which can be easily derived from the Shannon capacity, i.e., $b = 1, c = 2$. However, we are also discussing

$$\psi^{\mathrm{poly}}(r) = b\,r^c \quad b > 0, c > 1$$

as generalized candidate from [10].

III. Resource Allocation in OFDM Systems

In this section we briefly introduce the so-called margin adaptive problem minimizing the total transmit power subject to some rate constraints in a multiuser OFDM downlink scenario. We consider a system with N subcarriers and K users. Each user $k \in \{1, \ldots, K\}$ has a requirement of R_k, $R_k \in \mathbb{R}_+$, bits per OFDM symbol. Each subcarrier can be used by only one user at any given time which is optimal, see [14].

Perfect channel state information (CSI) is assumed to be available during transmission. Let $h_{k,n}$ denote the channel gain of subcarrier n for user k and $\sigma^2_{k,n}$ the according noise power. Hence, $u_{k,n} = h_{k,n}/\sigma^2_{k,n}$ is the carrier-to-noise ratio (CNR). If power $p_{k,n}$ is expended on subcarrier n for the transmission to user k, then, $p_{k,n}\,u_{k,n}$ is the received SNR. Using the interrelation between received SNR and rate described given by rate-power functions ψ_k, see Section II, $r_{k,n} = \psi_k^{-1}(p_{k,n}\,u_{k,n}) \geq 0$ is the corresponding rate.

The objective is to find a subcarrier assignment of minimum total transmit power such that each user receives his required data rate. In mathematical terms this reads as

$$\begin{aligned} \min \sum_{k=1}^{K}\sum_{n=1}^{N} & \frac{\psi_k(r_{k,n})}{u_{k,n}} && (1)\\ \text{s.t.} \sum_{n=1}^{N} & r_{k,n} \geq R_k, \quad k = 1, \ldots, K \\ \sum_{n=1}^{N} & r_{k,n}\,r_{\ell,n} = 0, \quad k, \ell = 1, \ldots, K,\ k \neq \ell \end{aligned}$$

Problem (1) is a complicated mixed continuous and combinatorial optimization problem since a joint decision on subcarrier and rate allocation has to be made. As we are interested in approximation errors on ψ_k we decouple the problem into K independent single-user problems by fixing the subcarrier assignment. This will be done by using BABS from [10] to assess the number of subcarriers per user and a simple greedy allocation scheme. Afterwards, the rate and power allocation is performed independently for all users. Consequently, we give a detailed investigation of single-user resource allocation in the next section.

IV. Single-User Resource Allocation

In this section we introduce a general scheme to solve a feasible single-user resource allocation problem with a discrete set of rates $\mathcal{R}$. First, we discuss under which conditions the final (discrete) resource allocation is optimal. Additionally, a lower and upper bound is given in (4). The theoretical insight will motivate the following resource allocation scheme. The proposed scheme enables for initialization which can speed up the computation time. We briefly describe optimal water-filling for two continuous rate-power functions. Those functions are used for some of the initialization strategies which are discussed at the end of this section.

A. Optimality of Discrete Single-User Resource Allocation

For determination of optimality the concept of the following property of a rate vector plays an important role. This property was introduced in [5] for *equidistant* rates, which means $\eta_k - \eta_{k-1} \equiv a$, $k = 1, \ldots, S$ holds. A rate vector $\mathbf{r}$ is called *efficient*, if

$$L_-(r_n, u_n) \leq L_+(r_m, u_m) \quad \forall\, n, m = 1, \ldots, N \qquad (2)$$

holds, where

$$L_+(\eta, u) = \begin{cases} \frac{\psi^{\mathrm{d}}(\mathrm{inc}(\eta)) - \psi^{\mathrm{d}}(\eta)}{u(\mathrm{inc}(\eta)-\eta)} & \mathrm{inc}(\eta) > \eta \\ \infty & \text{elsewise} \end{cases}$$

$$L_-(\eta, u) = \begin{cases} \frac{\psi^{\mathrm{d}}(\eta) - \psi^{\mathrm{d}}(\mathrm{dec}(\eta))}{u(\eta-\mathrm{dec}(\eta))} & \mathrm{dec}(\eta) < \eta \\ 0 & \text{elsewise} \end{cases}, \text{ with}$$

$$\begin{aligned} \mathrm{inc}(\eta) &= \min\{r \in \mathcal{R} \mid r > \eta \vee r = \eta_R\}, \\ \mathrm{up}(\eta) &= \min\{r \in \mathcal{R} \mid r \geq \eta \vee r = \eta_R\}, \\ \mathrm{dec}(\eta) &= \max\{r \in \mathcal{R} \mid r < \eta \vee r = \eta_0 = 0\}, \text{ and} \\ \mathrm{down}(\eta) &= \max\{r \in \mathcal{R} \mid r \leq \eta \vee r = \eta_0 = 0\}. \end{aligned}$$

Note that L_+ and L_- indicate the transmit power per rate, needed to increase or decrease to the next in size rate. A related property is exploited in [6] for integer rates. Evaluating (2) for $n = m$ with $\eta \in \mathcal{R} \setminus \{0, \eta_S\}$, it follows

$$\frac{\psi^{\mathrm{d}}(\eta) - \psi^{\mathrm{d}}(\mathrm{dec}(\eta))}{\eta - \mathrm{dec}(\eta)} \leq \frac{\psi^{\mathrm{d}}(\mathrm{inc}(\eta)) - \psi^{\mathrm{d}}(\eta)}{\mathrm{inc}(\eta) - \eta}. \qquad (3)$$

A discrete rate-power function ψ^{d} is called *discrete convex* if (3) holds for all $\eta \in \mathcal{R} \setminus \{0, \eta_S\}$. Hence, efficiency can only be used as criterion for optimality, if the rate-power function in use is discrete convex. If it is non-discrete convex, however, it is reasonable to neglect the rates which violate condition (3) as they occur in optimal rate allocation in exceptional cases only. Thus, for the applied example MIMO-STBC, see Table I, we take out and mark in red the rates 0.5, 1.5, and 6. In [15] it is stated that a rate allocation $\mathbf{r}$ is optimal with respect to a single-user problem with equidistant rates if and only if $\mathbf{r}$ is efficient and *tight*, i.e., the rate constraint $\sum_{n=1}^{N} r_n = R$ is exactly fulfilled. It is easy to see and proof that equidistant rates are necessary for this optimality criterion.

For not equidistant rates and a discrete convex rate-power function it holds that if $\mathbf{r}$ is efficient and tight the rate allocation is optimal. But there might be rate demands to which no tight and efficient rate allocation exists. However, to every rate demand R, which can be fulfilled, there exist efficient rate allocations $\mathbf{r}^-$ and $\mathbf{r}^+$ with $\sum_{n=1}^{N} r_n^- \leq R \leq \sum_{n=1}^{N} r_n^+$. This insight is used for the single-user resource allocation in next section as well as for the initialization procedure.

If using Algorithm 1 from next section with rate initialization $\mathbf{r} = \mathbf{0}$ all intermediate rate allocations are efficient such that the next to last and last rate allocation provide *lower and upper bounds* p^- and p^+ which are close to the optimal power allocation p^*.

$$p^- \leq p^* \leq p^+ \qquad (4)$$

B. Discrete Single-User Resource Allocation

We assume to have a discrete convex rate-power function in the following which is motivated in the last section. We suggest Algorithm 1 for rate allocation which enables for rate initialization in the discrete set of rates $\mathcal{R}$. Within the outer repeat-loop the rate on the subcarrier with highest power per rate L_- is decreased while the rate demand is fulfilled. Then, the rate on the subcarrier with lowest power per rate L_+ is increased until the rate demand is fulfilled again. As L_- and L_+ are increasing in r and both $L_-(r, u) = L_+(\mathrm{dec}(r), u)$ and $L_-(\mathrm{inc}(r), u) = L_+(r, u)$ hold for $r \neq 0$ and $r \neq \eta_S$, respectively, the alternating procedure of decreasing and increasing will converge to an efficient solution. Note that the selection of argmin and argmax should be unique in order to achieve the same solution independent on the initialization. A simple, sub-optimal, greedy approach for ReduceRates($\cdot$) is implemented which is executed only if the rate demand

Algorithm 1 EfficientRateAdmission(R, $\mathcal{R}$, $\mathbf{u}$)

> $\mathbf{r} \leftarrow$ InitializeRates(R, $\mathcal{R}$, $\mathbf{u}$)
> **repeat**
>   **while** $\sum_{n=1}^{N} r_n \geq R$ **do**
> $m \leftarrow \mathrm{argmax}_{n=1,\ldots,N}\ L_-(r_n, u_n)$
> $r_m \leftarrow \mathrm{dec}(r_m)$
>   **end while**
>   **repeat**
> $m \leftarrow \mathrm{argmin}_{n=1,\ldots,N}\ L_+(r_n, u_n)$
> $r_m \leftarrow \mathrm{inc}(r_m)$
>   **until** $\sum_{n=1}^{N} r_n \geq R$
> **until** IsEfficient($\mathbf{r}$)
> **return** ReduceRates($\mathbf{r}$, R, $\mathcal{R}$, $\mathbf{u}$)

is exceeded. This function might improve the allocation slightly. Common approaches do not decrease and increase the rate allocation, but do solely increase or decrease it, [5], [7]. We refer to the solely increasing approach as *greedy*. The usage of the continuous approximation of the rate-power function for initialization is only reasonable if the solution can be computed efficiently which is the case for ψ^{pot} and ψ^{poly}, see next section.

C. Optimal Water-Filling

The classical water-filling problem, see [16], is formulated for continuous rates. For the optimal resource allocation the water level, i.e., the quotient of the gradient of the continuous rate-power function and the CNR $\psi'(r_n)/u_n$, needs to be constant for all subcarriers with positive rate. The analogon for the discrete case is that all secants $L_+(r_n, u_n)$ should be the same. As this cannot be achieved in general this is relaxed to the efficiency condition (2).

In [17] it is shown that for ψ^{pot} and equidistant rates the optimal discrete solution is between the rounded down and rounded up continuous solution if the approximation is perfect, i.e., the discrete rate-power function ψ^{d} and the approximation ψ^{pot} are identical on the set of rates $\mathcal{R}$ i.e, $\psi^{\mathrm{d}}(\eta) = \psi^{\mathrm{pot}}(\eta)\ \forall\ \eta \in \mathcal{R}$, see also [5] for a special case. Statements on optimality are unknown to us if the assumption of perfect approximation is violated.

According to [9], the optimal water-filling solution using ψ^{pot} can be directly computed as

$$r_n^* = \frac{R}{N} + \log_c\left(\frac{u_n}{\bar{g}(\mathbf{u})}\right), \qquad (5)$$

where $\bar{g}(\mathbf{u})$ denotes the geometric mean of the CNRs. While there exist negative rates, (5) needs to be evaluated excluding the subcarriers with negative rates, cf. [17].

In the case of ψ^{poly} the water-filling solution may be computed efficiently as well. It is given as

$$r_n^* = \frac{R}{N} \frac{\sqrt[c-1]{u_n}}{\bar{a}\left(\sqrt[c-1]{\mathbf{u}}\right)},$$

where $\bar{a}\left(\sqrt[c-1]{\mathbf{u}}\right)$ is the arithmetic mean of the component wise $(c-1)$-th root of the CNRs. For this rate-power function all rates are nonnegative.

Note that for both rate-power functions the optimal rate allocation is independent of the parameter b such that we use $b = 1$ in the following.

D. Strategies for Initialization

We distinguish initialization with and without usage of (continuous) water-filling. The optimal rate allocation for the relaxed optimization problem using an adequate continuous rate-power function as approximation is denoted as $\mathbf{r}^*$. Then the types of initialization are given in Table II, where the evaluation on vectors is performed component wise. The function $\mathrm{rd} : \mathbb{R}_+ \to \mathcal{R}$ returns for all nonnegative rates the nearest one of the discrete set $\mathcal{R}$, i.e.,

Table II
Different initialization variants for InitializeRates(·) .

Name	Definition	Name	Definition
empty	$\mathbf{r} = \mathbf{0}$	full	$\mathbf{r} = \eta_S \mathbf{1}$
rd_avg	$\mathbf{r} = \mathrm{rd}(R/N)$	down_opt	$\mathbf{r} = \mathrm{down}(\mathbf{r}^*)$
rd_opt	$\mathbf{r} = \mathrm{rd}(\mathbf{r}^*)$	up_opt	$\mathbf{r} = \mathrm{up}(\mathbf{r}^*)$

$$\mathrm{rd}(r) = \max\left\{\eta_k \in \mathcal{R} \mid r \geq (\eta_{k-1} + \eta_k)/2, k = 0, \ldots, S\right\},$$

where $\eta_{-1} = 0$ holds.

As we suggest to make the solution efficient, see (2) and Algorithm 1, if possible, it is obviously clever to come as close as possible to an efficient solution during initialization already. This can be achieved using Algorithm 2 called with the optimal continuous solution $\mathbf{r}^*$. In $\mathbf{x}$ the secant of power per rate is stored. Those values depict L_+ of the final initialization if the rate is decreased and L_- if the rate is increased, respectively. In order to keep the maximal L_- value after initialization as low as possible and the minimal L_+ as high as possible the subcarriers with lowest x_n are rounded up and with highest x_n are decreased. During the while-loop the sum rate is kept next to the target rate R while rounding the rate of each subcarrier exactly once. Note that the asymmetric usage of up and dec ensures that there is no rate $r \in \mathcal{R}$ with $\mathrm{up}(r_n) < r < \mathrm{dec}(r_n)$.

In case of equidistant rates and a perfect approximating rate-power function ψ^{pot} this initialization leads to the optimal solution, see Section IV-C. Hence, this approach should be the most promising one. Note that the calculation of L_- and L_+ needs to be performed in Algorithm 1 anyway such that the computational effort against other initializations is very low.

Algorithm 2 EfficientInit(R, $\mathcal{R}$, $\mathbf{u}$, $\mathbf{r}$)

Requires: $\mathbf{0} \leq \mathbf{r} \leq \eta_S \mathbf{1}$
$A \leftarrow \{1 \leq n \leq N \mid 0 < r_n < \eta_S\}$
$\mathbf{x} \leftarrow \left(\frac{\psi^{\mathrm{d}}(\mathrm{up}(r_n) - \psi^{\mathrm{d}}(\mathrm{dec}(r_n))}{\mathrm{up}(r_n) - \mathrm{dec}(r_n)}\right)_{n \in A}$
while $A \neq \emptyset$ **do**
  if $\sum_{n=1}^{N} r_n \leq R$ **then**
    $m \leftarrow \mathrm{argmin}_{n \in A}\, x_n$
    $r_m \leftarrow \mathrm{up}(r_m)$
  else
    $m \leftarrow \mathrm{argmax}_{n \in A}\, x_n$
    $r_m \leftarrow \mathrm{dec}(r_m)$
  end if
  $A \leftarrow A \setminus \{m\}$
end while
return $\mathbf{r}$

V. Evaluation

The great benefit of Algorithm 1 is that for a discrete convex rate-power function the same resource allocation is achieved independently from initialization. Hence, the number of rate adaptations in Algorithm 1 is used as quality measure. Before discussing the results the simulation setup is outlined.

A. Simulation Setup

Users are equally distributed on a circular ring with radius from 20 to 100 m. They experience a path loss proportional to $d^{-\alpha}$. The individual channel gains per subcarriers are independent and identical exponentially distributed. The traffic type is randomly chosen according to the parameters from Table III. Video and audio traffic have a constant, normalized rate demand, while data rates are exponentially distributed. Hence, the sum rate of scenarios varies, even if the number of users and subcarriers is fixed. However, to enable repetitions of the experiments for same sum rate, the rates are uniformly adapted. The subcarrier assignment is based on BABS, see [10], to determine the number of assigned subcarrier per user. Afterwards the subcarrier assignment is performed in a simple greedy manner.

Table III
Simulation parameters.

Parameter	Values
Exponent of radial attenuation	$\alpha = 2.5$
User distances to BS [m]	$d \in [20, 100]$
Video/Audio/mean data rate [Bits/s]	$(16, 4, 8)$
Probability of video, audio, and data	$(0.1, 0.4, 0.5)$

B. Results

a) Greedy against Algorithm 1: Obviously the number of rate adaptations for Algorithm 1 is higher as in the greedy approach where rates are increased only. However, even for the initialization down_opt the sum power can be improved up to 4% on average for scenarios with an average rate demand of 6 per subcarrier, i.e., the cell load is $L = 6/9 = 2/3$ as 9 is the maximal rate, see Table I.

b) Best Approximation of Rate-Power Function: Comparison of the rate-power functions shows that ψ^{pot} outperforms ψ^{poly}. Firstly, the optimal parameter over different loaded scenarios is less volatile. Secondly, the number of adaptations is significantly lower. Although the best approximation parameter c depends on the load we suggest to take $c = 2$ resulting in $\psi^{\text{pot}}(r) = 2^r - 1$. This is the best parameter on average in simulation as well as the value motivated by the Shannon capacity.

c) Non-Discrete Convex Against Discrete Convex: Simulative investigations have shown that the number of adaptations and the sum power are both higher if using the non-discrete convex rate-power function instead of the discrete convex rate-power function skipping the non-discrete convex rates, see Table I. Obviously, the number of adaptations should be lower, if there are less rates. Furthermore, the algorithm is unaware of the fact that increasing (decreasing) the rate two steps would be better in terms of the transmit power per rate as increasing (decreasing) just one step if a non-discrete convex rate is met. The number of operations for the discrete convex function reduces up to 40% whereas the power reduction is roughly 1%. However, reduction strongly depends on the fact how many rates on the subcarriers are in the region of non-discrete convex rates.

d) Impact of Initialization: As depicted in Table IV Algorithm 2 clearly outperforms the other initializations while not being more complex than the ones using a continuous rate-power function.

Table IV

Average number of adaptations for $K = 1$, $N = 100$, $c = 2$, and load $L = 3/9 = 1/3$ using various initializations.

Name	# of adapt.	Name	# of adapt.
empty	303.13	full	398.19
avg_round	129.09	down_opt	43.70
rd_opt	27.14	up_opt	56.91
Algorithm 2	4.53		

e) Choice of Algorithms: Some simulative studies using $\psi^{\text{pot}}(r) = 2^r - 1$ suggest that if Algorithm 2 is used Algorithm 1 may be skipped if the power loss of circa 1% on average and about 3% at maximum is tolerable.

VI. Conclusion

In this paper we presented a near optimal single-user resource allocation algorithm for OFDM systems which produces the same allocation independent of the initialization. The latter holds for discrete convex power functions with arbitrary rate steps. If the given power function is non-discrete convex, which is the case for some practical systems, see Table I, we recommend to skip the non-discrete rates generally leading to better results. We presented tight lower and upper bounds for the optimum. Furthermore, we suggested an algorithm for initialization which is based on a continuous approximation of the rate-power function which comes close to the optimal solution.

Acknowledgment

This work was partially supported by *UMIC*, a research project in the framework of the German excellence initiative. It is based on a thesis of Alexander Schäper [18].

References

[1] C. Dubuc, D. Starks, T. Creasy, and Y. Hou, "A MIMO-OFDM prototype for next-generation wireless WANs," *IEEE Commun. Mag.*, vol. 42, no. 12, pp. 82–87, Dec. 2004.

[2] A. Jamalipour, T. Wada, and T. Yamazato, "A tutorial on multiple access technologies beyond 3G mobile networks," *IEEE Commun. Mag.*, vol. 43, no. 2, pp. 110–117, Feb. 2005.

[3] Y. J. Zhang and K. B. Letaief, "An efficient resource-allocation scheme for spatial multiuser access in MIMO/OFDM systems," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 107–116, Jan. 2005.

[4] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol. 50, no. 13, pp. 2127–2159, 2006.

[5] J. Campello, "Practical bit loading for DMT," in *Proceedings: IEEE International Conference on Communications (ICC)*, vol. 2, Vancouver, Canada, Nov. 1999, pp. 801–805.

[6] B. S. Krongold, K. Ramchandran, and D. L. Jones, "Computationally efficient optimal power allocation algorithms for multicarrier communication systems," *IEEE Trans. Commun.*, vol. 48, no. 1, pp. 23–27, Jan. 2000.

[7] A. Fasano, "On the optimal discrete bit loading for multicarrier systems with constraints," in *Proceedings: IEEE Conference on Vehicular Technology (VTC)*, Jeju Island, Korea, Apr. 2003, pp. 915–919.

[8] R. G. Gallager, *Information Theory and Reliable Communications*. New York: John Wiley & Sons Ltd., 1968.

[9] A. Feiten, R. Mathar, and M. Reyer, "Rate and power allocation for multiuser OFDM: An effective heuristic verified by branch-and-bound," *IEEE Trans. Wireless Commun.*, vol. 7, no. 1, pp. 60–64, Jan. 2008.

[10] D. Kivanc, G. Li, and H. Liu, "Computationally efficient bandwidth allocation and power control for OFDMA," *IEEE Trans. Wireless Commun.*, vol. 2, no. 6, pp. 1150–1158, Nov. 2003.

[11] C. Mohanram and S. Bhashyam, "A sub-optimal joint subcarrier and power allocation algorithm for multiuser OFDM," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 685–687, Aug. 2005.

[12] "IEEE 802.16e-2005 IEEE standard for local and metropolitan area networks, part 16: Air interface for fixed and mobile broadband wireless access systems," IEEE, 2005.

[13] C.-B. Chae, A. Forenza, R. W. Heath, M. R. McKay, and I. B. Collings, "Adaptive MIMO transmission techniques for broadband wireless communication systems," *IEEE Trans. Commun.*, vol. 48, no. 5, pp. 112–118, May 2010.

[14] J. Jang and K. B. Lee, "Transmit power adaption for multiuser OFDM systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 2, pp. 171–178, Feb. 2003.

[15] J. Campello, "Optimal discrete bit loading for multicarrier modulation systems," in *Proceedings: IEEE International Symposium on Information Theory (ISIT)*, Cambridge, USA, Aug. 1998, p. 193.

[16] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York, New York, USA: John Wiley & Sons, 1991.

[17] M. Reyer, "Optimierungsalgorithmen zur Ressourcenallokation in OFDMA-Systemen," Ph.D. dissertation, RWTH Aachen, 2008.

[18] A. Schäper, "Analyse und Bewertung von Approximationsfehlern bei Verwendung stetiger Ratenleistungsfunktionen in OFDM-Systemen," bachelor thesis, RWTH Aachen, Sep. 2010.