Robust Planning of Green Wireless Networks

Grit Claßen
RWTH Aachen University
Lehrstuhl II für Mathematik and
UMIC Research Centre
Aachen, Germany
Email: classen@umic.rwth-aachen.de

Arie M.C.A. Koster
RWTH Aachen University
Lehrstuhl II für Mathematik
Aachen, Germany
Email: koster@math2.rwth-aachen.de

Anke Schmeink
RWTH Aachen University
UMIC Research Centre
Aachen, Germany
Email: schmeink@umic.rwth-aachen.de

***Abstract*—Current methods dealing with the energy efficient wireless network planning problem require a static model. However, bit rate requirements vary and rise over time, users move around and path gain fluctuates. In this paper, robust optimization is applied to deal with demand uncertainty. We present a robust optimization model for the energy-efficient planning of wireless networks and apply cutting planes to reduce the complexity of the model. Furthermore, a case study is performed to compare the robust formulation to its deterministic counterpart and to conventional network planning.**

I. Introduction

Future radio networks are obliged to cope with drastically increasing user demands. This leads inevitably to a significantly increasing energy consumption (e. g., the energy consumption of Vodafone Group already increased about 27 % from 2006/07 to 2009/10 [20]). The high user demands (compared to the requirements for ordinary telephony and short message services) result from, e. g., traffic-intensive smartphone applications. Even though user demand and resource restrictions have been considered in the planning of third generation (3G) networks [2, 8, 18], these networks reach the limits of their capacity. High data rates can be offered, but only for some users and with limited coverage. Thus, more base stations (BSs) are needed and the energy consumption rises (not only by signal power but also by air-conditioning etc. [7]). To tackle the problem of inferior network performance, future networks utilize a couple of advanced techniques, e. g., Orthogonal Frequency Division Multiple Access (OFDMA). Nevertheless, an optimal planning of wireless networks is of utmost importance to fully utilize the technological gains and to reduce the energy consumption.

Energy efficient wireless network planning has recently attracted a great deal of attention: [10, 16, 17] to name just three. Current wireless network planning [8, 10, 16, 18] requires a static model of the problem. However, many factors of wireless networks are non-deterministic. Fluctuating bit rate requirements and channel conditions are just two of prominent examples of uncertain parameters.

The robust optimization approach by Bertsimas and Sim [3] deals with uncertainty by limiting the number of uncertain factors by a robustness parameter. The robustness of the solution can be adapted by varying the parameter.

In this paper, robust optimization is applied to deal with demand uncertainty. In Section II, we develop a static system model and formulate the problem as an integer linear program (ILP) based on [9] for the energy efficient planning of wireless networks. A simplified inter-cell interference model is used to unveil the novelty of the robust optimization model presented in Section III. The robust formulation is enhanced by the application of three types of cutting planes in Section IV. In Section V, we perform a case study for a realistic wireless network scenario to compare the robust ILP to its deterministic counterpart and to conventional wireless network planning.

II. System Model and Problem Formulation

First, we introduce the system model considering a future wireless network and downlink (DL) data transmission based on the model presented in [9]. Let S be a set of candidate sites (more precisely, site configurations) for BSs. A deployed BS $s \in S$ consumes power p_s and provides a total DL bandwidth b_s. Furthermore, let T be a set of traffic nodes (TNs) to be assigned to the deployed BSs. Each $t \in T$ has a demand w_t and can be assigned to at most one BS (i. e., no soft handover), which applies, e. g., to 4G networks.

By considering OFDMA as DL transmission technology, we assume that no intra-cell interference occurs. A simplified inter-cell interference model is used requiring the selected BSs to constitute an *independent set* in a predefined conflict graph $G = (S, E)$, i. e., a subset of S such that $ij \notin E \; \forall i, j \in S$. Additionally, for each (s, t) pair a value e_{st} called spectral efficiency is defined to incorporate, e. g., the modulation and coding scheme that is supported by the associated signal-to-noise ratio (SNR). To establish a transmission link, it must exceed a certain threshold $e_{\min}$. (Note that this restriction implicitly also guarantees a minimum SNR.)

If TN t is assigned to BS s, the established transmission link from s to t occupies a certain amount of the available DL bandwidth b_s. This amount can be computed by division of demand w_t by spectral efficiency e_{st} (provided $e_{st} \geq e_{\min}$).

To model the energy efficient planning problem as an ILP, we introduce the following auxiliary sets of indices to

The research described in this paper was supported by the excellence initiative of the German federal and state governments and by the UMIC Research Centre at RWTH Aachen University.

incorporate the constraint on the sprectral efficiency.

$$\begin{aligned} S*T &:= \{(s,t) \in S \times T \ : e_{st} \geq e_{\min}\}, \\ S_t &:= \{s \in S \ : (s,t) \in S*T\} \quad \forall t \in T, \\ T_s &:= \{t \in T \ : (s,t) \in S*T\} \quad \forall s \in S. \end{aligned}$$

Let $x_s \in \{0,1\}$ denote whether or not BS $s \in S$ is deployed and $z_{st} \in \{0,1\}$ whether $t \in T_s$ is assigned to s. We introduce a slack variable $u_t \in \{0,1\}$ that denotes whether t is **not** assigned to any BS. The main objective is to minimize the total power consumption of the network while minimizing the number of TNs not served by any BS, i.e., maximizing the number of covered TNs (without this part, the optimal solution would be zero). To combine these conflicting objectives, the parameter λ is introduced. The optimization model is defined as follows.

$$\min \sum_{s\in S} p_s x_s + \lambda \sum_{t \in T} u_t \tag{1a}$$

$$\text{s.t.} \sum_{s \in S_t} z_{st} + u_t = 1 \qquad \forall\, t \in T \tag{1b}$$

$$x_i + x_j \leq 1 \qquad \forall\, ij \in E \tag{1c}$$

$$\sum_{t \in T_s} \frac{w_t}{e_{st}} z_{st} \leq b_s x_s \qquad \forall\, s \in S \tag{1d}$$

$$x_s,\, z_{st},\, u_t \in \{0,1\} \tag{1e}$$

Constraints (1b) ensure that a TN is covered by at most one BS. To limit inter-cell interference, constraints (1c) guarantee an independent set of deployed BSs. The capacity constraints (1d) ensure that the total bandwidth allocated does not exceed the total available DL bandwidth. Furthermore, these constraints implicitly make sure that a TN can be assigned to a BS if and only if this BS is deployed.

III. Robust Formulation

Although many parameters in a wireless network are uncertain, such as channel conditions, interference etc., we only consider demand uncertainty in our model. We apply the robust optimization approach presented in [3] to formulation (1). The demand values are now modelled as symmetric and bounded random variables that take values in $[\bar{w}_t - \hat{w}_t, \bar{w}_t + \hat{w}_t]$, where $\bar{w}_t$ denotes a nominal value and $\hat{w}_t$ its highest deviation. We assume that at most $\Gamma \in \{0, \ldots, |T|\}$ demand values per BS deviate from the default value $\bar{w}_t$ simultaneously (in the worst case towards $\bar{w}_t + \hat{w}_t$). Hence, constraints (1d) are replaced by:

$$\sum_{t \in T_s} \frac{\bar{w}_t}{e_{st}} z_{st} + \max_{T' \subseteq T_s, |T'| \leq \Gamma} \sum_{t \in T'} \frac{\hat{w}_t}{e_{st}} z_{st} \leq b_s x_s \ \forall\, s \in S \tag{2}$$

By exploiting LP duality (introducing dual variables μ_s and ν_{st}), the maximum in (2) can be linearized, resulting in the following ILP as robust counterpart of (1) (see [3]).

$$\min \ (1a)$$

$$\text{s.t.} \ (1b), (1c), (1e)$$

$$\sum_{t \in T_s} \frac{\bar{w}_t}{e_{st}} z_{st} + \Gamma \mu_s + \sum_{t \in T_s} \nu_{st} \leq b_s x_s \quad \forall\, s \in S \tag{3a}$$

$$\mu_s + \nu_{st} \geq \frac{\hat{w}_t}{e_{st}} z_{st} \qquad \forall (s,t) \in S*T \tag{3b}$$

$$\mu_s \geq 0, \ \nu_{st} \geq 0 \tag{3c}$$

IV. Cutting Planes

The performance of the branch-and-bound algorithm for solving ILPs can significantly be improved by cutting planes, i. e., inequalities that are valid for all integer points but not for some linear relaxation solutions. State-of-the-art ILP solvers like CPLEX [11] generate cutting planes internally but cannot take advantage of the particular problem structure known to the user. For the problem at hand, we identified a number of problem-specific inequalities, partly well-known for substructures contained in (3). Without these cuts our computations would have been far less competitive.

A. Variable Upper Bounds [19]

Constraints (3a) implicitly state that a TN can be assigned to a BS iff this BS is deployed. It is well-known that (3) can be strengthened by adding these constraints explicitly:

$$z_{st} \leq x_s \qquad \forall (s,t) \in S*T \tag{4}$$

B. Maximal Clique Inequalities [15]

A clique of a graph $G = (S, E)$ is a subset $U \subseteq S$ for which holds: $u, v \in U \Rightarrow uv \in E$, i. e., a clique is a complete subgraph. A clique is maximal if it is not included in a larger clique. We replace constraints (1c) by all maximal clique inequalities for the independent set polytope:

$$\sum_{s \in U} x_s \leq 1 \quad \forall U \subset S,\ U \text{ is a maximal clique in } G = (S,E).$$

Though NP-complete, all maximal cliques can be computed by the Bron-Kerbosch algorithm [4] without much effort.

C. Cover Inequalities

Constraints (2) are closely related to the *robust knapsack problem*. The general structure of (2) can be written as

$$\sum_{i \in I} \bar{a}_i y_i + \max_{I' \subseteq I : |I'| \leq \Gamma} \sum_{i \in I'} \hat{a}_i y_i \leq bx\,. \tag{5}$$

where I denotes the set of items (TNs), $\bar{a}_i$ the default knapsack weight, $\hat{a}_i$ its deviation, and b the knapsack capacity. If $x = 1$, (5) describes a robust knapsack. Since $x = 0$ implies $y_i = 0$ for all $i \in I$, every valid inequality for the robust knapsack problem can be adapted to a valid inequality for (5) (and thus for (3)) by multiplying its right hand side with x.

In [12] the well-known *cover inequalities* for the knapsack problem have been generalized to the robust knapsack problem. A set $(C \cup J) \subseteq I$ is a *robust cover* if

$$|J| \leq \Gamma,\ |C| \geq 0 \text{ and } \sum_{i \in C} \bar{a}_i + \sum_{i \in J} (\bar{a}_i + \hat{a}_i) > b$$

For any robust cover $C \cup J$, the *robust cover inequality*

$$\sum_{i \in C \cup J} y_i \leq (|C \cup J| - 1)x \tag{6}$$

is valid for (5). Such a cutting plane can be strenghtened by means of the concept of an *extended cover*, cf. e. g., [5, 6]: selecting an item i whose default weight $\bar{a}_i$ is greater than or equal to the maximum default weight in C and whose peak weight $\bar{a}_i + \hat{a}_i$ is greater than or equal to the maximum peak weight in J implies that at most $|C \cup J| - 2$ items of the robust cover can be taken. Hence,

$$\sum_{i \in E(C,J)} y_i \leq (|C \cup J| - 1)x \tag{7}$$

is also valid, with $E(C, J) := E \cup (C \cup J)$ defined by

$$E := \left\{ i \in I : \bar{a}_i \geq \max_{j \in C} \bar{a}_j,\ \bar{a}_i + \hat{a}_i \geq \max_{j \in J} (\bar{a}_j + \hat{a}_j) \right\} \tag{8}$$

Since there exist (exponentially) many robust cover inequalities, these are not added to (3) in advance, but are separated on the fly. The separation problem is NP-hard in itself. Therefore, we adapt the separation heuristic given in [12] for robust cover inequalities. The main steps of this heuristic can be summarized as follows.

The items are sorted according to the smallest ratios of profit (i.e., in the objective of the separation problem) to peak demand in the first step. Then the set J, consisting of all TNs considered with their peak demands, is determined. If the capacity is already exceeded, we stop. Otherwise, the set C is filled with TNs, for which the nominal demand is considered, until the capacity is exceeded. Finally, the constructed robust cover is extended by means of (8), if possible.

V. Case Study

In this section, we describe a case study to reveal the added value of the robust optimization approach. After the description of the scenario, we discuss the optimization results and propose a way to determine a good choice for Γ. Finally, we compare the solutions with static network planning.

A. The Scenario

The proposed model is tested for a planning scenario based on signal propagation data for Munich, available at [1] with 40 BS candidate sites and 450 randomly distributed TNs. The signal prediction is done by a cube oriented ray launching algorithm [13]. Two BSs are adjacent in the conflict graph iff the distance between them is at most 500 m. The resulting graph is illustrated in Fig. 1. Furthermore, we use the following scenario parameters: $b_s = 10\,\text{MHz}$, $p_s = 4000\,\text{W}\ \forall s \in S$ (based on [7]), $e_{\min} = 0.5\,\text{bps/Hz}$.

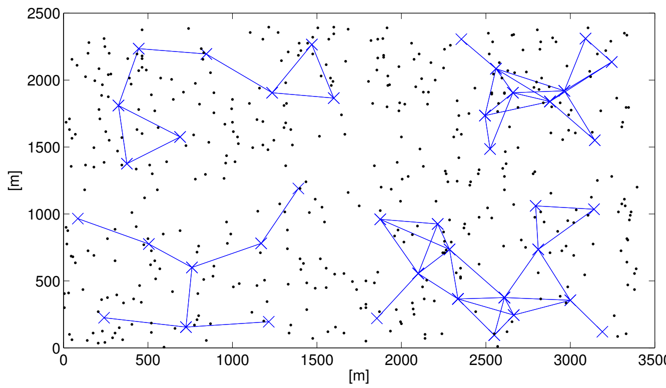


Fig. 1. The scenario. BSs are denoted by (blue) crosses. Each line is an edge in the conflict graph. (Black) dots stand for TNs.

TABLE I
Profiles for TNs

service	percentage [%]	bit rate [kbps]
data	[10,20]	[512,2000]
web	[20,40]	[128,512]

For each $t \in T$, $\bar{w}_t$ is computed by randomly generating user profiles from Table I: for both data and web services a percentage and bit rate is uniformly drawn from the intervals. The remaining percentage is used for Voice-over-IP (VoIP) with a bit rate of 64 kbps. The value $\hat{w}_t$ is computed such that $\bar{w}_t + \hat{w}_t = 643.2\,\text{kbps}$, simulating a worst case demand.

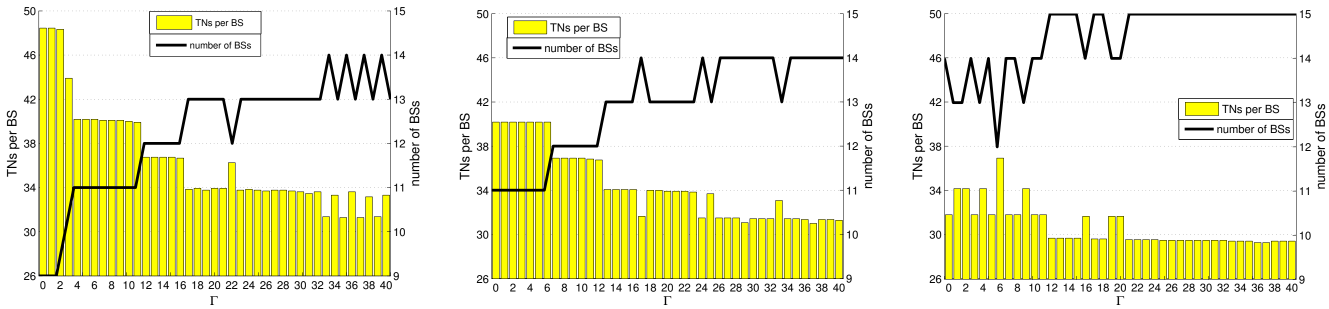
Since the solution depends on the choice of λ, we tested three values: 1000, 2000, 4000. Hence, based on the value of the total power consumption of a BS, it becomes beneficial to deploy an additional BS if more than four, two, or one TN(s) cannot be covered.

We used a Linux machine with a 2.93 GHz Intel Xeon W3540 processor, 12 GB RAM, and CPLEX 12.2 [11]. A CPU time limit of 4 h is set. Note, without the cutting planes described in Section IV the computations would have been far less competitive.

B. Γ-Robust Wireless Network Planning

For $\Gamma \in \{0, 1, \ldots, 40\}$ and $\lambda \in \{1000, 2000, 4000\}$, Fig. 2 shows the best solution value (primal bound) and a lower bound on the optimal solution value (dual bound) found within the given time limit. In about 59% of the cases both bounds match, hence an optimal solution is found. We observe that the optimality gap increases with increasing Γ (the course of the bounds is similar, independent of λ). The increase in the objective value is the *price of robustness*, i. e., more robust solutions come at a price.

Figure 3 shows for all λ the number of deployed BSs as well as the number of served TNs per BS. As expected, with increasing Γ the number of deployed BSs increases stepwise, whereas the TNs per BS decreases. Deviations from this rule can be explained by the non-optimality of the solutions.



(a) $\lambda = 1000$. (b) $\lambda = 2000$. (c) $\lambda = 4000$.

Fig. 3. Number of covered TNs averaged over the number of BSs and deployed BSs.

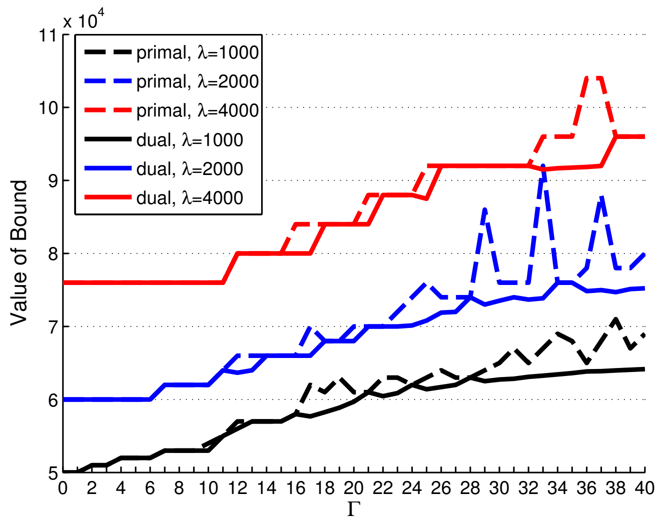


Fig. 2. Primal and dual bounds.

C. Evaluation of Robust Planning

To evaluate the robustness of the different solutions, we computed 1000 snapshots to simulate various traffic demand scenarios. In each snapshot, the demand of the TNs is binomially distributed between $\bar{w}_t$ and $\bar{w}_t + \hat{w}_t$ with a probability of 0.5 (this value is chosen rather high to increase the exploratory power of the evaluation). From this, we compute the (over)load for each deployed BS in a solution: the allocated bandwidth divided by the DL bandwidth. Fig. 4 shows the maximum load of the BSs for each λ and Γ averaged over the snapshots. A value less than 1 means that there is no overload; the TNs can be served at their required bit rates and it is even possible to serve more than the assigned TNs.

With increasing Γ the average overload decreases. Since the snapshots are representative for our model, Fig. 4 suggests to set $\Gamma = 16$ for a robust solution, independent of λ.

D. Conventional Wireless Network Planning

In conventional wireless network planning any uncertainties of parameters are ignored; all parameters are assumed to be static. To be able to compensate demand fluctuations, a network operator should plan with values equal or close to the peak demand values [14]. To compare our results with such a conventional planning approach, we run model (1) with $w_t = \bar{w}_t + \hat{w}_t$ for all $t \in T$. The best/optimal solution value ranges from $66,000$ ($\lambda = 1000$) to $96,000$ ($\lambda = 4000$). By definition of the demand values, no overload can occur in these solutions.

In comparison to the solutions for $\Gamma = 16$, the conventional solutions for $\lambda = 1000, 2000$ require one more BS (13, 14 in total). For $\lambda = 4000$, the same number of BSs is deployed (15), but less TNs can be covered (441 compared to 443). Stated otherwise, in all cases, a solution with higher energy efficiency could be found by the robust optimization approach, either by deploying less BSs, or serving more TNs with the same amount of energy.

VI. Concluding Remarks

We have introduced an optimization model for the energy efficient planning of future wireless networks which considers technical system characteristics such as OFDMA. Applying Γ-robustness, we incorporated demand uncertainty. Network operators can assess the trade-off between robustness and energy consumption by varying the robustness parameter and the λ. In our case study, we observed energy savings either by deploying less BSs or serving more TNs with the same number of BSs.

The above results naturally depend on the probability of high traffic volumes. If deviations in the snapshots simulated in Section V-C are less likely to happen, lower Γ values will suffice for planning a wireless network without overload situations. Ideally, snapshots based on real-traffic data are used to determine the value of Γ and the corresponding design.

Acknowledgement

The authors would like to thank Alexander Engels for providing us with realistic network data and Manuel Kutschka for assisting in algorithmic implementations.

References

[1] COST 231. Urban micro cell measurements and building data. [Online]. Available: http://www2.ihe.uni-karlsruhe.de/forschung/cost231/cost231.en.html

[2] E. Amaldi, A. Capone, and F. Malucelli, "Radio planning and coverage optimization of 3G cellular networks," *Wirel. Netw.*, vol. 14, pp. 435–447, 2008.

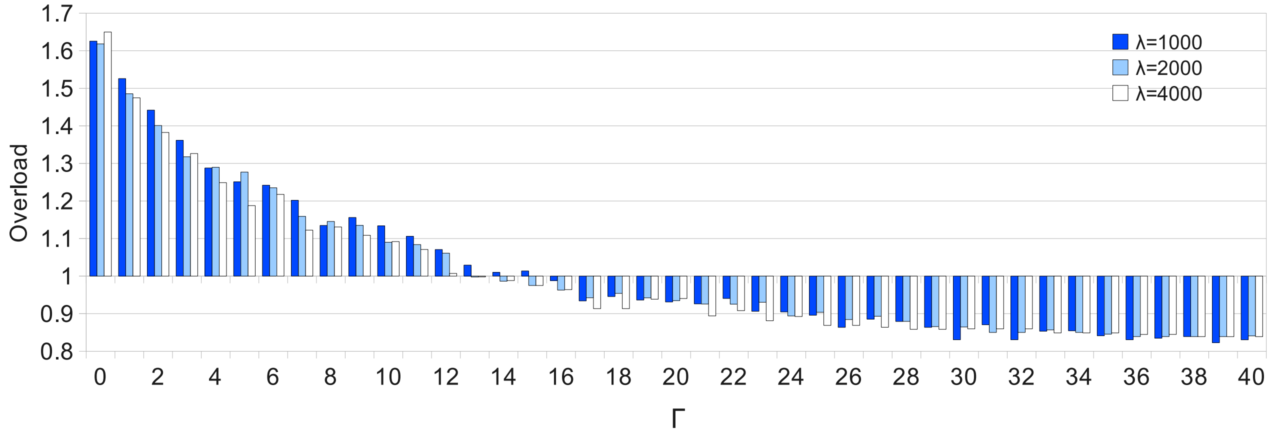


Fig. 4. Overload for robust wireless network planning.

[3] D. Bertsimas and M. Sim, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004.

[4] C. Bron and J. Kerbosch, "Algorithm 457: finding all cliques of an undirected graph," *Commun. ACM*, vol. 16, pp. 575–577, 1973.

[5] C. Büsing, "Recoverable robustness in combinatorial optimization," Ph.D. dissertation, Technische Universität Berlin, 2011.

[6] C. Büsing, A. M. C. A. Koster, and M. Kutschka, "Recoverable Robust Knapsacks: the Discrete Scenario Case," *Optimization Letters*, vol. 5, no. 3, pp. 379–392, 2011.

[7] M. Deruyck, W. Vereecken, E. Tanghe, W. Joseph, M. Pickavet, L. Martens, and P. Demeester, "Comparison of power consumption of mobile WiMAX, HSPA and LTE access networks," in *9th Conference on Telecommunications Internet and Media Techno Economics (CTTE)*, 2010, pp. 1–7.

[8] A. Eisenblätter, A. Fügenschuh, H. Geerdes, D. Junglas, T. Koch, and A. Martin, "Integer Programming Methods for UMTS Radio Network Planning," *Proc WiOpt04*, 2004.

[9] A. Engels, M. Reyer, and R. Mathar, "Profit-oriented combination of multiple objectives for planning and configuration of 4G multi-hop relay networks," in *7th Int. Symp. on Wireless Communication Systems (IEEE ISWCS)*, 2010, pp. 330–334.

[10] A. Fehske, F. Richter, and G. Fettweis, "Energy efficiency improvements through micro sites in cellular mobile radio networks," in *IEEE GLOBECOM Workshops*, 2009, pp. 1–5.

[11] IBM – ILOG. CPLEX Optimization Studio 12.2. [Online]. Available: http://www.ilog.com/products/cplex

[12] O. Klopfenstein and D. Nace. Cover inequalities for robust knapsack sets - Application to the robust bandwidth packing problem. [Online]. Available: http://perso.rd.francetelecom.fr/klopfenstein/Papers/

[13] R. Mathar, M. Reyer, and M. Schmeink, "A cube oriented ray launching algorithm for 3D urban field strength prediction," in *IEEE ICC*, 2007.

[14] E. Olinick, "Mathematical programming models for third generation wireless network design," in *Wireless Network Design: Optimization Models and Solution Procedures*, J. Kennington, E. Olinick, and D. Rajan, Eds. Springer, 2011.

[15] M. Padberg, "On the facial structure of set packing polyhedra," *Math. Program.*, vol. 5, pp. 199–215, 1973.

[16] F. Richter, A. Fehske, and G. Fettweis, "Energy efficiency aspects of base station deployment strategies for cellular networks," in *IEEE 70 th Vehicular Technology Conference Fall (VTC 2009-Fall)*, 2009, pp. 1–5.

[17] F. Richter, A. J. Fehske, P. Marsch, and G. P. Fettweis, "Traffic demand and energy efficiency in heterogeneous cellular mobile radio networks," in *IEEE 71st Vehicular Technology Conference (VTC 2010-Spring)*, 2010, pp. 1–6.

[18] I. Siomina, P. Varbrand, and D. Yuan, "An effective optimization algorithm for configuring radio base station antennas in UMTS networks," in *IEEE 64th Vehicular Technology Conference (VTC 2006-Fall)*, 2006, pp. 1–5.

[19] T. J. van Roy, "A cross decomposition algorithm for capacitated facility location," *Operations Research*, vol. 34, no. 1, pp. 145–163, 1986.

[20] Vodafone Group, "Sustainability report," 2010. [Online]. Available: http://www.vodafone.com/content/index/about/sustainability/publications.html