Autoregressive Modeling of Frequency Selective Channels for Synchronized OFDM Systems

Xiang Xu and Rudolf Mathar
Institute for Theoretical Information Technology, RWTH Aachen University
Aachen, Germany 52074
Email: {xu,mathar}@ti.rwth-aachen.de

***Abstract*—In this paper, an autoregressive (AR) channel model for simulating frequency selective channels in synchronized orthogonal frequency division multiplexing (OFDM) systems is proposed. By frequency domain AR filtering, the number of Rayleigh processes for generating the channel frequency response (CFR) is reduced. Thus, this model is able to achieve a low complexity.**

I. INTRODUCTION

Many different techniques have been proposed to model and simulate mobile radio channels with Rayleigh fading. The most representative models from early years are from Clarke and Gans based on sum of sinusoids [1] [2]. A simplified model proposed by Jakes have been widely used for decades [3]. However, Jakes' model is a deterministic model, and it has difficulty in creating multiple uncorrelated fading waveforms for frequency selective channels. A randomized simulator, which can solve this problem, is proposed by Pop and Beaulieu [4]. Pop and Beaulieu's model also solves the stationarity problem of Jakes' model, but higher-order statistics of this model may not match the desired ones. This deficiency can be overcome by introducing randomness in path gain, initial phase and Doppler frequency for all individual sinusoids [5].

Other than sum of sinusoids, inverse discrete Fourier transform (IDFT) and autoregressive (AR) models are also proposed to create Rayleigh processes [6] [7]. However, both of them have limitations on applications. IDFT method can only work with a relatively large Doppler frequency and FFT size. Whereas AR models have severe numerical problems when the Doppler frequency is small.

As a promising candidate for the future broadband wireless system, OFDM has become a hot topic for a long time [8]. Many efforts, devoted to perfect OFDM system performance, are tested by simulations. Particularly, in research areas of channel estimation and resource allocation, frequency selective channels with Rayleigh fading need to be simulated [9]. Due to the summation of large number of sinusoids, the complexity of generating such channels could be a burden. Nevertheless, the generation of channel coefficients should not be the most time consuming part of a simulation.

In this paper, a novel channel model based on frequency domain AR filtering is proposed. Using the decoupled frequency correlation function, only one Rayleigh process is needed for generating a frequency selective channel. Thus, the proposed method is computationally efficient. For notation, $(\cdot)^{-1}$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{*}$ represent matrix inverse, transpose and conjugate, respectively.

II. BACKGROUND

Consider a perfectly synchronized OFDM system. The frequency domain received signal Y at kth subcarrier of nth OFDM symbol can be written as

$$Y[n,k] = H[n,k]X[n,k] + W[n,k] \tag{1}$$

where H is the CFR, X is the transmitted symbol and W is the additive noise.

The CIR can be written as

$$h(t,\tau) = \sum_{l=0}^{L-1} \gamma_l(t)\delta(\tau - \tau_l), \tag{2}$$

where $L-1$ is the channel memory length, τ_l is the delay of the lth path, and $\gamma_l(t)$ is the corresponding complex amplitude. The CFR is the Fourier transformation of channel impulse response (CIR)

$$H[n,k] = \sum_{l=1}^{L} \gamma_l(nT) W_K^{kl}. \tag{3}$$

where $W_K = \exp -\jmath 2\pi/K$. Moreover, $\gamma_l(t)$'s are wide-sense stationary (WSS) complex Gaussian processes and independent for different paths with average power σ_l^2. The channel is normalized such that $\sum_{l=1}^{L}\sigma_l^2 = 1$. Furthermore, $\gamma_l(t)$'s are assumed to have the same correlation function $r_t(\Delta t)$.

Therefore, the correlation function of CFR for OFDM systems with K subcarriers and block length of T can be written as

$$\begin{aligned} r[\Delta n, \Delta k] &= \mathrm{E}\{H[n+\Delta n, k+\Delta k]H^*[n,k]\} \\ &= r_t(\Delta nT)\sum_{l=0}^{L-1}\sigma_l^2 W_K^{\Delta kl}. \end{aligned} \tag{4}$$

Define the time and frequency domain correlation functions as

$$r_t[\Delta n] \triangleq r_t(\Delta nT), \quad r_f[\Delta k] \triangleq \sum_{l=0}^{L-1}\sigma_l^2 W_K^{\Delta kl}. \tag{5}$$

This research was partly supported by the UMIC excellence cluster of RWTH Aachen University.

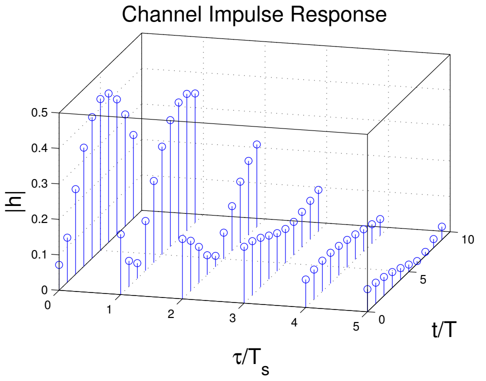

Fig. 1. Channel impulse response.

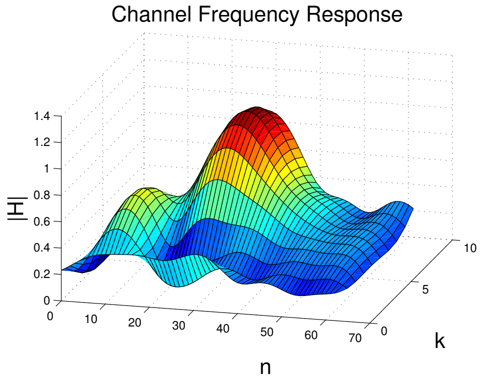

Fig. 2. Channel frequency response as Fourier transform of channel impulse response

The correlation function can be decoupled into time correlation and frequency correlation

$$r[\Delta n, \Delta k] = r_t[\Delta n] r_f[\Delta k]. \tag{6}$$

The time correlation of channel coefficients follows Jakes' isotropic scattering model [3]:

$$r_t[\Delta n] = J_0(2\pi f_{\mathrm{d}} T \Delta n), \tag{7}$$

where J_0 is the zeroth order Bessel function of the first kind and f_{d} is the maximum Doppler frequency. And the frequency correlation is determined by the power delay profile (PDP).

The conventional procedure of generating random variables following this correlation function is first to generate the CIR, namely, L independent temporally correlated Rayleigh process, each with power σ_l^2, as shown in Fig. 1. And use Fourier transform to get the CFR afterwards, as shown in Fig. 2.

III. AUTOREGRESSIVE MODELING

Since the correlation function can be decoupled, an alternative way to generate CFR for OFDM system is to generate one Rayleigh process in time domain and use frequency domain AR filtering to get the desired frequency correlation. A complex AR process of order p can be generated as

$$H[k] = -\sum_{\kappa=1}^{p} a_\kappa H[k-\kappa] + w[k], \tag{8}$$

where time index n is omitted for simplicity. $w[k]$ is a complex white Gaussian noise process with uncorrelated real and imaginary parts. The filter coefficients $\mathbf{a} = [a_1, a_2, \cdots, a_p]^{\mathrm{T}}$ follows the well known Yule-Walker equations

$$\mathbf{R}\mathbf{a} = -\mathbf{v}, \tag{9}$$

where $\mathbf{R}$ and $\mathbf{v}$ are the correlation matrix and vector, respectively. For a given PDP, $r_f[\Delta k]$ can be derived using (5). Therefore, the correlation matrix and vector can be written as

$$\begin{aligned} \mathbf{R} &= \begin{bmatrix} r_f[0] & r_f[-1] & \cdots & r_f[-p+1] \\ r_f[1] & r_f[0] & \cdots & r_f[-p+2] \\ \vdots & \vdots & \ddots & \vdots \\ r_f[p-1] & r_f[p-2] & \cdots & r_f[0] \end{bmatrix} \\ \mathbf{v} &= [r_f[1] \quad r_f[2] \quad \cdots \quad r_f[p]]^{\mathrm{T}}. \end{aligned} \tag{10}$$

The correlation matrix $\mathbf{R}$ is positive semidefinite. Therefore, the Yule-Walker equations have the unique solution

$$\mathbf{a} = -\mathbf{R}^{-1}\mathbf{v}. \tag{11}$$

Furthermore, the variance of $w[k]$ can be calculated as

$$\sigma_w^2 = r_f[0] + \mathbf{a}^{\mathrm{T}}\mathbf{v}^*. \tag{12}$$

Using this AR model, the desired frequency autocorrelation function can be perfectly matched up to lag p. As a matter of fact, many OFDM applications consider only a fairly small order of correlation [9]. That is, the AR filter order p is not necessarily a large number. Comparing to calculating a large number of sinusoids, the small order AR filtering could be faster. Thus, this approach is generally more computationally efficient than the conventional methods.

IV. SIMULATION RESULTS

In the simulation, the method in [5] is adopted to generate the initial Rayleigh process. The number of sinusoids is 128. An exponential PDP is applied. In total $K = 256$ subcarriers and $N = 10^3$ OFDM symbols are simulated. Standard MATLAB function `fft` is used for Fourier transform in the reference model, and `filter` is used for AR filtering. AR filters with order $p = 10, 20, 50$ are simulated. The frequency correlation functions of AR models and theoretical values for $L = 20$ are given in Fig. 3. As shown in Fig. 4, the proposed AR model is advantageous in computational time comparing to the conventional model using FFT, especially when the delay spread is large. Even when a large AR filter order $p = 50$ is chosen, the proposed model still offers a lower complexity than the FFT method.

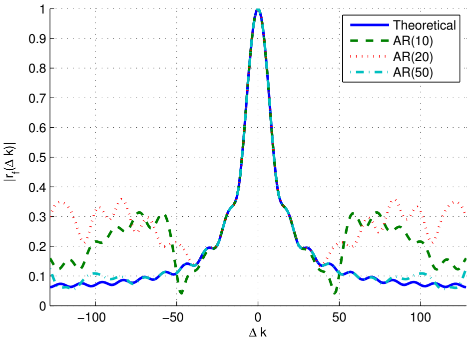

Fig. 3. Frequency correlation functions of AR models

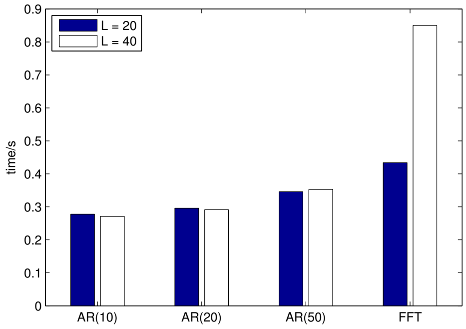

Fig. 4. Execution time per iteration for AR and FFT channel models

V. CONCLUSION

In this paper, an AR model for simulating frequency selective OFDM channels is proposed. Using AR filtering, only one Rayleigh process needs to be generated. The correlation function can be well matched. Furthermore, a lower complexity comparing to the conventional methods can be obtained.

REFERENCES

[1] R. H. Clarke, "A statistical theory of mobile radio reception," *Bell Labs System Technical Journal*, vol. 47, pp. 957–1000, Jul-Aug 1968.

[2] M. Gans, "A power-spectral theory of propagation in the mobile-radio environment," *Vehicular Technology, IEEE Transactions on*, vol. 21, no. 1, pp. 27–38, Feb 1972.

[3] W. C. Jakes, *Microwave Mobile Communications*. Wiley, 1974, re-issued by IEEE Press, 1994.

[4] M. Pop and N. Beaulieu, "Limitations of sum-of-sinusoids fading channel simulators," *Communications, IEEE Transactions on*, vol. 49, no. 4, pp. 699–708, Apr 2001.

[5] C. Xiao, Y. R. Zheng, and N. Beaulieu, "Novel sum-of-sinusoids simulation models for Rayleigh and Rician fading channels," *Wireless Communications, IEEE Transactions on*, vol. 5, no. 12, pp. 3667 –3679, Dec 2006.

[6] D. Young and N. Beaulieu, "The generation of correlated Rayleigh random variates by inverse discrete Fourier transform," *Communications, IEEE Transactions on*, vol. 48, no. 7, pp. 1114–1127, Jul 2000.

[7] K. Baddour and N. Beaulieu, "Autoregressive modeling for fading channel simulation," *Wireless Communications, IEEE Transactions on*, vol. 4, no. 4, pp. 1650–1662, Jul 2005.

[8] Y. Li, J. Winters, and N. Sollenberger, "MIMO-OFDM for wireless communications: signal detection with enhanced channel estimation," *Communications, IEEE Transactions on*, vol. 50, no. 9, pp. 1471–1477, Sep 2002.

[9] X. Xu and R. Mathar, "Low complexity joint channel estimation and decoding for LDPC coded MIMO-OFDM systems," in *Vehicular Technology Conference (VTC Spring), 2011 IEEE 73rd*, Budapest, Hungary, May 2011.