Analysis of a Serial UWB based Sensor Network Topology in Fading Channels

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***Abstract*—One of the challenging issues in design of wireless sensor networks (WSNs) is the energy consumption of the sensor nodes which is an important factor and should be as low as possible. In this paper, we present a power assignment strategy along with a routing path selection for distributed detection in serial decision fusion WSNs. A Nakagami-m fading model is considered for the communication channel and an impulse radio ultra wideband (IR-UWB) technology for the transceiver circuit of the sensor nodes. The objective is to minimize the global probability of detection error of the serial network under a total network power constraint. Simulation results are provided for the global probability of detection error to evaluate the performance of our power assignment strategy in different fading intensities.**

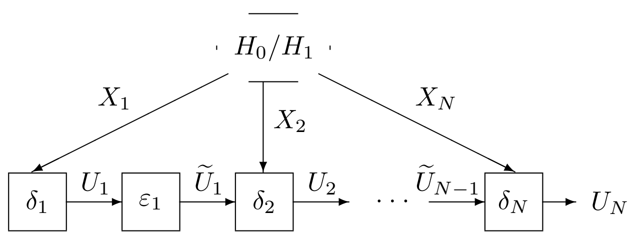

Fig. 1. Serial network with noisy channels.

I. INTRODUCTION

The problem of binary distributed detection in wireless sensor networks (WSNs) has attracted attention in recent years [1], [2]. Most research in this area has been focused on designing optimum detection techniques by different criteria for different WSN architectures, e.g., serial and parallel sensor topologies. In serial distributed detection, the sensors successively form decisions that depend on the received decision of a preceding sensor as well as the own local observation until a final detection result is reached [3]. As the communication channels between sensor nodes are subject to noise and interference, it becomes necessary to take wireless channel conditions into account in order to optimally design the distributed detection system [4]. Yet, the channel quality can be controlled, e.g., by appropriate assignment of transmission power levels. In wireless sensor networks deployed for distributed detection, the power assignment eventually should be adapted to optimize application-specific metrics, thus exploiting dependencies between signal processing and wireless networking [5].

For serial distributed detection in sensor networks we have to establish a routing path, which determines the order in which the local decisions are routed in the network from sensor to sensor until the final node is reached. The final node delivers the final detection result to the external observer.

In [6], authors consider a sensor network with serial architecture to solve a binary distributed detection problem. Assuming a Neyman-Pearson formulation, the optimal decision rules associated with each fusion stage are derived. It is shown that the optimal Neyman-Pearson tests comply with the optimal fusion rules of a serial configuration with dependent observations. This work takes a different approach than our work and it does not consider the effect of wireless channel fading in the analysis. In [7], a combined routing path selection and power allocation strategy is presented that is especially designed for distributed detection in sensor networks with a serial topology. The objective is to minimize the global probability of detection error of the serial network under a total network power constraint. After the routing path is established, a subsequently performed power allocation algorithm aims to optimally distribute a total power budget with respect to the detection performance of the sensor network. Similarly, this work does not take into account the effect of fast channel fading which is an important issue in designing the distributed power allocation algorithms and can affect the detection error probability. Our previous work in [8] proposes an opportunistic power assignment strategy for distributed detection in parallel fusion WSNs which considers a Nakagami-m fading model for the communication channel and time-hopping (TH) ultra wideband (UWB) for the transmitter of the sensor nodes. This works investigated the parallel network topology of WSNs while in this research we study a serial topology.

In this paper, we present a combined routing path selection and power allocation strategy for distributed detection in wireless sensor networks in the serial topology as can be seen in Fig. 1. In fact, we generalize the work in [7] and [8] for the serial sensor networks and under the assumption of Nakagami-

m fading. The objective is to establish a routing path through the network and subsequently allocate transmission power to the sensor nodes in order to minimize the global probability of detection error under a total network power constraint. The strategy for the selection of a routing path is based on the local observation SNR of the sensors as well as fading conditions. Our power allocation algorithm aims to optimally distribute a total power budget among the nodes with respect to the detection performance of the sensor network. TH-UWB is employed as the physical layer technology of the sensor nodes. Simulation results for this serial topology are derived based on detection error probability for different fading intensities, m.

II. Distributed Detection in Serial Networks

The problem of distributed detection in serial networks can be stated as follows (see Fig. 1) [7]. We consider a binary hypothesis testing problem with hypotheses H_0, H_1 indicating the state of the observed environment and associated prior probabilities $\pi_0 = P(H_0)$, $\pi_1 = P(H_1)$. In order to detect the true state of nature, a network of N sensors $S_1, \dots, S_N \in \mathcal{N}$ collects random observations $X_1, \dots, X_N$, which are assumed to be conditionally independent across sensors given the underlying hypothesis, i.e., the joint conditional probability density function (pdf) of the observations factorizes to

$$f(x_1, \dots, x_N|H_k) = \prod_{i=1}^{N} f_i(x_i|H_k), \quad k = 0, 1.$$

In the serial network, the first sensor makes a decision $U_1 = \delta_1(X_1)$ which only depends on its own observation and subsequently transmits it to its neighbor. The succeeding sensors form decisions

$$U_j = \delta_j(\widetilde{U}_{j-1}, X_j), \quad j = 2, \dots, N, \tag{1}$$

which depend on the received and potentially corrupted decision $\widetilde{U}_{j-1}$ of the preceding sensor as well as the own

In the case that every wireless sensor is allowed to transmit only one bit per observation, the sensor decisions are binary-valued random variables $U_j \in \{0, 1\}$, $j = 1, \dots, N$. The local detection error probabilities for each sensor are given by the local probability of false alarm P_{f_j} and the local probability of miss P_{m_j} according to

$$P_{f_j} = P(U_j = 1|H_0), \quad P_{m_j} = P(U_j = 0|H_1) \tag{2}$$

for $j = 1, \dots, N$. Due to fading effects and noisy channels, the received decisions $\widetilde{U}_1, \dots, \widetilde{U}_N$ are potentially corrupted. We model the communication link of the jth sensor by a binary symmetric channel with bit-error probability ε_j, i.e.

$$\varepsilon_j = P(\widetilde{U}_j = 1|U_j = 0) = P(\widetilde{U}_j = 0|U_j = 1) \tag{3}$$

for $j = 1, \dots, N$. The resulting modified error probabilities $\widetilde{P}_{f_j} = P(\widetilde{U}_j = 1|H_0)$ and $\widetilde{P}_{m_j} = P(\widetilde{U}_j = 0|H_1)$ can be calculated as

$$\begin{aligned} \widetilde{P}_{f_j} &= P_{f_j} + \varepsilon_j(1 - 2P_{f_j}), \\ \widetilde{P}_{m_j} &= P_{m_j} + \varepsilon_j(1 - 2P_{m_j}). \end{aligned} \tag{4}$$

The overall performance metric for the serial network is the probability of error P_{e_N} of the Nth sensor according to

$$P_{e_N} = \pi_0 P_{f_N} + \pi_1 P_{m_N}. \tag{5}$$

Applying locally optimal detection at each sensor, the probability of error P_{e_N} of the last node can be calculated iteratively. Performing optimal detection at the Nth sensor, we obtain

$$\begin{aligned} P_{e_N} =& \pi_0 \big[(1 - \widetilde{P}_{f_{N-1}})(1 - F_{L_N}(\tau_N^{(0)}|H_0)) \\ &\quad + \widetilde{P}_{f_{N-1}}(1 - F_{L_N}(\tau_N^{(1)}|H_0))\big] \\ &+ \pi_1 \big[\widetilde{P}_{m_{N-1}} F_{L_N}(\tau_N^{(0)}|H_1)) \\ &\quad + (1 - \widetilde{P}_{m_{N-1}}) F_{L_N}(\tau_N^{(1)}|H_1)\big], \end{aligned} \tag{6}$$

where $F_{L_N}(\cdot|H_k)$ is the conditional cumulative distribution function (cdf) of the log-likelihood ratio L_N under H_k and

$$\tau_N^{(0)} = \log \frac{\pi_0(1 - \widetilde{P}_{f_{N-1}})}{\pi_1 \widetilde{P}_{m_{N-1}}}, \; \tau_N^{(1)} = \log \frac{\pi_0 \widetilde{P}_{f_{N-1}}}{\pi_1(1 - \widetilde{P}_{m_{N-1}})}. \tag{7}$$

III. Routing Path Selection and Power Allocation

In this section, we propose a cross-layer routing path selection strategy. It determines the order in which the detection results $U_1, \dots, U_N$ are transmitted from sensor to sensor by choosing a routing path through the network.

Later, the detection performance can be further improved by a subsequently performed application-specific power assignment to the selected nodes by simultaneously taking into account the fading conditions as well.

A. Cross-Layer Sensor Ordering

The strategy for the determination of the routing path is based on the local observation SNR of the sensors. Furthermore, fading conditions in terms of path gain are used as weighting factor. The intention of the approach is to order the sensors in the serial network from lowest weighted observation SNR to highest weighted observation SNR. The rationale behind this ordering is based on the observation, that for

Algorithm 1 Algorithm for determination of the routing path

```
Initialize:
D ← N \ S_N;
c ← |N|;
Route ← zeros(|N|, 1);
Route[c] ← N;
μ(S_j) ← h_Nj g_Nj · SNR_j;    S_j ∈ D
while D ≠ ∅ do
    c ← c − 1;
    if c < 1 then
        break;
    end if
    S_k = arg max_{S_j ∈ D} μ(S_j);
    Order[c] ← k;
    D ← D \ S_k;
    μ(S_j) ← h_Nj g_kj · SNR_j;    S_j ∈ D
end while
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serial networks with perfect communication channels it is advantageous to sort the nodes with an ascending observation SNR. Hence, nodes with high local detection performance according to high observation SNR should be used in the later part of the route. The path gain as a weighting factor accounts for the fact that in case of noisy communication channels a transmission with a high path gain results in a low bit-error probability ε_j of the channel. Especially in the later part of the serial network the bit-error probabilities of the communication channels should be low, such that the probability to corrupt high quality decisions during the transmission is also low.

While a sensor ordering based only on the observation SNR could simply be accomplished by a sorting algorithm, the problem becomes combinatorial by additionally considering fading conditions. As an efficient heuristic approach we propose to implement the routing path selection strategy by a backward greedy algorithm. The determination of the routing path starts with the final node S_N and then iteratively each node S_k determines its predecessor in the route. The decision is based on the application-specific node measure μ calculated for all nodes not yet included in the route. It is defined by

$$\mu(S_j) = h_{kj} g_{kj} \cdot \mathrm{SNR}_j, \tag{8}$$

where SNR_j is the observation SNR of node S_j, g_{kj} and h_{kj} are the path gain and the fading coefficient between S_j and its potential successor S_k in the route, respectively. Please note that h_{kj} follows a Nakagami-m distribution with parameters (m_{kj}, Ω_{kj}). Nakagami-m distribution model is given by:

$$f_{H_{kj}}(h_{kj}) = \frac{2}{\Gamma(m_{kj})} \left(\frac{m_{kj}}{\Omega_{kj}}\right)^{m_{kj}} h_{kj}^{2m_{kj}-1} e^{-\frac{m_{kj} h_{kj}^2}{\Omega_{kj}}}, \tag{9}$$

where $\Gamma(.)$ is the complex Gamma function, $E\left[h_{kj}^2\right] = \Omega_{kj}$, with $\mathbf{E}[.]$ denoting the expectation, and $m_{kj} > \frac{1}{2}$. We assume that all the nodes experience the same fading and $\Omega_{kj} = \Omega$ for $j = 1, ..., N$.

In our backward greedy algorithm the nodes iteratively choose among all available nodes the one with the maximum node measure μ as predecessor until all nodes are included in the routing path. Note, that this algorithm results in a sensor ordering with ascending observation SNR for perfect communication channels. A formal description of the routing algorithm including the sensor selection strategy as described in the following subsection is given in Algorithm 1. In the algorithm it is assumed that every node is identified by a unique number $j = 1, \ldots, N$. The routing path which is described by a permutation of these numbers is stored in the array *Route*. Set $\mathcal{D}$, which includes all nodes not yet included in the routing path and the counter c are auxiliary values. Our routing algorithm aims to establish a routing path with ascending weighted observation SNRs of the nodes. This means that the first nodes in the route might only provide a limited contribution on the quality of the final detection result while still consuming transmission power.

B. Power Control Strategy

The objective of the opportunistic power assignment strategy is to optimize the sensor network detection performance in terms of the global probability of error (5) given a budget of total transmission power. Based on the modified error probabilities (4), we define the effective sensor weights

$$\widetilde{\lambda}_j = \log\left(\frac{(1-\widetilde{P}_{f_j})(1-\widetilde{P}_{m_j})}{\widetilde{P}_{f_j}\widetilde{P}_{m_j}}\right) \tag{10}$$

for $j = 1, \ldots, N$. Note that for $P_{f_j}, P_{m_j} \in [0, \frac{1}{2}]$, and an arbitrary bit-error rate $\varepsilon_j \in [0, 1]$, the effective sensor weight $\widetilde{\lambda}_j$ is always less than or equal to the initial sensor weight λ_j, which is given as

$$\lambda_j = \log\left(\frac{(1-P_{f_j})(1-P_{m_j})}{P_{f_j}P_{m_j}}\right). \tag{11}$$

We employ a marginal analysis approach and assign the SINR for which the slope of the effective sensor weight $\widetilde{\lambda}$ with respect to γ falls under a predetermined threshold ϱ. Eventually, we determine the designated SINR γ_j of S_j as

$$\gamma_j = \left(\frac{\partial \widetilde{\lambda}_j}{\partial \gamma}\right)^{-1}(\varrho). \tag{12}$$

where ϱ is the threshold value which is used as a trade-off parameter to balance total transmission power $p_{\text{tot}} = \sum_{j=1}^{N} p_j$ and global probability of error P_e.

IV. Numerical Results

In this section, we investigate the performance of the proposed strategies from Section III. The scenario is generated by randomly deploying sensor nodes uniformly in a rectangular area of size A. The final node is supposed to be located in the middle of the scenario. The considered observation model is the problem of detecting the presence or absence of a deterministic signal in Gaussian noise [9]. The observation SNRs of the sensors are independent and uniformly distributed between 0 and 10 dB. As transmission technology we consider impulse radio ultra-wideband with binary pulse-position modulation, since it is considered to be an enabling technology for wireless sensor networks. The transmission power to achieve the SINR γ_j for an IR-UWB node is given by

$$p_j = \frac{\gamma_j \eta}{g_j N_j T_f}, \tag{13}$$

where in each time frame of length T_f exactly one impulse is transmitted. The information bits are transmitted by a number of N_j equally modulated impulses and η denotes the energy of the additional noise. More details can be found in [10].

Assuming that the communication channel is affected by fading, its probability of error (ε_j in equation (3)) is given by

$$\varepsilon_{j|h_j} = Q(h_j\sqrt{2\gamma_j}), \tag{14}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-a^2/2} da$ is the Gaussian Q-function. After averaging over h_j, ε_j is given by

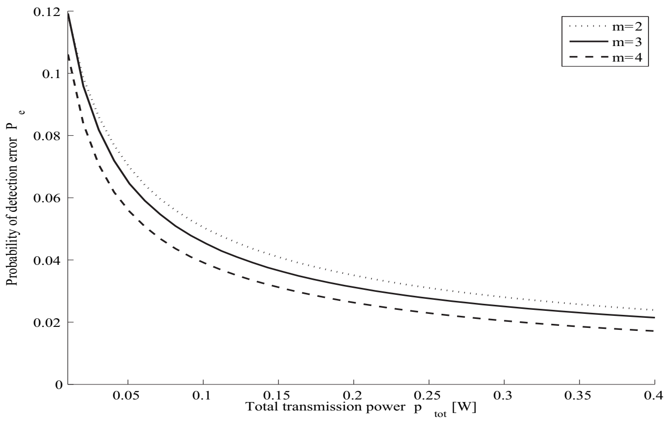

Fig. 2. Global probability of detection error P_e at the final node versus total transmit power, p_{tot}, for different fading intensities m.

$$\begin{aligned}\varepsilon_j &= \mathbf{E}_{h_j}\{\varepsilon_{j|h_j}\}\\ &= \frac{2}{\Gamma(m_j)}\left(\frac{m_j}{\Omega_j}\right)^{m_j}\int_0^{\infty} x^{2m_j-1}e^{-\frac{m_j x^2}{\Omega_j}}Q(x\sqrt{2\gamma_j})dx.\end{aligned} \quad (15)$$

To simplify the numerical analysis, we use the approximation in [11] for the Q-function according to

$$Q^n(x) \approx \frac{1}{2^n}\sum_{k=0}^{n}\left(\begin{array}{c} n \\ k \end{array}\right)\left((-1)^{n-k}e^{-A(n-k)\frac{x}{\sqrt{2}}}\right)\frac{e^{-An\frac{x}{\sqrt{2}}}}{(B\sqrt{\pi x})^n}, \quad (16)$$

where $A = 1.98$ and $B = 1.135$. By replacing equation (16) in (15), we obtain:

$$\begin{aligned}\varepsilon_j = \frac{I^{m_j}b^{-m_j}}{2\Gamma(m_j)B\sqrt{2\pi\gamma_j}}\Bigg(& b^{1/2}\Gamma(m_j-1/2)\\ &-\sqrt{b}\Gamma(m_j-1/2)_1F_1(m_j-1/2;1/2;\frac{a^2}{4b})\\ &+a\Gamma(m_j)_1F_1(m_j;3/2;\frac{a^2}{4b})\Bigg),\end{aligned} \quad (17)$$

where $I = \frac{m_j}{\Omega_j}$, $a = A\sqrt{\gamma_j}$ and $b = A\sqrt{2}\gamma_j + I$. Please note that, the channel fading effect is inherently taken into account in updating the sensor effective weights, $\widetilde{\lambda}_j$, through incorporating (17) into (4) and then (10). It is also taken into account in updating the assigned transmit power (Eq. (13)) for the sensor nodes through the sensors's target designated SINR in (12), by incorporating (17) into (4), (10) and then (12).

The system parameters for our network topology and the TH-UWB transceivers are summarized in Table I.

Fig. 2 illustrates the absolute detection performance of the system in terms of the global probability of detection error P_e at the final node as a function of the total transmission power p_{tot} for different fading intensities m. It can be observed that the detection performance is almost uniform for different fading intensity m. This is contrary to the result that we obtained for a parallel topology model in [8] where for low levels of p_{tot} the detection performance is almost independent of the fading intensity m and for high levels, however, the influence of the fading conditions becomes more important.

TABLE I
PARAMETERS USED IN THE SIMULATION

parameter	value
N	15
A	100 m × 100 m
N_j	10
T_f	100 ns
η	10^{-11} J

V. Conclusion

This paper addressed the detection error minimization problem in UWB-based wireless sensor networks with serial topology. In fact, we proposed a power assignment and a routing algorithm for distributed detection in serial fusion networks under Nakagami-m fading. Our results show that for low levels of total assigned transmit power, the detection performance is almost independent of the fading intensity m but for high levels, this influence becomes more important.

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