Optimum Power Allocation for Sensor Networks that Perform Object Classification

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***Abstract*—This publication analyzes the power allocation problem for a distributed sensor network. We consider a network that may have power-limited sensor nodes and is used for target object classification. In the classification process, the absence, the presence, or the type of a target object is observed by the sensor nodes independently. Since the observations are noisy, and are thus unreliable, they are fused together as a reliable global decision in order to increase the overall classification probability. The global decision is performed at a remotely located fusion center, after combining the local observations. The combiner uses the best linear unbiased estimator in order to estimate the reflection coefficient of the present object accurately. By using the proposed system architecture, we are able to optimize the power allocation analytically in order to maximize the classification performance if the total power of the sensor network is limited. Two different cases of power constraints are discussed and compared with each other. The corresponding results are valid for additive white Gaussian channels as well as for frequency-flat slow-fading channels.**

I. Introduction

In this paper, a sensor network is considered where each of N nodes individually transmits a signal and receives the reflected echo from a jointly observed target object. The object may be of K different types. It should be noted that sheer detection may be treated as the special case of $K = 2$ which corresponds to the decision '*some object is present*' versus '*there is no object*'. The particular information at each node is then sent to a fusion center which combines the local observations into a single decision. This setup is illustrated in Fig. 1, whose technical components will be specified in detail later. Both the sensing and communication channels are subject to additive noise. Additionally, we assume that the sensor nodes (SN) have only limited power available for sensing and communication. A potential application of our approach is radar sensing, where an unknown target object is observed for classification. Instead of using a single high-power radar system, this task is carried out by a network of cheap and energy-efficient SNs. To achieve comparable system performance, the fusion center combines a multitude of local observations into a single reliable one.

Quite naturally the problem arises how to allocate the power for first transmitting the signal for sensing and secondly communicating the message to the fusion center. The problem of finding an optimum power allocation for classification and a closed form of the objective function is extremely hard, and it is even harder to determine optimal points under certain constraints. The main difficulty is associated with finding an explicit representation of the objective function as mentioned in [1]. For a Bayesian hypothesis test criterion, the overall classification probability cannot be analytically evaluated [2]. This limits the usability of this criterion for optimizing the power allocation scheme. Bounds, such as the Bhattacharyya bound [3], are also difficult to use for optimizing multidimensional problems. In the present paper the objective function is the average deviation between the estimated and true reflection coefficients which are assumed to uniquely characterize the type of the target object. The suggested objective function together with the proposed system architecture enable us to derive an analytical solution to the power allocation problem. Total and individual power constraints are considered. Both lead to explicit policies for the power allocation. These are the main contributions of the present work.

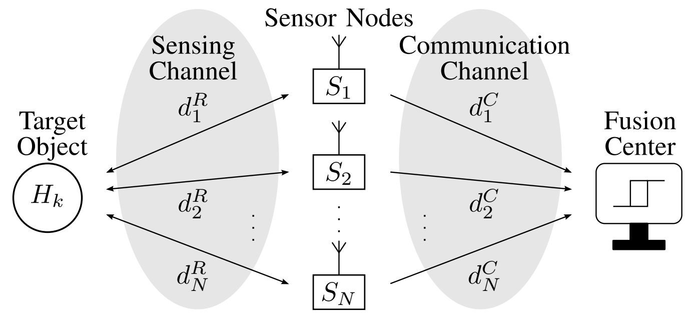

Fig. 1. System model of the distributed wireless sensor network.

Research on distributed detection originated from the attempt to combine signals of different radar devices [4]. Currently, distributed detection is rather discussed in the context of wireless sensor networks, where the sensor units may also be radar nodes [5]–[7]. In [8], the power allocation problem for distributed wireless sensor networks, which perform object detection and classification, is only treated for ultra-wide bandwidth (UWB) technology. Other applications, which require or benefit from detection and classification capabilities, are localization and tracking [9] or through-wall surveillance [10]. In [11], an approximate solution of the power allocation problem is proposed, which allows for an analytical treatment of power limitation per sensor node. However, the fusion rule for the global decision has not yet been investigated in the context of object classification.

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At this point, we have to mention that a similar allocation problem was treated in [12]. The main difference arises from the application of the sensor networks; in case of object classification, a part of the total power is consumed for object sensing. Due to this fine distinction, the optimization problem is no longer convex and thus quite hard to solve.

The present paper is organized as follows. We start with a detailed description of the underlying technical system in the next section. Subsequently, the power allocation problem is specified and analytically solved. The results are then carefully compared and discussed.

Mathematical Notations:

Throughout this paper we denote the set of natural, integer, real, and complex numbers by $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$, respectively. Note that the set of natural numbers does not include the element zero. Moreover, $\mathbb{R}_+$ denotes the set of non-negative real numbers. Furthermore, we use the subset $\mathbb{F}_N \subseteq \mathbb{N}$ which is defined as $\mathbb{F}_N := \{1, \ldots, N\}$ for any given natural number N. We denote the absolute value of a real or complex-valued number z by $|z|$ while $\bar{h}$ represents the conjugate complex of some complex function h. The expected value of a random variable v is denoted by $\mathcal{E}[v]$.

II. Overview and Technical System Description

Distributed *target object* classification can be formally modeled as a multiple hypotheses testing problem with hypotheses H_k, $k \in \mathbb{F}_K$, for a specific number $K \in \mathbb{N}$, $K \geq 2$, of different object types. Each H_k corresponds to an object of the same size, shape and alignment, but different material and, hence, complex-valued reflection coefficients $r_k \in \mathbb{C}$, $k \in \mathbb{F}_K$. Thus, the reflection coefficients are the only recognition features in this work. The a-priori probability of occurrence for each object type is denoted by $\pi_k \in \mathbb{R}_+$, $k \in \mathbb{F}_K$, with $\sum_{k=1}^{K} \pi_k = 1$.

At any instance of time, a network of $N \in \mathbb{N}$ independent and spatially distributed sensors, as shown in Fig. 1, receives random observations. If a target object is present, then the received energy at the SN S_n is a part of its own radiated energy which is back-reflected from the object's surface and is weighted by its reflection coefficient. The corresponding random observations are assumed to be conditionally independent, given any of the underlying hypotheses. We denote all parameters which refer to the sensing channel between the sensors and the target object by superscript R.

In general, the observations are not identically distributed because the SNs have different distances d_n^R from the target object and their radiated powers are also different. Thus, the signal-to-noise ratio (SNR) varies between the SNs. Due to the distributed nature of the problem, the n^{th} sensor S_n performs an independent observation and transmits this observation to a fusion center located at a remote location. The communication quality between the SN S_n and the fusion center is determined by the corresponding distance d_n^C and assigned transmission power. We denote all parameters which refer to the communication channel between the SNs and the fusion center by superscript C. Note that the data transmission to the fusion center may also be performed by a wired communication that does not have any influences on the purpose of the present work. Furthermore, we assume that the propagation channels may be frequency-flat slow-fading channels and all data transmissions are disturbed by additive white Gaussian noise (AWGN). We disregard time delays within all transmissions and assume synchronized data communication. We use two distinct waveforms for each SN so as to distinguish its communication links from the communication links of other SNs. Each waveform has to be suitably chosen in order to suppress inter-user (inter-node) interference at each receiver. Hence, the N received signals at the fusion center are assumed to be conditionally independent, given any of the underlying hypotheses. The received random signals correspond to the local observations at the SNs are also not identically distributed because of different distances and assigned transmission powers. Unlike the local independent observations, the global decision rule depends on all observations in order to increase the overall classification probability.

In the following subsections, we describe the underlying system architecture in detail, whose model is depicted in Fig. 2. The continuous-time system is modeled by its discrete-time equivalent, where the sampling rate of the corresponding signals is equal to the object observation rate, for the sake of simplicity.

A. Sensor nodes

Each SN generates two streams of data symbols s_n^R and $s_{n|k}^C$ which are associated to each other due to the local observation, as described further below.

In order to eliminate collisions caused by multiple access, distinct waveforms $h_n^C(t)$ and $h_n^R(t)$ are assigned to each stream. Furthermore, they are used as matched-filter to limit the bandwidth of the signal. They also have to fulfill the Nyquist criterion to avoid intersymbol interferences (ISI).

The symbol stream s_n^R establishes the radiation to the target object and uses always the same data symbol. We assume, without loss of generality, that this stream is real-valued. The power of this stream is variable in order to adjust the power and to enable distributed power allocation. The mean value of the instantaneous transmission power $\left|s_n^R\right|^2$ for each observation event is given by

$$P_n^R := \mathcal{E}\left[\left|s_n^R\right|^2\right], \; n \in \mathbb{F}_N \; . \tag{1}$$

After irradiation of the present target object, a part of the radiated signal s_n^R is reflected from the target surface back to the antenna. The received signal, together with additive noise, passes through the matched-filter and is sampled uniformly at the right time. We denote the corresponding samples of the received signal by $x_{n|k}^R \in \mathbb{C}$ which include the corresponding samples $w_n^R \in \mathbb{C}$ of the noise. We assume that the noise has a standard deviation of $\sigma^R/\sqrt{2} > 0$ in each direction. If all receiver components are linear, then for all $k \in \mathbb{F}_K$ and $n \in \mathbb{F}_N$ we can describe each sample by

$$x_{n|k}^R := s_n^R \, r_k \, g_n^R + w_n^R \; , \tag{2}$$

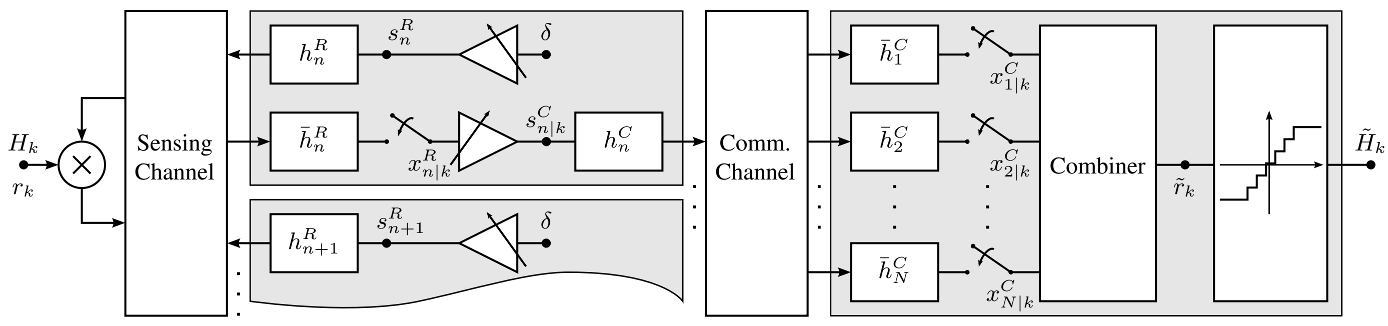

Fig. 2. System architecture.

where the transmitted signal is weighted by the product of the factors r_k and $g_n^R \in \mathbb{C}$. The time-dependent function g_n^R includes the radar cross section, the influence of the antenna and the propagation channel, the impact of the filters, and all additional attenuation of the transmitted signal. This function is usually a decreasing function of the distance $2 \cdot d_n^R$ between transmitter and receiver. Here, the factor of two in the distance results from that back and forth transmission between the transceiver and the object.

After sampling the received signal, each sample is amplified by the factor $v_n \in \mathbb{R}_+$ to generate the symbol stream $s_{n|k}^C \in \mathbb{C}$. We assume that the amplifier is linear over the whole bandwidth and power range so that

$$s_{n|k}^C := x_{n|k}^R \, v_n, \; n \in \mathbb{F}_N. \tag{3}$$

The amplification factor is an unknown parameter for the time being and will be determined by the power allocation procedure later.

The symbol stream $s_{n|k}^C$ is used to transmit the local observations to the fusion center. The power of this stream is also variable due to the amplification factor. The mean value of its instantaneous transmission power $\left|s_{n|k}^C\right|^2$ for each observation event is given by

$$P_{n|k}^C := \mathcal{E}\left[\left|s_{n|k}^C\right|^2\right], \; k \in \mathbb{F}_K, n \in \mathbb{F}_N \; . \tag{4}$$

Note that the instantaneous power fluctuates from observation to observation depending on the present target object.

B. Fusion center

After transmission of the stream $s_{n|k}^C$ by the SN S_n, the signal is attenuated depending on the distance and it reaches the antenna at the fusion center. The received signal, together with additive noise, is matched-filtered and sampled at the right time similar to the received signals at the SNs. We do not consider inter-user interferences at the fusion center because of the distinct waveform choices.

We denote the corresponding samples of the received signal, which include the corresponding samples $w_n^C \in \mathbb{C}$ of the noise, by $x_{n|k}^C \in \mathbb{C}$. We assume that the noise has a standard deviation of $\sigma^C/\sqrt{2} > 0$ in each direction. If all receiver components are linear, then we can describe each sample of the received signal by

$$x_{n|k}^C := s_{n|k}^C \, g_n^C + w_n^C, \; n \in \mathbb{F}_N \; , \tag{5}$$

where the function $g_n^C \in \mathbb{C}$ is defined analogous to the definition of g_n^R in Section II-A.

C. Fusion of local observations

The fusion of the local observations is performed by weighting and combining the received samples. We use a linear combiner which reduces the computational effort and is a key idea in this work for an analytical solution of the power allocation problem. We denote the output value of the combiner by

$$\tilde{r}_k := \sum_{n=1}^{N} u_n \, x_{n|k}^C, \; k \in \mathbb{F}_K, \; u_n \in \mathbb{C}, \; n \in \mathbb{F}_N \, . \tag{6}$$

The value $\tilde{r}_k$ should be a good estimate for the true reflection coefficient r_k of the present target object. Thus, we have to optimize the sensing powers P_n^R, the amplification factors v_n and the weights u_n in order to minimize the average deviation between $\tilde{r}_k$ and the present r_k. This will be extensively explained later. After determining P_n^R, v_n and u_n, the fusion center observes a disturbed version of the true reflection coefficient r_k at the input of its decision unit. Hence, by using the present system architecture, we are able to separate the power allocation problem from the classification problem and optimize both independently of each other.

D. Some remarks on the system model

All described assumptions are necessary to obtain a framework suitable for analyzing the power allocation problem, without studying problems of different classification methods in specific systems and their settings.

The knowledge of the distances d_n^R and d_n^C is necessary for a successful classification. They can be estimated by a tracking algorithm before applying the classification process.

The accurate estimation of the channel state is also necessary for the classification process and for the power allocation. We are not often able to estimate the transmission channels, consequently the parameters g_n^R and g_n^C remain unknown. In such cases, the allocation problem cannot be solved exactly.

In general, SNs have only one power amplifier and a single antenna. The antenna is usually connected to a circulator in order to separate the signal of the transmitter to the antenna from the signal of the antenna to the receiver, which is not depicted in Fig. 2. The power amplifier is also shared for sensing and communication tasks, but not considered in this work.

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In order to increase the available power range at each SN, time-division multiple-access (TDMA) can be used to separate s_n^R and $s_{n|k}^C$ into different time slots and to periodically share the same power amplifier.

The introduced system architecture describes a baseband communication system without considering time, phase and frequency synchronization problems.

III. Power Allocation

In this section, we present an analytical method to solve the power allocation problem subject to limited total power, which can be arbitrarily allocated to the radar task and the communication task. We denote the expected value of the total power by $P_{\text{tot}} \in \mathbb{R}_+$. In practice, each SN has its own constraint for the transmission power. The expected value of the power-limitation per SN is denoted by $P_{\text{max}} \in \mathbb{R}_+$. In the next subsection, we assume that the power-limitation P_{max} of each SN is greater than the total power P_{tot}. Thus, none of the SNs will exceed its own power limitation because no more power than P_{tot} is available. In Subsection III-B, we will consider the other case in which the power-limitation P_{max} is less than the total power P_{tot}.

In general, the objective is to maximize the overall classification probability, however, a direct solution to the allocation problem does not exist, since no analytical expression for the overall classification probability is available. Instead, we minimize the average deviation between $\tilde{r}_k$ and r_k, in order to determine the power allocation. The motivation for this method is the separation of the power allocation problem from the object classification procedure, as described in the last section.

We now summarize notations that we will use hereinafter and are needed for the description of each observation process:

- P_{tot}: total transmission power of the network;
- P_{max}: power-constraint per SN;
- P_n^R and $P_{n|k}^C$: mean values of the sensing and communication power for the n^{th} sensor;
- π_k and r_k: probability of occurrence and reflection coefficient of the k^{th} object type;
- $\tilde{r}_k$: the estimate of r_k;
- g_n^R and g_n^C: channel coefficients including additional attenuations of transmitted signals;
- w_n^R, w_n^C, $\frac{\sigma^R}{\sqrt{2}}$ and $\frac{\sigma^C}{\sqrt{2}}$: zero-mean AWGN and corresponding standard deviations in each direction;
- v_n: amplification factor of the n^{th} sensor;
- $x_{n|k}^C$ and u_n: input samples and weights of the combiner.

A. No power limitation per sensor node

For each new classification process, the limitation of the total power is an upper bound for the sum

$$\underbrace{\sum_{n=1}^{N} \underbrace{P_n^R}_{\text{Radar task}} + \underbrace{P_n^C}_{\text{Data communication}}}_{\text{Transmission power of one sensor for a single observation}} \leq P_{\text{tot}} \,, \tag{7}$$

where P_n^C denotes the expected value of the communication power. The value of P_n^C follows from the equations (1)–(4) and is given in terms of P_n^R by

$$P_n^C := \sum_{k=1}^{K} \pi_k P_{n|k}^C = P_n^R \left| r_{\text{rms}} \, v_n \, g_n^R \right|^2 , \; n \in \mathbb{F}_N . \tag{8}$$

The quantity r_{rms} is the root mean squared value of the reflection coefficients which is given by the equation

$$r_{\text{rms}} := \sqrt{\sum_{k=1}^{K} \pi_k \left| r_k \right|^2} \; . \tag{9}$$

After incorporating (8) into (7), we obtain the modified constraint

$$\sum_{n=1}^{N} P_n^R \cdot \left[1 + \left| r_{\text{rms}} \, v_n \, g_n^R \right|^2 \right] \leq P_{\text{tot}} \tag{10}$$

which will be our first constraint for solving the allocation problem.

As mentioned in the last section, we aim at finding estimators $\tilde{r}_k$ of minimum mean squared error in the class of unbiased estimators for each k. By using the equations (1)–(3), (5), and (6) we may write $\tilde{r}_k$ as

$$\tilde{r}_k = r_k \sum_{n=1}^{N} u_n v_n g_n^C g_n^R \sqrt{P_n^R} + \sum_{n=1}^{N} u_n \left[v_n g_n^C w_n^R + w_n^C \right] . \tag{11}$$

The estimate $\tilde{r}_k$ is hence unbiased simultaneously for each k if $\mathcal{E}[\tilde{r}_k - r_k] = 0$, i.e.,

$$\sum_{n=1}^{N} u_n v_n g_n^C g_n^R \sqrt{P_n^R} = 1 \; . \tag{12}$$

This equality is our second constraint in what follows. Note that the mean of the second term in (11) vanishes since the noise is zero-mean. Furthermore, we do not consider the impact of the random numbers g_n^C and g_n^R in our calculations because we assume that the coherence time of channels is much longer than the observation time.

Since the objective is to minimize the mean squared error, we use (11) and (12) in order to write the objective function as

$$\mathcal{E}\left[\left| \tilde{r}_k - r_k \right|^2 \right] = \sum_{n=1}^{N} \left| u_n \right|^2 \left[\left| v_n g_n^C \sigma^R \right|^2 + \left(\sigma^C \right)^2 \right] \; . \tag{13}$$

Note that (13) is only valid if w_n^R and w_n^C are white and jointly uncorrelated.

In summary, the optimization problem is to minimize the mean squared error in (13) with respect to u_n, v_n, and P_n^R, subject to constraints (10) and (12). Note that the optimization problem is a *signomial program*, which is a generalization of *geometric programming*, and is thus non-convex in general, see [13].

1) The solution: By using the method of Lagrangian multipliers we are able to find a unique solution for the optimization problem. In order to present the solution, we have to re-index the SNs so that the inequality chain

$$c_n := \frac{|g_n^R\, g_n^C|}{\sigma^R\ |g_n^C| + r_{\text{rms}}\, \sigma^C\ |g_n^R|} > c_{n+1}\ ,\ n \in \mathbb{F}_{N-1}\ , \quad (14)$$

holds. Then, we obtain the solution in which only the first SN is active and consumes the whole power P_{tot}. All other SNs participate neither in the data communication, nor in the classification of the target object. The corresponding optimal weight, amplification factor, and transmission powers are

$$u_1 = \sqrt{\frac{r_{\text{rms}}\ |g_1^R|}{\sigma^C\ P_{\text{tot}}}}\ \frac{\sqrt{\sigma^R\ |g_1^C| + r_{\text{rms}}\, \sigma^C\ |g_1^R|}}{g_1^R\, g_1^C}\ , \quad (15)$$

$$v_1 = \sqrt{\frac{\sigma^C}{r_{\text{rms}}\, \sigma^R\ |g_1^R\, g_1^C|}}\ , \quad (16)$$

$$P_1^R = \frac{\sigma^R c_1 P_{\text{tot}}}{|g_1^R|} \quad \text{and} \quad P_1^C = \frac{r_{\text{rms}}\, \sigma^C c_1 P_{\text{tot}}}{|g_1^C|}\ . \quad (17)$$

By incorporating these results into the objective function (13) we obtain the absolute variance of $\tilde{r}_k$ as

$$\mathcal{E}\left[|\tilde{r}_k - r_k|^2\right] = \frac{1}{P_{\text{tot}}\, c_1^2}\ . \quad (18)$$

Note that by using the above results, the corresponding fusion rule is simplified by discarding the influence of inactive SNs from the fusion rule. The fusion rule (6) becomes

$$\tilde{r}_k = u_1\, x_{1|k}^C,\ k \in \mathbb{F}_K\ . \quad (19)$$

Note that $\tilde{r}_k$ is an unbiased estimator for r_k due to constraint (12). By similar methods we can also minimize the mean squared error without restricting ourself to unbiased estimators. Obviously, the optimal value will be smaller in that case.

2) Interpretation of the solution: The proposed power allocation has the following interpretation. The sensor S_n with the largest c_n consumes the whole power P_{tot} because its communication channels are possibly the best. Thus, the observation of the target object is less interfered by noise and consequently results in better data communication. The other SNs do not get any transmission power at all since their information reliability is too poor to be considered for data fusion.

B. Supplementary power limitation per sensor node

In this section, we assume that all SNs have the same transmission power-constraint $P_{\max}$ and we consider the case in which the power-constraint is less than the total power P_{tot}. Thus, it is not always possible to assign the total power only to a single SN. Hence, a subset of $\tilde{N} \leq N$ sensors will be active. We retain the condition (7) that the sum power is limited and, in addition to the constraints (10) and (12), the constraints

$$P_n^R + P_n^C \leq P_{\max},\ n \in \mathbb{F}_N\ , \quad (20)$$

are included into the optimization problem.

We remark that the described method can also be extended to individual power constraints per SN.

1) The solution: Similar to the solution in the last subsection, the optimization problem has a unique solution that can also be determined by the method of Lagrangian multipliers. In order to present the solution, we have to re-index the SNs as in (14). Then, for the first $\tilde{N} - 1$ nodes the solutions for the transmission powers are

$$P_n^R = \frac{\sigma^R c_n P_{\max}}{|g_n^R|} \quad \text{and} \quad P_n^C = \frac{r_{\text{rms}}\, \sigma^C c_n P_{\max}}{|g_n^C|}\ . \quad (21)$$

One can easily see, that for the first $\tilde{N} - 1$ nodes the sum $P_n^R + P_n^C$ is equal to $P_{\max}$. The last SN obtains the remaining power which is given by

$$P_{\text{remain}} := P_{\text{tot}} - (\tilde{N} - 1) P_{\max}\ . \quad (22)$$

Its transmission powers can be calculated as

$$P_{\tilde{N}}^R = \frac{\sigma^R c_{\tilde{N}} P_{\text{remain}}}{|g_{\tilde{N}}^R|} \quad \text{and} \quad P_{\tilde{N}}^C = \frac{r_{\text{rms}}\, \sigma^C c_{\tilde{N}} P_{\text{remain}}}{|g_{\tilde{N}}^C|}\ . \quad (23)$$

In order to satisfy the inequality

$$0 < P_{\text{remain}} \leq P_{\max}\ , \quad (24)$$

which must be fulfilled for the last SN and is necessary for a successful power allocation, we deduce the number $\tilde{N}$ of active SNs as well as a lower bound for $P_{\max}$. The number $\tilde{N} \leq N$ of active SNs is the smallest integer number for which the inequality

$$\tilde{N} \geq \frac{P_{\text{tot}}}{P_{\max}} \quad (25)$$

holds. From $N \geq \tilde{N}$ follows the lower bound of $P_{\max}$, where the corresponding range is given by

$$\frac{P_{\text{tot}}}{N} \leq P_{\max} \leq P_{\text{tot}}\ . \quad (26)$$

In case that the inequality $P_{\max} < \frac{P_{\text{tot}}}{N}$ is valid, all SNs attain their power constraints and a part of the total power remains unallocated.

The optimized values of the amplification factors v_n and the weights u_n are given by the equations

$$v_n = \sqrt{\frac{\sigma^C}{r_{\text{rms}}\, \sigma^R\ |g_n^R\, g_n^C|}}\ ,\ n \in \mathbb{F}_{\tilde{N}}\ , \quad (27)$$

and

$$u_n = \frac{\frac{c_n}{g_n^R\, g_n^C} \sqrt{\frac{r_{\text{rms}} |g_n^C|\, |g_n^R|^3 P_n^R}{\sigma^R \sigma^C}}}{P_{\text{tot}}\, c_{\tilde{N}}^2 + P_{\max} \sum_{n=1}^{\tilde{N}} (c_n^2 - c_{\tilde{N}}^2)},\ n \in \mathbb{F}_{\tilde{N}}\ . \quad (28)$$

By inserting (21), (23), (27) and (28) into the objective function (13) we obtain the absolute variance of $\tilde{r}_k$ as

$$\mathcal{E}\left[|\tilde{r}_k - r_k|^2\right] = \frac{1}{P_{\text{tot}}\, c_{\tilde{N}}^2 + P_{\max} \sum_{n=1}^{\tilde{N}} (c_n^2 - c_{\tilde{N}}^2)}\ . \quad (29)$$

Note that for $\tilde{N} > 1$, the value in (29) is larger than that in (18) for the inequality chain $c_n > c_{n+1}$, $n \in \mathbb{F}_{N-1}$.

In the considered case, the fusion rule is not as simple as (19) since more SNs are active in general. The fusion rule (6) becomes

$$\tilde{r}_k = \sum_{n=1}^{\tilde{N}} u_n \, x_{n|k}^C, \; k \in \mathbb{F}_K \; . \tag{30}$$

2) Comparison of the solutions: We make the following observation on the results by comparing the solutions which are described in Section III-A and III-B. In the case $P_{\text{max}} < P_{\text{tot}}$ the overall classification performance is reduced because of two reasons. First, the SNR of each SN is reduced due to the power limitation P_{max}. Second, the larger the number of active SNs is, the more noise power is summed up by the combiner at the fusion center, as we have learned from the absolute variances in (18) and (29). This behavior is not surprising and the performance reduction was predictable since we have included more restrictions into the optimization problem.

Note that the solutions in Subsection III-A are different from the *water-filling* results while the solutions in Subsection III-B are similar to water-filling, see [14].

IV. Classification

As we have seen in the last section, we are able to optimize P_n^R, v_n and u_n such that the estimate $\tilde{r}_k$ is unbiased for each k. The input of the decision unit is hence a disturbed version of the true reflection coefficient r_k with additive zero-mean Gaussian noise. If we denote the result in (18) or in (29) by σ^2, and if we use the identities (11) and (12), then the corresponding covariance matrix of $\tilde{r}_k$ is determined by

$$\frac{\sigma^2}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{31}$$

Hence, the conditional probability density of $\tilde{r}_k$ is given by

$$f_k(r) = \frac{1}{\pi\sigma^2}\exp\!\left(-\frac{|r - r_k|^2}{\sigma^2}\right), \; r \in \mathbb{C}, k \in \mathbb{F}_K . \tag{32}$$

Due to the simple form of the conditional densities and equal covariance matrices for all k, we are able to use a distance classifier for the global classification rule. Distance classifiers are easily implementable, because in the present case we deal with linear discriminant functions. Furthermore, they yield high classification performances [2] because they coincide with the Bayes classifier. Their average probability of correct classification can be calculated by

$$\int\limits_{r \in \mathbb{C}} \max_{k \in \mathbb{F}_K} \big(\pi_k f_k(r)\big) \, \mathrm{d}r \, . \tag{33}$$

In general, it is challenging to evaluate the above integral analytically. Instead, numerical computations can be used to evaluate the probability.

Note that the outcome of the last integral must finally be averaged over the corresponding random variables, for example the position of the target object or the realization of channel coefficients g_n^R and g_n^C.

V. Conclusion

For the classification of a target object we have introduced a system architecture for a distributed sensor network and its corresponding power allocation scheme. We have shown that by applying a simple fusion rule for combining independent sensor observations, we are able to solve the power allocation problem analytically. Furthermore, we have shown that the proposed allocation procedure works with and without a transmission power constraint per sensor node. We have also demonstrated that by considering an extra individual power constraint at each sensor node, the classification performance is reduced, as expected. Moreover, the proposed solutions are valid for AWGN channels as well as for frequency-flat slow-fading channels with channel state information at each receiver. The proposed method also supports selecting the right number of sensor nodes which transmit information more reliably than other ones. This selection method allows us to decrease the number of active sensor nodes. It subsequently increases the classification performance while the computational complexity is simultaneously decreased. Furthermore, the proposed method enables the application of simple distance classifiers which are easy to implement and achieve high classification performance.

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