Complexity-Reduced Optimal Power Allocation in Passive Distributed Radar Systems

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***Abstract*—In this paper, we provide an alternative derivation of the optimal power allocation for distributed passive radar systems in closed-form. Our approach provides new insights to the nature of the power allocation problem and extracts some optimality conditions which are in turn used to achieve a new algorithm with reduced complexity for a reliable sensor selection. Finally, we show the computational complexity and the run-time of the proposed algorithm against the previously available one by analytic and simulative comparisons.**

I. Introduction

Nowadays, many applications benefit from the idea of distributed sensor networks for purposes of observation and communication. The range of these applications especially covers health care, traffic monitoring, radio astronomy, particle physics, and military applications [1], [2], where a distributed passive multiple-radar system (DPMRS) is needed. As an interesting example, we can mention the '*IceCube Neutrino Observatory*' at the south pole, where a sensor network with more than 5000 sensor nodes (SNs) is deployed to observe certain characteristics of sub-atomic particles [3]. Since the operation of the whole sensor network is mostly intended to consume minimum resources while keeping the individual cost and maintenance of SNs low, an energy efficient operation is highly desirable. Hence, the related problem of optimal power allocation and corresponding energy-aware system design has been addressed in many works, e.g., in [4], [5] and [6]. In particular, the optimal power allocation scheme for DPMRSs is provided by [1] which is the main platform for the present work.

As it is studied in [1], the optimal power allocation problem for DPMRSs results in a similar solution as the well-known method of water-filling (WF) solution [7]. This method requires an iterative search to obtain the optimal operation mode of each SN. The main contribution of the present work is to derive a sorting mechanism that maps this iterative process into an incremental search over the reliability of each SN. As a result, optimal separation of the sensor nodes can be achieved via an iterative bi-section search which significantly reduces the required number of iteration-steps. Finally, we discuss the computational complexity and the run-time of the proposed algorithm against the previously available one by analytic and simulative comparisons.

The remaining part of the present paper is organized as follows. In Section II, we describe the investigated system model. Section III presents a short overview of the previously existing optimal solution to the power allocation problem. In Section IV we extend the known proof of the optimization problem by a new alternative proof to extract the novel iterative algorithm for optimal sensor selection. An extensive complexity analysis is performed in Section V. An overview of the key results of the present work is summarized in Conclusion.

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II. System Model

We investigate a network of K amplify-and-forward (AF) passive SNs, cooperating to achieve a single global observation via a fusion center (FC). Both communication and sensing channels (Rayleigh frequency-flat fading) are assumed to be wireless and static during the observation process. In order to incorporate the limitations of the network and single node power consumption, we apply a total as well as individual power constraints on the function of SNs. The final goal of each observation process is to classify (or detect) a target signal $r \in \mathbb{C}$. Each observation process can be segmented into three parts: sensing process, communication process and information fusion. Table 1 represents the used notations for different signals and system parameters. The detailed description of the system function has been explained in [1, Section II]. io

A. Operation of SNs

If a target signal r is present, each SN receives and amplifies the incoming signal by an amplification coefficient $u_k \in \mathbb{C}$. The communication with FC is performed by using orthogonal waveforms for each SN so that data from the different SNs can be separated and processed in FC. The process of each SN can be described as

$$x_k := (rg_k + m_k)\, u_k \tag{1}$$

and

$$X_k := \mathcal{E}\{|x_k|^2\} = |u_k|^2 \left(R|g_k|^2 + M_k\right), \quad R := \mathcal{E}\{|r|^2\}, \tag{2}$$

where $\mathcal{E}\{\cdot\}$ represents mathematical expectation. The sensing channel coefficient, transmit signal and its power from the SN with index k is respectively denoted by g_k, $x_k \in \mathbb{C}$ and X_k, and $m_k \in \mathbb{C}$ represents the additive white noise on the sensing process with variance M_k. The limit of each SN power is modeled as

$$X_k \leq P_k \Leftrightarrow |u_k|^2 \left(M_k + R|g_k|^2\right) \leq P_k, \tag{3}$$

TABLE I. USED SYMBOLS AND NOTATIONS

Notation	Description
K	number of all nodes
$r,\ R$	target signal and its power
$\tilde{r}$	the estimate of r
g_k, h_k	complex-valued sensing and communication channel coefficients
m_k, n_k	complex-valued zero-mean AWGN at each SN and FC
$M_k,\ N_k$	variances of m_k and n_k
u_k, v_k	complex-valued amplification factors and fusion weights
X_k	communication power of k^{th} SN
P_{tot}, P_k	sum and individual output power-range constraint for SNs
$\mathbb{F}_K$	the index-set of all K nodes
$\mathbb{K}_0,$	the index-set of all inactive nodes
$\mathbb{K}_{\text{sat}}$	the index-set of all nodes operating with maximum power
$\mathbb{K}$	the index-set of all active nodes (not saturated and not inactive)

while the limit of the network power consumption is described by

$$\sum_{k=1}^{K} X_k \leq P_{\text{tot}} \Leftrightarrow \sum_{k=1}^{K} |u_k|^2 \left(M_k + R|g_k|^2\right) \leq P_{\text{tot}}. \quad (4)$$

B. Fusion Center

The transmitted signal of each SN passes through the communication channel, $h_k \in \mathbb{C}$, and is combined with additive white Gaussian noise $n_k \in \mathbb{C}$ at the FC. A linear combination rule with weights $v_k \in \mathbb{C}$ is then applied at the FC to achieve an estimation $\tilde{r}$ from the observed target signal r by the SNs. This is described as

$$y_k := (h_k x_k + n_k)\, v_k, \quad (5)$$

and results in

$$\tilde{r} := \sum_{k=1}^{K} y_k = r \sum_{k=1}^{K} g_k u_k h_k v_k + \sum_{k=1}^{K} (m_k u_k h_k + n_k)\, v_k. \quad (6)$$

While linear processing and fusion strategies are not necessarily optimal, they are very simple and facilitate a feasible analytic approach to an optimum solution. In the present work, we assume availability of the perfect channel information for both sensing and communication channels. In general, it is rather difficult to estimate the sensing channel in an accurate way. Hence the presented results can be treated as theoretical limits for the operation of a realistic system.

III. Optimal Power Allocation with Iterative Set Examination

In this section we define our optimization strategy and discuss the available solution to the defined problem.

A. Optimization problem

Our objective and system constraints are identical to those in [1]. In order to achieve a good estimation of the observed signal, the estimate must be as close as possible to the actual target signal. Hence the objective V is chosen as the minimum mean squared error (MMSE) and is given by

$$V := \mathcal{E}\{|\tilde{r} - r|^2\} = \sum_{k=1}^{K} |v_k|^2 \left(M_k |u_k|^2 |h_k|^2 + N_k\right). \quad (7)$$

Furthermore, we limit our solution set to unbiased estimators which by considering (6) follow the identity

$$\sum_{k=1}^{K} c_k = 1, \quad c_k := g_k h_k u_k v_k. \quad (8)$$

By including the defined power constraints (3) and (4) we may now formulate our optimization problem as

$$\begin{aligned} &\min_{u_k, v_k,\ k \in \mathbb{F}_K} V \\ &\quad \text{s.t.} \sum_{k \in \mathbb{F}_K} c_k = 1, \quad \sum_{k \in \mathbb{F}_K} X_k \leq P_{\text{tot}}, \\ &\quad\quad X_k \leq P_k, \quad \forall k \in \mathbb{F}_k. \end{aligned} \quad (9)$$

B. Available solution

The optimal solution to the above-mentioned problem is presented in [1], assuming real-valued amplification coefficients ($u_k \in \mathbb{R}^+$). For the detailed description of the solution please refer to [1, Section III], equations (11-58). As it is been explained, the solution relies on the separation of the function of SNs into the nodes with zero allocated power (*inactive* status, $X_k = 0$), the nodes with maximum allocated power (*saturated*, $X_k = P_k$) and nodes with *active* status ($0 < X_k < P_k$). While the presented procedure provides the optimal solution in closed form, the separation of nodes into correct status is only performed via an iterative search and examining the nodes combination. This process gets more complicated and occasionally leads to long searching periods as the number of the nodes increases. In the following we try to address this problem accordingly.

IV. Optimal Power Allocation with Sorting Mechanism

We start our solution by studying the optimality conditions on the phase of the involved variables. As an initial result we show that the real-valued assumption in [1] does not reduce the optimality of the solution. For a given (feasible) power allocation ($X_k,\ k \in \mathbb{F}_k$), our problem can be accordingly formulated as

$$\min_{u_k, v_k, k \in \mathbb{F}_K} V, \quad \text{s.t.} \sum_{k=1}^{K} c_k = 1, \quad (10)$$

where $c_k := g_k h_k u_k v_k$ as in (8). The following lemma provides important information on the phase of our system parameters in the optimum point:

Lemma 1: For any optimal choice of system parameters ($u_k, v_k, \forall k \in \mathbb{F}_K$), the following parameter update is feasible and does not degrade (increase) the objective value in (10):

$$\forall k \in \mathbb{F}_k: \quad v_{k,\text{new}} := |v_k| \frac{(g_k h_k)^*}{|g_k h_k| \left(\sum_{k \in \mathbb{F}_K} |g_k h_k u_k v_k|\right)}, \quad u_{k,\text{new}} := |u_k|, \quad (11)$$

where $(\cdot)^*$ represents conjugation.

Proof: It is clear that (11) does not violate the power constraints (3) and (4) as the absolute value of amplification factor u_k and consequently X_k are kept constant. Furthermore it is easily verified that the unbiased condition (8) still holds:

$$\sum_{k\in\mathbb{F}_K} g_k h_k u_{k,\mathrm{new}} v_{k,\mathrm{new}} = \frac{\sum_{k\in\mathbb{F}_K} |g_k h_k u_k v_k|}{\sum_{k\in\mathbb{F}_K} |g_k h_k u_k v_k|} = 1. \quad (12)$$

On the other hand, due to (8) and triangular inequality we have:

$$\sum_{k\in\mathbb{F}_K} |g_k h_k u_k v_k| \geq \sum_{k\in\mathbb{F}_K} g_k h_k u_k v_k = 1, \quad (13)$$

which shows that the variable update (11) does not increase the norms of v_k and u_k and hence does not increase the objective value (7). ■

The above lemma provides us with few useful results. Firstly, it shows that the real-valued assumption for $u_k,\ k \in \mathbb{F}_K$ does not reduce the optimality. Secondly, it provides us with an optimal choice of $\angle v_k$ and simplifies our optimization problem into finding $|u_k|, |v_k|, k \in \mathbb{F}_K$ by assuming

$$u_k \in \mathbb{R}^+, \ \ v_k = |v_k| \angle \left(g_k h_k\right)^*, \quad (14)$$

where $\angle(\cdot)$ represents the phase. We continue our solution by recalling the identity (2) which together with (14) result in a unique $u_k \in \mathbb{R}^+$

$$u_k = \sqrt{\frac{X_k}{R|g_k|^2 + M}}. \quad (15)$$

This accordingly simplifies our problem into

$$\min_{|v_k|, k\in\mathbb{F}_K} V, \ \text{ s.t. } \sum_{k=1}^{K} |g_k h_k| \cdot u_k \cdot |v_k| = 1, \quad (16)$$

which is now a convex optimization problem over $|v_k|,\ \forall k \in \mathbb{F}_k$. By relaxing the range of $|v_k|$ and formulating the respective Lagrangian function we have

$$L\left(|v_k|, \lambda\right) = \sum_{k\in\mathbb{F}_K} |v_k|^2 \left(M_k u_k^2 |h_k|^2 + N_k\right) + \lambda \left(1 - \sum_{k=1}^{K} |g_k h_k| \cdot u_k \cdot |v_k|\right). \quad (17)$$

We should note once again that above formulation follows assuming a fixed (pre-defined) values for $X_k,\ k \in \mathbb{F}_k$. In any stationary point of the defined Lagrangian function the first derivative must vanish with respect to $|v_k|$:

$$\frac{\partial L}{\partial |v_k|} = 2|v_k| \left(M_k u_k^2 |h_k|^2 + N_k\right) - \lambda |g_k h_k| u_k = 0, \quad (18)$$

which by multiplying the value $|v_k|$ and summing up over $k \in \mathbb{F}_k$ and incorporating (8) we achieve

$$\lambda = 2V, \ \ |v_k| = V \frac{|g_k h_k| u_k}{M_k u_k^2 |h_k|^2 + N_k}. \quad (19)$$

Similarly, by multiplying the term $\frac{|g_k h_k| u_k}{M_k u_k^2 |h_k|^2 + N_k}$ and summing up over $k \in \mathbb{F}_k$ and incorporating (8) we obtain

$$V = \left(\sum_{k=1}^{K} \frac{|g_k h_k|^2 u_k^2}{M_k u_k^2 |h_k|^2 + N_k}\right)^{-1}. \quad (20)$$

At this point by recalling the given power values, $X_k, k \in \mathbb{F}_k$, and incorporating (15), we obtain a direct relation between the allocated power values and the resulting objective as

$$V^{-1} = \sum_{k=1}^{K} J_k(X_k), \ \ J_k(X_k) := \frac{X_k \alpha_k^2}{X_k + \beta_k^2}, \quad (21)$$

where $\alpha_k := \sqrt{\frac{|g_k|^2}{M_k}}$ and $\beta_k := \sqrt{\frac{N_k(R|g_k|^2 + M_k)}{M_k |h_k|^2}}$. Note that each $J_k(X_k)$ can be interpreted as the the independent contribution of each SN to the objective value (V^{-1}). Due to the fact that (21) holds for any feasible power allocation scheme, our original problem (9) is now reduced to finding the optimal power assignment for each node:

$$\max_{X_k,\, k\in\mathbb{F}_K} \sum_{k=1}^{K} J_k(X_k), \ \text{ s.t. } \sum_{k=1}^{K} X_k \leq P_{\mathrm{tot}}, \ \ 0 \leq X_k \leq P_k. \quad (22)$$

In the following we provide some insights into the problem (22) which will be later used for a more efficient design.

Lemma 2: The objective in (22) is increasing and jointly concave with respect to $X_k,\ k \in \mathbb{F}_k$.

Proof: Since the contribution of the each SN's power on the objective is independent of the power of other nodes (i.e., $\frac{\partial J_k(X_k)}{\partial X_l} = 0,\ l \neq k$) we have:

$$\hat{J}_k\left(X_k\right) := \frac{\partial \left(V^{-1}\right)}{\partial X_k} = \frac{\partial J_k(X_k)}{\partial X_k} = \frac{\alpha_k^2 \beta_k^2}{\left(X_k + \beta_k^2\right)^2} > 0, \quad (23)$$

and

$$\frac{\partial^2 \left(V^{-1}\right)}{\partial {X_k}^2} = \frac{\partial \hat{J}_k}{\partial X_k} = \frac{-2\alpha_k^2 \beta_k^2}{\left(X_k + \beta_k^2\right)^3} < 0, \quad (24)$$

$$\frac{\partial^2 \left(V^{-1}\right)}{\partial X_k \partial X_l} = 0 \quad k \neq l, \quad (25)$$

where (23) explains the increasing nature of the objective with respect to each X_k. Furthermore, equations (24) and (25) show that the Hessian matrix of the objective (22) with respect to allocated power values $(X_1, \cdots, X_K)$ is a diagonal matrix with non-positive elements. This is the certificate for jointly concave nature of V^{-1} with respect to the $X_k,\ \forall k \in \mathbb{F}_K$. ■

Lemma 3: For an optimal power allocation $(X_1, \cdots, X_K)$, the values of $\hat{J}_k(X_k),\ k \in \mathbb{K}$ are equivalent for all active nodes. We call this value as water level and denote it as $\mathcal{L}$: $\hat{J}_k(X_k) = \mathcal{L},\ \forall k \in \mathbb{K}$.

Proof: Active status is defined with the power range $0 < X_k < P_k$ for the corresponding node. If there exists a pair of active nodes where $\hat{J}_k(X_k) < \hat{J}_l(X_l),\ k, l \in \mathbb{K}$, then we can enhance the objective while preserving the feasibility. This can be done by decreasing an arbitrarily small power from the node with smaller slope in the objective (node k) and equally increase the power at the other one (node l), so that the total power consumption is remained constant. Hence unequal values of $\hat{J}_k(X_k),\ k \in \mathbb{K}$ may never happen in an optimum point. ■

Lemma 4: In the optimality, node with index k is inactive if $\hat{J}_k\left(0\right) \leq \mathcal{L}$.

Proof: Proof is achieved via contradiction. If a node with $\hat{J}_k(0) \leq \mathcal{L}$ is active in the optimality, due to Lemma 3 there will be a power value, $X_k > 0$, which satisfies $\hat{J}_k(X_k) = \mathcal{L}$. On the other hand, due to Lemma 1, J_k is concave over X_k. As the result we have $\hat{J}_k(X_k) < \hat{J}_k(0)$ which together with the initial assumption, $\hat{J}_k(0) \leq \mathcal{L}$, leads to the following contradicting statement:

$$\mathcal{L} = \hat{J}_k(X_k) < \hat{J}_k(0) \leq \mathcal{L}. \tag{26}$$

This concludes that the aforementioned node can not be active. On the other hand, if the node with $\hat{J}_k(0) \leq \mathcal{L}$ is saturated ($X_k = P_k$) in the optimality, due to Lemma 2 we have:

$$\hat{J}_k(P_k) < \hat{J}_k(0) \leq \mathcal{L}, \tag{27}$$

which shows that the slope of the objective with respect to the power of the discussed node is smaller than all active nodes. Hence we can reduce power from this node and add to the currently active nodes and enhance the objective value. Together with the last argument, this concludes the inactive status of the node where $\hat{J}_k(0) \leq \mathcal{L}$. ■

Lemma 5: In the optimality, node with index k is saturated if $\hat{J}_k(P_k) \geq \mathcal{L}$.

Proof: The proof is similar to Lemma 4, with reversed arguments and inequality signs. ■

In the following we define an iterative search procedure, by choosing $\mathcal{L}$ as our search parameter applying the results of the above Lemmas. The significance of the Lemmas 4, 5 lies in the fact that they define clear boarders on how variation of water-level ($\mathcal{L}$) affects the segmentation of the nodes into different status (inactive, active, saturated). In the other words, we observe that variation of $\mathcal{L}$ only affects the segmentation of the nodes if it passes the boarders defined in Lemmas 4, 5 ($\hat{J}_k(0), \hat{J}_k(P_k), \ \forall k \in \mathbb{F}_K$). Hence by sorting all $2K$ values of $\hat{J}_k(0), \hat{J}_k(P_k), \ \forall k \in \mathbb{F}_K$ into a single monotonic sequence (namely, the sequence $\boldsymbol{B}$ as: $B_1 \leq B_2 \cdots \leq B_{2k}$), we have $2K+1$ incremental regions to examine for the valid $\mathcal{L}$. This incremental search can be efficiently done via a bi-section search. As it has been explained, for each step of this examination if the $\mathcal{L}$ is located on a node's inactive region, i.e., $\hat{J}_k(0) \leq \mathcal{L}$, we have $X_k = 0$ and we have $X_k = P_k$ in case of saturation ($\hat{J}_k(P_k) \geq \mathcal{L}$). For the nodes with active status we have

$$\mathcal{L} = \hat{J}_k(X_k) = \frac{\alpha_k^2 \beta_k^2}{\left(X_k + \beta_k^2\right)^2}$$
$$\Rightarrow X_k = \frac{\alpha_k \beta_k}{\sqrt{\mathcal{L}}} - \beta_k^2, \tag{28}$$

which by taking a sum over all active nodes results in

$$\mathcal{L} = \left(\frac{\sum_{k \in \mathbb{K}} \alpha_k \beta_k}{P_{\text{tot}} - \sum_{k \in \mathbb{K}_{\text{sat}}} P_k + \sum_{k \in \mathbb{K}} \beta_k^2} \right)^2. \tag{29}$$

The above equality provides us with a unique $\mathcal{L}$, and consequently from (30) the power allocation values X_k in each step. In each step of our bi-section search, if the assumed region in the sequence $\boldsymbol{B}$ (and consequently the separation of the nodes into active, inactive, and saturated status) is correct, the resulted $\mathcal{L}$ will also point into the same region. If this is not the case, whether the resulted water-level is pointing to the higher or lower region determines the direction of update for the next bi-section search step. The detailed description of this process is presented in Algorithm 1 which provides us with an optimal power allocation strategy. The optimal set of $u_k, v_k, \ k \in \mathbb{K}$ is accordingly achieved via (15), (19) and (20) with a similar formulation as in [1].

Algorithm 1 Efficient classification process for SNs via proposed sorting mechanism.

$B_1 \leq \cdots \leq B_{2K} \leftarrow \text{sort}\left\{\hat{J}_k(0), \hat{J}_k(P_k), \ \forall k \in \mathbb{F}_K\right\}$ ▷ see (23)
$i_{\min} \leftarrow 1, \ \ i_{\max} \leftarrow 2K+1$
repeat
  $i \leftarrow \lfloor \frac{i_{\min}+i_{\max}}{2} \rfloor$
  $\mathbb{K}_0 \leftarrow \left\{k \in \mathbb{F}_K | B_i \geq \hat{J}_k(0)\right\}$ ▷ see Lemma 4
  $\mathbb{K}_{\text{sat}} \leftarrow \left\{k \in \mathbb{F}_K | B_{i+1} \leq \hat{J}_k(P_k)\right\}$ ▷ see Lemma 5
  $\mathbb{K} \leftarrow \mathbb{F}_K \setminus (\mathbb{K}_{\text{sat}} \cup \mathbb{K}_0)$
  $P_{\text{remain}} \leftarrow P_{\text{tot}} - \sum_{k \in \mathbb{K}_{\text{sat}}} P_k$
  if $\mathbb{K} = \emptyset$ **then**
    if $P_{\text{remain}} = 0$ **then**
      break
    else if $P_{\text{remain}} < 0$ **then**
      $i_{\min} \leftarrow i$
    else if $P_{\text{remain}} > 0$ **then**
      $i_{\max} \leftarrow i$
    end if
  end if
  $\mathcal{L} \leftarrow \left(\frac{\sum_{k \in \mathbb{K}} \alpha_k \beta_k}{P_{\text{tot}} - \sum_{k \in \mathbb{K}_{\text{sat}}} P_k + \sum_{k \in \mathbb{K}} \beta_k^2} \right)^2$ ▷ see (29)
  if $\mathcal{L} > B_{i+1}$ **or** $P_{\text{tot}} < \sum_{k \in \mathbb{K}_{\text{sat}}} P_k$ **then**
    $i_{\min} \leftarrow i$
  else
    $i_{\max} \leftarrow i$
  end if
until ($\mathcal{L} > B_i$ **and** $\mathcal{L} < B_{i+1}$)
$X_k \leftarrow \frac{\alpha_k \beta_k}{\sqrt{\mathcal{L}}} - \beta_k^2, \ k \in \mathbb{K}$ ▷ see (28)
return ($\mathbb{K}_{\text{sat}}, \mathbb{K}_0, \ X_k, k \in \mathbb{K}$)

V. Complexity Analysis and Comparison

In this part we compare the solution offered in [1] with the proposed Algorithm 1 in terms of the computational complexity and convergence speed. While the method in [1] also achieves an optimal solution in closed form, the resulted solution has the similar structure to the famous water-filling [7] solution for power allocation. Hence it requires an iterative examination of the nodes in order to obtain their optimal status (see Section III). The convergence speed of this process suffers as the number of SNs increase. Furthermore, due to the absence of the achieved boarders in Section IV, Lemma 4,5, the corresponding convergence speed depends on the accuracy of the decisions in each iteration. In its extreme case, this leads to a significantly slower convergence if the nodes active region (i.e., $[\hat{J}_k(0), \hat{J}_k(P_k)], k \in \mathbb{F}_K$) are not overlapping and hence the corresponding decision in each iteration is valid for only one node. In such situation, the effectiveness of the proposed sorting mechanism in Algorithm 1 becomes more clear since it facilitates a bi-section search with a guaranteed

TABLE II. ANALYTIC COMPLEXITY COMPARISON FOR TWO ALGORITHMS.

K	Algorithm 1	Proc. in [1]
Add/Subtract	$\xi_1 (3K+2) + 3K + 1$	$\xi_2 (3K+1) + K$
Multiplication	$\xi_1 + 8K + 1$	$\xi_2 K + 6K$
Division	$2\xi_1 + 5K$	$\xi_2 + 2K$
Square-root	$K + 1$	K
Compare	$\xi_1 (3K+7) + K \log_2 K$	$\xi_2 (10K+4)$

convergence speed (upper-bounded by $\log_2(K)+1$ iterations). In the following we study the computational complexity gains achieved via Algorithm 1. As the first step we study the required examination iterations to achieve correct classification of the nodes, denoted as ξ_1 for Algorithm 1 and as ξ_2 for the process in [1] (see figure 1). For the proposed method in [1], this is done both by averaging ξ_2 for 1000 channel realizations as a Monte Carlo simulation ([1]-Average), and as the achieved ξ_2 for the most computationally demanding choice of channel coefficients and power constraints which leads to the lengthiest search period ([1]-WorstCase). The same simulation approach is also applied to the Algorithm 1 (Alg.1-Average, Alg.1-WorstCase, respectively). Unless otherwise is stated, we use $R = 1,\ N_k = 1,\ M_k = 1$ and assume $P_k = 1W, k \in \mathbb{F}_K$ as the values for our system parameters. We also define the power ratio $\rho := \frac{K}{P_{\text{tot}}}$ which is an indication of how tight the network operation is bounded by the SNs collective power consumption (P_{tot}). As we observe from Fig.1, while the required iterations is rather constant in different scenarios for Algorithm 1, the different system parameters lead to significantly lengthier and variable search iterations for the process in [1]. A deeper view of the required computational load for the two algorithms is presented in Table II. The comparison is made with respect to the required processor instructions (addition/subtraction, multiplication, division, comparison). Note that the values of ξ_1, ξ_2 are largely different ($\xi_1 \lll \xi_2$) and dependent on the system parameters (see Fig.1). As another comparison, the required CPU time on a system with Core i5 processor and 8 GB of RAM is also reported for the corresponding algorithm implementations. Please see [8] for the detailed implementation of two algorithms. The benefit of the proposed sorting mechanism in providing a reduced complexity and robust process (with respect to different system parameters) is observed from both simulations in Fig.1 and Fig.2.

VI. Conclusion

For achieving a high performance with a distributed radar system an allocation of each sensor node to its correct operating status is necessary. It is apparent that sensor nodes with a better situation of sensing and communication channels should be allocated with more power compared to others. In order to optimally separate all sensor nodes into the three possible operating modes (inactive, active and saturated) and select the most reliable sensor nodes, we have derived a new algorithm which is based on a simple sorting mechanism of sensor nodes. The new algorithm maps the problem under consideration onto an incremental and iterative search over defined quality regions. This results in the fast convergence of the separation process and significantly reduces the required computational complexity.

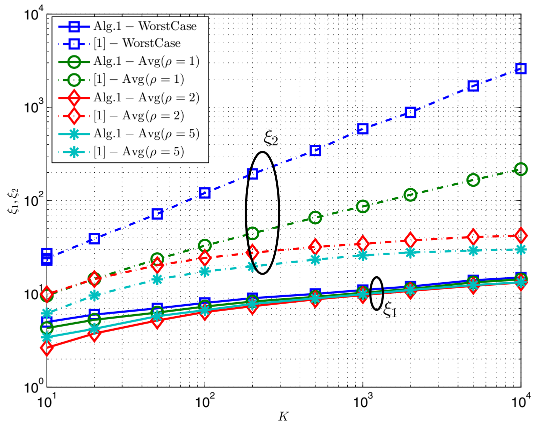

Fig. 1. Required search iterations for optimal node separation (ξ_1, ξ_2) vs. number of SNs (K). A higher convergence speed is observable for the procedure in Algorithm 1.

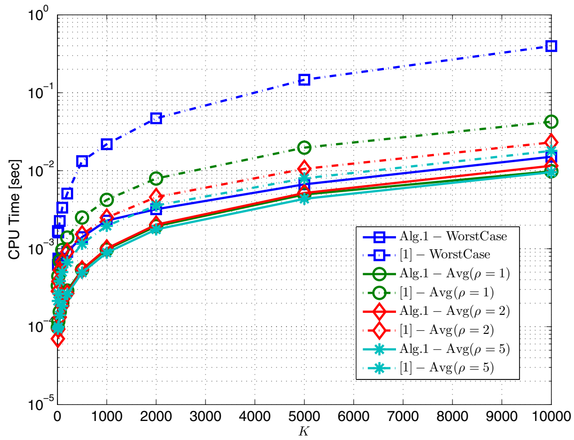

Fig. 2. Required CPU time [sec] vs. number of SNs (K). CPU time is reduced as a result of smaller required iterations in Algorithm 1.

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