User-Relay Duality in Cyclic Interference Alignment for Cascaded Two-Way Relay Networks

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***Abstract*—We investigate a cascaded two-way relay network with K users and K relays alternately arranged in terms of a conceptual channel model based on polynomial rings. This channel model is inspired by cyclic codes and can be applied to time interference alignment by propagation delay for instance. A multiple-unicast problem for both the open-loop case, i. e., a straight line network of alternately arranged users and relays, and the corresponding closed-loop topology is considered. We provide an upper bound on the degrees of freedom which is achieved by means of signal alignment and interference alignment. Moreover, we observe a dual relationship for the closed-loop network if the roles of users and relays are swapped.**

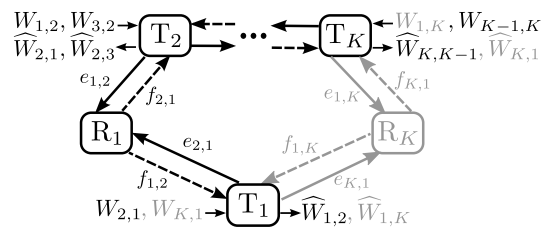

Fig. 1. A cascaded two-way relay network: K users and K relays are alternately arranged. User T_i, is only connected to its predecessor T_{i-1} and its successor T_{i+1} via two-way relays R_{i-1} and R_i, respectively, for each $i = 1, \dots, K$. The uplink channel $\boldsymbol{E}$ is described by solid lines and the downlink channel $\boldsymbol{F}$ by dashed lines. The open-loop case neglects Relay R_K.

I. Introduction

It is a well-known problem that the exact capacity analysis of many multi-user networks is utterly challenging. As multiple independent signals from different transmitters share common resources in a wireless channel, the emerging interference must be taken into account and should be avoided by a proper signalling strategy. Interference Alignment (IA) [1] focuses on designing transmission signals such that multiple interference signals overlap in a minimal signal space at undesired receivers while dedicated signals remain interference-free at the desired receivers. A particular benefit of IA is the linear scaling property of the degrees of freedom (DoF). In the K-user interference channel [1], the DoF achieved by IA scales with $\frac{K}{2}$. In contrast, the DoF achieved by a multiple-access scheme scales reciprocally with K, leading to $\frac{1}{K}$ DoF per user.

In the particular setup of two-way relay communication systems with full-duplex users and full-duplex intermediate relays, all devices concurrently exchange messages with each other in a bidirectional manner. The elementary network problem of the well-known two-way relay channel [2] considers a pair of two users communicating via one relay, that is used to forward the bidirectional messages. Achievable rate regions for cooperation schemes are provided in [3], [4] for two-way decode-and-forward or two-way compress-and-forward, respectively. An approximate capacity analysis of the two-way relay channel within 3 bits, involving the conceptual linear shift deterministic channel model (LDCM) [5], is provided in [6]. Generalizations to $K \geq 2$ user-pairs communicating over a single relay are considered in [7], [8]. A multi-hop scheme for a single user-pair on a line of K two-way relays is presented in [9]. The given scheme avoids the regeneration of back-propagated self-interference at the relays. Another generalization of two-way relaying concerns multiple-unicast transmissions per user. For the case $K = 3$, this is called a Y- channel [10]–[12]. In that case, each of the 3 users intends to transmit a single message to the other two users over a single two-way relay. The communication schemes achieving the upper bounds on the DoF involve Signal Alignment (SA) and the cancellation of back-propagated self-interference.

The cyclic polynomial channel model (CPCM) introduced in [13] is a conceptual model describing the impact of interference in multi-user networks. It is inspired by the algebraically simple polynomial representation of cyclic codes similar to [14]. Furthermore, the time-unrolled CPCM can be applied to the periodic time interference alignment by propagation delay in line-of-sight channels [15]. Based on the CPCM, elementary Cyclic IA schemes on the 2-user X- channel and the K- user interference channel were introduced in [13]. Closely related separability/feasibility conditions are also observed in a specific MIMO OFDM 3-user interference channel with two orthogonal subcarriers in [16] and in the pre-coding based network alignment scheme of a finite field X- channel as considered in [17]. A polynomial decomposition of IA in the time-frequency domain for OFDM systems is described in [18] guiding to practical systems.

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Similar to the LDCM, the results of the CPCM mainly intend to provide a conceptual view on deriving DoF-optimal achievable IA schemes. A particular benefit of the CPCM concerns the algebraic accessibility, even for K users and asymmetric and non-reciprocal channel gains.

Contributions. In this work, we investigate K cascading concatenated two-way relay channels w. r. t. the CPCM depicted in Fig. 1. This is a particular generalization of a two-way relay network that has not been considered in the literature, to the best of our knowledge. We consider two closely related systems with a number of K users and K or $K-1$ relays forming either an open-loop line network or a closed-loop ring network, respectively. The transmissions of each user are involved in two neighbouring two-way relay channels. We provide an upper bound and an achievable scheme that is based on both SA and IA and the cancellation of back-propagated self-interference. Furthermore, we observe for the closed-loop case that switching the roles of users and relays yields a dual network with reciprocal channels.

Organization. The CPCM of the considered channel is provided in Section II. Upper bounds on the DoF are derived in Section III. The Cyclic IA schemes for the open-loop and the closed-loop cases are presented in Sections IV-A and IV-B, respectively. In Section V, we discuss the dual case of the closed-loop network. We conclude in Section VI.

Notation. Italic letters represent scalars, e. g., a, b, and bold letters are vectors/matrices $\boldsymbol{a}, \boldsymbol{A}$. Sets are denoted by calligraphic capital letters $\mathcal{A}$. The transposed vector of $\boldsymbol{a}$ is denoted by $\boldsymbol{a}^\mathsf{T}$. $\mathbf{1}_q$ describes a $q \times q$ matrix of ones and $\boldsymbol{I}_q$ a $q \times q$ identity matrix. A circulant $k \times k$ matrix is denoted by $\boldsymbol{Z}_k = (z_{j,i})_{0 \le i,j \le k-1}$ and has circulant entries $z_{j,i} = z_{i-j \,(\mathrm{mod}\, k)}$. A polynomial in an indeterminate x is denoted by $p(x) = \sum_{k=0}^{n-1} p^{[k]} x^k$ with coefficients $p^{[k]}$. We define the element-wise product of two polynomials $p(x)$ and $q(x)$, with maximal degree $n-1$, by:

$$p(x) \circ q(x) = \sum_{k=0}^{n-1} p^{[k]} q^{[k]} x^k.$$

II. System Model

The sets of user-indices $\mathcal{K}_\mathrm{T}$ and relay-indices $\mathcal{K}_\mathrm{R}$ are both defined by $\mathcal{K}_\mathrm{T} = \mathcal{K}_\mathrm{R} = \mathcal{K} := \{1, 2, \dots, K\}$. We consider a full-duplex two-way relay network with K users T_i, $i \in \mathcal{K}_\mathrm{T}$, and K relays R_j, $j \in \mathcal{K}_\mathrm{R}$, $K \ge 3$, as depicted in Figure 1.

There are $M = 2K$ independent messages W_{ji}. Each user T_i, $i \in \mathcal{K}$, intends to convey one message $W_{i-1,i}$ to its predecessor T_{i-1} over relay R_{i-1} and another message $W_{i+1,i}$ to its successor T_{i+1} over relay R_i. For notational simplicity, indices $K+i$ correspond to i for a closed-loop of K users in a circular indexation. The messages $\widehat{W}_{ji}$ to be decoded at each T_j are denoted with a hat. To compactly describe the number of messages $m_{j,i} \in \mathbb{N}$ from T_i to T_j for each communication involved, we define a messaging matrix $\boldsymbol{M} = (m_{j,i})_{j,i \in \mathcal{K}}$ and set its entries to $m_{i,i+1} = m_{i,i-1} = 1$ and $m_{j,i} = 0$, else. The total number of messages is $M = \|\boldsymbol{M}\|_1 = 2K$.

Now we take a closer look at the signalling and the communication channel itself. In the CPCM [13] we consider polynomial rings $F(x)$ modulo $x^n - 1$ with the indeterminate x and $n \in \mathbb{N}$. The channel access at each T_i and R_j is partitioned into n equally sized dimensions, each normalized to length one. A single dimension in the period of n dimensions is addressed by one of the *offsets* $x^0, x^1, \dots, x^{n-1}$. A transmitter T_i can allocate coded messages to each coefficient of a polynomial in x. A message is a binary string $W_{ji} \in \mathbb{B}^t = \{0,1\}^t$ with $t \in \mathbb{N}$ symbols. The transmitted signal from T_i is a polynomial with messages $W_{ji} \in \mathbb{B}^t$ for each intended receiver T_{i-1} and T_{i+1}:

$$u_i(x) \equiv W_{i-1,i} x^{p_{i-1,i}} + W_{i+1,i} x^{p_{i+1,i}} \mod (x^n - 1). \quad (1)$$

The parameters $p_{j,i} \in \{0, \dots, n-1\}$ allocate the message W_{ji} to a particular offset within n dimensions.

The impact of the channel is modelled by a cyclic shift, i. e., a transmitted codeword $u_i(x)$, is multiplied by x^k for some $k \in \mathbb{Z}^+$ which is imposed by the channel[1], and then reduced modulo $x^n - 1$. This simple algebraic operation is well-known and often used in cyclic codes as in [14].

In our case of the cascaded two-way network, the uplink (UL) channel matrix describes all subchannels from the users T_i to the relays R_i. It is defined by $\boldsymbol{E} = (e_{j,i})_{j,i \in \mathcal{K}}$ and has the elements $e_{j,i} \in \mathcal{D}$ with the set of monomials $\mathcal{D} = \{x^k \mid k \in \mathbb{Z}^+\}$ describing the individual shifts from T_i to R_j. In $\boldsymbol{E}$, most elements are zero, except $e_{i,i+1}$ and $e_{i+1,i}$ for all $i \in \mathcal{K}$. The downlink (DL) channel matrix $\boldsymbol{F} = (f_{j,i})_{j,i \in \mathcal{K}}$ for subchannels from R_i to T_j is defined accordingly for $f_{j,i} \in \mathcal{D}$ with non-zero $f_{i,i+1}$ and $f_{i+1,i}$ for all $i \in \mathcal{K}$. All channel gains are globally known. In Fig. 1, the UL subchannels are depicted by solid lines and the DL subchannels by dashed lines. The channel transfer function of the UL is the congruence:

$$\boldsymbol{r}^\mathsf{T} \equiv \boldsymbol{E} \boldsymbol{u}^\mathsf{T} \mod (x^n - 1), \quad (2)$$

with the $1 \times K$ input vector $\boldsymbol{u}$ and the $1 \times K$ output vector $\boldsymbol{r}$:

$$\boldsymbol{u} = (u_1(x), \dots, u_K(x)), \ \boldsymbol{r} = (r_1(x), \dots, r_K(x)). \quad (3)$$

The modulo operation is taken element-wise. The signals $r_i(x)$ received at R_i are further processed as follows. The *filter polynomial* $z_i(x)$ has well-chosen coefficients $z_i^{[k]} \in \{0,1\}$ for all $k \in \{0, 1, .., n-1\}$. The entries in $z_i(x)$ are chosen such that undesired interference terms in $r_i(x)$ at relay R_i are removed (by multiplying zero) in the element-wise product $r_i(x) \circ z_i(x)$ whereas dedicated signals in $r_i(x)$ remain unchanged (by multiplying one). The filtered polynomial may furthermore be cyclically shifted by x^{γ_i} with offset $\gamma_i \in \mathbb{N}$. The resulting polynomial after filtering and shifting is forwarded by R_i and denoted:

$$v_i(x) \equiv x^{\gamma_i} (r_i(x) \circ z_i(x)) \mod (x^n - 1). \quad (4)$$

The channel transfer function of the DL is the congruence:

$$\boldsymbol{t}^\mathsf{T} \equiv \boldsymbol{F} \boldsymbol{v}^\mathsf{T} \mod (x^n - 1), \quad (5)$$

for the $1 \times K$ input vector $\boldsymbol{v}$ and $1 \times K$ output vector $\boldsymbol{t}$:

$$\boldsymbol{v} = (v_1(x), \dots, v_K(x)), \ \boldsymbol{t} = (t_1(x), \dots, t_K(x)). \quad (6)$$

[1]For a time-unrolled shift, this can be interpreted as a discrete delay.

For the rest of the paper, we neglect the element-wise modulo $x^n - 1$ term in congruences for brevity.

To ensure interference-free decodability of all dedicated signals, we consider the following *separability conditions* characterizing the particular type of interference. The *separability conditions* for $i \in \mathcal{K}$ as defined in [13] are adapted to this particular communication problem as follows:

- *Multiple-access interference conditions*: Dedicated messages to T_i transmitted by different sources T_{i-1} and T_{i+1} must be separable to each other at destination T_i:

$$f_{i,i-1} x^{\gamma_{i-1}} e_{i,i-1} x^{p_{i,i-1}} \not\equiv f_{i,i+1} x^{\gamma_i} e_{i,i+1} x^{p_{i,i+1}}. \qquad (7)$$

- *Intra-user interference conditions*: Messages from the same source T_i, but dedicated for different destinations must be distinct:

$$x^{p_{i+1,i}} \not\equiv x^{p_{i-1,i}}. \qquad (8)$$

- *Inter-user interference conditions*: The dedicated messages from T_{i-1} to T_{i-2} and from T_{i+1} to T_{i+2} may not interfere with any of the dedicated signals for T_i:

$$f_{i,i-1} x^{\gamma_{i-1}} e_{i,i-1} x^{p_{i-2,i-1}} \not\equiv f_{i,i+1} x^{\gamma_i} e_{i,i+1} x^{p_{i,i+1}}, \qquad (9)$$
$$f_{i,i+1} x^{\gamma_i} e_{i,i+1} x^{p_{i+2,i+1}} \not\equiv f_{i,i-1} x^{\gamma_{i-1}} e_{i,i-1} x^{p_{i,i-1}}. \qquad (10)$$

III. Upper Bounds

To evaluate the achieved data-rate, we use the DoF measure, defined by the quotient of the number of interference-free messages M conveyed within n dimensions [13]:

$$\mathrm{DoF} = \frac{M}{n}. \qquad (11)$$

This measure is closely related to the DoF described in [1], [19] that approximate the multi-user capacity in Gaussian channels in the high SNR regime within $o(\mathrm{SNR})$.

For a fixed messaging matrix $\boldsymbol{M}$ the minimal number of dimensions necessary [13] is bounded by:

$$n \geq \max_{m_{ji}} \left(\sum_{i=1}^{K} m_{ji} + \sum_{j=1}^{K} m_{ji} - m_{ji} \right). \qquad (12)$$

In other words, the bound on n is determined by the row j and column i of messaging matrix $\boldsymbol{M}$ that maximizes the sum in (12). For $K \geq 3$, the messaging matrix $\boldsymbol{M}$ will always provide $n = 2$ for each $i, j \in \mathcal{K}$ and hence:

$$\mathrm{DoF} \leq \frac{2K}{2} = K. \qquad (13)$$

Note that the case $K = 2$ corresponds to the elementary two-way relay channel which is also covered by this bound, since for $\boldsymbol{M} = \mathbf{1}_2 - \boldsymbol{I}_2$, (12) provides $n = 1$ and hence the upper bound (11) yields $\mathrm{DoF} \leq \frac{M}{n} = \frac{2}{1} = 2$.

IV. Achievability

To achieve the upper bound (13) provided above, we propose a forwarding scheme including both Cyclic SA and IA.

UL-phase: We first consider the alignment schemes used in the UL at the receiving relays R_i for all $i \in \mathcal{K}$. The two

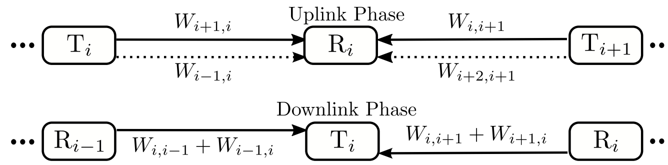

Fig. 2. Uplink phase: Signal alignment (solid arrows) of dedicated signals and interference alignment (dashed arrows) is performed at each relay R_i. Downlink phase: Superimposed dedicated messages at T_i are allocated by multiple-access and the self-interference is successively cancelled at T_i.

dedicated signals to be bidirectionally exchanged between T_i, T_{i+1} are aligned at R_i by SA:

$$e_{i+1,i} x^{p_{i+1,i}} \equiv e_{i,i+1} x^{p_{i,i+1}}. \qquad (14)$$

The dedicated signals to be forwarded to T_i and T_{i+1} are superimposed at R_i and hence yield network-coded messages, i. e., $W_{i+1,i} + W_{i,i+1}$. Contrarily, the interfering messages at R_i, not dedicated for T_i or T_{i+1} are aligned likewise by IA:

$$e_{i+1,i} x^{p_{i-1,i}} \equiv e_{i,i+1} x^{p_{i+2,i+1}}. \qquad (15)$$

DL-phase: Now, we consider the alignment scheme used in the DL at the receiving users T_i for all $i \in \mathcal{K}$. The interference aligned in (15) is nulled by a filter polynomial as in (4):

$$z_i(x) = e_{i,i+1} x^{p_{i,i+1}} \equiv e_{i+1,i} x^{p_{i+1,i}}. \qquad (16)$$

As a result, no inter-user interference is forwarded by R_i so that the inter-user interference conditions (9), (10) always hold, if this alignment is feasible. The feasibility of this alignment will be proven in Theorem 1 and Theorem 2 for the open and the closed-loop cases, respectively. The remaining superimposed dedicated signals between T_i and T_{i+1} are forwarded by R_i to both users. This is a multicast problem: Each relay demands to convey a single albeit network-coded message to two neighbouring users, simultaneously. Note that forwarded signals received at each user must yet satisfy the multiple-access conditions in (7) to be decodable.

T_i also receives back-propagated self-interference from R_i and also from R_{i-1} due to the network-coded signals. We may apply self-interference cancellation at each T_i to remove the corresponding self-interference $W_{i-1,i}$ and $W_{i+1,i}$ that is known a priori from the previous UL transmission. Then, only the dedicated messages $W_{i,i-1}$ and $W_{i,i+1}$ remain at each T_i and are decoded interference-free. The described scheme is illustrated in Fig. 2.

A. Case 1 - Open-Loop Line Network

We first discuss the open-loop case, discarding R_K w.l.o.g., so that the links between T_1 and T_K are severed and $p_{K,1}$, $p_{1,K}$ omitted. The grey links in Fig. 1 are discarded. We use the same $\boldsymbol{M}$ as provided in the system model, except that $m_{1,K} = m_{K,1} = 0$. Then, the total number of messages is only $M = \|\boldsymbol{M}\|_1 = 2K - 2$. Despite the slightly changed $\boldsymbol{M}$, the minimal number of dimensions necessary is still bounded by $n \geq 2$. The upper bound of (11) reduces to $\frac{2K-2}{2} = K - 1$ DoF.

Theorem 1. *A combined Cyclic IA and SA scheme achieves $K-1$ DoF on the given line network of $K \geq 3$ users and $K-1$ relays with a number of $n = 2$ dimensions.*

Proof:

(a) *Necessity of $n \geq 2$*:
The conditions in (7) demand that two dedicated signals are decodable in the DL at each T_i, $i \in \mathcal{K}$. These two signals must be received in separate dimensions due to the multiple-access interference condition, so that $n \geq 2$ dimensions are necessary. Note that the edge-users T_1 and T_K are an exception demanding only $n \geq 1$.

(b) *Sufficiency of Cyclic IA*:
UL-phase: We may fix the UL transmission parameter $p_{2,1}$ at T_1 w.l.o.g. At T_2, we use the given SA in (14), to obtain the parameters $p_{1,2}$. As T_2 also has a dedicated message intended for T_3, we fix the parameter $p_{3,2}$ at T_2, satisfying the intra-user interference conditions (8). For $n = 2$ dimensions, parameter $p_{3,2}$ occupies the complementary offset to $p_{1,2}$. At T_3, we use the given SA in (14) and the IA condition (15) to compute the parameters $p_{2,3}$ and $p_{4,3}$ respectively. Both satisfy the intra-user interference conditions (8).

The allocation of parameters $p_{i-1,i}$ and $p_{i+1,i}$ is continued analogously for each subsequent user $i = 3, \ldots, K-1$. At T_K, only the signal alignment in (14) is needed to determine the parameters $p_{K-1,K}$, as $p_{1,K} = 0$ produces no interference at T_K. There are no constraints on the channel matrix $\boldsymbol{E}$ of the UL and each parameter can be determined uniquely.

DL-phase: The coefficients of the filter polynomial $z_i(x)$ applied at each R_i as given in (16), are non-zero for the offsets of the dedicated signals. As a result, the complementary offset $e_{i+1,i}x^{p_{i-1,i}} \equiv e_{i,i+1}x^{p_{i+2,i+1}}$ containing the aligned interference $W_{i-1,i}+W_{i+2,i+1}$ in $r_1(x)$ is removed by the zero-coefficients of $z_1(x)$. The filtered signal $r_1(x) \circ z_1(x)$ only contains the superimposed dedicated signals at R_i. Due to the multiple-access interference conditions in (7), the forwarded signal from R_1 to T_2 may not align to the forwarded signal from R_2 to T_2, i.e., $x^{\gamma_1}f_{2,1} \not\equiv x^{\gamma_2}f_{2,3}$ must hold. We may fix the parameter γ_1 for R_1 w.l.o.g., and a unique solution for γ_2 exists for $n = 2$. Analogously, $x^{\gamma_i}f_{i+1,i} \not\equiv x^{\gamma_{i+1}}f_{i+1,i+2}$ must hold for all other γ_i with $i = 2, \ldots, K-2$. Each γ_i can be determined uniquely for $n = 2$. Thus, there are no further constraints on $\boldsymbol{F}$ in the DL either. The back-propagated self-interference is known at each T_i a priori and cancelled from the received signal.

Altogether, each T_i, with $2 \leq i \leq N-1$, sends two dedicated messages, and the edge-users T_1 and T_K send one dedicated message each, and thus a total number of $M = 2K-2$ messages. All messages are conveyed interference-free over $n = 2$ dimensions and yield $\frac{2K-2}{2} = K-1$ DoF. ■

B. Case 2 - Closed-Loop Ring Network

The closed-loop case is depicted in Fig. 1 by including the grey links with an active R_K and the messages between T_1 and T_K exist. Furthermore, UL and DL channels are called reciprocal if:

$$e_{i,j}^{-1} \equiv f_{j,i} \mod (x^n - 1) \tag{17}$$

holds for all $i \neq j \in \mathcal{K}$.

Theorem 2. *A Cyclic SA and IA scheme achieves* DoF $\leq K$ *for $n = 2$ dimensions on the given closed-loop network with K users and K relays, if the following UL and DL conditions for the channel matrices $\boldsymbol{E}$, $\boldsymbol{F}$ hold:*

$$\prod_{i=1}^{K} e_{i+1,i}^{-1}e_{i,i+1} \equiv 1 \mod (x^n - 1), \tag{18}$$

$$\prod_{i=1}^{K} f_{i,i+1}^{-1}f_{i+1,i} \equiv 1 \mod (x^n - 1). \tag{19}$$

Proof:

(a) *Necessity of $n \geq 2$*:
Cf. proof in Theorem 2(a), but without edge-users.

(b) *Necessity of the UL condition* (18):
For a fixed $p_{1,K}$ in the SA condition (14), we obtain:

$$x^{p_{1,K}} \equiv e_{1,K}^{-1}e_{K,1}x^{p_{K,1}}. \tag{20}$$

Then, we substitute the parameter $x^{p_{K,1}}$ by $x^{p_{K,1}} \equiv e_{2,1}^{-1}e_{1,2}x^{p_{3,2}}$ from the IA condition (15) into (20):

$$x^{p_{1,K}} \equiv e_{1,K}^{-1}e_{K,1}e_{2,1}^{-1}e_{1,2}x^{p_{3,2}}.$$

In the next step, $x^{p_{3,2}}$ is substituted by using the SA condition (14). By a simple unrolling of the dependencies with an alternating application of the IA and SA conditions in (14) and (15) for all $1 < i < K$, we obtain an additional product-term of $e_{i,i+1}^{-1}e_{i+1,i}$ for each i. In the last step, $x^{p_{1,K}}$ appears on both sides of the congruence. Clearly, the congruence is only true if the UL condition (18) on $\boldsymbol{E}$ holds.

(c) *Sufficiency of Cyclic SA and IA in the UL*:
The achievable scheme is analogous to Theorem 1 for the allocation of the parameter values $p_{i,i+1}$ and $p_{i+1,i}$ for all $i \in \{1, 2, \ldots, K-1\}$ for $n = 2$. A feasible solution for the two remaining parameters $p_{K,1}$ and $p_{1,K}$, satisfying the separability conditions, only exists if (18) holds.

(d) *Necessity of the DL condition* (19):
For an arbitrary DL matrix $\boldsymbol{F}$, a reciprocal UL matrix $\boldsymbol{E}_{\mathrm{rpr}}$ can be computed easily by (17). If $\boldsymbol{E}_{\mathrm{rpr}}$ satisfies (18), then $\boldsymbol{F}$ will satisfy (19). As already given by parts (a), (b), and (c), a feasible Cyclic SA/IA scheme for the reciprocal UL with $\boldsymbol{E}_{\mathrm{rpr}}$ exists. For the multicast transmission in the reciprocal DL with $\boldsymbol{F}$, the superimposed dedicated signals are send back over exactly the same dimensions over which they have been received in the reciprocal UL with $\boldsymbol{E}_{\mathrm{rpr}}$. As the intra-user interference conditions already hold in the reciprocal UL, the multiple-access conditions hold analogously in the DL by channel reciprocity. And conversely, if (19) is violated, the parameters γ_i can not be determined uniquely for all $i = 1, \ldots, K$.

(e) *Sufficiency of Cyclic SA in the DL*:
The inter-user interference conditions (9), (10) always hold since the inter-user interference is removed at each R_i by using the corresponding filtering polynomial. Now the superimposed dedicated signals are multicast by each R_i and received at

T_i and T_{i+1}. We may fix parameter γ_1. Again, $x^{\gamma_i} f_{i+1,i} \not\equiv x^{\gamma_i} f_{i+1,i+1}$ must hold for all γ_i with $i = 1,\ldots,K$, to ensure that the multiple-access conditions are not violated. This is valid for any $\boldsymbol{F}$ that satisfies (19). In the last step, the back-propagated self-interference is cancelled. Altogether, a total number of $\frac{2K}{2} = K$ DoF is achieved. ■

Note that if the given matrices $\boldsymbol{E}$ and $\boldsymbol{F}$ are reciprocal, conditions (18) and (19) become equivalent.

V. Discussion: User-Relay Duality

A *dual network* of the closed-loop case is physically the same network as the primary network with $\boldsymbol{E}$ and $\boldsymbol{F}$, but with former users operating as relays and former relays operating as users instead. Although the indexation of such a dual network is not unique for the closed-loop case, it is obvious that the resulting dual network is indeed unique, since the equivalent indexations do not change the setup of dedicated messages between neighbouring users. W.l.o.g., we may relabel $\mathrm{R}_i \rightarrow \mathrm{T}_i$ and $\mathrm{T}_{i+1} \rightarrow \mathrm{R}_i$, i. e., the labels are cyclically right-shifted by one position. Let $\tilde{\boldsymbol{Z}}_K$ denote a particular circulant matrix with $\tilde{z}_{K-1} = 1$, and all other $\tilde{z}_j = 0$. The UL and DL matrices of the dual network are cyclically rotated versions $\boldsymbol{E}$ and $\boldsymbol{F}$ from the primary network:

$$\boldsymbol{E}_{\mathrm{dual}} = \tilde{\boldsymbol{Z}}_K \boldsymbol{F}, \tag{21}$$

$$\boldsymbol{F}_{\mathrm{dual}} = \boldsymbol{E} \tilde{\boldsymbol{Z}}_K^{\mathsf{T}}. \tag{22}$$

The corresponding dual UL and DL conditions yield:

$$\prod_{i=1}^{K} f_{i,i+1} f_{i+1,i}^{-1} \equiv 1 \bmod (x^n - 1), \tag{23}$$

$$\prod_{i=1}^{K} e_{i+1,i} e_{i,i+1}^{-1} \equiv 1 \bmod (x^n - 1). \tag{24}$$

Hence, if $\boldsymbol{F}$ satisfies the primary DL condition, then $\boldsymbol{E}_{\mathrm{dual}}$ satisfies the dual UL condition, and analogously if $\boldsymbol{E}$ satisfies the primary UL condition, then $\boldsymbol{F}_{\mathrm{dual}}$ satisfies the dual DL condition. Any cyclically (right-)shifted relabelling by an odd number of positions will yield the same dual conditions as given in (23) and (24). As a result, we observe the following:

User-Relay Duality: *If the Cyclic SA and IA scheme is feasible on the primary network, it will also be feasible on the dual network.*

VI. Conclusion

In the present work, we propose a combined Cyclic Interference Alignment and Signal Alignment scheme to show that a total number of $K-1$ DoF is achievable for the open-loop cascaded two-way relay network and a number of K DoF is achievable for a closed-loop cascaded network. The communication strategy comprises an uplink phase where users transmit signals to the relays, and a downlink phase where relays transmit signals to the users. In the uplink phase, the basic idea is to align bidirectional dedicated signals at each relay into a dedicated signal space. On the other hand, the interference is similarly aligned at each relay into an interference signal space that is complementary to the dedicated signal space. In the downlink phase, the interference signals are omitted and the superimposed dedicated signals are multicast from the relays to the users such that the two dedicated signals from each neighbouring relay is received separately and hence decodable. The back-propagated self-interference of the network-coded dedicated signals is known a priori and cancelled.

Moreover, we consider the dual network, i. e., the roles of users and relays are swapped. We observe that, a user-relay duality holds: If the primary network is feasible for the alignment scheme, then the dual network is also feasible.

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