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Exercise 1
- Proposed Solution -

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Solution of Problem 1

a) Straight-forward from (b) by setting k to 1.

b) *(Taken directly from the lecture's script)* Given two matrices $\mathbf{M} \in \mathbb{R}^{n\times m}$ and $\mathbf{N} \in \mathbb{R}^{m\times r}$, the goal is to compute $\mathbf{MN}$. Map function generates the following key-value pairs:

- For each element m_{ij} of $\mathbf{M}$ produce r key-value pairs $((i,k),(j,m_{ij}))$ for $k = 1,\dots,r$.
- For each element n_{jk} of $\mathbf{N}$ produce n key-value pairs $((i,k),(j,n_{jk}))$ for $i = 1,\dots,n$.

The Reduce function computes the multiplication as follows:

- For each key (i,k), find the values with the same j.
- Multiply m_{ij} and n_{jk} to get $m_{ij}n_{jk}$.
- Sum up all $m_{ij}n_{jk}$ over j to get $\sum_{j=1}^{m} m_{ij}n_{jk}$.

Solution of Problem 2

a) Given two vectors $\mathbf{v} \in \mathbb{R}^n$ and $\mathbf{w} \in \mathbb{R}^n$, the goal is to compute $\mathbf{v}+\mathbf{w}$. The map function generates the following key-value pairs:

- For each element v_i of $\mathbf{v}$ produce 1 key-value pair $((i),(v_i))$.
- For each element w_j of $\mathbf{w}$ produce 1 key-value pair $((j),(w_j))$.

The Reduce function computes the multiplication as follows:

- For all i, find all the key-value pairs with the same i.
- Sum v_i and w_i to get either $v_i + w_i$, v_i, or w_i.

b) Given one sparse vector $\mathbf{v} \in \mathbb{R}^n$, the goal is to compute $\frac{1}{n}\sum_{i=1}^{n} v_i$. The map function generates the following key-value pairs:

- For each element v_i of $\mathbf{v}$ produce 1 key-value pair $((i),(v_i))$.

The Reduce function computes the multiplication as follows:

- For all i, fetch v_i.
- Sum up all v_i and divide by n to get $\frac{1}{n}\sum_{i=1}^{n} v_i$.