Problem 1. (Support Vector Machine with Only One Member per Class) Let the dataset consist of only two points, \((x_1, y_1 = +1)\) and \((x_2, y_2 = -1)\). Find the SVM classifier and its parameters.

Problem 2. (Support Vector Machine Margin) Let the dataset consist of points, \((x_i, y_i = +1)\), \(i = 1, 2\) and \((x_3, y_3 = -1)\). Suppose that these points are linearly separable.

a) Show that if these points form an obtuse triangle, the maximum margin of the SVM classifier is obtained by the minimum of \(\|x_1 - x_3\|\) and \(\|x_2 - x_3\|\).

b) Discuss the case where the points form an acute triangle and argue why the margin cannot be as the above case.

Problem 3. SVM equivalent formulations: Consider the SVM problem formulated as follows for a linearly separable dataset:

\[
\arg \min_{a \in \mathbb{R}^p, b \in \mathbb{R}} \frac{1}{2} \|a\|^2 \quad \text{s.t.} \quad y_i(a^T x_i + b) \geq 1, \ i = 1, \ldots, n
\]

Show that this problem is equivalent to the following problems.

a)

\[
\arg \max_{a \in \mathbb{R}^p, b \in \mathbb{R}, \|a\| = 1} \min_{i \in \{1, \ldots, n\}} y_i(a^T x_i + b)
\]

b)

\[
\arg \max_{a \in \mathbb{R}^p, b \in \mathbb{R}, \|a\| = 1} \min_{i \in \{1, \ldots, n\}} |a^T x_i + b| \quad \text{s.t.} \quad y_i(a^T x_i + b) > 0, \ i = 1, \ldots, n
\]