Lecture Notes on

## **Information Theory**

Univ.-Prof. Dr. rer. nat. Rudolf Mathar RWTH Aachen University Institute for Theoretical Information Technology Kopernikusstr. 16, 52074 Aachen, Germany





WS 18/19

 $\bigcirc$ 

 $\bigcirc$ 



Information Theory

Prof. Dr. Rudolf Marthar	www.fi.rwth-eacher.de
o Register at L2P /	RWTH online
Exercises	Mo. 14:30-16:00 15:30-17:00 Mo. 16:30 17:15 17:15-18:00 rishnan, Emilio Balda, G. Alirezaei)
Mo. 8.10.18 Mo. 15.10.18 Mo. 22.10.18	
Mo. 29.10.18 (?) Mo. 5.11.18 Mo. 12.11.18	T. (13:00-10:00)
10. 19.11:18 Mo. 7.1.19 Mo. 14.1.19 Mo. 21.1.19 Mo. 21.1.19	Time: (13:00-18:00) We. 9.1.19 (15:30-17:00) We. 16.1.19 (15:30-17:00) We. 23.1.19 (15:30-17:00) We. 30.1.19 (15:30-17:00)

o Written examination: 23 March 2019, 10:30-12:00

Thinking the Future Zukunft denken



Univ.-Prof. Dr. Rudolf Mathar

## **Information Theory**

Textbooks, WS 2018/19

- [1] Thomas M. Cover and Joy A. Thomas: Elements of Information Theory. Second edition. Wiley, New York, 2006.
- [2] Imre Csiszar, Janos Körner: Information Theory: Coding Theorems for Discrete Memoryless Systems. Cambridge University Press, 2011.
- [3] Abbas El Gamal, Young-Han Kim: Network Information Theory. Cambridge University Press, 2011.
- [4] Rudolf Mathar: Informationstheorie. Teubner Studienbücher, Stuttgart, 1996.
- [5] David J. C. MacKay: Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003.
- [6] Raymond W. Yeung: A First Course in Information Theory. Kluwer Academic Publishers, New York, 2002.



WS 2018/19

Information Theory

Table of Cantents (provisional)

- 1. Introduction
- 2. Fundamentals of Information Theory
  - 2.1. Information Measures
  - 2.2. luequalities
  - 2.3. Information Measures for Random Sequences
  - 2.4. Asyupuptotic Equipartition Property (AED)
  - 2.5. Differential Entropy
  - 3. Source Coding
  - 4. Information Channels and Capacity
  - 5. Rate Distortion Theory
  - 6. Information Theory and Machine Learning
  - 7. Biological Inhormution Processing

 $\bigcirc$ 

 $\bigcirc$ 

## **1** Introduction

According to Merriam-webster.com "Information is any entity or form that provides the answer to a question of some kind or resolves uncertainty. It is thus related to data and knowledge, as data represents values attributed to parameters, and knowledge signifies understanding of real things or abstract concepts.".

However, modern *Information Theory* is not a theory which deals with the above on general grounds. Instead, information theory is a mathematical theory to model and analyze how information is transferred. Its starting point is an article by Claude E. Shannon, "A Mathematical Theory of Communication", Bell System Technical Journal, 1948.



Figure 1.1: Claude Elwood Shannon (1916 – 2001)

Quoting from the introduction of this article provides insight into the main focus of information theory: "The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have meaning. ... These semantic aspects of communications are irrelevant to the engineering problem. ... The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design."

Later, in 1964 a book by Claude E. Shannon and Warren Weaver with a slightly modified title "The Mathematical Theory of Communication" appeared at University of Illinois Press, emphasizing the



Figure 1.2: The general model of a communciation system.

## generality of this work.

*Information Theory* provides methods and analytical tools to design such systems. The basic components of a communication are shown in Fig. 1.2. Although the starting point of information theory was in electrical engineering and communications, the theory turned out to be useful for modeling phenomena in a variety of fields, particularly in physics, mathematics, statistics, computer science and economics. It cannot be regarded only as a subset of communication theory, but is much more in general. In recent years, its concepts were applied and even further developed in biological information processing, machine learning and data science.

This lecture focuses on the latter. We first provide the basic concepts of information theory and prove some main theorems in communications, which refer to source coding, channel coding and the concept of channel capacity. This will be followed by the relation between rate distortion theory and autoencoders. Biological information processing will be modeled and analyzed by the concept of mutual information. This will finally lead to artificial neural networks and contributions of information theory to understanding how such networks learn in the training phase.