Exercise 22. There are four so called weak DES keys. One of those is the key

\[ K = 00011111\ 00011111\ 00011111\ 00011111\ 00001110\ 00001110\ 00001110\ 00001110. \]

(a) What happens if you use this key?
(b) Can you find the other three weak keys?

Exercise 23. Let \( \varphi : \mathbb{N} \rightarrow \mathbb{N} \) be the Euler \( \varphi \)-function, i.e., \( \varphi(n) = |\mathbb{Z}_n^*| \).

(a) Determine \( \varphi(p) \) for a prime \( p \).
(b) Determine \( \varphi(p^k) \) for a prime \( p \) and \( k \in \mathbb{N} \).
(c) Determine \( \varphi(p \cdot q) \) for two different primes \( p \neq q \).
(d) Determine \( \varphi(4913) \) and \( \varphi(899) \).

Exercise 24.

(a) Use the Miller-Rabin Primality Test to prove that 341 is composite.
(b) The Miller-Rabin Primality Test comprises a number of successive squarings. Suppose a 300-digit number \( n \) is given. How many squarings are needed in worst case during a single run of this primality test?