Problem 1. (Dividers) Let $a, b, c, d \in \mathbb{Z}$. The integer $a$ divides $b$ if and only if there exists a $k \in \mathbb{Z}$ such that $a \cdot k = b$. This property is denoted by $a \mid b$. Prove the following implications:

a) $a \mid b$ and $b \mid c \implies a \mid c$.

b) $a \mid b$ and $c \mid d \implies (ac) \mid (bd)$.

c) $a \mid b$ and $a \mid c \implies a \mid (xb + yc) \forall x, y \in \mathbb{Z}$.

Problem 2. (Permutation Cipher) The plaintext is an English sentence. A permutation cipher with blocklength 8 revealed the following ciphertext

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a) Decrypt the ciphertext and explain your approach.

b) Determine the corresponding permutations $\pi$ and $\pi^{-1}$.

Problem 3. (GCD Multiplicativity) Let $a, b, m \in \mathbb{Z}$. Show that if $\gcd(a, b) = 1$, then $\gcd(ab, m) = \gcd(a, m) \gcd(b, m)$. 