Homework 8 in Optimization in Engineering

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Exercise 1. (norm reformulation) Linear optimization problems are convex optimization problems for which the objective function and all constraint functions are affine. Let $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ and $\boldsymbol{b} \in \mathbb{R}^m$ be given. Recall that for $\boldsymbol{x} \in \mathbb{R}^n$,

$$||\boldsymbol{x}||_\infty = \max_{i=1,\dots,n} |x_i| \quad \text{and} \quad ||\boldsymbol{x}||_1 = \sum_{i=1}^{n} |x_i|$$

holds. Find an equivalent linear formulation for the following optimization problems.

a) minimize $||\boldsymbol{Ax} - \boldsymbol{b}||_\infty$

b) minimize $||\boldsymbol{Ax} - \boldsymbol{b}||_1$

c) minimize $||\boldsymbol{Ax} - \boldsymbol{b}||_1$ subject to $||\boldsymbol{x}||_\infty \le 1$

d) minimize $||\boldsymbol{x}||_1$ subject to $||\boldsymbol{Ax} - \boldsymbol{b}||_\infty \le 1$

e) minimize $||\boldsymbol{Ax} - \boldsymbol{b}||_1 + ||\boldsymbol{x}||_\infty$

Exercise 2. (Lagrangian and dual function) Consider the optimization problem

$$\begin{array}{rc} \text{minimize} & x^2 + 1 \\ \text{subject to} & (x-2)(x-4) \le 0 \end{array}$$

with optimization variable $x \in \mathbb{R}$.

a) Plot the objective function. Describe the feasible set and find the optimizer x^* and the optimal value p^*.

b) Compute the Lagrangian $L(x, \lambda)$ and plot it (as a function of x) for $\lambda \in \{1, 2, 3\}$.

c) Plot the Lagrange dual function $g(\lambda) = \inf_x L(x, \lambda)$. Verify that the lower bound property $p^* \ge g(\lambda)$ holds.