# Homework 7 in Optimization in Engineering

Prof. Dr. Anke Schmeink, Michael Reyer, Alper Tokel

01.12.2014

**Exercise 1.** (Norm reformulation) Linear optimization problems are convex optimization problems for which the objective function and all constraint functions are affine. Let $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ and $\boldsymbol{b} \in \mathbb{R}^m$ be given. Recall that for $\boldsymbol{x} \in \mathbb{R}^n$,

$$||\boldsymbol{x}||_\infty = \max_{i=1,\ldots,n} |x_i| \quad \text{and} \quad ||\boldsymbol{x}||_1 = \sum_{i=1}^{n} |x_i|$$

holds. Find an equivalent linear formulation for the following optimization problems.

(a) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_\infty$

(b) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_1$

(c) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_1$ subject to $||\boldsymbol{x}||_\infty \leq 1$

(d) minimize $||\boldsymbol{x}||_1$ subject to $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_\infty \leq 1$

(e) minimize $||\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}||_1 + ||\boldsymbol{x}||_\infty$

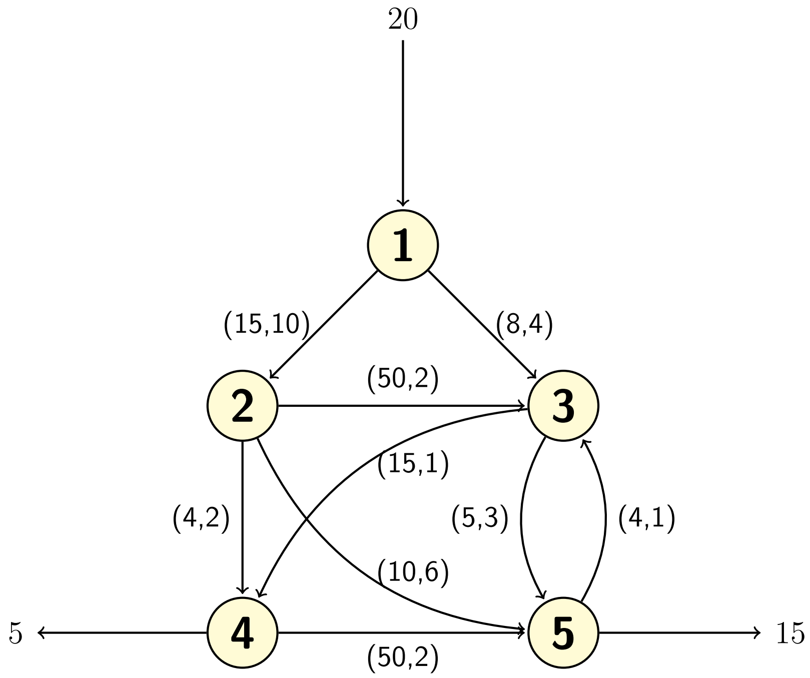
- Please scroll -

**Exercise 2.** (Network flow problem) Consider a network of $n$ nodes. The variables in the problem are the flows on each link, where $x_{ij}$ denotes the flow from node $i$ to node $j$. The cost of the flow along the link from node $i$ to node $j$ is given by $c_{ij}x_{ij}$, where $c_{ij}$ is given. Each link flow $x_{ij}$ is also subject to a given lower bound $l_{ij}$ and an upper bound $u_{ij}$.

The external supply at node $i$ is given by $b_i$, where $b_i > 0$ means that an external flow enters the network at node $i$, and $b_i < 0$ means that at node $i$, an amount of $|b_i|$ flows out of the network. We assume that $\mathbf{1}^T \boldsymbol{b} = 0$, i.e., the total external supply equals total external flow. At each node we have conservation of flow through the network: the total flow into node $i$ along links and the external supply, minus the total flow out along links, equals zero.

The problem is to minimize the total cost of flow through the network, subject to the constraints described above.

(a) Formulate this problem as a linear optimization problem.




(b) Find the minimum total cost of flow for the network in the graph above (by means of the cvx-package in Matlab). The capacity $u_{ij}$ and the cost of the flow $c_{ij}$ are given next to each link as $(u_{ij}, c_{ij})$. All lower bounds $l_{ij}$ are assumed to be 0. External supply and flows are shown at nodes 1, 4 and 5 with corresponding direction and amount of the flows.

(c) Is the solution unique?