Exercise 1. (Geometric Programming) Let $p(x)$ and $q(x)$ be posynomials defined as

\[ p(x) = x_1^2 x_2 + \frac{1}{x_1 x_2} \quad \text{and} \quad q(x) = \frac{x_1^2}{x_2} + \frac{x_2^2}{x_1}, \]

and the monomial $r(x)$ is defined as

\[ r(x) = 2x_1 x_2. \]

Express the following problems as geometric programming problems in $\mathbb{R}^2_{>0}$ and find the optimal solutions by means of cvx.

(a) Minimize $\max \{ p(x), q(x) \}$.

(b) Minimize $\frac{p(x)}{r(x) - q(x)}$ subject to $r(x) > q(x)$.

Remark: To solve geometric programming problems in monomial and posynomial form in cvx, the cvx_begin gp command must be used.
Exercise 2. (Optimal transmitter power allocation) Consider a wireless network as discussed in the lecture with $m$ users/transmitters and $n$ receivers. The signal-to-interference ratio of user $i$ is

$$SIR_i = \frac{h_{ii}p_i}{\sum_{j \neq i} h_{ij}p_j + \sigma_i^2}$$

where $p_i$ is the transmit power of user $i$, $h_{ij}$ is the channel gain from user $j$ to the home receiver of user $i$, and $\sigma_i^2$ is the noise power of receiver $i$. In this exercise we consider a network with 4 users and 4 base stations. The channel gain matrix $H$ is given as

$$H = (h_{ij}) = \frac{1}{100} \begin{bmatrix} 37 & 2 & 1 & 6 \\ 10 & 30 & 3 & 6 \\ 1 & 14 & 354 & 3 \\ 10 & 8 & 6 & 171 \end{bmatrix}$$

It is assumed that $\sigma_i^2 = 1$ for all users.

Formulate the following optimization problem as a geometric programming problem and solve it using cvx.

$$\text{maximize} \quad \min_{1 \leq i \leq 4} SIR_i$$
$$\text{subject to} \quad 0 \leq p_i \leq 30, \quad i = 1, \ldots, 4.$$ 

Remark: To solve geometric programming problems in monomial and posynomial form in cvx, the cvx_begin gp command must be used.